Research Article

Novel Joint-Financing Model for Bilateral Capital Restricted Supply Chains under Cap-and-Trade Regulation

Yanfang Huo, Xize Wang, Quan Deng, and Peng Han

College of Management and Economics, Tianjin University, Tianjin 300072, China

Correspondence should be addressed to Yanfang Huo; yanfanghuo@tju.edu.cn

Received 9 March 2018; Accepted 13 August 2018; Published 30 August 2018

1. Introduction

Cap-and-trade regulation is generally accepted as one of the most effective market-based mechanisms to curb corporate carbon emissions [1] and is currently widely implemented across the globe [2, 3]. Under a cap-and-trade system, a government agency allocates a predetermined amount of carbon emissions (i.e., “carbon cap”) to a company; the company is then free to buy or sell carbon credit according to its actual amount of carbon emissions on a carbon trading market such as the European Emissions Trading System [4]. In April 2002, Great Britain established the world’s first carbon emissions trading market; the United States, Canada, Australia, and other countries followed suit. The Chinese government has also adopted a variety of management tools and policies to reduce greenhouse gas emissions. The Shenzhen Carbon Emissions Exchange was officially put into use on June 18, 2013, which marked the official start of carbon trading in China.

SMEs account for over 90% of Chinese enterprise. They are inherently capital-limited and may struggle financially to implement carbon-cutting initiatives. They also struggle to secure individual loan applications due to a lack of sufficient credit or collateral. There is substantial research significance in designing new financing mechanisms to help Chinese SMEs obtain the start-up capital necessary for emissions reduction while also giving consideration to the benefits and risks of the banks supporting them.

Carbon rights can be considered a tradable resource similar to other traditional resources. New regulations offer SMEs a feasible channel to obtain capital support. The trading market provides carbon credits not only for traditional resources such as inventory and notes, but also for financial attributes such as investment and financing. The trade credit is an important short-term financing source for retailers [5]. Petersen and Rajan [6] estimate that 70% of small firms in the US provide trade credit to their customers; Ge and Qiu found that 27% of total sales in China are based on trade credit [7]. Introducing carbon credits to a pledge model not only provides the realistic possibility for innovation within the traditional supply chain financing model but also provides a new way for SMEs to solve financing problems.

Commercial banks have also come to recognize the remarkable potential of carbon trading and low-carbon...
financing. Industrial Bank launched China's first low-carbon credit card as a financial channel for carbon management in SMEs, such as energy-saving service providers financing models, energy-saving emissions reduction technology loans, emissions pledge financing projects, and other low-carbon financing models. China Merchants Bank sets up a special low-carbon product research and development (R&D) center which supports green equipment buyer’s credits and other low-carbon and emissions reduction financing products. Minsheng Bank, Shanghai Pudong Development Bank, and others are also actively exploring low-carbon financial products and programs to provide enterprises the support they need for energy-saving and carbon-related credit support.

Carbon trade financing is yet a relatively unsophisticated practice despite considerable progress both in actual application and in theoretical research. There has been very little research on this subject in regard to SMEs specifically. Providing a new type of financing for SMEs based on carbon assets is an important research subject in the carbon trading field. This paper presents a novel financing pattern for bilateral capital restricted supply chains per the practical characteristics of SMEs, in which both the manufacturer and the retailer have insufficient capital for a carbon emission cutting project. An analysis of bank decisions broadens the supply chain financing data. The trend of total carbon emissions in the supply chain system is discussed as it relates to carbon trading prices. A combination of penalties and incentives is a suitable choice for the Chinese government to control carbon emissions rather than a simple regime of harsh punishment under the joint-financing pattern; such a combination will effectively promote emissions reduction in SMEs.

2. Literature Review

In practice, SMEs often face much more difficulties in accessing credit than larger ones. Supply chain finance (SCF) was proposed to solve the short plate effect caused by shortage of funds, which has obtained a rapidly development in terms of both theoretical and practical aspects. Shenzhen Development Bank took the lead in proposing and conducting supply chain financing business in China, and other commercial banks have also launched their own supply chain financing business model. It is expected that, by 2020, the capacity of supply chain financing will reach RMB14.98 trillion Yuan. Meanwhile, research shows that nearly 60% of enterprises have the subjective intention of supply chain financing [8].

As for the theoretical researches [9, 10], other researchers showed that financing services can create new supply chain value and help the supply chain parties to achieve optimal decision. The innovation adoption of SCF could optimize working or operational capital and reduce capital costs or financial costs along the supply chain [11]. Buzacott and Zhang, in the context of inventory management earlier, explored the problem of capital-constrained business operations and financing decisions when banks were strategic decision-makers [12]. Randall demonstrates how supply chain financial management techniques, such as cash-to-cash cycles, reduce the financial costs and gain a potential supply chain improvement through collaborative management [13]. Dada and Hu established a Stackelberg game model of strategic banks and capital-constrained retailers, given the game equilibrium and the financing system coordination contract [14]. Li et al. studied the pledge rate decision of inventory pledge financing in logistics finance through the construction of capital constraints [15]. Further, Kouvelis and Zhao studied the design of supply chain coordination contracts in financing model [16]. Jin and Luo [17] investigate the equilibrium financing portfolio strategies of trade credit and bank financing for a supply chain where the upstream and the downstream firms were both capital-constrained and got the optimal wholesale price, order quantity, and loan interest rate under different combinations. Nearly all the existing studies are generally assumed that only one party has the financial constraints. Meanwhile there always exists a focus firm who can improve the inferior partners' credit level through its own good credit. As a return, the larger participant might receive financial rewards or information as to how the others operate [18]. Thereby, the supply chain can operate in a more efficient way to make all the parties involved more profitable. Studies have proved that when the retailer is facing the problem of capital restriction, financial service from the internal help in a chain could not only create new profits but optimize decision-making as a whole [19, 20]. Yet, it could be possible, even more common, that both sides are weaker and have no enough money to take low-carbon technologies. In the case, the two SMEs need finance jointly to raising their credit levels, which is an emerging area deserving more focuses [21].

In addition, there are few papers discussing the financing problem on cap-and-trade systems faced by SMEs. Studies on modelling and quantitative analysis are especially limited. In a low-carbon supply chain, carbon credits could be used as a pledge by SMEs when loaning from banks. Carbon credit is a new property right based on the carbon emission quota allocated by the government. Its exchange value accounts for its security interest as collateral of pledge [22]. Wang and Song created several carbon finance methods to drive the development of emissions reductions [23]. New monetary systems for carbon emissions credits are questioned on stability, causing the current gap in low-carbon investment. Under certain economic conditions, banks would shy away from lending to low-carbon activities even in the presence of a carbon price [24]. Yet, proposals have been discussed on the issue of unified relative price, the stability of the currency, and exchange rate determination to solve the dilemma [25]. The carbon-based monetary instrument has already been delineated and applied, which can lower the risk levers of enterprises and banks, enhancing their attractiveness for the reduction project [26].

We focus on existing carbon credit loans as the first step in expanding the loaning services channel for SMEs. The objective of this study was to design a novel joint-financing mechanism for SMEs which helps them to overcome capital constraints in the emissions reduction process. The main innovation presented in this paper is the definition of bilateral capital-constrained supply chain financing under cap-and-trade systems. Considering the financial characteristics
of carbon credits in a low-carbon environment, a joint-financing mode for the upstream and downstream enterprises is established with carbon “rights” as bank collateral. The impact of this mode on each participant’s decision-making process is analyzed. The upstream and downstream SMEs of the supply chain are jointly financed by the bank to enhance the overall strength of their credit. Carbon resources in the low-carbon environment can serve as collateral to expand the financing channels available to SMEs. The risk incurred by the bank in the transaction can also be defined to provide theoretical support for low-carbon financing operations under cap-and-trade systems.

Various contracts are implemented to coordinate supply chain systems, such as the revenue-sharing contract [27], buy-back contract [28], two-part tariff contract [29], all-unit quantity discount contract [30], and revenue- and cost-sharing contract [31]. This paper centers on a buy-back contract, which can maximize profits across the whole supply chain; said profits can be distributed appropriately by adjusting the contract parameters [32]. We also introduce a cost-sharing contract to the joint-financing model to ensure feasible investments despite high abatement cost, which is a technique commonly used in supply chains involving significant investment [1].

The remainder of this paper is organized as follows. Section 3 shows the parameters and assumptions of the proposed model. Sections 4 and 5 discuss the mathematical formulation of the basic model with capital constraints and the proposed joint-financing model. Section 6 presents a numerical analysis which we conducted to examine how the joint-financing model affects total emissions and provides managerial recommendations to climate policy-makers. The final section summarizes our main findings.

3. Problem Setting and Assumptions

We examined a dual-echelon supply chain in this study wherein a dominant manufacturer and a following retailer are combined into a Stackelberg game model. Both are small-to-medium enterprises with limited capital for carrying out carbon reduction activities and little probability of obtaining financial support from a bank separately, so they may instead choose joint-financing with the expectation of a higher credit rating. Both enterprises also agree to a buy-back contract which encourages cooperation between them as the retailer continually orders products.

In cap-and-trade systems, the supply chain must control CO$_2$ emissions as necessary under the pressure of consumer preferences and government-mandated policies. The manufacturer asks the retailer to share the cost of emissions reduction based on their dominant position. Assume that the manufacturer takes on $\lambda$ ($0 < \lambda < 1$) of the cost; the retailer retains $1-\lambda$ through negotiation and consensus. The manufacturer determines the wholesale price $w$, the sharing rate of reduction investment $\lambda$, and the unit emissions reduction $\Delta e_m$ before the sales season begins. The retailer then gives an order quantity $q$ under which the manufacturer must arrange their production.

The model also involves customers, the carbon trading market, carbon emissions management verification agencies, and third-party financial institutions such as banks. They do not directly make decisions related to the supply chain system, but the product prices, bank loan interest rates, carbon quota, and carbon emissions trading prices do directly affect the supply chain joint loan game, as shown in Figure 1.

Like the traditional supply chain financing model, commercial banks are the ultimate source of funding for low-carbon joint-financing mechanisms for SMEs. When approving an SME joint-financing application, the bank evaluates the carbon assets of the whole supply chain and entrusts the low-carbon service provider to manage and run the pledged carbon assets. If any supply chain node is unable to repay the loan on schedule, the loss can be resolved by turning the carbon assets into cash through the trading platform. The carbon trading market allows for carbon trading and realization for the main players involved in the game. The carbon emissions management and verification platform is responsible for allocating initial emissions quotas to each firm ($E_0$) and issuing a traction price ($p_0$) at the beginning of the period. The term $p_0 E_0$ represents the carbon asset of the manufacturer. At the end of the term, the carbon footprint of the supply chain node enterprises is verified. The platform
4. Basic Model without Fund Support

The no-funding-support model is a special-case joint-financing pattern. The original model provides some basic theoretical support for the proposed model including a lemma which applies to both. The retailer’s expected sales function is $S(q) = q - \int_0^q F(x)dx$, the expected leftover inventory is $L(q) = \int_0^q F(x)dx$, and the lost-sales is $L(q) = \mu - q + \int_0^q F(x)dx$. The buy-back contract signed between the manufacturer and the retailer can be defined as follows:

$$T = wq - bI(q).$$  \hspace{1cm} (1)

Without funding support, the supply chain does not have enough capital to obtain emissions reduction technology; $\Delta e_m = 0$ and the order quantity of the retailer is $q = q_0 - aw$. Then the retailer’s profit function can be expressed as follows:

$$\pi^0 = pS(q) + bI(q) - c_q q - gL(q) - wq.$$  \hspace{1cm} (2)

Under government scrutiny, the manufacturer decides whether to buy or sell out carbon quotas according to its actual carbon emissions. The cost for excess carbon is $p_0(e_m q - E_0)$. So the manufacturer’s profit function is

$$\pi^0_m = wq + c_e I(q) - bI(q) - p_0(e_m q - E_0) - c_m q$$

$$- gL(q).$$  \hspace{1cm} (3)

The manufacturer and the retailer are concerned only by maximizing their own profits when decision-making is decentralized. In the Stackelberg game, the manufacturer is in a dominant position over the retailer. We first solve the retailer’s optimal order by employing a backward solution method:

$$q^* = F^{-1}\left(\frac{p + g_r - w - c_r}{p + g_r - b}\right).$$  \hspace{1cm} (4)

Put $q^*$ into the manufacturer’s profit function and compute its first-order derivative at the wholesale price $w$, so that

$$w^0 = \frac{\left[(b + g_m - c_m) F(q^*_m) + p_0 e_m + c_m - g_m + q_0/\alpha\right]}{2}.$$  \hspace{1cm} (5)
Then the retailer’s optimal order quantity and the manufacturer’s optimal wholesale price can be solved as follows:

\[ w^0 = \frac{[b + g_m - c_r] \left( (p + g - c_r - c_m - p_0 e_m - q_0/\alpha) / (2p + g + g_r - c_r - b) \right) + p_0 e_m + c_m - g_m + q_0/\alpha]}{2} \]

\[ q_r^0 = F^{-1} \left( \frac{2p + g - 2c_r - c_m - p_0 e_m - q_0/\alpha}{2p + g - c_r - b + g_r} \right). \]

Under the centralized decisions condition, the total profit of the supply chain system, i.e., the summation of the profits of the retailer and the manufacturer, is

\[ \pi^0 = pS(q) + c_r l(q) - c_m q - c_r q - gL(q) - p_0 (e_m q - E_0). \]

and the optimal order quantity of the supply chain is

\[ q^0 = F^{-1} \left( \frac{p + g - c_m - c_r - p_0 e_m}{p + g - c_r} \right). \]

Obviously, \( F^{-1}(q_r^0) < F^{-1}(q^0) \). The supply chain system is uncoordinated.

Reexpressing the original profit functions (2), (3), and (8) yields the following formulas:

\[ \pi_r = (p - b + c_r) S(q) + (b - w - c_r) q - g_r \mu, \]

\[ \pi_m^0 = (b - c_r + g_m) S(q) + (w + c_r - c_m - b - p_0 e_m) q + p_0 E_0 - g_m \mu, \]

\[ \pi^0 = (p - c_r + g) S(q) + (c_r - c_m - c_r - p_0 e_m) q + p_0 E_0 - g_m \mu. \]

Suppose there exist supply chain contract parameters \( \phi_r \) and \( \phi_m \) satisfying \( 0 < \phi_r, \phi_m < 1 \); then, formula (11) is consistently workable:

\[ p - b + g_r = \phi_r (p - c_r + g) \]

\[ b - w - c_r = \phi_r (c_r - c_m - c_r - p_0 e_m) \]

\[ b - c_r + g_m = \phi_m (p - c_r + g) \]

\[ w + c_r - c_m - b - p_0 e_m = \phi_m (c_r - c_m - c_r - p_0 e_m) \]

and the following formula can be derived:

\[ \pi_m^0 = \phi_m \pi^0 + [\phi_m g_\mu + p_0 E_0 (1 - \phi_m) - g_m \mu] \]

\[ \pi_r^0 = \phi_r \pi^0 + (\phi_r g_\mu - p_0 E_0 \phi_r - g_r \mu). \]

Only if the profit functions of the retailer and manufacturer are affine functions of the supply chain will formula (9) hold. At this point, decisions made in the decentralized supply chain and centralized supply chain are balanced: the summation of the manufacturer’s optimal production quantity and the retailer’s order quantity under decentralized decisions are equal to the optimal order quantity of the supply chain; namely, the whole supply chain is indeed coordinated.

When formula (12) is satisfied, the profit functions of the retailer and the manufacturer are affine functions of the profit function of supply chain system. At this time, the optimal throughput of the manufacturer and the optimal order quantity of the retailer under decentralized decision-making are equal to the optimal order quantity of the supply chain system under centralized decision-making, which means all members of the supply chain are in coordination and Lemma 5 holds.

**Lemma 5.** With buy-back contact, only when the profit functions of the retailer and the manufacturer are affine functions of the supply chain system, the supply chain system be coordinated.

### 5. Joint-Financing Decision Model under Cap-and-Trade System

The manufacturer must take action to reduce carbon emissions under pressure from both consumers and government mandates. The manufacturer requires that the retailer share abatement costs. As mentioned above, the manufacturer is dominant between them; further, both firms have capital constraints and jointly apply for bank loans to cover the cost of emissions reduction.

The bank dominates the financial transaction. The bank sets the loan interest rate; then the manufacturer and retailer ally to make a final decision to accept it. In the Stackelberg game of the manufacturer and retailer, similar to the no-funding-support model described above, the manufacturer gives the wholesale price \( w \), sharing rate of reduction investment \( \lambda \), and unit reduction quantity \( \Delta e_m \) first. The retailer’s order quantity is \( q = q_0 - \alpha w + \beta \Delta e_m \).

#### 5.1. Centralized Supply Chain Decisions

The retailer and the manufacturer share the cost of emissions reduction. Due to capital constraints, they must jointly seek a bank loan and bear the financial cost of the loan together. The total reduction cost is \((1/2)a\Delta e_m^2 + (1/2)(1 - \lambda)a\Delta e_m^2 \) [34]. The manufacturer and the retailer take on amounts of \((1/2)\lambda a\Delta e_m^2\) and \((1/2)(1 - \lambda)a\Delta e_m^2\), respectively. They apply for bank loans of \( L_m(q) = (1/2)(\lambda a\Delta e_m^2 - O_m) \) and \( L_r(q) = (1/2)[(1 - \lambda) a\Delta e_m^2 q - O_r] \) according to their own funds and bear the corresponding financial cost \( rL_m(q) \) and \( rL_r(q) \). If the same buy-back
contract $T = wq - bI(q)$ is maintained, the profit functions above transform as follows:

$$\pi^F = (p - b + g_r) S(q) + (b - w - c_r) q$$
$$- \frac{1}{2} (1 - \lambda) a \Delta e_m^2 (1 + r) + r O_r - g_r \mu.$$  

$$\pi^F = (p - c_r + g_m) S(q)$$
$$+ (w + c_r - c_m - b - p_0 e_m + p_0 \Delta e_m) q$$
$$- \frac{1}{2} \lambda a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o - g_m \mu,$$

where

$$\pi^F = \frac{(p - c_r + g_m)(w + c_r - c_m - b - p_0 e_m + p_0 \Delta e_m) q}{2}$$
$$- \frac{1}{2} \lambda a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o - g_m \mu.$$  

Referring to formula (11),

$$p - b + g_r = \varphi_r (p - c_r + g)$$
$$b - w - c_r = \varphi_r (c_r - c_m - c_r - p_0 e_m + p_0 \Delta e_m)$$
$$b - c_r + g_m = \varphi_m (p - c_r + g)$$
$$w + c_r - c_m - b - p_0 e_m + p_0 \Delta e_m$$

Thus, the profit functions of the retailer and manufacturer are

$$\pi^F_m = \varphi_m \pi^F + [p_0 E_o (1 - \varphi_m) + \mu (\varphi_m g - g_m)$$
$$+ r (O_m - \varphi_m O_m - \varphi_m O_r)] + \frac{1}{2} a (1 + r) (\varphi_m - \lambda)$$
$$\cdot \Delta e_m (q)^2.$$  

$$\pi^F_r = \varphi_r \pi^F + [p_0 E_o \varphi_r + \mu (\varphi_r g - g_r)$$
$$+ r (O_r - \varphi_r O_r - \varphi_r O_m)] + \frac{1}{2} a (1 + r) (\varphi_r + \lambda - 1)$$
$$\cdot \Delta e_m (q)^2.$$  

The profit functions of the manufacturer and the retailer are not affine functions of the supply chain in this scenario. The supply chain is uncoordinated, and the optimal order quantity of the retailer is not equivalent to the optimal production of the manufacturer or the optimal order quantity of the supply chain. We need to modify the contract to re-coordinate the supply chain via Proposition 6 and Corollary 7.

**Proposition 6.** When the buy-back contract is modified as $T = wq - b^F I(q)$, the supply chain can achieve coordination under a joint-financing pattern, where $b^F = b + a \Delta e_m^2 (1 + r)/2 \cdot (\lambda - \varphi_m)/(S(q) - q)$. Proof. Under the new buy-back contracts of $b^F = b + a \Delta e_m^2 (1 + r)/2 \cdot (\lambda - \varphi_m)/(S(q) - q)$, the manufacturer’s profit function can be accommodated as follows:

$$\pi^F_m = \left[ b^F - c_r + g_m \right] S(q)$$
$$+ \left[ w + c_r - c_m - b - p_0 e_m + p_0 \Delta e_m \right] q + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \lambda a \Delta e_m^2 (1 + r) + r O_m,$$

which can be simplified as

$$\pi^F_m = \varphi_m (p - c_r + g) S(q) + \varphi_m (c_r - c_m - c_r - p_0 e_m$$
$$+ p_0 \Delta e_m) q - \frac{1}{2} \varphi_m \lambda a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o$$
$$- g_m \mu - \frac{1}{2} \varphi_m a \Delta e_m^2 (1 + r) + r O_m + p_0 E_o.$$

Then the profit function of the retailer can be solved:

$$\pi^F_r = \pi^F - \pi^F_m = \varphi_r \pi^F + [\mu (\varphi_r g - g_r) - p_0 E_o \varphi_r$$
$$+ r (O_r - \varphi_r O_r - \varphi_r O_m)].$$

It is clear that the profit functions of the retailer and the manufacturer are both affine functions of the profit function of the supply chain system with the new parameters, so the supply chain members are recoordinated. Proposition 6 holds.

**Corollary 7.** The optimal order quantity of the retailer with financial support ($q^F_r$) is greater than that of the no-funding-support model ($q^0_r$).

Proof. For the second-order condition of supply chain’s profit function $\pi^F$ at the order quantity $q^F$ is less than 0; i.e., $d^2 \pi^F/dq^2 < 0$; there is a point with maximum value. Solve the first-order derivative of the supply chain’s profit function, which is the optimal order quantity of the supply chain system with financial support, to obtain the following:

$$q^F_r = F^{-1} \left[ \frac{p + g - c_m - c_r - p_0 e_m + p_0 \Delta e_m + p_0 (q_0 - \alpha w + \beta \Delta e_m) / \beta - a \Delta e_m (1 + r) / \beta}{p + g - c_r} \right].$$  

(19)
Let Corollary 7 hold.

Always held in practice because the decision-making condition can be transformed as follows: Thus, the manufacturer's unit product emissions \( \Delta e_m \) can decrease significantly, so the manufacturer's unit product emissions \( \Delta e_m \) can decrease significantly, so

\[
\pi_m = wS(q) + \left( c - b \right) I(q) - \frac{1}{2} \alpha \Delta e_m^2 (1 + r) + g_m L(q)
\]

The distribution function \( F(x) \) strictly increases monotonically, so

\[
q^F > q^0.
\]

Corollary 7 holds.

According to Corollary 7, with the joint-finance funding, the supply chain system can not only effectively reduce the manufacturer's unit product emissions \( \Delta e_m > 0 \), but also expand the production scale \( q^1 > q^0 \). It shows that the new financing model can bring a good low-carbon effect and production effect as well.

5.2. Decentralized Supply Chain Decisions. With the new contract, the buy-back price \( b \) has been modified to \( b^F \) and the profit functions of the retailer and the manufacturer under joint-financial model can be reexpressed as follows:

\[
\begin{align*}
\pi_r^F &= pS(q) + b^F I(q) - wq - c_q - g_q L(q) \\
& \quad - \frac{1}{2} \alpha \Delta e_m^2 (1 + r) + g_m L(q)
\end{align*}
\]

\[
\begin{align*}
\pi_m^F &= wS(q) + \left( c - b^F \right) I(q) - p_0 q \left( e_m - \Delta e_m \right) \\
& \quad - \frac{1}{2} \alpha \Delta e_m^2 (1 + r) + g_m L(q) + p_0 E_0 - \frac{1}{2} \alpha \Delta e_m^2 (1 + r)
\end{align*}
\]

In accordance with the incentive compatibility principle, the profits of the participants should be not less than those before joint-financing. Thus, the manufacturer's decision-making condition can be transformed as follows:

\[
\begin{align*}
\Delta e_m(q^F, w, \lambda, r) &= \frac{\left[ (w + g_m - p_0 e_m - c_m) - F(q^F) \left( b^F - c_v + g_m \right) \right] \cdot \left( dq^F / d\Delta e_m \right)}{\lambda a (1 + r) - p_0 \left( dq^F / d\Delta e_m \right)}
\end{align*}
\]

Analyse the retailer’s new decision first. Assume that the retailer’s opportunity cost is \( \pi_{r \text{min}} = O_\alpha \), where \( \alpha \) is the retailer’s opportunity cost per unit fund. Generally, the retailer applies for the loan only if his profit from the loan is better than its opportunity cost. The new decision with the new contract is

\[
q^r = \text{argmax}_{q} \pi^F_r (\Delta e_m, \lambda, w)
\]

\[\text{s.t. } \pi^F_r (\Delta e_m, \lambda, w) > \pi_{r \text{min}}.\]

Compute the first-order conditions of formula (25) at \( q \) to obtain

\[
q^F_r = F^{-1} \left[ \frac{\beta (p + g_r - w - c_v) - (1 - q_m) a \Delta e_m (1 + r)}{\beta (p + g_r - b^F)} \right].
\]

For \( d^2 \pi^F_r / dq^2 < 0 \), \( q^F_r \) should be the maximum value point of the function.

Next, consider the manufacturer’s new decision. The Hessian matrix of the manufacturer’s profit function is negative definite, so there must be a unique \( w \) and \( \lambda \) allowing the manufacturer to secure the maximum possible profit. Substitute \( q^F_r \) into the manufacturer’s profit function and solve its partial derivative with respect to \( w \) and \( \lambda \):

\[
\begin{align*}
\frac{\partial \pi^F_m}{\partial w} &= \left( b^F - c_v + g_m \right) + \left( p_0 e_m + c_m - w - g_m \right) \cdot \frac{dq^F_r}{dw} + \frac{dq^F_r}{dw}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \pi^F_m}{\partial \lambda} &= \left( w + p_0 e_m + g_m - p_0 c_m \right) \cdot \frac{dq^F_r}{d\lambda} = 0
\end{align*}
\]

Substitute \( q^F_r \) for the manufacturer’s profit function expression (25) and compute the first-order derivative at \( \Delta e_m \) such that

\[
\{ w, \Delta e_m, \lambda \} = \text{argmax} \pi^F_m (w, \Delta e_m, \lambda | q^F_r)
\]

\[\text{s.t. } \pi^F_m (q^F_r) > \pi^0_m (q^0_r) \quad (P1)\]

\[\pi^F_r (q^F_r) > \pi^0_r (q^0_r).\]
The manufacturer also must ensure that his own profit is greater than the opportunity cost; i.e., \( \pi'_m > \pi_{\text{min}} = O_m \beta_m \), where \( \alpha'_m \) is the manufacturer's opportunity cost of unit fund invested. The optimal emissions reduction \( \Delta e^*_m \), optimal wholesale price \( w^* \), and optimal cost-sharing coefficient \( \lambda^* \) can be determined by solving (27), (28), and (29) in the financing model to satisfy the above constraints.

**Proposition 8.** In the joint-financing model, there are negative correlations between the manufacturer's emissions reductions of unit product \( \Delta e^*_m \) with the sharing coefficient of emissions reduction cost \( \lambda \), loan rate \( r \), and wholesale price \( w \).

**Proof.** We first analyze the relationship between the manufacturer's optimal emissions reductions per unit product \( \Delta e_m \) and the retailer's optimal order quantity \( q^*_r \).

Assume that \( dq^*_r/d\Delta e_m > 0 \), so \( d^2q^*_r(\Delta e_m) > dq^*_r(0) \). The distribution function \( F(x) \) strictly increases monotonically, so \( F[q^*_r(\Delta e_m)] > F[q^*_r(0)] \) is held; that is,

\[
\frac{p + g_r - w - c_r}{p + g_r - b^2} - \frac{(1 - \lambda) a \Delta e_m (1 + r)}{\beta^2 (p + g_r - b^2)} > \frac{p + g_r - w - c_r}{p + g_r - b^2}. \tag{30}
\]

For \( p > b^2 \) and \( 0 < \lambda < 1 \), the above inequality is obviously false. Therefore, \( dq^*_r/d\Delta e_m < 0 \), which means \( \Delta e_m \) and \( q^*_r \) are negatively correlated.

For \( d^2 \pi'_m/d\Delta e_m^2 < 0 \), the manufacturer's profit function is convex with respect to \( \Delta e_m \). Then there must be a unique \( \Delta e^*_m \) under the joint-financing pattern according to (32) per the first-order condition of the manufacturer's profit function.

Compute the first-order derivative of (32) at the wholesale price \( w \) so that

\[
d\Delta e_m (q^*_r, w, \lambda, r) = \frac{dq^*_r/d\Delta e_m}{\lambda a (1 + r) - p_0 (dq^*_r/d\Delta e_m)} \tag{31}
\]

Thus, \( \Delta e^*_m \) is negatively correlated with \( w \).

Similarly, we can prove that \( \Delta e^*_m \) are negatively correlated with \( \lambda \) and \( r \) as well.

Proposition 8 holds.

\( \Delta e^*_m \) and \( w \) are negatively correlated because a higher wholesale price means a lower retailer order quantity and thus a lower investment in carbon reduction. If the manufacturer seeks to further reduce carbon emissions, he must reduce his wholesale price. \( \Delta e^*_m \) is related to \( \lambda \) and \( r \) are due to the fact that an increase in either directly impacts the emissions costs and financing costs of lending to the manufacturer in addition to a decline in emissions reduced.

### 5.3. Bank Decisions.

The bank decision involves evaluating the risk of the loan and setting a suitable loan rate \( r \). In this model, risk is incurred in lending to both the manufacturer and the retailer. Here, we first discuss the risk from lending to the manufacturer.

The manufacturer pays for the principal and interest as long as it receives payment for goods from the retailer. He then manages the transaction of carbon emission quotas and payments under the buy-back contract. The manufacturer's payment ability is not influenced by the cash flow of the carbon emission quota transaction and repurchasing of products. That is, the manufacturer can repay the loan on time in terms of his own ability.

Now consider the default probability of the manufacturer. Generally speaking, when the trading value of the original quota is greater than the sum of the principal and interest of the loan, the manufacturer will not default as a rational being. The manufacturer makes payments on time as long as

\[
p_0 E_0 - \left( \frac{\lambda a \Delta e^2_m}{2} - O_m \right) (1 + r) > 0. \tag{32}
\]

The manufacturer's original carbon emission quota is greater than half of the carbon emissions reduction of the unit product; i.e., \( E_0 > \Delta e^2_m/2 \). Because \( 0 < \lambda < 1 \) and \( p_0 > a \Delta e_m \), then \( p_0 E_0 > \lambda a \Delta e^2_m/2 \). If \( O_m(1 + r) \geq r \cdot \lambda a \Delta e^2_m/2 \), then formula (32) must be established. That is, when the manufacturer's own capital and interest is sufficient to cover the investment, he usually does not default. Otherwise, he is penalized for carbon excess or suffers the loss of buying the same carbon quotas. In reality, the manufacturer meets the condition in most cases. Even if he does not meet the condition and chooses to default, the bank can auction the pledge of carbon emission quotas. For \( p_0 E_0 > \lambda a \Delta e^2_m/2 \), the bank can cover the principal at least with slight surpluses.

Next, consider the risks from loaning to the retailer. The bank will not release the pledged carbon quotas to the manufacturer until the retailer has repaid it, so the manufacturer takes advantage of the position of supervisor over the retailer to repay the loan; the retailer's default risk is not taken into account. The risks from the retailer mainly result from demand uncertainty. When suffering a severe downturn, the retailer facing low market demands is unable to pay for the principal and interest to the bank using its operating income; i.e., \( ps(q) + b'q < [(1/2)(1 - \lambda)a \Delta e^2_m - O_q](1 + r) \). At this point, he is in danger of bankruptcy. Retailer ruin probability may reverse with the buy-back price \( b \), as the manufacturer bears jointly more risk of sales caused by uncertain demands and relatively high buy-back prices. Consider an extreme scenario in which the buy-back price is the same as the sales price: the manufacturer bears the retailer's sales risk and the retailer's bankruptcy risk drops to zero. The retailer's bankruptcy probability is positively related to \( r \) and negatively to \( \lambda \).

In practice, the bank can assess the risk of lending based on the buy-back price \( b \) and cost-sharing ratio \( \lambda \). If necessary, he can even require the manufacturer to increase the buy-back price and cost-sharing ratio so as to reduce the risk of the retailer's bankruptcy and thus minimize the risk of lending. The bank can adopt the risk side management mode \((\theta, \gamma)\) to control the loan risk from the retailer. A stop-loss point \( \theta \) can be preset as desired under the control standard \( \gamma \).
When the probability of loan loss greater than \( \theta \) is less than \( \gamma \), i.e., \( P((1 - \lambda)a\Delta e_m^2 - O_r - pS(q) - b'I(q) > \theta) < \gamma \), the bank will refuse the loan request. When equivalent to \( P([(1 - \lambda)a\Delta e_m^2/2 - O_r - b'q - \theta]/(p - b')) < \gamma \), the credit ceiling of the retailer is \((1/2)(1 - \lambda)a\Delta e_m^2 - O_r\), where \( \Delta e_m^2 = 2[(p - b')F^{-1}(\gamma) + \theta + O_r - b'q]/(1 - \lambda)a\). Thus, the side-risk management model allows banks to review lending risks by controlling the lending limit. When the retailer and the manufacturer make a joint loan request, they provide information such as sales price \( p \), buy-back price \( b \), cost-sharing factor \( \lambda \), and order quantity \( q \); the bank determines the loan limit accordingly. When the retailer applies for a loan larger than that value, the bank will refuse the application or require contract modification within the supply chain.

**Proposition 9.** The manufacturer and the retailer jointly apply for a loan from the bank only if the bank’s interest rate satisfies

\[
r < \min \left\{ \frac{ps(q) + b'I(q) - wq - c_r - g_rL(q)}{(1/2)(1 - \lambda)a\Delta e_m^2 - O_r)}/2 - 1, \right. \\
\left. \frac{wq - (b' - c_r)I(q) - p_0(e_{a0}q - \Delta e_m - E_0) - c_mq - g_mL(q)}{(1 + \lambda)a\Delta e_m^2 - O_m)/2} - 1, \right. \\
\]

The bank lends to the joint firms only if \( \{(wq - O_r)(1 + r) - b'q)/(p - b')\} > \int_0^{x} F(x) \) holds, where \( z = \{(wq - O_r)(1 + r) - b'q)/(p - b')\). \( \Box \)

**Proof.** The bank should ensure their profits after loaning \( \pi_m^F \) and \( \pi_r^F \) are greater than zero when determining the interest rate \( r \) to guarantee that this rate is accepted by the retailer and manufacturer. He must find a suitable \( r \) which makes both (24) and (25) greater than zero. The acceptable range of \( r \) to the joint loan-seekers can be solved as follows:

\[
r < \min \left\{ \frac{ps(q) + b'I(q) - wq - c_r - g_rL(q)}{(1/2)(1 - \lambda)a\Delta e_m^2 - O_r)/2} - 1, \right. \\
\left. \frac{wq - (b' - c_r)I(q) - p_0(e_{a0}q - \Delta e_m - E_0) - c_mq - g_mL(q)}{(1 + \lambda)a\Delta e_m^2 - O_m)/2} - 1, \right. \\
\]

The retailer and the manufacturer will actively accept financing together to carry out emissions reduction.

Next, consider the condition under which the bank is willing to make this transaction. The bank always expects that his profit from the transaction is greater than the average return on investment (ROI):

\[
\pi_e > (L_m - L_r)r',
\]

where \( \pi_e \) is the bank’s profit function in the end of the sales period which can be calculated by \( \pi_e = B_m + B_r - L_m - L_r \), where \( B_m = L_m(1 + r) = ((1/2)\lambda a\Delta e_m^2 - O_m)(1 + r) \) is the bank’s revenue from the manufacturer and \( B_r = \min|ps(q) + b'I(q), [(1/2)(1 - \lambda)a\Delta e_m^2 - O_r](1 + r) \) is the revenue from the retailer. The condition that the bank willing to lend can be solved as follows:

\[
\frac{cq - O_m + wq - O_r}{p - b'}(r - r') > \int_0^z F(x),
\]

where \( z = ((wq - O_r)(1 + r) - b'q)/(p - b') \). Proposition 9 holds.

**6. Numerical Analysis and Managerial Implications**

We conducted a numerical analysis to verify the aforementioned theorems and to determine managerial suggestions for the government according to which policies may be more effective under the cap-and-trade system.

**6.1. Numerical Analysis.** We assume that the demand of products follows a normal distribution \( N(1000, 50^2) \) and the initial market size is \( q_0 = 1000 \). The value of other parameters are \( p = 150 \) Yuan, \( p_0 = 20 \) Yuan, \( e_m = 2.5 \) ton, \( O_r = O_m = 80 \) Yuan, \( E_0 = 100 \) ton, \( g_r = g_m = 20 \) Yuan, \( \alpha = 5, \beta = 10, b = 5 \) Yuan, \( q_m = 0.2, C_m = 60 \) Yuan, and \( C_r = 5 \) Yuan, \( C_0 = 0 \). According to actual conditions in China, we take \( 5\% \) to \( 6\% \) as the reasonable zone of \( r \) \[34\]. The optimal emissions reduction level and order quantity can be solved numerically and the above theorems were verified accordingly.

Table 1 shows how the total CO\(_2\) emissions fluctuate with loan interest rate \( r \) and emissions reduction cost coefficient \( a \) under the joint-financing model. The total emissions decrease dramatically to the quota as \( a \) and \( r \) decrease, which suggests that the supply chain members prefer to pay the transaction fee for carbon credits rather than seek loans collaboratively for energy-savings and carbon emissions reduction when the loan interest rates (or costs of green investment) are relatively high. This tendency grows more intense as the cost efficiency increases from 30 to 100, at which point there is a larger

<table>
<thead>
<tr>
<th>( r )</th>
<th>( a )</th>
<th>( \lambda )</th>
<th>( \Delta e_m )</th>
<th>( b^F )</th>
<th>( q_r^F )</th>
<th>( (e_m - \Delta e_m)q_r^F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6%</td>
<td>200</td>
<td>1</td>
<td>0.9401</td>
<td>2.5462</td>
<td>965.64</td>
<td>1506.30</td>
</tr>
<tr>
<td>6%</td>
<td>100</td>
<td>0.432</td>
<td>1.1963</td>
<td>4.1016</td>
<td>971.97</td>
<td>1267.16</td>
</tr>
<tr>
<td>6%</td>
<td>30</td>
<td>0.0456</td>
<td>2.3933</td>
<td>5.0791</td>
<td>937.40</td>
<td>93.74</td>
</tr>
<tr>
<td>4%</td>
<td>30</td>
<td>0.0136</td>
<td>2.3974</td>
<td>5.3073</td>
<td>974.31</td>
<td>99.96</td>
</tr>
<tr>
<td>3%</td>
<td>30</td>
<td>0.0064</td>
<td>2.3975</td>
<td>5.3347</td>
<td>974.44</td>
<td>99.88</td>
</tr>
</tbody>
</table>
amount of emissions in the supply chain. Mandatory emissions trading schemes are established to control emissions and avoid penalties. The government thus must also resolve any unexpected phenomena by setting higher transaction prices or supporting banks in lowering their interest rates to motivate firms to invest in environmental friendliness.

In fact, if the bank receives a government subsidy reified as a reduction of \( r \) from 6% to 3%, the retailer’s order quantity and emissions reduction per unit rise gradually when \( a \) is fixed. The joint-financing pattern has a verifiable comparative advantage over singular financing in terms of emissions reduction per unit and would in practice stimulate growth in order quantities from the market. However, higher production quantities under this financing pattern themselves represent an increase in carbon emissions. There is an assumed increase in market demand when consumers who prefer “green” products become aware of the supply chain’s emissions reduction strategies. A rational retailer would order more from the manufacturer in this case, thus the increase in production which creates more emissions.

We also explored the influence that \( CO_2 \) transaction price exerts on total carbon emissions. The cost per unit emission imposed by the government varies depending on the national policy, e.g., $2/ton in Japan under their cap-and-trade system (World Bank 2014), $7.54/ton on average in the European Union under their emission tax system in 2014 (European Energy Exchange, 2015), or $3.8/ton in the US under a voluntary offset credit system. In certain pilot cities in China such as Shenzhen, Shanghai, and Beijing, the transaction price of carbon credit ranges from approximately ¥10/ton to ¥50/ton ($1.53/ton to $7.66/ton). Here, we consider the range to extend from ¥20/ton to ¥70/ton. To simplify the analysis, we let \( r = 4\% \).

Figures 2 and 3 show the emissions status of the manufacturer with the carbon transaction prices under different patterns, which can be regarded as the supply chain’s total emissions. Figure 2 shows the no-funding pattern and Figure 3 shows the joint-financing pattern. The latter results in lower emissions under the same level of regulations; the government may impose lesser penalties on companies who benefit from joint loans to achieve the same (or better) emissions reduction effects than unfunded companies. In short, the joint-financing pattern appears to improve regulatory efficiency. The proposed model also results in larger ordering quantities, which benefit both environmental and economic sustainability.

In Figure 2, there is a break-point which marks a decline towards the bottom of the graph followed by a rebound. All markets have a range of carbon credits they may decide to remit in exchange for excess emissions. Once the credits exceed the threshold, the manufacturer must expand his production quantity to balance the heavy spending on carbon credits.

Figure 4 shows the trend in emissions according to cost coefficient \( a \) and transaction price \( p_0 \) together under the joint-financing pattern. When \( a \) is negligible compared to \( p_0 \), the manufacturer can easily cut his emission to 100 tons. (The left part of the graph marks the given carbon emissions quota.) The retailer takes charge of the whole cost of emissions reduction under the increasing demands of environmentalists. When the coefficient is too high and the credit for abatement is expensive, the manufacturer may exceed the emissions quota and pay handsomely for carbon credits while neglecting other tools for emissions reduction despite the expectations of both the enterprises and the government. The manufacturer bears the majority of this cost. Figure 4 also shows that a heavier punishment may better mitigate emissions when green R&D cost is lower. If the cost is fairly high, the transaction price trend may reverse to the point where green-innovation subsidies are necessary.

6.2 Managerial Implications. Each party involved can benefit from the joint-financing pattern. For the manufacturer, a lower loan rate leads to higher demand for production orders. Though the retailer must take responsibility for a portion of the emissions abatement cost, he receives in exchange a higher buy-back price which diminishes his risk of bankruptcy. The bank certainly secures beneficial revenue if the transaction succeeds. If the bank also receives government subsidies, the lower interest rates would bring collective
benefits to the entire supply chain. If the joint-financing pattern is well-designed and implemented effectively, a dramatic reduction in carbon emissions will substantially enhance the environmental sustainability of the industries involved. This, of course, requires effort from all parties and especially on the part of policy-makers. Below, we provide several managerial insights based on this analysis.

For the Supply Chain Members. The joint-financing pattern provides SMEs more opportunities to implement green technology, which benefits the entire country’s environmental sustainability. The supply chain members are more effectively coordinated under the joint-financing model as they are free to modify buy-back prices and sharing ratios when necessary. When given a suitable loan rate, supply chain members can receive financial support from the bank as a joint unit; and \( a \) and \( p_0 \) are critical in their decisions to enact emissions reduction strategies. The cost coefficient for reduction \( a \) is the most important parameter for green investments. If \( a \) is reasonable, supply chain members are incentivized to develop green technologies and reduce carbon emissions. Otherwise, they may prefer to break quotas unless they receive subsidies from the government. Carbon transaction price \( p_0 \) is also a key factor in green investment. In the case of joint-financing, it is better for the manufacturer to reduce his emissions to the quota \( E_0 \) with increase in \( p_0 \).

For the Commercial Bank. The joint-financing pattern minimizes the risk of loans where the carbon quota of the supply chain serves as a pledge. The bank can secure the desired revenue from the transaction. Setting an optimal loan interest rate \( r \) is his primary task in this process as he seeks to maximize benefits but also help the supply chain decrease its carbon emissions. The bank may set a penalty if the financial ally fails to reach the quota to further drive the supply chain’s emissions reduction efforts. The bank can actively seek government subsidies to improve his ability to issue green loans. Lower interest rates bring collective benefits to the entire supply chain.

For the Government Climate Policy-Maker. The government must lead efforts to reduce carbon emissions. In special cases when loan rates cannot be cut, the state may modify its emissions abatement policies to directly promote green incentives.

The government and various companies may have opposing attitudes towards carbon emissions. The government seeks plainly to minimize carbon emissions while the companies also seek to maximize profits. The government can encourage emissions reduction practices while maximizing social welfare by controlling the transaction price of the carbon trading market. Figures 2 and 3 show that fair transaction prices are crucial for controlling carbon emissions regardless of the funding model utilized. The government should not excessively interfere with profits but instead seek to minimize carbon emissions via mandatory emissions trading schemes; both the environment and the economy suffer when carbon credits are taxed too heavily. Even if the cost coefficient is too high to hold the firms back from emissions reduction, the government can provide motivation in the form of subsidies for green investment. If the ROI of banks is relatively high, subsidies to the banks can enhance the efficacy of joint-financing patterns.

7. Conclusions

Today’s SMEs are faced with the troublesome task of reducing carbon emissions due to increasingly stringent government policies as well as demand from consumers. Their inherently limited capital may render them simply unable to invest in green technologies as they navigate the carbon trading system. This paper proposed a joint-financing pattern for bilateral capital constraint supply chains wherein carbon credits serve as the monetary instruments. We demonstrated the advantages of this pattern over the traditional, nonfunded pattern in terms of per unit emissions reduction, increase in order quantities, and enhanced regulatory efficiency.

We also provide suggestions for policy-makers as they establish emissions management systems. Under WTO law, cap-and-trade regulations are equivalent to emission taxes in single industries when competition is imbalanced; enterprises are assumed to be the price takers for emission permits [35]. A higher tax (or allowance price) on carbon does not always result in greater reduction in emissions as it also alters order quantities [35–38]. Increasing the magnitude of the transaction price per unit emissions can actually increase the emissions. Previous researchers have also found that imposing strict climate policies may be counterproductive within certain ranges [39]. When the carbon credit is too heavily taxed, both the environment and economy suffer. The government must ensure an effective trade-off between green-innovation subsidies and penalties to minimize environmental effects while maximizing societal benefits. A balance among penalties and incentives under the joint-financing pattern makes a suitable choice for the government.

In this study, we assumed a stable emissions trading price without risk of fluctuation, which may have influenced the estimated loan risks and potential emissions reduction. This limitation should be duly noted. Our specific parameter values reflect real-world situations, which allowed us to establish a reliable approach to resolving economic problems.
preventing SMEs from green investment. In the future, it may be helpful to extend this model to analyze dynamic carbon credit price changes to more comprehensively assess the joint-financing pattern.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Humanity and Social Science Foundation of the Ministry of Education of China (no. 16YJAZH010) and the Innovation Method Fund of the Ministry of Science and Technology of China (no. 2015IM020500).

References


