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*Research Article*

# Mechanical Analysis of the Circular Tunnel considering the Interaction between the Ground Response Curve and Support Response Curve

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The viewpoint that the ground initial elastic displacement and the interaction between the ground response curve (GRC) and support response curve (SRC) in the surrounding rock should be considered at the same time in the mechanical analysis of the circular tunnel is proposed, and its solution method is also established. Meanwhile, in order to consider the effect of the intermediate principle stress, Drucker-Prager criterion is introduced to describe the yield property of the surrounding rock. The calculation example indicates that the final radial displacement of the tunnel circumference will increase when the ground initial elastic displacement in the surrounding rock is considered before the support structure is applied, which indicates that it is necessary to consider the ground initial elastic displacement in the surrounding rock before the support structure is applied. With increasing the support resistance force and the initial field stress, the plastic zone radius in the surrounding rock and the radial displacement of the tunnel circumference will decrease and increase, respectively, while with increasing the rock internal friction angle and cohesion, the plastic zone radius in the surrounding rock and the radial displacement of the tunnel circumference both decrease. Meanwhile, with the stress Lode parameter increasing from −1 to 1, the plastic zone radius in the surrounding rock and the radial displacement of the tunnel circumference both greatly decrease and then slightly increase. It indicates that the intermediate principle stress has some effect on the calculation results.

## 1. Introduction

Many researches have been done on the tunnel engineering [1–6], of which the deformation and stress responses of the ground caused by tunnel excavation and the interaction with the support structure are the theoretical basis for guiding the design of tunnel excavation and support [4–6]. Li et al. [4] investigated the undrained responses for deep circular tunnels with reinforcement in saturated ground analytically and found that, with the decrease of internal support pressure, the plastic zone may firstly appear not only in the reinforced ground or in the natural ground, but also in the two zones simultaneously. Ghorbani and Hasanzadehshooiili [5] present a simple novel approach based on evolutionary polynomial regression to determine ground reaction curve of rock masses obeying both Mohr-Coulomb and Hoek-Brown criteria and strain-softening behaviors. The proposed models accurately present support pressures based on radial displacement, rock mass strength, and softening parameter. Zou et al. [6] investigate a new procedure for the GRC in strain-softening surrounding rock for a circular opening. By assuming different plastic radii (using a plastic radius increment), GRC, the evolution curve of plastic radius and internal support pressure can be obtained analytically. However, the mechanical behavior of the surrounding rock is related to not only the physical and mechanical properties of the rock, field stress, and geological environment, but also the external factors such as the tunnel section shape, excavation, and support type, so it is a very complicated process. Therefore, many scholars have carried out a profound study on it. First from the viewpoint of the internal factors, the existing research mainly focuses on the selection of rock constitutive model and strength criterion. Due to the different rock types and loading conditions, the surrounding rock will exhibit different mechanical responses such as elasticity, plasticity, and viscosity. Therefore, many scholars have studied the mechanical property of the tunnel surrounding rock based on different rock constitutive models. For example, Wang et al. [7, 8] obtained the stress and displacement solutions for deep buried double tunnel excavation based on the viscoelastic rock constitutive model. Paraskevopoulou et al. [9] adopted the isotropic axisymmetric finite difference model to study and analyze the total displacement around a circular tunnel in viscoelastic medium. Cui et al. [10] put forward the elasto-plastic softening model considering softening coefficient and then adopted it to calculate the surrounding rock stress and strain state of a circular tunnel. And the influence of the softening coefficient on the deformation responses of surrounding rock and the support pressure is studied. Birchall and Osman [11] proposed a time-varying response method for deeply buried and semi-infinite circular tunnel based on the three-dimensional energy solution. Zhu et al. [12] proposed a new constitutive model based on the generalized three-dimensional Hoek-Brown strength criterion.

Meanwhile, the effect of rock strength criterion on the ground stress response in the surrounding rock also attracts much attention. At present, the scholars all assumed that the intermediate principal stress has some effect on the stress field and the plastic zone in the surrounding rock. Therefore, the strength criterion considering intermediate principal stress, such as Drucker-Prager criterion, Mises criterion, or double shear strength criterion, should be adopted to replace some other strength criteria such as Mohr-Coulomb one, which does not consider the intermediate principal stress. On the condition that the rock elastoplastic model is adopted to study the plastic zone in the surrounding rock, it is assumed that the strength of the surrounding rock can be fully activated to reduce the support workload when the intermediate principal stress is taken into account. And many researchers [13–17] have conducted lots of works on it. Meanwhile, the shape of the tunnel section, the time of support applied, and the external stress field also have much influence on the displacement and stress in the surrounding rock. Due to the limitation of theory, the existing research mostly focuses on the circular section of the tunnel. However, there are also some studies on the noncircular sections of the tunnel: for example, Wang et al. [18] proposed a method to map a noncircular tunnel to a circular unit in the complex plane and then spread the analytic function into a Laurent series. The stress unified solution of oval and horseshoe cross section can be determined with Muskhelishvili's complex variables function method. Subsequently, the solution can be taken into the Griffith strength failure criterion and determine the scale and shape of the plastic zone in the tunnel surrounding rock. Fotieva [19] proposed a method for determining the stresses in tunnel linings of an arbitrary cross section. But at present, the mechanical analysis of the noncircular tunnel is still based on the model test, field test, and numerical calculation. Zhou and Shou [20] proposed a new non-Euclidean dynamic model to investigate the zonal disintegration mechanism of isotropic rock masses around a deep circular tunnel subjected to dynamic unloading, in which the defect parameter $R$ is introduced to describe effects of microdefects on the deformation and failure of deep rock masses.

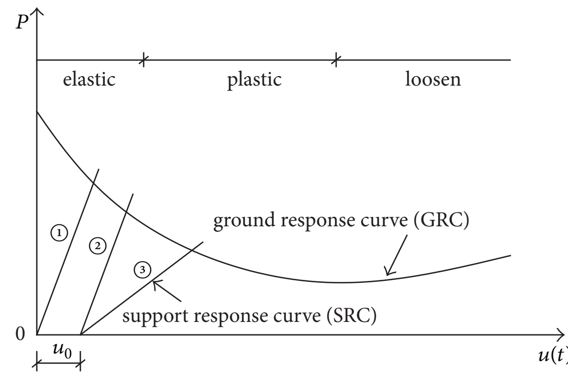


Figure 1: The interaction between GRC and SRC. ① and ② are the same kind of support structure, but their construction time is different. ③ is another kind of support structure. The support structure ① is constructed immediately after the tunnel is excavated and accordingly its support workload is more than that of the support structures ②. And the support structures ② is constructed after a while when the tunnel is excavated and some displacement occurs in the surrounding rock, accordingly the support workload will be less.

However, although many scholars have made a thorough analysis of the mechanical responses of the circular tunnel and obtained a lot of research results, there are still many problems to solve. For example, the actual construction process of the tunnel is not sufficiently considered in the existing models, which will lead to the waste of the support workload or the occurrence of engineering accidents. The interaction between GRC and SRC is shown in Figure 1. It can be seen that the support structure especially the rockbolts should not be applied immediately after the excavation of the tunnel. If it is carried out after a certain ground initial deformation in the surrounding rock, we can make full use of the self-bearing capacity of the surrounding rock to reduce the amount of support workload [21]. The general tunnel construction sequence can be summarized as: excavation → first support → second support. In order to protect the self-bearing capacity of the surrounding rock, the first support work should be carried out as soon as possible. Therefore, the time of the first support is very important, and in general, it should be completed within half time of the self-stability of the surrounding rock after the tunnel excavation. That is to say, before carrying out the first support, the tunnel has already had a certain deformation. The support is carried out in a tunnel which has already deformed, rather than in the initial undeformed tunnel. However, nearly none of the existing researches consider this issue. Therefore, based on this viewpoint, we make a profound study on the ground stress response in the tunnel surrounding rock. For the sake of simplification, the following four assumptions are made here. ① Only the first support is considered, here the rockbolts are taken for example in this study. ② The

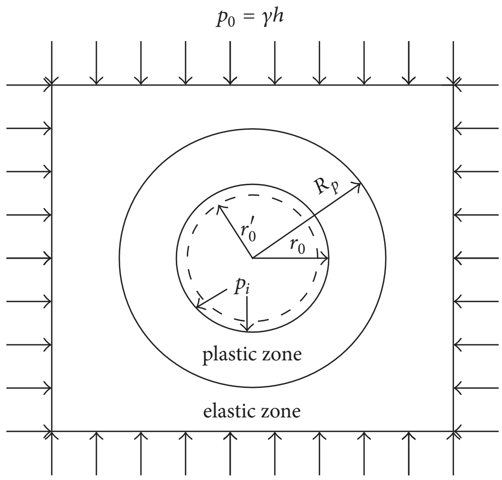


Figure 2: The calculation model of circular tunnel. $r_0$ is the initial radius of the tunnel, $r_0'$ is the radius of the tunnel after the initial elastic deformation, and $R_p$ is the radius of the plastic zone. $\gamma$ is gravitational density, and $h$ is depth from the ground surface to the top of the calculation model.

study on the mechanical behavior of the tunnel is divided into the following two stages: namely, one is that the tunnel excavation is completed but the support is not applied, and the other is that the tunnel excavation is completed and the support is applied. ③ In order to protect the self-bearing capacity of the surrounding rock, it is assumed that only the elastic deformation occurs in the surrounding rock before the application of the support structure, and therefore there is no plastic zone. ④ In order to study the effect of the intermediate principal stress on the ground stress response and plastic zone in the tunnel surrounding rock, Drucker-Prager criterion is adopted as the yield criterion of rock strength.

Based on the actual construction process of the tunnel, we put forward the stress analysis method of the underground circular tunnel considering the interaction between GRC and SRC, which can provide a reference for other engineering projects.

## 2. Elastic Mechanical Analysis of the Tunnel before Support

The surrounding rock is assumed to be homogeneous and isotropic. Meanwhile, because of the deeply buried and lengthwise direction of the tunnel, the influence of gravity gradient can be ignored within the scope of the tunnel, and therefore it can be regarded as a planar strain issue. The calculation model is shown in Figure 2. In order to solve the problem, the polar coordinate system is adopted. It is assumed that $\sigma_\theta, \sigma_r, \sigma_z, \varepsilon_\theta, \varepsilon_r$, and $\varepsilon_z$ are the tangential, radial, and axial stress and strain of the circular section, respectively, and the stress satisfies $\sigma_\theta > \sigma_z > \sigma_r$ (according to the rule of rock mechanics, the compressive stress is positive and the tensile stress is negative). Then the unsupported tunnel can be assumed to be a thick wall cylinder which is only subjected to the external pressure, and the original ground stress is assumed to be $p_0$.

The basic equations are as follows. The equilibrium differential equation is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{1}$$

The geometric equation is

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \frac{u}{r}. \tag{2}$$

The elastic constitutive equation is

$$\varepsilon_r = \frac{1-\nu^2}{E}\left(\sigma_r - \frac{\nu}{1-\nu}\sigma_\theta\right), \qquad \varepsilon_\theta = \frac{1-\nu^2}{E}\left(\sigma_\theta - \frac{\nu}{1-\nu}\sigma_r\right), \tag{3}$$

where $u$ is the radial displacement of the tunnel surrounding rock and $E$ and $\nu$ are the rock elastic modulus and Poisson's ratio, respectively.

The stress and displacement of the tunnel surrounding rock are as follows (the subscript "1" and the following subscript "2" indicate the physical variables of the tunnel surrounding rock before and after support, resp. The subscripts "$e$" and "$p$" represent physical variables of elastic and plastic zones, resp.):

$$\sigma_{re1} = p_0\left(1 - \frac{r_0^2}{r^2}\right), \qquad \sigma_{\theta e1} = p_0\left(1 + \frac{r_0^2}{r^2}\right), \qquad u_{e1} = \frac{p_0}{E}\left[\frac{(1+\nu)\,r_0^2}{r} + (1+\nu)\,r\right]. \tag{4}$$

We can obtain the radius of the circular tunnel after the initial elastic deformation $r_0'$:

$$r_0' = r_0 - u_{e1}. \tag{5}$$

## 3. Mechanical Analysis of the Circular Tunnel after Support

*3.1. Drucker-Prager Strength Criterion.* The Mohr-Coulomb strength criterion has been widely applied in geomaterials. It preferably embodies their compressive and shear failure essence. However, it cannot reflect the influence of the intermediate principal stress on their failure. Therefore, it cannot explain the yield damage of the rock under the high confining pressure and the hydrostatic pressure. The Drucker-Prager strength criterion [22–24] proposed in 1950s is a good way to overcome this shortage, which is the extension of the Mohr-Coulomb strength criterion and the Mises yield criterion:

$$f = \alpha I_1 + \sqrt{J_2} - k = 0, \tag{6}$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $J_2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]/6$, $\alpha = (2\sin\varphi)/\sqrt{3}(3 - \sin\varphi)$, $k = (6c\cos\varphi)/\sqrt{3}(3 - \sin\varphi)$.

$\sigma_1$, $\sigma_2$, and $\sigma_3$ are first, intermediate, and third principal stresses, respectively, and $c$ and $\varphi$ are the rock cohesion and internal friction angel, respectively.

The stress Lode parameter $\mu_\sigma$ can be expressed as

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}. \tag{7}$$

Then the intermediate principal stress is

$$\sigma_2 = \frac{1}{2}\left[\sigma_1\left(1 + \mu_\sigma\right) + \sigma_3\left(1 - \mu_\sigma\right)\right]. \tag{8}$$

Then $I_1$ and $J_2$ can be represented by $\mu_\sigma$

$$\begin{aligned} I_1 &= \frac{1}{2}\left[\sigma_1\left(3 + \mu_\sigma\right) + \sigma_3\left(3 - \mu_\sigma\right)\right], \\ J_2 &= \frac{\left(\sigma_1 - \sigma_3\right)^2}{4}\left(1 + \frac{{\mu_\sigma}^2}{3}\right). \end{aligned} \tag{9}$$

Substituting (9) into (6), we can obtain the Drucker-Prager strength criterion expressed by the Lode parameter (expressed in polar coordinates):

$$\frac{\sigma_\theta - \sigma_r}{2}\left(\sqrt{1 + \frac{\mu_\sigma^2}{3}} - \frac{\alpha\mu_\sigma}{2}\right) - \frac{3\alpha\left(\sigma_\theta + \sigma_r\right)}{2} - k = 0. \tag{10}$$

Mohr-Coulomb criterion does not consider the effect of the intermediate principal stress, however, which is well taken into account in Drucker-Prager one. Drucker-Prager gave a three-dimensional failure criterion for geomaterials, which is widely used in numerical analysis. The simplicity and smooth curvature in $\pi$-plane are the prominent advantage of the Drucker-Prager criterion. The Drucker-Prager criterion is divided into three types: Inscribe Drucker-Prager, Middle Circumscribe Drucker-Prager, and Circumscribe Drucker-Prager based on their failure surface with respect to the failure surface of Mohr-Coulomb criterion, shown in Figure 3.

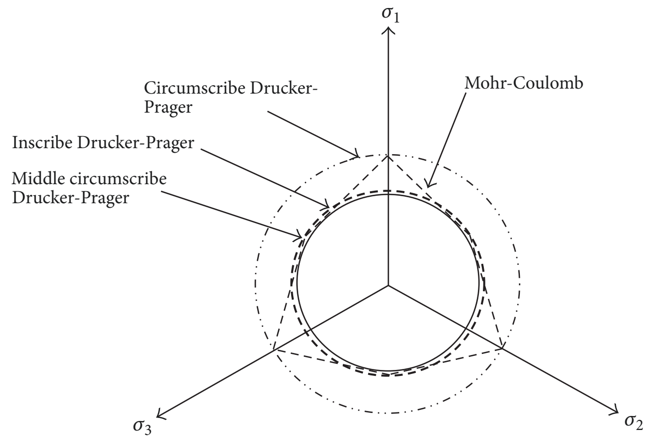


FIGURE 3: The failure surface of Mohr-Coulomb and Drucker-Prager criteria in $\pi$-plane.

*3.2. Stress Analysis in Plastic Zone.* According to the interaction between GRC and SRC shown in Figure 1, the surrounding rock should be allowed to produce some elastic deformation in order to fully mobilize its self-bearing capacity and finally reduce the support workload. As a result of the ground deformation in the surrounding rock, the circular tunnel radius is changed from $r_0$ to $r_0'$ (Figure 2). It is assumed that the support is applied in a tunnel with a radius of $r_0'$ and then continues to adjust with the surrounding rock stress field and interaction of the support structure. When the surrounding rock in the stress field satisfies the rock failure criterion, the circular plastic zone with radius $R_p$ in the surrounding rock will occur, outside which there is still the elastic zone. Therefore, the stress state of the plastic zone is firstly analyzed, and Drucker-Prager strength criterion shown as (10) is chosen as the rock yield criterion.

On the inner wall of the surrounding rock, namely, when $r = r_0'$, $\sigma_{rp} = p_s$ ($p_s$ is the support force), and then the stress components in the plastic zone can be obtained with (1) and (10)

$$\begin{aligned} \sigma_{rp2} &= \left(\frac{r}{r_0'}\right)^m\left(p_s + \frac{n}{m}\right) - \frac{n}{m}, \\ \sigma_{\theta p2} &= \left(\frac{r}{r_0'}\right)^m\left(\left(m+1\right)p_s + \frac{\left(m+1\right)n}{m}\right) - \frac{n}{m}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} m &= \frac{6\alpha - \sqrt{1 + \mu_\sigma^2/3}}{\sqrt{1 + \mu_\sigma^2/3} - \alpha\mu_\sigma/2 - 3\alpha}, \\ n &= \frac{2k}{\sqrt{1 + \mu_\sigma^2/3} - \alpha\mu_\sigma/2 - 3\alpha}. \end{aligned} \tag{12}$$

*3.3. Stress Analysis in Elastic Zone.* The outside of the plastic zone is still the elastic zone, and its stress boundary condition is as follows. When $r \to \infty$, $\sigma_{re2} = p_0$, and when $r = R_p$, $\sigma_{re2} = \sigma_R$, where $\sigma_R$ is the radial stress at the interface of elastic and plastic zone. Under the axisymmetric stress state, the stress of the elastic zone can be obtained from the internal and external stress boundary conditions and the single value condition of the displacement

$$\begin{aligned} \sigma_{re2} &= p_0\left(1 - \frac{{R_p}^2}{r^2}\right) + \sigma_R\frac{{R_p}^2}{r^2}, \\ \sigma_{\theta e2} &= p_0\left(1 + \frac{{R_p}^2}{r^2}\right) - \sigma_R\frac{{R_p}^2}{r^2}. \end{aligned} \tag{13}$$

The corresponding displacement component of the elastic zone in the surrounding rock is

$$u_{e2} = \frac{1+\nu}{E}\cdot\frac{{R_p}^2}{r}\left(p_0 - \sigma_R\right). \tag{14}$$

*3.4. The Plastic Zone Radius $R_p$.* It is known that the stress at the interface of the elastic and plastic zones should satisfy the stress conditions of the elastic and plastic zones at the same time. For the elastic region, (13) should be satisfied, and we obtain

$$\sigma_{re2} + \sigma_{\theta e2} = 2p_0. \tag{15}$$

The stress is also located in the plastic zone, so Drucker-Prager strength criterion should be satisfied at the same time, so there is

$$\frac{\sigma_{\theta p2} - \sigma_{rp2}}{2}\left(\sqrt{1+\frac{\mu_\sigma^2}{3}} - \frac{\alpha\mu_\sigma}{2}\right) - \frac{3\alpha\left(\sigma_{\theta p2} + \sigma_{rp2}\right)}{2} - k = 0. \tag{16}$$

Meanwhile, because of $\sigma_{re2} = \sigma_{rp2}$, $\sigma_{\theta e2} = \sigma_{\theta p2}$, combining with (15), (16), we obtain

$$\sigma_{rp2} = p_0 - \frac{3\alpha p_0 + k}{\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2}. \tag{17}$$

When $r = R_p$, $\sigma_R = \sigma_{rp2} = \sigma_{re2}$, and then we can obtain from (11) and (17) that

$$R_p = r_0'\left(\frac{p_0 - (3\alpha p_0 + k)/\left(\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2\right) + n/m}{p_s + n/m}\right)^{1/m}. \tag{18}$$

Therefore, the plastic zone radius $R_p$ can be solved.

*3.5. The Displacement $u_0$ around the Tunnel.* The displacement of the elastic zone in the surrounding rock can be obtained from (14). When $r = R_p$, the displacement at the interface of the elastic zone and the plastic one is

$$\left(u_{e2}\right)_{r=R_p} = \frac{1+\nu}{E} \cdot R_p \frac{3\alpha p_0 + k}{\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2}. \tag{19}$$

In the plastic zone, the displacement of the plastic zone $u_p = C/r$ can be obtained, where $C$ is integral constant which can be determined by the boundary conditions. When $r = R_p$, $u_{e2} = u_p$, and then the integral constant $C$ can be determined. So we can obtain the displacement of plastic zone

$$u_p = \frac{1+\nu}{Er} \cdot R_p^2 \frac{3\alpha p_0 + k}{\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2}. \tag{20}$$

When $r = r_0'$, the displacement $u_0'$ of the tunnel circumference with the elastic deformation is

$$u_0' = \frac{1+\nu}{Er_0'} \cdot R_p^2 \frac{3\alpha p_0 + k}{\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2}. \tag{21}$$

However, in the actual engineering, it is necessary to obtain the ground displacement $u_0$ in the surrounding rock with initial excavation radius $r_0$; then according to the relationship between the deformations, we obtain

$$u_0 = u_0' + u_{e1}. \tag{22}$$

Table 1: The rock physical and mechanical property.

| $\rho$/kg/m$^3$ | $E$/GPa | $\nu$ | $c$/MPa | $\varphi$/° |
|---|---|---|---|---|
| 2500 | 10 | 0.25 | 1.0 | 30 |

$\rho$: mass density, $E$: elastic modulus, $\nu$: Poisson ratio, $c$: cohesion, and $\varphi$: internal friction angle.

## 4. Example Analysis

The radius $R_0$ of a deeply buried tunnel is 3 m, the initial gravitational stress $p_0$ of the rock is 30 MPa, and the support resistance $p_s$ is 1 MPa. The rock physical and mechanical property is shown in Table 1.

According to (4) and the rock property shown in Table 1, the ground maximum radial elastic displacement in the surrounding rock at the inner diameter of the tunnel is 0.0225 m. For simplicity, it is considered that the maximum elastic displacement 0.0225 m of the tunnel surrounding rock occurs before the support. Then the support is actually applied in a tunnel with a radius $r_0' = 2.9775$ m. First, the calculation results obtained with the proposed method are compared with that without considering the initial elastic deformation produced by the ground in the surrounding rock before the support structure is applied, which illustrate the difference between these two methods. Secondly, the influences of the rock strength and the intermediate principal stress on the ground plastic zone and the displacement in the surrounding rock are studied by the single factor sensitivity analysis. That is to say, only one parameter is changed each time and the other parameters remain unchanged.

*4.1. The Comparison and Analysis of Two Different Methods of the Ground Displacement in the Surrounding Rock.* The ground displacement in the surrounding rock with the proposed method can be calculated with (22). When the ground initial elastic displacement in the surrounding rock is not taken into account, according to the calculation method similar to the proposed method, we obtain

$$u_e = \frac{r_0(1+\nu)A}{E}\left(\frac{p_0 - A + n/m}{p_s + n/m}\right)^{2/m}, \tag{23}$$

where $A = (3\alpha p_0 + k)/(\sqrt{1+\mu_\sigma^2/3} - \alpha\mu_\sigma/2)$.

The calculation results obtained with these two different methods are compared in Figure 4. The following conclusions can be obtained. When the elastic displacement occurring before the support structure is applied in the tunnel is considered, the calculation results of ground ultimate convergence displacement in the surrounding rock is larger than that without considering the elastic displacement occurring before the support structure is applied in the tunnels, which indicates that it is necessary to consider the effect of the actual construction process of the tunnel on its convergent displacement. From the variation extent, when considering the elastic displacement produced before support, it is equivalent to apply support in a tunnel whose radius reduces by 22.5 mm (its decrease rate of 0.75%). The ground ultimate convergence displacement in the surrounding rock increases

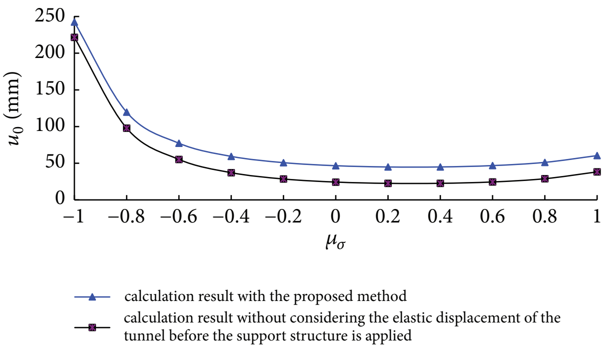


Figure 4: Comparison of the radial displacement of the tunnel circumference obtained with two methods.

from the original 38.19 mm to 60.4 mm (here $\mu_\sigma = 1$ is taken for example); namely, the increase rate is 58.16%. It indicates that the influence of the ground initial elastic deformation in the surrounding rock on the calculation results is great. Meanwhile, from the influence of $\mu_\sigma$ on the ground displacement in the surrounding rock, when $\mu_\sigma$ increased from −1 to 1, the ground convergence and displacement in the surrounding rock firstly decreased drastically and then increased slightly, which means that the surrounding rock stress state has great influence on its convergence and displacement. As the intermediate principal stress increases with increasing $\mu_\sigma$, the ground convergence displacement in the tunnel surrounding rock basically decreases with increasing the intermediate principal stress. It shows that the intermediate principal stress has some influence on the ground convergence displacement in the surrounding rock.

*4.2. The Plastic Zone Radius $R_p$.* Figure 5 shows the effect of support resistance, initial field stress, rock internal friction angle, and cohesion on the ground plastic zone radius in the surrounding rock. The following conclusions can be obtained. ① With increasing the support resistance, the ground plastic zone radius in the surrounding rock decreases gradually, which indicates that the higher support resistance can limit the ground convergence and deformation in the surrounding rock. With increasing the initial field stress, the ground plastic zone radius in the surrounding rock increases gradually. This is because the initial field stress is the external load to destroy the surrounding rock. Accordingly the ground plastic zone in the surrounding rock will increase. ② With increasing the rock internal friction angle and cohesion, the ground plastic zone in the surrounding rock gradually decreases. This is because, with increasing the rock internal friction angle and cohesion, the ability of the rock to resist plastic damage increases, and then the ground plastic zone radius decreases. Meanwhile, it can be found that the influence of rock internal friction angle on the ground plastic zone radius is larger than that of the cohesion. ③ From the effect of $\mu_\sigma$ on the ground plastic zone radius in the surrounding rock, it can be seen that the effect law in Figure 4 is consistent. That is when $\mu_\sigma$ increased from −1 to 1, the ground plastic zone radius in the surrounding rock decreases sharply first and then increased slightly, it also shows that, with increasing the intermediate principal stress, the ground plastic zone radius in the surrounding rock basically decreases; namely, the intermediate principal stress has some influence on the ground plastic zone radius in the surrounding rock.

*4.3. The Ground Radial Displacement $u_0$ of the Tunnel Circumference.* Figure 6 shows the influence of support resistance, initial field stress, rock internal friction angle, and cohesion on the ground radial displacement around the tunnel. It can be seen that the variation law is basically the same as that in Figure 4. That is to say, with increasing the support resistance and the initial field stress, the ground radial displacement of the tunnel circumference decreases and increases, respectively. And with increasing the rock internal friction angle and cohesion, the ground radial displacement around the tunnel decreases gradually. Meanwhile, the influence of the rock internal friction angle on the ground radial displacement of the tunnel circumference is greater than that of the cohesion. The effect of $\mu_\sigma$ on the ground radial displacement around the tunnel is similar to that in Figure 5.

## 5. Conclusion

(1) According to the actual construction process of the tunnel and the requirement of fully taking advantage of the self-support ability of the surrounding rock, a support design method for the surrounding rock is put forward which considers the interaction between GRC and SRC in the design of the circular tunnel support. Meanwhile, in order to consider the influence of the intermediate principal stress on the mechanical property of the tunnel surrounding rock, Drucker-Prager strength criterion of the rock is introduced into the proposed calculation method.

(2) Examples show that when the ground elastic deformation in the surrounding rock is considered or not before the support is applied, the final ground radial displacement of the tunnel circumference of the former is larger than that of the latter. It indicates that it is very necessary to consider the ground elastic deformation in the surrounding rock before the support structure is applied.

(3) The influence of the support force, original field stress, rock internal friction angle, and cohesion on the ground plastic zone radius and radial displacement of the tunnel circumference is studied. The results show that, with increasing the support force and original ground stress, the ground plastic zone radius and radial displacement in the surrounding rock will decrease and increase, respectively. And with increasing the rock friction angle and cohesion, the ground plastic zone radius and radial displacement in the surrounding rock will both decrease. Meanwhile, with increasing $\mu_\sigma$ from −1 to 1, the ground plastic zone radius and radial displacement of the tunnel circumference sharply decrease first and then slightly increase, which indicates that the intermediate principal stress has some influence on the calculation results.

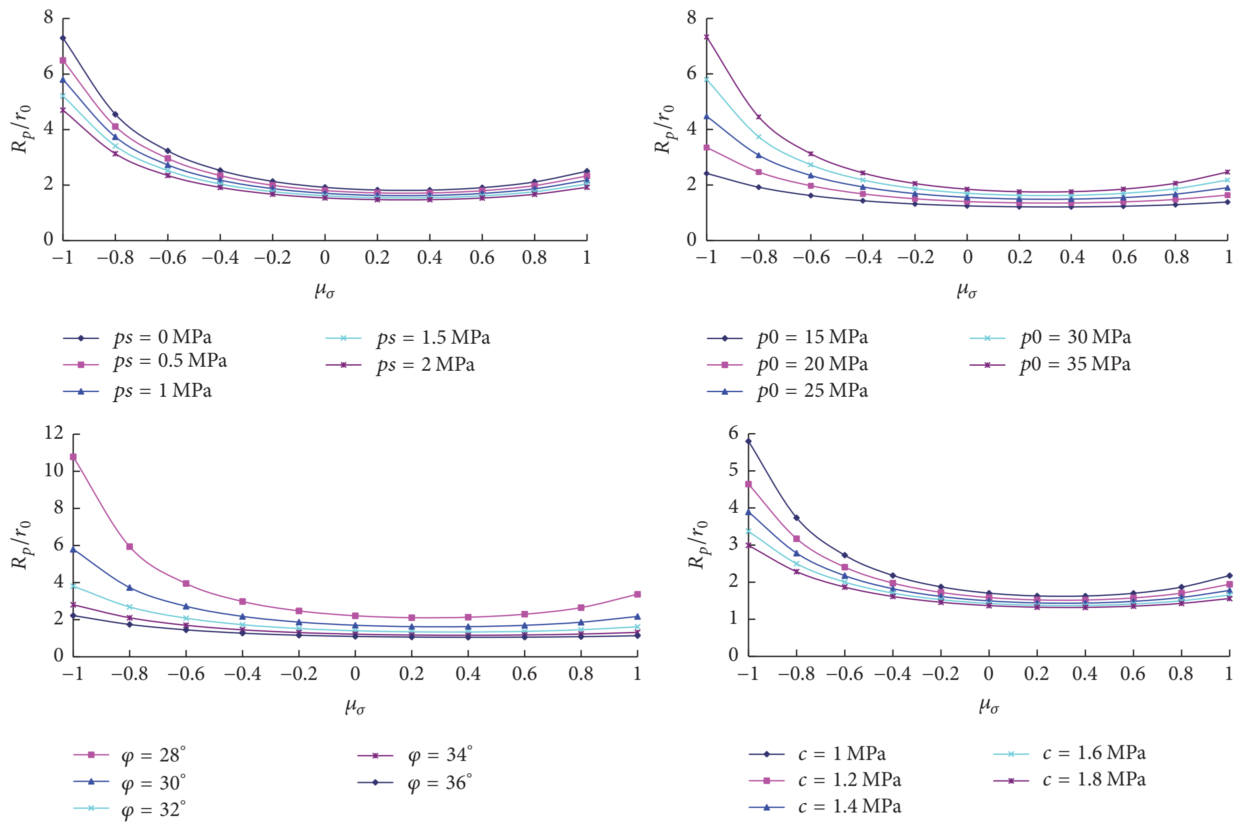


FIGURE 5: The effect of $\mu_\sigma$ on the radius of the ground plastic zone.

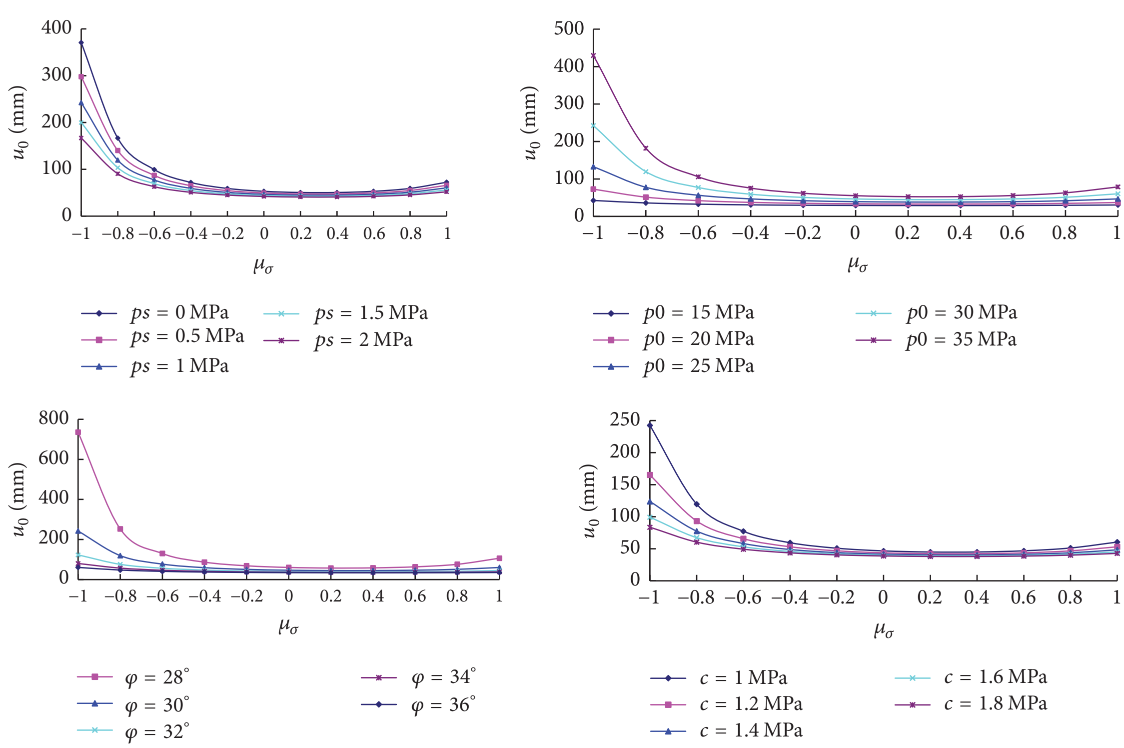


FIGURE 6: The effect of $\mu_\sigma$ on the ground radial displacement of the tunnel circumference.

(4) Finally, it should be noted that the proposed method is more suitable for the hard rock for which the time effect can be ignored.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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