Research Article

Strong Tracking Filter for Nonlinear Systems with Randomly Delayed Measurements and Correlated Noises

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This paper proposes a novel strong tracking filter (STF), which is suitable for dealing with the filtering problem of nonlinear systems when the following cases occur: that is, the constructed model does not match the actual system, the measurements have the one-step random delay, and the process and measurement noises are correlated at the same epoch. Firstly, a framework of decoupling filter (DF) based on equivalent model transformation is derived. Further, according to the framework of DF, a new extended Kalman filtering (EKF) algorithm via using first-order linearization approximation is developed. Secondly, the computational process of the suboptimal fading factor is derived on the basis of the extended orthogonality principle (EOP). Thirdly, the ultimate form of the proposed STF is obtained by introducing the suboptimal fading factor into the above EKF algorithm. The proposed STF can automatically tune the suboptimal fading factor on the basis of the residuals between available and predicted measurements and further the gain matrices of the proposed STF tune online to improve the filtering performance. Finally, the effectiveness of the proposed STF has been proved through numerical simulation experiments.

1. Introduction

Over the past decades, the filtering problem of nonlinear systems has been an active field of research on account of its widespread applications, for example, dynamic target tracking [1, 2], signal processing [3], and integrated navigation [4]. As a general rule, in view of the fact that the Bayesian estimator of nonlinear systems in minimum mean square error (MMSE) sense is usually faced with intractable computation [5], consequently using approximation methods to design cost-efficient estimators has received much attention. One approximation method is to use piecewise and time-varying linear functions to approximate nonlinear functions, for instance, the extended Kalman filter (EKF) [5] derived from Taylor expansion, the interpolation-based central difference Kalman filter (CDKF) [6], and the divided difference filter (DDF) [7] by using the polynomial interpolation of Stirling. These methods usually have better computational efficiency but are sensitive to linearization errors or differential operations. Another approximation method is to use Gaussian or

Gaussian mixture distribution to represent the conditional state probability density function [8, 9]. By utilizing different numerical integration technologies with cost-effective and acceptable accuracy, this approximation method yields the framework of different Gaussian approximation filters (GAFs), such as unscented transform- (UT-) based unscented Kalman filter (UKF) [10], the Gauss-Hermite filter (GHF) [11] based on the rule of Gaussian-Hermite quadrature, the Gaussian sum-quadrature Kalman filter (GS-QKF) [12] based on statistical linear regression and Gauss-Hermite quadrature, the square-root quadrature Kalman filter (SRQKF) [13] on the basis of triangular decomposition of matrix, the cubature Kalman filter (CKF) [14] according to the rule of third-degree spherical radial cubature, the high-degree CKFs [2] based on the rule of arbitrary-degree spherical radial cubature, and the sparse-grid quadrature filter (SGQF) [15] derived from sparse-grid theory.

In general, all the above approximation methods can obtain better filtering accuracy in case the models of state-space and measurement have sufficient accuracy. However,

in practical applications, the above two models may have model uncertainties; that is, the model does not match the actual system, and the main reasons for the model uncertainties are as follows: model simplification, inaccurate description of noise characteristic, original condition, or system parameter. For the sake of improving the filtering performance of nonlinear systems with model uncertainties, the STF, which can timely alter the matrices of predicted state error covariance and gain by introducing the time-varying suboptimal fading factors, was first proposed in [16]. Subsequently, the numerous transformations of STF have been presented. In [17], the sampling strong tracking nonlinear UKF was proposed for use in eye tracking. An adaptive UKF [18] based on STF and wavelet transform was presented to further enhance the tracking performance and robustness of standard UKF. The authors in [19] combine particle filter (PF) with the idea of STF and proposed an adaptive PF with strong tracking ability in the case of particle degeneracy and target state mutation. For the sake of tracking maneuvering target, a strong tracking spherical simplex-radial CKF has been designed in [20]. All the aforementioned strong tracking filtering methods are formulated in case the measurements can be reached on time and the process noise is not correlated with the measurement noise. But the following cases may occur in practical situations; that is, the measurements received may be affected by the random delay, or the process noise may be correlated with the measurement noise. For the first case, literatures [21, 22] have, respectively, proposed the networked STF and STF with randomly delayed measurements (STF/RDM). For the second case, a novel nonlinear filter derived from square-root CKF and the idea of STF was proposed in [23]. However, when the above two cases exist simultaneously, these existing STFs [17–23] are not suitable for dealing with the filtering problem in the above two coupled cases, and little attention has been paid to the study of deriving the corresponding STF. Consequently, there is a great demand to further improve the STF for the nonlinear discrete-time stochastic dynamic systems with randomly delayed measurements and correlated noises, which motivate this study.

In this paper, a novel filter, which we have called the strong tracking filter with randomly delayed measurements and correlated noises (STF/RDMCN), is proposed. Basically, the novel contributions of the paper are composed of the following. Under the idea of eliminating correlated noises at the same epoch, a general decoupling filter (DF) is derived through an equivalent transformation of the system model. The implementation of DF can thus be transformed into calculating the Gaussian-weighted integrals, which is achieved through the application of the first-order linearization approximation method to develop a new EKF algorithm. In the sense of extended orthogonality principle (EOP), the adaptive adjustment formula of the suboptimal fading factor is derived, introducing the suboptimal fading factor into the new EKF algorithm to make EKF adjusting the gain matrices in real time. This results in the ultimate form of STF/RDMCN. Numerical simulation experiments with nonlinear state estimation illustrate the effectiveness of the proposed algorithm.

The rest of this paper is arranged as follows. Section 2 gives the problem that needs to be investigated. Then, in Section 3, the general DF with one-step randomly delayed measurement is designed, and the EKF based on the first-order linearization approximation method is developed as an implementation of the proposed DF. Thereafter, in Section 4, according to the EOP, the formula of the suboptimal fading factor is derived, and it is combined with the above EKF to form the STF/RDMCN. In Section 5, the analysis of numerical simulation experiments is provided. Finally, some conclusions are supplied in Section 6.

2. Problem Formulation

Firstly, the nonlinear system model, which has one-step randomly delayed measurements and correlated noises, is formulated. Secondly, a new system model transformed from the former is given.

Consider the discrete-time, nonlinear stochastic system

$$\begin{aligned} x_{k+1} &= f_k(x_k) + w_k, \quad k \geq 0, \\ z_k &= h_k(x_k) + v_k, \quad k \geq 1, \\ y_k &= \begin{cases} (1 - \gamma_k) z_k + \gamma_k z_{k-1}, & k > 1, \\ z_k, & k = 1, \end{cases} \end{aligned} \quad (1)$$

where $\{x_k; k \geq 0\}$ represents the $n \times 1$ state vector, $\{z_k; k \geq 1\}$ represents the $m \times 1$ real measurement vector, $\{y_k; k \geq 1\}$ represents the $m \times 1$ available measurement vector, nonlinear mappings $f_k(\bullet)$ and $h_k(\bullet)$ are infinitely continuously differentiable, $\{w_k; k \geq 0\}$ and $\{v_k; k \geq 1\}$ are sequences of correlated zero-mean Gaussian white noises with covariance matrices $E[w_k w_k^T] = Q_k \delta_{kl}$, $E[v_k v_k^T] = R_k \delta_{kl}$, and $E[w_k v_l^T] = S_k \delta_{kl}$, with δ_{kl} representing the Kronecker delta function, the initial state x_0 , which is independent of $\{w_k; k \geq 0\}$ and $\{v_k; k \geq 1\}$, denotes a random Gaussian variable having mean $E[x_0] = \hat{x}_{0|0}$ and covariance $E[x_0 x_0^T] = P_{0|0}$, and $\{\gamma_k; k > 1\}$ represents a sequence of uncorrelated Bernoulli random variables that can take the value 0 or 1 with

$$\begin{aligned} p(\gamma_k = 1) &= E[\gamma_k] = p_k, \\ p(\gamma_k = 0) &= 1 - E[\gamma_k] = 1 - p_k, \\ E[(\gamma_k - p_k)^2] &= (1 - p_k) p_k, \\ E[(\gamma_k - p_k)] &= 0, \end{aligned} \quad (2)$$

where p_k denotes the delay probability.

Remark 1. In fact, the Bernoulli random variable $\{\gamma_k; k > 1\}$ describes the random delay characteristic of the available measurement vector; that is, when $\gamma_k = 0$, therefore $y_k = z_k$ which indicates that the available measurement vector is not affected by the random delay and is updated based on probability $1 - p_k$; when $\gamma_k = 1$, therefore $y_k = z_{k-1}$ which indicates that the available measurement vector randomly delays one sampling time based on probability p_k .

Remark 2. Without considering the model mismatch, a general framework of GF applied in the system shown in (1) has been proposed in [24]. Here, the two-step predictive probability density function (PDF) $p(x_{k+1} | Y_{k-1})$ of the state is assumed to be Gaussian, and this assumption faces complicated computation procedures in the state prediction phase. According to the literature [25], we can reconstruct the nonlinear state function in (1) for decoupling the correlation between process noise and measurement noise. Afterwards, the DF can be obtained by applying the general framework of GF with one-step random delay measurements in [26], which can surmount the defect of complicated computation procedures in the state prediction phase for the method mentioned above. This means that the derivation of the STF/RDMCN is based on DF.

At the same epoch, for decoupling the correlation of the process and measurement noises, the literature [27] introduced a positive definite matrix

$$U_k = \begin{bmatrix} I & -S_k R_k^{-1} \\ 0 & I \end{bmatrix}. \quad (3)$$

Here, I represents the unit matrix and R_k and S_k denote the covariance of the measurement noise v_k and the cross-covariance of the process noise w_k and the measurement noise v_k , respectively. Thus, we get

$$\begin{aligned} U_k \begin{bmatrix} w_k \\ v_k \end{bmatrix} &= \begin{bmatrix} I & -S_k R_k^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} = \begin{bmatrix} w_k - S_k R_k^{-1} v_k \\ v_k \end{bmatrix} \\ &= \begin{bmatrix} \bar{w}_k \\ v_k \end{bmatrix}, \end{aligned} \quad (4)$$

where $\bar{w}_k$ is pseudo-process noise satisfying $E[\bar{w}_k] = 0$ and $E[\bar{w}_k \bar{w}_l^T] = (Q_k - S_k R_k^{-1} S_k^T) \delta_{kl}$. It is easy to find that the pseudo-process noise and measurement noise are uncorrelated with each other because $E[\bar{w}_k v_l^T] = E[w_k v_l^T] - S_k R_k^{-1} E[v_k v_l^T] = 0$. Putting equation $w_k = \bar{w}_k + S_k R_k^{-1} v_k$ into the expression of x_{k+1} in (1) yields

$$x_{k+1} = f_k(x_k) + S_k R_k^{-1} v_k + \bar{w}_k. \quad (5)$$

Define $J_k = S_k R_k^{-1}$, $F_k(x_k) = f_k(x_k) + J_k v_k$. Then, the discrete-time nonlinear stochastic system shown by (1) is transformed into the following form:

$$x_{k+1} = F_k(x_k) + \bar{w}_k, \quad k \geq 1, \quad (6)$$

$$z_k = h_k(x_k) + v_k, \quad k \geq 1, \quad (7)$$

$$y_k = \begin{cases} (1 - \gamma_k) z_k + \gamma_k z_{k-1}, & k > 1, \\ z_k, & k = 1, \end{cases} \quad (8)$$

where x_0 , $\{\bar{w}_k; k \geq 1\}$, $\{v_k; k \geq 1\}$, and $\{\gamma_k; k > 1\}$ are mutually independent.

3. DF with One-Step Randomly Delayed Measurements

Firstly, considering literature [26], the framework of DF is given, due to the fact that nonlinear system model as shown in (6)–(8) satisfies the condition; that is, the process noise and measurement noise are uncorrelated. Secondly, a new EKF algorithm with first-order linear approximations is developed on the basis of this framework.

3.1. The Framework of DF. Continue to consider the nonlinear system model as shown in (6)–(8). Substituting (7) into (8), we have

$$\begin{aligned} y_{k+1} &= (1 - \gamma_{k+1}) [h_{k+1}(x_{k+1}) + v_{k+1}] \\ &\quad + \gamma_{k+1} [h_k(x_k) + v_k]. \end{aligned} \quad (9)$$

According to (9), we can find that the first two moments of $p(x_{k+1} | Y_{k+1})$ and $p(v_{k+1} | Y_{k+1})$ in the MMSE sense need to be obtained in deducing the framework of DP. Thus, it is necessary to define an augmented state vector as follows:

$$x_{k+1}^a = \begin{bmatrix} x_{k+1} \\ v_{k+1} \end{bmatrix}, \quad (10)$$

where the first two moments of $p(x_{k+1}^a | Y_{k+1})$ in the MMSE sense have the following expression:

$$\begin{aligned} \hat{x}_{k+1|k+1}^a &= \begin{bmatrix} \hat{x}_{k+1|k+1} \\ \hat{v}_{k+1|k+1} \end{bmatrix}, \\ P_{k+1|k+1}^a &= \begin{bmatrix} P_{k+1|k+1} & P_{k+1|k+1}^{xv} \\ (P_{k+1|k+1}^{xv})^T & P_{k+1|k+1}^{vv} \end{bmatrix}. \end{aligned} \quad (11)$$

In (11), $(\hat{x}_{k+1|k+1}^a, P_{k+1|k+1}^a)$, $(\hat{x}_{k+1|k+1}, P_{k+1|k+1})$, and $(\hat{v}_{k+1|k+1}, P_{k+1|k+1}^{vv})$ are the filtering estimation and the covariance at time $k + 1$ of the augmented state, the state, and the measurement noise, respectively; $P_{k+1|k+1}^{xv}$ is the cross-covariance at time $k + 1$ of the state and the measurement noise. Considering the independence of v_{k+1} with y_k and x_{k+1} , the augmented state prediction and the covariance are

$$\begin{aligned} \hat{x}_{k+1|k}^a &= \begin{bmatrix} \hat{x}_{k+1|k} \\ 0_{m \times 1} \end{bmatrix}, \\ P_{k+1|k}^a &= \begin{bmatrix} P_{k+1|k} & 0_{n \times m} \\ 0_{m \times n} & R_{k+1} \end{bmatrix}, \end{aligned} \quad (12)$$

$$k \geq 0.$$

Define the mean, covariance, and cross-covariance

$$\begin{aligned} \hat{x}_{k+1|k} &= E[x_{k+1} | Y_k], \\ P_{k+1|k} &= E[\tilde{x}_{k+1|k} \tilde{x}_{k+1|k}^T | Y_k], \\ \hat{y}_{k+1|k} &= E[y_{k+1} | Y_k], \\ P_{k+1|k}^{yy} &= E[\tilde{y}_{k+1|k} \tilde{y}_{k+1|k}^T | Y_k], \\ \hat{z}_{k+1|k} &= E[z_{k+1} | Y_k], \end{aligned}$$

$$\begin{aligned}
P_{k+1|k}^{zz} &= E \left[\tilde{z}_{k+1|k} \tilde{z}_{k+1|k}^T \mid Y_k \right], \\
\hat{z}_{k|k} &= E \left[z_k \mid Y_k \right], \\
P_{k|k}^{zz} &= E \left[\tilde{z}_{k|k} \tilde{z}_{k|k}^T \mid Y_k \right], \\
P_{k+1|k}^{xy} &= E \left[\tilde{x}_{k+1|k} \tilde{y}_{k+1|k}^T \mid Y_k \right], \\
P_{k+1|k}^{yy} &= E \left[\tilde{y}_{k+1|k} \tilde{y}_{k+1|k}^T \mid Y_k \right], \\
P_{k+1|k}^{xz} &= E \left[\tilde{x}_{k+1|k} \tilde{z}_{k+1|k}^T \mid Y_k \right], \\
P_{k+1,k|k}^{xz} &= E \left[\tilde{x}_{k+1|k} \tilde{z}_{k|k}^T \mid Y_k \right], \\
P_{k+1|k}^{vz} &= E \left[\tilde{v}_{k+1|k} \tilde{z}_{k+1|k}^T \mid Y_k \right], \\
P_{k+1,k|k}^{vz} &= E \left[\tilde{v}_{k+1|k} \tilde{z}_{k|k}^T \mid Y_k \right],
\end{aligned} \tag{13}$$

where $Y_k = \{y_i\}_{i=1}^k$ is the set of the available measurements in (8). According to literature [26], the equations describing the framework of DF are as follows.

Step 1 (state prediction). Putting (6) into the expression of $\hat{x}_{k+1|k}$ and $P_{k+1|k}$ in (13) yields

$$\begin{aligned}
\hat{x}_{k+1|k} &= E \left[(F_k(x_k) + \bar{w}_k) \mid Y_k \right], \\
P_{k+1|k} &= E \left[x_{k+1} x_{k+1}^T \mid Y_k \right] - \hat{x}_{k+1|k} \hat{x}_{k+1|k}^T \\
&= E \left[F_k(x_k) F_k^T(x_k) \mid Y_k \right] + E \left[\bar{w}_k \bar{w}_k^T \mid Y_k \right] \\
&\quad - \hat{x}_{k+1|k} \hat{x}_{k+1|k}^T.
\end{aligned} \tag{14}$$

Under the known $(\hat{x}_{k|k}^a, P_{k|k}^a)$, considering that $\bar{w}_k$ is independent of v_k and Y_k , we have

$$\begin{aligned}
\hat{x}_{k+1|k} &= \int [f_k(x_k) + J_k v_k] N(x_k^a; \hat{x}_{k|k}^a, P_{k|k}^a) dx_k^a, \\
P_{k+1|k} &= \int [f_k(x_k) + J_k v_k] [f_k(x_k) + J_k v_k]^T \\
&\quad \times N(x_k^a; \hat{x}_{k|k}^a, P_{k|k}^a) dx_k^a - \hat{x}_{k+1|k} \hat{x}_{k+1|k}^T \\
&\quad + Q_k - J_k R_k J_k^T.
\end{aligned} \tag{15}$$

Putting (15) into (12), the predictive estimates $(\hat{x}_{k+1|k}^a, P_{k+1|k}^a)$ of the augmented state can be obtained.

Step 2 (state correction). Under the known $(\hat{x}_{k|k}^a, P_{k|k}^a)$ and $(\hat{x}_{k+1|k}^a, P_{k+1|k}^a)$, considering that $\bar{w}_k$, v_k , y_k , and Y_k are mutually independent, we have

$$\hat{x}_{k+1|k+1}^a = \hat{x}_{k+1|k}^a + K_k (y_{k+1} - \hat{y}_{k+1|k}), \tag{16}$$

$$P_{k+1|k+1}^a = P_{k+1|k}^a - K_k P_{k+1|k}^{yy} K_k^T, \tag{17}$$

$$\begin{aligned}
K_k &= \begin{pmatrix} K_k^x \\ K_k^y \end{pmatrix} = P_{k+1|k}^{x^a y} (P_{k+1|k}^{yy})^{-1} \\
&= \begin{pmatrix} P_{k+1|k}^{xy} \\ P_{k+1|k}^{yy} \end{pmatrix} (P_{k+1|k}^{yy})^{-1},
\end{aligned} \tag{18}$$

$$\hat{y}_{k+1|k} = (1 - p_{k+1}) \hat{z}_{k+1|k} + p_{k+1} \hat{z}_{k|k}, \tag{19}$$

$$\begin{aligned}
P_{k+1|k}^{yy} &= (1 - p_{k+1}) P_{k+1|k}^{zz} + p_{k+1} P_{k|k}^{zz} \\
&\quad + (1 - p_{k+1}) p_{k+1} (\hat{z}_{k+1|k} - \hat{z}_{k|k}) (\hat{z}_{k+1|k} - \hat{z}_{k|k})^T,
\end{aligned} \tag{20}$$

$$\begin{aligned}
P_{k+1|k}^{x^a y} &= \begin{pmatrix} P_{k+1|k}^{xy} \\ P_{k+1|k}^{yy} \end{pmatrix} \\
&= \begin{pmatrix} (1 - p_{k+1}) P_{k+1|k}^{xz} + p_{k+1} P_{k+1,k|k}^{xz} \\ (1 - p_{k+1}) P_{k+1|k}^{vz} + p_{k+1} P_{k+1,k|k}^{vz} \end{pmatrix},
\end{aligned} \tag{21}$$

where K_k is the gain matrix of the augmented state and

$$\hat{z}_{k+1|k} = \int h_{k+1}(x_{k+1}) N(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k}) dx_{k+1}, \tag{22}$$

$$\begin{aligned}
P_{k+1|k}^{zz} &= \int h_{k+1}(x_{k+1}) h_{k+1}^T(x_{k+1}) \\
&\quad \cdot N(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k}) dx_{k+1} - \hat{z}_{k+1|k} \hat{z}_{k+1|k}^T \\
&\quad + R_{k+1},
\end{aligned} \tag{23}$$

$$\hat{z}_{k|k} = \int [h_k(x_k) + v_k] N(x_k^a; \hat{x}_{k|k}^a, P_{k|k}^a) dx_k^a, \tag{24}$$

$$\begin{aligned}
P_{k|k}^{zz} &= \int [h_k(x_k) + v_k] [h_k(x_k) + v_k]^T \\
&\quad \cdot N(x_k^a; \hat{x}_{k|k}^a, P_{k|k}^a) dx_k^a - \hat{z}_{k|k} \hat{z}_{k|k}^T,
\end{aligned} \tag{25}$$

$$\begin{aligned}
P_{k+1|k}^{xz} &= \int x_{k+1} h_{k+1}^T(x_{k+1}) \\
&\quad \cdot N(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k}) dx_{k+1} \\
&\quad - \hat{x}_{k+1|k} \hat{z}_{k+1|k}^T,
\end{aligned} \tag{26}$$

$$\begin{aligned}
P_{k+1,k|k}^{xz} &= \int [f_k(x_k) + J_k v_k] [h_k(x_k) + v_k]^T \\
&\quad \times N(x_k^a; \hat{x}_{k|k}^a, P_{k|k}^a) dx_k^a - \hat{x}_{k+1|k} \hat{z}_{k|k}^T,
\end{aligned} \tag{27}$$

$$\begin{aligned}
P_{k+1|k}^{vz} &= R_{k+1}, \\
P_{k+1,k|k}^{vz} &= 0.
\end{aligned} \tag{28}$$

3.2. Implementation of the DF. The cruxes of implementing the DF in (14) and (16)–(21) are to calculate the Gaussian-weighted integrals in (15) and (22)–(27). Owing to the nonlinearity of $f_k(\bullet)$ and $h_k(\bullet)$, the analytical calculation of the above integrals is intractable and infeasible. Therefore,

some technologies of numerical approximation are needed, for example, the first-order linear approximations. Here, we use the EKF with one-step randomly delayed measurements based on the first-order linearization to implement the DF in (14) and (16)–(21).

Given the filtering estimates $(\hat{x}_{k|k}^a, P_{k|k}^a)$, $f_k(x_k)$ and $h_k(x_k)$ are linearized about $x_k = \hat{x}_{k|k}$; that is,

$$x_{k+1} \approx f_k(\hat{x}_{k|k}) + \bar{F}_k(x_k - \hat{x}_{k|k}) + J_k v_k + \bar{w}_k, \quad (29)$$

$$z_k \approx h_k(\hat{x}_{k|k}) + H_k(x_k - \hat{x}_{k|k}) + v_k, \quad (30)$$

where $\bar{F}_k = \partial f_k(x_k)/\partial x_k|_{x_k=\hat{x}_{k|k}}$ and $H_k = \partial h_k(x_k)/\partial x_k|_{x_k=\hat{x}_{k|k}}$. Equations (15) are approximated as follows:

$$\hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}) + J_k \hat{v}_{k|k}, \quad (31)$$

$$P_{k+1|k} = \bar{F}_k P_{k|k} \bar{F}_k^T + \bar{F}_k P_{k|k}^{xv} J_k^T + (\bar{F}_k P_{k|k}^{xv} J_k^T)^T + J_k P_{k|k}^{vv} J_k^T + Q_k - J_k R_k J_k^T. \quad (32)$$

Further, the predictive estimates $(\hat{x}_{k+1|k}^a, P_{k+1|k}^a)$ can be calculated by putting (31)–(32) into (12).

Given the predictive estimates $(\hat{x}_{k+1|k}^a, P_{k+1|k}^a)$, we linearize $h_{k+1}(x_{k+1})$ about $x_{k+1} = \hat{x}_{k+1|k}$; that is,

$$z_{k+1} \approx h_{k+1}(\hat{x}_{k+1|k}) + H_{k+1}(x_{k+1} - \hat{x}_{k+1|k}) + v_{k+1}, \quad (33)$$

where $H_{k+1} = \partial h_{k+1}(x_{k+1})/\partial x_{k+1}|_{x_{k+1}=\hat{x}_{k+1|k}}$. Then, (22)–(27) are approximated as follows:

$$\hat{z}_{k+1|k} = h_{k+1}(\hat{x}_{k+1|k}), \quad (34)$$

$$P_{k+1|k}^{zz} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1}, \quad (35)$$

$$\hat{z}_{k|k} = h_k(\hat{x}_{k|k}) + \hat{v}_{k|k}, \quad (36)$$

$$P_{k|k}^{zz} = H_k P_{k|k} H_k^T + H_k P_{k|k}^{xv} + (H_k P_{k|k}^{xv})^T + P_{k|k}^{vv}, \quad (37)$$

$$P_{k+1|k}^{xz} = P_{k+1|k} H_{k+1}^T, \quad (38)$$

$$P_{k+1,k|k}^{xz} = \bar{F}_k P_{k|k} H_k^T + \bar{F}_k P_{k|k}^{xv} + J_k P_{k|k}^{vx} H_k^T + J_k P_{k|k}^{vv}. \quad (39)$$

Putting (34)–(39) into (16)–(21), the filtering estimates $(\hat{x}_{k+1|k+1}^a, P_{k+1|k+1}^a)$ of the augmented state can be calculated.

4. Derivation of the STF/RDMCN

In [16], the standard STF for nonlinear systems is proposed, and it has the following advantages: (1) when the model is uncertain due to the simplification of the system model, the uncertainty of the noise characteristics and initial conditions, or variation of system parameters, it has strong robustness, (2) it has an outstanding ability to track the state, regardless of the sudden or slow change in the state and even the system achieving a stable state or not, and (3) it adds a small amount of data overhead, and the computation complexity does not increase significantly. As a result, we consider that STF is especially suitable for the nonlinear state estimation in

these cases, namely, model uncertainties, randomly delayed measurements, and correlated noises.

However, the above STF cannot be straightforwardly employed in the nonlinear system shown in (1), owing to the fact that the discretionarily chosen pairs of residuals according to the orthogonality principle are computed on the basis of measurements without random delay. Therefore, an EOP, which is applied in the nonlinear system shown in (1), is given.

$$E \left\{ (x_{k+1}^a - x_{k+1|k+1}^a) (x_{k+1}^a - x_{k+1|k+1}^a)^T \right\} = \min, \quad (40)$$

$$E \left\{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \right\} = 0, \quad (41)$$

$$k = 0, 1, 2, \dots; j = 1, 2, \dots$$

Equation (40) is the performance index of the proposed EKF, and the corresponding derivation process can refer to [22]. Equation (41) means that the discretionarily chosen pairs of residuals which are calculated based on (9) and (19) are mutually orthogonal.

4.1. Derivation of the Suboptimal Fading Factor. It is not difficult to find that the proposed EKF offers a suboptimal estimation of the augmented state by using the given available measurements $Y_k = \{y_i\}_{i=1}^k$ when the system model is exact. However, when the model with uncertainty is developed, the estimation performance of the EKF will be poor or even divergent. The fundamental problem is that the gain matrix shown in (18) is not able to adapt to the change of the residuals between the available measurements and predicted measurements. In order to overcome this problem and make the proposed EKF have the excellent characteristics of STF, a natural idea is to combine the EOP with the proposed EKF to derive an STF/RDMCN by introducing a suboptimal fading factor into filtering estimate $P_{k|k}^a$ of the augmented state. The modified filtering estimate $P_{k|k}^{m,a}$ is as follows:

$$P_{k|k}^{m,a} = \begin{bmatrix} \lambda_{k+1} & 0 \\ 0 & \lambda_{k+1} \end{bmatrix} \begin{bmatrix} P_{k|k} & P_{k|k}^{xv} \\ (P_{k|k}^{xv})^T & P_{k|k}^{vv} \end{bmatrix} = \begin{bmatrix} P_{k|k}^m & P_{k|k}^{m,xv} \\ (P_{k|k}^{m,xv})^T & P_{k|k}^{m,vv} \end{bmatrix}, \quad (42)$$

where λ_{k+1} ($\lambda_{k+1} \geq 1$) denotes the suboptimal fading factor.

Remark 3. Substituting (42) into (32), we can find that the predictive estimate $P_{k+1|k}$ of the state is also modified by the same suboptimal fading factor. Continuing to consider (35), (37)–(39), and (18), we also find that the proposed STF/RDMCN can undermine the impact of the insignificant past information by utilizing the time-varying suboptimal fading factor and adjust the gain matrix of the augmented state in real time with the aim of improving the tracking performance of the filter.

Then, the next work is to determine the suboptimal fading factor λ_{k+1} according to the EOP.

Considering (9), (19), (30), (33), (34), and (36), we get

$$\begin{aligned}\tilde{y}_{k+1|k} &= (1 - \gamma_{k+1}) (H_{k+1} \tilde{x}_{k+1|k} + v_{k+1}) \\ &+ \gamma_{k+1} (H_k \tilde{x}_{k|k} + \tilde{v}_{k|k}) \\ &+ (\gamma_{k+1} - p_{k+1}) (\tilde{z}_{k|k} - \tilde{z}_{k+1|k}),\end{aligned}\quad (43)$$

where $\tilde{y}_{k+1|k} = y_{k+1} - \hat{y}_{k+1|k}$, $\tilde{x}_{k+1|k} = x_{k+1} - \hat{x}_{k+1|k}$, $\tilde{x}_{k|k} = x_k - \hat{x}_{k|k}$, $\tilde{v}_{k|k} = v_k - \hat{v}_{k|k}$.

Using (29) minus (31) yields

$$\tilde{x}_{k+1|k} = \bar{F}_k \tilde{x}_{k|k} + J_k \tilde{v}_{k|k} + \bar{w}_k. \quad (44)$$

Putting (44) into (43) yields

$$\begin{aligned}\tilde{y}_{k+1|k} &= [(1 - \gamma_{k+1}) H_{k+1} \bar{F}_k + \gamma_{k+1} H_k] \tilde{x}_{k|k} \\ &+ [(1 - \gamma_{k+1}) H_{k+1} J_k + \gamma_{k+1}] \tilde{v}_{k|k} \\ &+ (1 - \gamma_{k+1}) (H_{k+1} \bar{w}_k + v_{k+1}) \\ &+ (\gamma_{k+1} - p_{k+1}) (\tilde{z}_{k|k} - \tilde{z}_{k+1|k}).\end{aligned}\quad (45)$$

Using a similar derivation procedure, we have

$$\begin{aligned}\tilde{y}_{k+j+1|k+j} &= [(1 - \gamma_{k+j+1}) H_{k+j+1} \bar{F}_{k+j} + \gamma_{k+j+1} H_{k+j}] \tilde{x}_{k+j|k+j} \\ &+ [(1 - \gamma_{k+j+1}) H_{k+j+1} J_{k+j} + \gamma_{k+j+1}] \tilde{v}_{k+j|k+j} \\ &+ (1 - \gamma_{k+j+1}) (H_{k+j+1} \bar{w}_{k+j} + v_{k+j+1}) \\ &+ (\gamma_{k+j+1} - p_{k+j+1}) (\tilde{z}_{k+j|k+j} - \tilde{z}_{k+j+1|k+j}).\end{aligned}\quad (46)$$

Putting (46) into (41) yields

$$\begin{aligned}E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= E \{ [[(1 - \gamma_{k+j+1}) H_{k+j+1} \bar{F}_{k+j} + \gamma_{k+j+1} H_{k+j}] \\ &\cdot \tilde{x}_{k+j|k+j} + [(1 - \gamma_{k+j+1}) H_{k+j+1} J_{k+j} + \gamma_{k+j+1}] \\ &\cdot \tilde{v}_{k+j|k+j} + (1 - \gamma_{k+j+1}) (H_{k+j+1} \bar{w}_{k+j} + v_{k+j+1}) \\ &+ (\gamma_{k+j+1} - p_{k+j+1}) (\tilde{z}_{k+j|k+j} - \tilde{z}_{k+j+1|k+j})] \tilde{y}_{k+1|k}^T \}.\end{aligned}\quad (47)$$

Considering that x_0 , $\{\bar{w}_k; k \geq 1\}$, $\{v_k; k \geq 1\}$, $\{\gamma_k; k > 1\}$, and Y_k are mutually independent and combining (2), (47) is simplified to the following form: that is,

$$\begin{aligned}E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= M_{k+j} E \{ \tilde{x}_{k+j|k+j} \tilde{y}_{k+1|k}^T \} \\ &+ W_{k+j} E \{ \tilde{v}_{k+j|k+j} \tilde{y}_{k+1|k}^T \},\end{aligned}\quad (48)$$

where

$$\begin{aligned}M_{k+j} &= (1 - p_{k+j+1}) H_{k+j+1} \bar{F}_{k+j} + p_{k+j+1} H_{k+j}, \\ W_{k+j} &= (1 - p_{k+j+1}) H_{k+j+1} J_{k+j} + p_{k+j+1}.\end{aligned}\quad (49)$$

Considering (16), (11), and (12), we have

$$\begin{aligned}\tilde{x}_{k+j|k+j} &= \tilde{x}_{k+j|k+j-1} - K_{k+j-1}^x \tilde{y}_{k+j|k+j-1}, \\ \tilde{v}_{k+j|k+j} &= v_{k+j} - K_{k+j-1}^v \tilde{y}_{k+j|k+j-1}.\end{aligned}\quad (50)$$

Putting (50) into $E \{ \tilde{x}_{k+j|k+j} \tilde{y}_{k+1|k}^T \}$ and $E \{ \tilde{v}_{k+j|k+j} \tilde{y}_{k+1|k}^T \}$ in (48), we have

$$\begin{aligned}E \{ \tilde{x}_{k+j|k+j} \tilde{y}_{k+1|k}^T \} &= E \{ (\tilde{x}_{k+j|k+j-1} - K_{k+j-1}^x \tilde{y}_{k+j|k+j-1}) \tilde{y}_{k+1|k}^T \}, \\ E \{ \tilde{v}_{k+j|k+j} \tilde{y}_{k+1|k}^T \} &= E \{ (v_{k+j} - K_{k+j-1}^v \tilde{y}_{k+j|k+j-1}) \tilde{y}_{k+1|k}^T \}.\end{aligned}\quad (51)$$

Based on (44), (46), and using a similar simplification method in (47), $E \{ \tilde{x}_{k+j|k+j} \tilde{y}_{k+1|k}^T \}$ and $E \{ \tilde{v}_{k+j|k+j} \tilde{y}_{k+1|k}^T \}$ can be simplified to

$$\begin{aligned}E \{ \tilde{x}_{k+j|k+j} \tilde{y}_{k+1|k}^T \} &= \alpha_{k+j-1} E \{ \tilde{x}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \} \\ &+ \beta_{k+j-1} E \{ \tilde{v}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \}, \\ E \{ \tilde{v}_{k+j|k+j} \tilde{y}_{k+1|k}^T \} &= \chi_{k+j-1} E \{ \tilde{x}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \} \\ &+ \delta_{k+j-1} E \{ \tilde{v}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \},\end{aligned}\quad (52)$$

where

$$\begin{aligned}\alpha_{k+j-1} &= \bar{F}_{k+j-1} - K_{k+j-1}^x M_{k+j-1}, \\ \beta_{k+j-1} &= J_{k+j-1} - K_{k+j-1}^x W_{k+j-1}, \\ \chi_{k+j-1} &= -K_{k+j-1}^v M_{k+j-1}, \\ \delta_{k+j-1} &= -K_{k+j-1}^v W_{k+j-1}.\end{aligned}\quad (53)$$

Putting (52) into (48), rearranging (48) yields

$$\begin{aligned}E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= \varepsilon_{k+j-1} E \{ \tilde{x}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \} \\ &+ \phi_{k+j-1} E \{ \tilde{v}_{k+j-1|k+j-1} \tilde{y}_{k+1|k}^T \},\end{aligned}\quad (54)$$

where

$$\begin{aligned}\varepsilon_{k+j-1} &= M_{k+j} \alpha_{k+j-1} + W_{k+j} \chi_{k+j-1}, \\ \phi_{k+j-1} &= M_{k+j} \beta_{k+j-1} + W_{k+j} \delta_{k+j-1}.\end{aligned}\quad (55)$$

According to (48) and (54), we can get the following form of $E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \}$ by using the iterative operation: that is,

$$\begin{aligned}E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= \varepsilon_{k+i} E \{ \tilde{x}_{k+i|k+i} \tilde{y}_{k+1|k}^T \} \\ &+ \phi_{k+i} E \{ \tilde{v}_{k+i|k+i} \tilde{y}_{k+1|k}^T \},\end{aligned}\quad (56)$$

where

$$\begin{aligned}
\varepsilon_{k+i} &= M_{k+i}, \\
\phi_{k+i} &= W_{k+i}, \\
i &= j, \\
\varepsilon_{k+i} &= \varepsilon_{k+i+1}\alpha_{k+i} + \phi_{k+i+1}\chi_{k+i}, \\
\phi_{k+i} &= \varepsilon_{k+i+1}\beta_{k+i} + \phi_{k+i+1}\delta_{k+i}, \\
\alpha_{k+i} &= \bar{F}_{k+i} - K_{k+i}^x M_{k+i}, \\
\beta_{k+i} &= J_{k+i} - K_{k+i}^x W_{k+i}, \\
\chi_{k+i} &= -K_{k+i}^v M_{k+i}, \\
\delta_{k+i} &= -K_{k+i}^v W_{k+i}, \\
i &= j-1, j-2, \dots, 1, \\
M_{k+i} &= (1 - p_{k+i+1}) H_{k+i+1} \bar{F}_{k+i} + p_{k+i+1} H_{k+i}, \\
W_{k+i} &= (1 - p_{k+i+1}) H_{k+i+1} J_{k+i} + p_{k+i+1}, \\
i &= j, j-1, \dots, 1.
\end{aligned} \tag{57}$$

For $i = 1$, the following form of (56) can be obtained: that is,

$$\begin{aligned}
E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= \varepsilon_{k+1} E \{ \tilde{x}_{k+1|k+1} \tilde{y}_{k+1|k}^T \} \\
&+ \phi_{k+1} E \{ \tilde{v}_{k+1|k+1} \tilde{y}_{k+1|k}^T \}.
\end{aligned} \tag{58}$$

Using (50) about $j = 1$ and the expression of $P_{k+1|k}^{xy}$ and $P_{k+1|k}^{yy}$ in (13), we can get the following form of (58): that is,

$$\begin{aligned}
E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= \varepsilon_{k+1} E \{ (\tilde{x}_{k+1|k} - K_k^x \tilde{y}_{k+1|k}) \tilde{y}_{k+1|k}^T \} \\
&+ \phi_{k+1} E \{ (v_{k+1} - K_k^v \tilde{y}_{k+1|k}) \tilde{y}_{k+1|k}^T \} \\
&= \varepsilon_{k+1} (P_{k+1|k}^{xy} - K_k^x V_{k+1}^0) \\
&+ \phi_{k+1} (P_{k+1|k}^{vy} - K_k^v V_{k+1}^0),
\end{aligned} \tag{59}$$

where $V_{k+1}^0 \triangleq E \{ \tilde{y}_{k+1|k} \tilde{y}_{k+1|k}^T \}$ is the covariance of the residuals. Putting (18) into (59), we obtain

$$\begin{aligned}
E \{ \tilde{y}_{k+j+1|k+j} \tilde{y}_{k+1|k}^T \} &= \varepsilon_{k+1} P_{k+1|k}^{xy} \left(I - (P_{k+1|k}^{yy})^{-1} V_{k+1}^0 \right) \\
&+ \phi_{k+1} P_{k+1|k}^{vy} \left(I - (P_{k+1|k}^{yy})^{-1} V_{k+1}^0 \right).
\end{aligned} \tag{60}$$

According to (60), we can find that if a suitable suboptimal fading factor λ_{k+1} in (42) is chosen to ensure

$$I - (P_{k+1|k}^{yy})^{-1} V_{k+1}^0 = 0, \tag{61}$$

then the EOP will be satisfied. Putting (20), (35), and (37) into (61), rearranging (61) yields

$$\begin{aligned}
(1 - p_{k+1}) H_{k+1} P_{k+1|k} H_{k+1}^T &+ p_{k+1} \left(H_k P_{k|k} H_k^T + H_k P_{k|k}^{xy} + (H_k P_{k|k}^{xy})^T + P_{k|k}^{vv} \right) \\
&= V_{k+1}^0 - (1 - p_{k+1}) \\
&\cdot \left[p_{k+1} (\hat{z}_{k+1|k} - \hat{z}_{k|k}) (\hat{z}_{k+1|k} - \hat{z}_{k|k})^T + R_{k+1} \right].
\end{aligned} \tag{62}$$

Putting (32) and (42) into (62), rearranging (62) yields

$$\begin{aligned}
\lambda_{k+1} \left[(1 - p_{k+1}) \left(H_{k+1} \bar{F}_k P_{k|k} \bar{F}_k^T H_{k+1}^T \right. \right. &+ H_{k+1} \bar{F}_k P_{k|k}^{xy} J_k^T H_{k+1}^T + H_{k+1} (\bar{F}_k P_{k|k}^{xy} J_k^T)^T H_{k+1}^T \\
&+ H_{k+1} J_k P_{k|k}^{vv} J_k^T H_{k+1}^T \left. \right) + p_{k+1} \left(H_k P_{k|k} H_k^T + H_k P_{k|k}^{xy} \right. \\
&+ (H_k P_{k|k}^{xy})^T + P_{k|k}^{vv} \left. \right) \left. \right] = V_{k+1}^0 - (1 - p_{k+1}) \\
&\cdot \left[p_{k+1} (\hat{z}_{k+1|k} - \hat{z}_{k|k}) (\hat{z}_{k+1|k} - \hat{z}_{k|k})^T + R_{k+1} \right. \\
&+ H_{k+1} Q_k H_{k+1}^T - H_{k+1} J_k R_k J_k^T H_{k+1}^T \left. \right].
\end{aligned} \tag{63}$$

Similar to the idea of the literature [22], for obtaining the suboptimal fading factor λ_{k+1} , the trace operation is introduced into both sides of (63) as follows:

$$\begin{aligned}
\text{tr} \left[\lambda_{k+1} \left[(1 - p_{k+1}) \left(H_{k+1} \bar{F}_k P_{k|k} \bar{F}_k^T H_{k+1}^T \right. \right. \right. &+ H_{k+1} \bar{F}_k P_{k|k}^{xy} J_k^T H_{k+1}^T + H_{k+1} (\bar{F}_k P_{k|k}^{xy} J_k^T)^T H_{k+1}^T \\
&+ H_{k+1} J_k P_{k|k}^{vv} J_k^T H_{k+1}^T \left. \right) + p_{k+1} \left(H_k P_{k|k} H_k^T + H_k P_{k|k}^{xy} \right. \\
&+ (H_k P_{k|k}^{xy})^T + P_{k|k}^{vv} \left. \right) \left. \right] = \text{tr} \left[V_{k+1}^0 - (1 - p_{k+1}) \right. \\
&\cdot \left[p_{k+1} (\hat{z}_{k+1|k} - \hat{z}_{k|k}) (\hat{z}_{k+1|k} - \hat{z}_{k|k})^T + R_{k+1} \right. \\
&+ H_{k+1} Q_k H_{k+1}^T - H_{k+1} J_k R_k J_k^T H_{k+1}^T \left. \right] \left. \right].
\end{aligned} \tag{64}$$

Define

$$\begin{aligned}
M_{k+1} &\triangleq (1 - p_{k+1}) \left(H_{k+1} \bar{F}_k P_{k|k} \bar{F}_k^T H_{k+1}^T \right. \\
&+ H_{k+1} \bar{F}_k P_{k|k}^{xy} J_k^T H_{k+1}^T + H_{k+1} (\bar{F}_k P_{k|k}^{xy} J_k^T)^T H_{k+1}^T \\
&+ H_{k+1} J_k P_{k|k}^{vv} J_k^T H_{k+1}^T \left. \right) + p_{k+1} \left(H_k P_{k|k} H_k^T + H_k P_{k|k}^{xy} \right. \\
&+ (H_k P_{k|k}^{xy})^T + P_{k|k}^{vv} \left. \right),
\end{aligned} \tag{65}$$

$$\begin{aligned}
N_{k+1} &\triangleq V_{k+1}^0 - (1 - p_{k+1}) \\
&\cdot \left[p_{k+1} (\hat{z}_{k+1|k} - \hat{z}_{k|k}) (\hat{z}_{k+1|k} - \hat{z}_{k|k})^T + R_{k+1} \right. \\
&+ H_{k+1} Q_k H_{k+1}^T - H_{k+1} J_k R_k J_k^T H_{k+1}^T \left. \right].
\end{aligned} \tag{66}$$

Hence, (66) can be simplified to

$$\text{tr} [\lambda_{k+1} M_{k+1}] = \text{tr} [N_{k+1}]. \quad (67)$$

Then, the suboptimal fading factor λ_{k+1} is described as follows:

$$\lambda_{k+1} = \frac{\text{tr} [N_{k+1}]}{\text{tr} [M_{k+1}]} \quad (68)$$

Nevertheless, the covariance of the residuals V_{k+1}^0 in (66) is unknown, which can be determined by the following rough method:

$$V_{k+1}^0 = \begin{cases} \hat{y}_{1|0} \hat{y}_{1|0}^T & k = 0 \\ \frac{\rho V_k^0 + \hat{y}_{k+1|k} \hat{y}_{k+1|k}^T}{1 + \rho} & k \geq 1, \end{cases} \quad (69)$$

where ρ ($0 < \rho \leq 1$) is a forgetting factor which is often selected as $\rho = 0.95$ according to [22]. For $\lambda_{k+1} \geq 1$, the suboptimal fading factor λ_{k+1} can take effect, so λ_{k+1} can be ultimately calculated via

$$\lambda_{k+1} = \max \left\{ 1, \frac{\text{tr} [N_{k+1}]}{\text{tr} [M_{k+1}]} \right\}. \quad (70)$$

4.2. Computational Procedure of the STF/RDMCN. Now, we apply the first-order linear approximation method to compute these integrals in (15) and (22)–(27) and develop a new STF. Further, the proposed STF/RDMCN for nonlinear system model (1) is summarized as follows.

(1) Initialization ($k = 0$) is as follows:

$$\begin{aligned} \hat{x}_{0|0}^a &= \begin{bmatrix} \hat{x}_{0|0} \\ 0 \end{bmatrix}, \\ P_{0|0}^a &= \begin{bmatrix} P_{0|0} & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \quad (71)$$

(2) For $k = 1$, one has the following.

Step 1 (calculation and introduction of suboptimal fading factor). $\hat{x}_{1|0}, \hat{y}_{1|0}, M_1$, and N_1 can be calculated as follows:

$$\begin{aligned} \hat{x}_{1|0} &= f_0(\hat{x}_{0|0}), \\ \hat{y}_{1|0} &= \hat{z}_{1|0} = h_1(\hat{x}_{1|0}), \\ M_1 &\triangleq H_1 \bar{F}_0 P_{0|0} \bar{F}_0^T H_1^T, \\ N_1 &\triangleq V_1^0 - R_1 + H_1 Q_0 H_1^T, \end{aligned} \quad (72)$$

where V_1^0 can be calculated by (69). Putting M_1 and N_1 into (70) obtains λ_1 . Then, introducing λ_1 into (42) obtains $P_{0|0}^{m,a}$.

Step 2 (state prediction). $P_{1|0}$ can be calculated by $P_{1|0} = \bar{F}_0 P_{0|0}^m \bar{F}_0^T + Q_0$. The predictive estimates $(\hat{x}_{1|0}^a, P_{1|0}^a)$ can be calculated by putting $\hat{x}_{1|0}$ and $P_{1|0}$ into (12).

Step 3 (state correction). $P_{1|0}^{yy}$ and $P_{1|0}^{x^a y}$ can be calculated as follows:

$$\begin{aligned} P_{1|0}^{yy} &= P_{1|0}^{zz} = H_1 P_{1|0} H_1^T + R_1, \\ P_{1|0}^{x^a y} &= \begin{pmatrix} P_{1|0}^{xy} \\ P_{1|0}^{vy} \end{pmatrix} = \begin{pmatrix} P_{1|0}^{xz} \\ P_{1|0}^{vz} \end{pmatrix} = \begin{pmatrix} P_{1|0} H_1^T \\ R_1 \end{pmatrix}. \end{aligned} \quad (73)$$

The filtering estimates $(\hat{x}_{1|1}^a, P_{1|1}^a)$ can be calculated by putting $\hat{y}_{1|0}$ and (73) into (16)–(18).

(3) For $k > 1$, one has the following.

Step 1 (calculation and introduction of suboptimal fading factor). Assume that, at time k , the filtering estimates $(\hat{x}_{k|k}^a, P_{k|k}^a)$ and the covariance of the residuals V_k^0 are all known. For time $k+1$, $\hat{x}_{k+1|k}, \hat{y}_{k+1|k}, V_{k+1}^0, M_{k+1}$, and N_{k+1} can be calculated by (31), (19), (69), (65), and (66), respectively. Putting M_{k+1} and N_{k+1} into (70) obtains λ_{k+1} . Then, introducing λ_{k+1} into (42) obtains $P_{k|k}^{m,a}$.

Step 2 (state prediction). $P_{k+1|k}$ can be calculated by

$$\begin{aligned} P_{k+1|k} &= \bar{F}_k P_{k|k}^m \bar{F}_k^T + \bar{F}_k P_{k|k}^{m,xv} J_k^T + (\bar{F}_k P_{k|k}^{m,xv} J_k^T)^T \\ &\quad + J_k P_{k|k}^{m,vv} J_k^T + Q_k - J_k R_k J_k^T. \end{aligned} \quad (74)$$

The predictive estimates $(\hat{x}_{k+1|k}^a, P_{k+1|k}^a)$ can be calculated by putting $\hat{x}_{k+1|k}$ and $P_{k+1|k}$ into (12).

Step 3 (state correction). $P_{k+1|k}^{zz}$ and $P_{k+1|k}^{x^a z}$ can be calculated by (35) and (38). $P_{k+1|k}^{zz}$ and $P_{k+1|k}^{x^a z}$ can be calculated as follows:

$$\begin{aligned} P_{k+1|k}^{zz} &= H_k P_{k|k}^m H_k^T + H_k P_{k|k}^{m,xv} + (H_k P_{k|k}^{m,xv})^T \\ &\quad + P_{k|k}^{m,vv}, \\ P_{k+1|k}^{x^a z} &= \bar{F}_k P_{k|k}^m H_k^T + \bar{F}_k P_{k|k}^{m,xv} + J_k P_{k|k}^{m,vx} H_k^T \\ &\quad + J_k P_{k|k}^{m,vv}. \end{aligned} \quad (75)$$

Once we obtain a new measurement y_{k+1} , putting $\hat{x}_{k+1|k}^a, P_{k+1|k}^a, \hat{y}_{k+1|k}, P_{k+1|k}^{zz}, P_{k+1|k}^{x^a z}$, and (75) into (16)–(21) can calculate the filtering estimates $(\hat{x}_{k+1|k+1}^a, P_{k+1|k+1}^a)$ at time $k+1$.

5. Simulation

To validate the effectiveness of the STF/RDM in nonlinear state estimation, the universal nonstationary growth model is used in the numerical simulation experiments. Meanwhile, we compare the performance of the three different filters, that is, the proposed filter, the STF/RDM in [22], and the existing EKF in the Appendix. The nonlinear systems model is as follows:

$$x_{k+1} = 0.5x_k + 25 \frac{x_k}{1 + x_k^2} + 8 \cos(1.2k) + w_k, \quad k \geq 0, \quad (76)$$

$$z_k = \frac{x_k^2}{20} + v_k, \quad k \geq 1,$$

TABLE 1: Mean of RMSE_k for $S_k = 0$.

One-step delay probability	STF/RDMCN	STF/RDM
$p_k = 0.2$	14.2186	14.2186

FIGURE 1: RMSE curves for $p_k = 0.2$ and $S_k = 0$.

where the true value of the initial state, x_0 , is zero, but, in simulation, the initial state estimate $\hat{x}_{0|0}$ is a random Gaussian variable with zero-mean and covariance which is a random number between zero and one, w_k and v_k are zero-mean Gaussian white noises satisfying $Q_k = 10$ and $R_k = 1$, and w_k and v_k are correlated with cross-covariance S_k . Assuming that the available measurements are one-step randomly delayed, then

$$y_k = (1 - \gamma_k) z_k + \gamma_k z_{k-1}, \quad k > 1; \quad y_1 = z_1, \quad (77)$$

where γ_k represents a sequence of uncorrelated Bernoulli random variables with $p(\gamma_k = 1) = p$ for all k .

The root mean square error (RMSE) is used as performance index for various nonlinear filters. The RMSE at time k is defined as

$$\text{RMSE}_k = \left(\frac{1}{\text{MC}} \sum_{s=1}^{\text{MC}} (x_k^{(s)} - \hat{x}_k^{(s)})^2 \right)^{1/2}, \quad 1 \leq k \leq 50, \quad (78)$$

where $\text{MC} = 1000$ represents the total number of the independent numerical simulation experiments and $x_k^{(s)}$ and $\hat{x}_k^{(s)}$, respectively, denote the true and estimated states at the s th numerical simulation experiment.

In case I, $p_k = 0.2$ and $S_k = 0$. Figure 1 shows the RMSE results of the proposed STF/RDMCN and the STF/RDM. The mean of RMSE_k from the two filters is computed in Table 1. As can be seen from Figure 1 and Table 1, the proposed STF/RDMCN estimate performs exactly the same as the STF/RDM. The reason for this is that $S_k = 0$ means that w_k is not correlated with v_k , and the proposed STF/RDMCN can

TABLE 2: Mean of RMSE_k for $S_k = 0.5$.

One-step delay probability	STF/RDMCN	STF/RDM	Existing EKF
$p_k = 0.5$	12.9925	13.8617	19.4514

FIGURE 2: RMSE curves for $p_k = 0.5$ and $S_k = 0.5$.

degrade to the STF/RDM. That is to say, regardless of whether the noises are correlated or not, the proposed STF/RDMCN can solve the filtering problem in these two cases; therefore it has a wider range of applications than the STF/RDM.

In case II, $p_k = 0.5$ and $S_k = 0.5$. The RMSE results of the proposed STF/RDMCN, the STF/RDM, and the existing EKF are shown in Figure 2, the mean of RMSE_k about the above three filters is shown in Table 2, and the mean of suboptimal fading factors derived from the STF/RDMCN and STF/RDM is given in Figure 3. According to Table 2 and Figures 2 and 3, the STF/RDMCN and STF/RDM outperform the existing EKF in estimation accuracy. This is due to the fact that the STF/RDMCN and STF/RDM can seasonably find out the increase of residuals and enhance the estimation precision via the suboptimal fading factors adaptively increasing while the existing EKF does not adapt to the increase of residuals. Moreover, the mean of RMSE_k and that of the suboptimal fading factor of STF/RDMCN are smaller than the STF/RDM. Compared with the STF/RDM, the proposed STF/RDMCN can reflect the change of the residuals by lesser adjusting of the suboptimal fading factors. This means that, unlike the STF/RDM, the proposed STF/RDMCN can weaken the effect of the accumulative estimation error through the smaller suboptimal fading factor to ensure better tracking accuracy.

In case III, $p_k = 0.1, 0.2, \dots, 0.9$ and $S_k = 0.5$. Figure 4 gives the mean of RMSE_k calculated by utilizing the proposed STF/RDMCN, the STF/RDM, and the existing EKF. As the value of p increases, the mean of the existing EKF is increased, and those of the proposed STF/RDMCN and STF/RDM

FIGURE 3: The mean of fading factor for $p_k = 0.5$ and $S_k = 0.5$.

FIGURE 4: The mean of $RMSE_k$ for $p_k = 0.1, 0.2, \dots, 0.9$ and $S_k = 0.5$.

are decreased. The average of the proposed STF/RDMCN is smaller than that of the STF/RDM and the existing EKF. This figure indicates that the proposed STF/RDMCN has the best filtering performance compared to the other two filters, in case the delay probability p is chosen from a greater value.

In case IV, $p_k = 0.5$ and $S_k = 0.1, 0.2, \dots, 0.9$. Figure 5 shows the mean of $RMSE_k$ about the three filters. As can be seen from Figure 5, the mean of existing EKF is the worst, meaning that the proposed STF/RDMCN and the STF/RDM have better tracking accuracy. Furthermore, the proposed STF/RDMCN is more promising, regardless of whether correlation parameter S_k has a large range of change or not.

FIGURE 5: The mean of $RMSE_k$ for $p_k = 0.5$ and $S_k = 0.1, 0.2, \dots, 0.9$.

6. Conclusion

In this paper, we have presented a strong tracking filter with randomly delayed measurements and correlated noises (STF/RDMCN). By reconstructing an equivalent nonlinear state function, the framework of decoupling filter (DF) is derived, which can eliminate the correlation between the process and measurement noises. Then, a new EKF with one-step randomly delayed measurements is developed by using first-order linearization approximation for calculating the Gaussian-weighted integrals in the DF framework. Further, in order to make the above EKF have a strong tracking ability, the suboptimal fading factor, which is derived in the sense of the extended orthogonality principle (EOP), is introduced. Finally, the STF/RDMCN, which can seasonably find the alteration of residuals between the available measurements and predicted measurements and keep well-tracking performance via changing the suboptimal fading factor in real time, is formed. The numerical experiment outcomes confirmed that, under the condition of selecting different delay probabilities and correlation parameters, the proposed STF/RDMCN exceeds the STF/RDM and the existing EKF in tracking accuracy. It is also demonstrated that the previously mentioned STF/RDM is the special case of the proposed method because the STF/RDMCN can degrade to the STF/RDM via setting the correlation parameter to zero.

Appendix

According to the literature [24], the existing EKF can be obtained by using first-order linearization approximation.

Step 1 (state prediction). One has

$$\hat{x}_{k+1|k} = \hat{x}_{k+1|k-1} + M_k (y_k - \hat{y}_{k|k-1}),$$

$$\begin{aligned}
P_{k+1|k} &= P_{k+1|k-1} - M_k P_{k|k-1}^{yy} M_k^T, \\
M_k &= P_{k+1,k|k-1}^{xy} (P_{k|k-1}^{yy})^{-1}, \\
\hat{y}_{k|k-1} &= (1 - p_k) \hat{z}_{k|k-1} + p_k \hat{z}_{k-1|k-1}, \\
P_{k|k-1}^{yy} &= (1 - p_k) P_{k|k-1}^{zz} + p_k P_{k-1|k-1}^{zz} + (1 - p_k) \\
&\quad \cdot p_k (\hat{z}_{k|k-1} - \hat{z}_{k-1|k-1}) (\hat{z}_{k|k-1} - \hat{z}_{k-1|k-1})^T, \\
P_{k+1,k|k-1}^{xy} &= (1 - p_k) P_{k+1,k|k-1}^{xz} + p_k P_{k+1,k-1|k-1}^{xz},
\end{aligned} \tag{A.1}$$

where M_k represents the gain matrix and

$$\begin{aligned}
\hat{x}_{k+1|k-1} &= f_k (\hat{x}_{k|k-1}), \\
P_{k+1|k-1} &= F_k P_{k|k-1} F_k^T + Q_k, \\
F_k &= \left. \frac{\partial f_k(x_k)}{\partial x_k} \right|_{x_k = \hat{x}_{k|k-1}}, \\
\hat{z}_{k|k-1} &= h_k (\hat{x}_{k|k-1}), \\
P_{k|k-1}^{zz} &= H_k P_{k|k-1} H_k^T + R_k, \\
H_k &= \left. \frac{\partial h_k(x_k)}{\partial x_k} \right|_{x_k = \hat{x}_{k|k-1}}, \\
\hat{z}_{k-1|k-1} &= h_{k-1} (\hat{x}_{k-1|k-1}) + \hat{v}_{k-1|k-1}, \\
P_{k-1|k-1}^{zz} &= H_{k-1} P_{k-1|k-1} H_{k-1}^T + H_{k-1} P_{k-1|k-1}^{xv} \\
&\quad + (H_{k-1} P_{k-1|k-1}^{xv})^T + P_{k-1|k-1}^{vv}, \\
P_{k+1,k|k-1}^{xz} &= F_k P_{k|k-1} H_k^T + S_k, \\
P_{k+1,k-1|k-1}^{xz} &= F_k P_{k,k-1|k-1}^{xx} H_{k-1}^T, \\
H_{k-1} &= \left. \frac{\partial h_{k-1}(x_{k-1})}{\partial x_{k-1}} \right|_{x_{k-1} = \hat{x}_{k-1|k-1}}, \\
P_{k,k-1|k-1}^{xx} &= P_{k,k-1|k-2}^{xx} - M_{k-1} P_{k-1|k-2}^{yy} (K_{k-1}^x)^T, \\
P_{k,k-1|k-2}^{xx} &= F_{k-1} P_{k-1|k-2}, \\
F_{k-1} &= \left. \frac{\partial f_{k-1}(x_{k-1})}{\partial x_{k-1}} \right|_{x_{k-1} = \hat{x}_{k-1|k-2}}.
\end{aligned} \tag{A.2}$$

For $k = 0$, $\hat{x}_{1|0} = f_0(\hat{x}_{0|0})$ and $P_{1|0} = F_0 P_{0|0} F_0^T + Q_0$. For $k = 1$, $y_1 = z_1$, $\hat{y}_{1|0} = \hat{z}_{1|0}$, $P_{1|0}^{yy} = P_{1|0}^{zz}$, $P_{2,1|0}^{xy} = P_{2,1|0}^{xz}$. The predictive estimates $\hat{x}_{k+1|k}^a$ and $P_{k+1|k}^a$ of the augmented state can be calculated by putting $\hat{x}_{1|0}$ and $P_{1|0}$ into (12).

Step 2 (state correction). One has

$$\begin{aligned}
\hat{x}_{k+1|k+1}^a &= \hat{x}_{k+1|k}^a + K_{k+1} (y_{k+1} - \hat{y}_{k+1|k}), \\
P_{k+1|k+1}^a &= P_{k+1|k}^a - K_{k+1} P_{k+1|k}^{yy} K_{k+1}^T,
\end{aligned}$$

$$\begin{aligned}
K_{k+1} &= \begin{bmatrix} K_{k+1}^x \\ K_{k+1}^y \end{bmatrix} = P_{k+1|k}^{x^a y} (P_{k+1|k}^{yy})^{-1} \\
&= \begin{bmatrix} P_{k+1|k}^{xy} \\ P_{k+1|k}^{yy} \end{bmatrix} (P_{k+1|k}^{yy})^{-1}, \\
P_{k+1|k}^{x^a y} &= \begin{bmatrix} P_{k+1|k}^{xy} \\ P_{k+1|k}^{yy} \end{bmatrix} \\
&= \begin{bmatrix} (1 - p_{k+1}) P_{k+1|k}^{xz} + p_{k+1} P_{k+1,k|k}^{xz} \\ (1 - p_{k+1}) P_{k+1|k}^{yz} + p_{k+1} P_{k+1,k|k}^{yz} \end{bmatrix},
\end{aligned} \tag{A.3}$$

where K_k represents the gain matrix and

$$\begin{aligned}
P_{k+1|k}^{xz} &= P_{k+1|k} H_{k+1}^T, \\
H_{k+1} &= \left. \frac{\partial h_{k+1}(x_{k+1})}{\partial x_{k+1}} \right|_{x_{k+1} = \hat{x}_{k+1|k}}, \\
P_{k+1,k|k}^{xz} &= F_k P_{k|k} H_k^T + F_k P_{k|k}^{xv} + J_k (H_k P_{k|k}^{xv})^T \\
&\quad + J_k P_{k|k}^{vv}, \\
F_k &= \left. \frac{\partial f_k(x_k)}{\partial x_k} \right|_{x_k = \hat{x}_{k|k}}, \\
H_k &= \left. \frac{\partial h_k(x_k)}{\partial x_k} \right|_{x_k = \hat{x}_{k|k}}, \\
P_{k+1|k}^{vz} &= R_{k+1}, \\
P_{k+1,k|k}^{vz} &= 0.
\end{aligned} \tag{A.4}$$

For $k = 0$, $y_1 = z_1$, $\hat{y}_{1|0} = \hat{z}_{1|0}$, $P_{1|0}^{yy} = P_{1|0}^{zz}$, $P_{1|0}^{xy} = P_{1|0}^{xz}$, $P_{1|0}^{vy} = P_{1|0}^{vz}$. Once y_{k+1} are obtained, the filtering estimates $\hat{x}_{k+1|k+1}^a$ and $P_{k+1|k+1}^a$ at time $k + 1$ of the augmented state can be calculated.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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