Research Article

Mobilizable Strength Design for Multibench Retained Excavation

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Multibench retaining systems can be used in large area excavations for the purpose of eliminating horizontal struts. However, there is no design method for this retaining system. Based on the mobilizable strength design (MSD) concept, a design procedure for a two-bench retaining system considering the interaction of the first and second retaining structures was proposed and tested. Based on an admissible strain field for a two-bench retained excavation in undrained condition, the shear strain in the superimposed strain and the lateral earth pressure distribution acting on the retaining structures can be determined. Then, the mobilized shear strength corresponding to the strain field could be calculated by the equations of force and moment equilibrium. Further, the crest displacements, earth pressures, and bending moment in a two-bench retained excavation can be calculated. The calculated results using MSD were verified by the finite difference analysis.

1. Introduction

A large and deep excavation in a soft soil area typically uses a support system consisting of a retaining wall with one or more levels of horizontal struts, which is costly and requires a long construction period [1]. Multibench retaining systems can be used to eliminate horizontal struts [2]. This retaining system has been used in several excavation projects in China [3, 4]. A typical two-bench retained excavation is shown in Figure 1. This retaining system consisted of two levels: double-row [5] and single-row [6, 7] contiguous pile walls were used as the first and second retaining structures, respectively.

Certain researches of multibench retaining system have been carried out in recent years. Zheng et al. [8] (2014) studied the stability and failure mechanism of the multibench retaining system using finite element method and material point method. Three failure modes of this retaining system were proposed based on the simulation results, that is, integrated failure, interactive failure, and disconnected failure. Further, Zheng et al. [2] designed and conducted a series of model tests to investigate the working and failure mechanism of the multibench retaining system and the three above-mentioned failure modes were also observed in these model tests.

Because the multibench retaining system does not have generally accepted design procedure and its geometry is similar to that of the earth berm supported wall/piles [9–14], the analysis method for earth berm [15] was usually applied in the design of the multibench retaining system in practice. However, this method neglects the restraint of the second retaining structure to the earth berm.

Bolton and Powrie [16] introduced the concept of “mobilizable soil strength (MSD)” to the design of cantilever retaining wall. Then, the MSD method was also applied to the prediction of the ground movements of braced excavations [17] and the design of the flexible walls propped at the crest [18]. Wang et al. [19] modified the MSD method by considering the undrained anisotropic shear strength of \( K_0 \) normally consolidated soft clay and bending strain energy of the retaining wall.
The mobilized shear strength corresponding to $\gamma$ matched.

This paper. The results calculated using this method and of the first and second retaining structures is proposed in a two-bench retaining system considering the interaction mobilized shear strain points near the wall to with rotation angle of strain field, the cantilever retaining wall rotates around a pivot was adopted in this method, as shown in Figure 2. In this method, as shown in Figure 2. In this strain field, the cantilever retaining wall rotates around a pivot point near the wall toe with a rotation angle of $\delta \theta$.

As the volumetric strain is 0 in undrained condition, the mobilized shear strain $\gamma_{\text{mob}}$ is equal to $2 \delta \theta [20]$.

Based on the stress-strain relation of the soil, the mobilized shear strength corresponding to $\gamma_{\text{mob}}$ can be determined and will be referred to as $c_{\text{mob}}$. Osman and Bolton [20] proposed that the corresponding active and passive pressures mobilized at depth $z$ can be given by

$$
\begin{align*}
\sigma_a &= \gamma z + q_0 - 2c_{\text{mob}} \\
\sigma_p &= \gamma z + q_0 + 2c_{\text{mob}} \\
qu_0 &= (K_0 - 1)(\gamma - \gamma_w)z,
\end{align*}
$$

where $\sigma_a$ and $\sigma_p$ are the active and passive pressures, respectively; $\gamma$ is the bulk unit weight of the soil; $\gamma_w$ is the unit weight of water; $q_0$ is the initially mobilized deviator stress; and $K_0$ is the initial lateral earth pressure coefficient at rest.

The idealized stress distribution can be assumed as shown in Figure 3 [20]. $z_c$ in Figure 3 is the depth of dry tension crack and can be calculated by

$$
z_c = \frac{2c_{\text{mob}}}{\gamma + (K_0 - 1)(\gamma - \gamma_w)}.
$$

If the wall height $D$, excavation height $H$, and bulk unit weight of soil are given, the mobilized shear strength $c_{\text{mob}}$ and the height of the pivot point above the toe $r$ can be determined by solving the equations of force and moment equilibrium. The stress-strain relationship at a representative point located at the middle height of the retaining wall is chosen to represent that in the entire admissible strain field [20]. Then, the corresponding $\gamma_{\text{mob}}$ and $\delta \theta$ can be calculated.

3. MSD Method for Multibench Retained Excavation

For convenience, the first and second retaining structures are represented by 1RS and 2RS, respectively. The experimental results showed that the deformation mode of the retaining structures in multibench retained excavations was similar to that of the cantilever retaining structures (Zhang et al., 2016). The assumed strain field of cantilever retaining wall in undrained condition was accepted for each retaining structure in multibench retained excavation.

The strain field of multibench retaining system is shown in Figure 4. As the bench width ($AB$) is limited in multibench retaining system, the strain fields of the 1RS and 2RS are overlapped in the soil between two retaining structures (i.e., bench zone soil). As shown in Figure 4, the superimposed strain field is a pentagon, whose edges were $AB$, $BC$, $CD$, $DE$, and $EA$.

3.1. Shear Strain in Superimposed Strain Field. The core issue of MSD method for multibench retained excavation is the calculation of the shear strain in superimposed strain field. The superimposed strain field can be divided into three parts according to different mobilized shear strains, as shown in Figure 5.

For a soil element of part $a$ in Figure 4, if the rotation angles of the 1RS and 2RS are $\delta \theta_1$ and $\delta \theta_2$, respectively, the horizontal displacements of the soil zone caused by the 1RS and 2RS are $h_1\delta \theta_1$ and $-h_2\delta \theta_2$ (the minus sign represents that the movement of the retaining structure tends to cause the soil zone to extend), respectively. $h_1$ and $h_2$ are the distance between the soil element and the toes of the 1RS and 2RS, respectively. The horizontal strain in part $a$ $\delta e_h^a$ can be calculated by

$$
\delta e_h^a = \frac{-h_2\delta \theta_2 + h_1\delta \theta_1}{B},
$$

where $B$ is the length of the bench width (the length of $AB$ in Figure 5).
Figure 3: Lateral earth pressure distribution for an embedded cantilever wall in undrained conditions [20].

Figure 4: Kinematically admissible strain field of multibench retaining system.

Figure 5: Different zones of the superimposed strain field for multibench retaining system.

\[
\delta \varepsilon_{v} = \delta \varepsilon_{vol} - \delta \varepsilon_{h} = -\frac{h_{2} \delta \theta_{2} - h_{1} \delta \theta_{1}}{B}, \tag{4}
\]

\[
\delta \varepsilon_{v} = \delta \varepsilon_{a} - \delta \varepsilon_{b} = \frac{2h_{2} \delta \theta_{2} - 2h_{1} \delta \theta_{1}}{B}. \tag{5}
\]

\[
\delta \varepsilon_{h} = \frac{h_{2} \delta \theta_{2}}{h_{2}} - \frac{h_{1} \delta \theta_{1}}{h_{1}} = -\delta \theta_{2} + \delta \theta_{1}. \tag{6}
\]

\[
\delta \varepsilon_{v} = \delta \varepsilon_{vol} - \delta \varepsilon_{h} = \frac{h_{2} \delta \theta_{2} - h_{1} \delta \theta_{1}}{B}, \tag{7}
\]

\[
\delta \varepsilon_{b} = \delta \varepsilon_{a} - \delta \varepsilon_{h} = 2\delta \theta_{2} - \frac{2h_{1} \delta \theta_{1}}{B}, \tag{8}
\]

where \( \delta \varepsilon_{vol} \) is the volume strain in part \( b \).

For a soil zone in part \( c \), the horizontal strain in part \( c \) \( \delta \varepsilon_{h} \) can be calculated by

\[
\delta \varepsilon_{h} = -\frac{h_{2} \delta \theta_{2}}{h_{2}} + \frac{h_{1} \delta \theta_{1}}{h_{1}} = -\delta \theta_{2} + \delta \theta_{1}. \tag{9}
\]

The vertical strain \( \delta \varepsilon_{v} \) and shear strain \( \delta \varepsilon_{s} \) in part \( b \) can be given by

\[
\delta \varepsilon_{v} = \delta \varepsilon_{vol} - \delta \varepsilon_{h} = \delta \theta_{2} - \frac{h_{1} \delta \theta_{1}}{B}, \tag{10}
\]

\[
\delta \varepsilon_{s} = \delta \varepsilon_{a} - \delta \varepsilon_{h} = 2\delta \theta_{2} - \frac{2h_{1} \delta \theta_{1}}{B}, \tag{11}
\]

where \( \delta \varepsilon_{vol} \) is the volume strain in part \( c \).

In (5), (8), and (11), the superimposed shear strains are related to the bench width and the distances between the soil zone and the toes of the 1RS and 2RS. For different geometry of multibench retaining system, the superimposed strain field and its shear strains are variable. As shown in Figure 6, there are 5 different superimposed strain fields (including that shown in Figure 4) according to the different interfaces of the superimposed strain fields and retaining
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![Diagram](image)

Figure 6: Different contact relations of the superimposed strain fields and retaining structures.

The weighted average superimposed shear strain $\delta \varepsilon^d_s$ is adopted to represent the mobilized shear strain in the entire superimposed strain field to simplify the analysis procedure. The superimposed shear strain can be calculated by

$$\delta \varepsilon^d_s = \frac{A_a \delta \varepsilon^a_s + A_b \delta \varepsilon^b_s + A_c \delta \varepsilon^c_s}{A_a + A_b + A_c}, \tag{12}$$

where $A_a$, $A_b$, and $A_c$ are the areas of parts $a$, $b$, and $c$ in the superimposed strain field, respectively. $A_a \delta \varepsilon^a_s$, $A_b \delta \varepsilon^b_s$, and $A_c \delta \varepsilon^c_s$ can be calculated through the integral along $h_1$ and $h_2$. According to the stress-strain relation of the soil, $\delta \theta_1$ and $\delta \theta_2$ can be expressed by the mobilized shear strength of the 1RS $c_{m1}$ and that of the 2RS $c_{m2}$, which is

$$\delta \varepsilon^d_s = f (c_{m1}, c_{m2}). \tag{13}$$

3.2. Earth Pressures Acting on 1RP and 2RP. In the multi-bench retaining system, the earth pressure exerted on the retaining structures by the soil in the normal strain field is equal to that of cantilever retaining wall, whereas the earth pressure exerted by the soil in the superimposed strain field is determined by the weighted average superimposed shear strain. Taking the pattern in Figure 4 as an example, because the embedded ratio of 2RP is smaller than that of 1RP, $\delta \theta_2$ may be larger than $\delta \theta_1$. Therefore, assume that $\delta \varepsilon^d_s$ is negative, and then the lateral earth pressure distribution acting on the 1RS can be illustrated as that in Figure 7.

In Figure 7, $z_1$ is the depth of dry tension crack for the 1RS and can be calculated by

$$z_1 = \frac{2c_{m1}}{\gamma - (1 - K_0)(\gamma - \gamma_w)} \tag{14}$$

The earth pressures in the active side of the 1RS in Figure 7 are given by

$$s_{11a} = \gamma_w (L_1 - R_1) + K_0 (\gamma - \gamma_w) (L_1 - R_1) - 2c_{m1}$$

$$s_{11p} = \gamma_w (L_1 - R_1) + K_0 (\gamma - \gamma_w) (L_1 - R_1) + 2c_{m1}$$

![Diagram](image)

Figure 7: Lateral earth pressure distribution for the 1RS.

$$s_{12p} = \gamma_w L_1 + K_0 (\gamma - \gamma_w) L_1 + 2c_{m1}, \tag{15}$$

where $L_1$ is the length of the 1RS and $R_1$ is the height of the pivot point above the toe of the 1RS.

For the passive side of the 1RS, as $\delta \varepsilon^d_s$ is assumed to be negative, which means that the soil in the superimposed strain field tends to be extended, the earth pressure exerted by the soil in the superimposed strain field tends to an active state and dry tension crack will form in the passive side of 1RP, as shown in Figure 7. If the stress-strain relationship for the representative point of the 1RS can be expressed by $\delta \varepsilon^d_s = f_1(c_{m1})$, the tension crack depth and the earth pressure acting on the passive side can be given by

$$z_2 = \frac{2f_1^{-1} \left[ f(c_{m1}, c_{m2}) \right]}{\gamma - (1 - K_0)(\gamma - \gamma_w)}$$

$$s_{21a} = \gamma_w (L_2 - B) + K_0 (\gamma - \gamma_w) (L_2 - B)$$

$$s_{21p} = \gamma_w (L_2 - B) + K_0 (\gamma - \gamma_w) (L_2 - B) + 2c_{m1}$$
The earth pressures acting on the passive side of the 2RS can be given by

\[ s_{21p} = 2c_{m2} \]
\[ s_{22p} = \gamma_w (L_2 - H_2 - R_2) + K_0 (y - \gamma_w) (L_2 - H_2 - R_2) + 2c_{m2} \]
\[ s_{22a} = \gamma_w (L_1 - H_1 - R_1) + K_0 (y - \gamma_w) (L_1 - H_1 - R_1) - 2c_{m1} \]
\[ s_{23a} = \gamma_w (L_1 - H_1) + K_0 (y - \gamma_w) (L_1 - H_1) - 2c_{m1} \]

where \( z_3 \) is the dry tension crack formed in the passive side of the 1RS, \( L_2 \) is the length of the 2RS, and \( H_1 \) is the height of the first excavation.

The lateral earth pressure distribution acting on the 2RS can be illustrated as that in Figure 8. The tension crack depth and the earth pressures acting on the active side of the 2RS can be given by

\[ z_3 = z_2 = \frac{2f_2^{-1} [f(c_{m1}, c_{m2})]}{\gamma - (1 - K_0)(y - \gamma_w)} \]
\[ s_{31p} = \gamma_w (L_1 - H_1 - B) + K_0 (y - \gamma_w) (L_1 - H_1 - B) - 2c_{m2} \]
\[ s_{31a} = \gamma_w (L_1 - H_1 - B) + K_0 (y - \gamma_w) (L_1 - H_1 - B) - 2c_{m2} \]
\[ s_{32p} = \gamma_w (L_2 - R_2) + K_0 (y - \gamma_w) (L_2 - R_2) - 2c_{m2} \]
\[ s_{32a} = \gamma_w (L_2 - R_2) + K_0 (y - \gamma_w) (L_2 - R_2) + 2c_{m2} \]
\[ s_{33p} = \gamma_w L_2 + K_0 (y - \gamma_w) L_2 + 2c_{m2} \]

where \( z_3 \) is the dry tension crack formed in the active side of the 2RS, \( R_2 \) is the height of the pivot point above the toe of the 2RS, and \( \delta \varepsilon = f_2(c_{m2}) \) is the stress-strain relationship for the representative point of the 2RS. As the lengths and embedded situations of the 1RS and 2RS are different, the representative points and their stress-strain relationships are different.

3.3. MSD Analysis Procedure. In (14)–(18), there are 4 unknown parameters, which are \( c_{m1}, c_{m2}, R_1, \) and \( R_2 \). Meanwhile, there are 2 force equilibrium equations and 2 moment equilibrium equations for the 1RS and 2RS. Therefore, the unknown parameters can be solved.

The analysis procedures of the multibench retaining system using MSD method can be summarized as follows.

1. Select the representative points of stress-strain relation for the 1RS and 2RS, respectively.
2. Calculate the weighted average superimposed shear strain \( \delta \varepsilon_s^d \).
3. Determine the relationship between \( \delta \varepsilon_s^d \) and \( c_{m1}, c_{m2} \), according to the representative stress-strain relationship of the 1RS and 2RS.
4. Determine the earth pressure distribution modes acting on the 1RS and 2RS based on the geometry of the superimposed strain field.
5. Calculate \( c_{m1}, c_{m2}, R_1, \) and \( R_2 \) by solving the equations of force and moment equilibrium of the 1RS and 2RS.
6. Calculate the mobilized shear strains of the 1RS and 2RS. Then, the crest displacements of the 1RS and 2RS can be calculated based on \( \delta \theta_1 \) and \( \delta \theta_2 \).

4. Finite Difference (FD) Analysis

To verify the accuracy of the MSD method for multibench retaining system, a two-dimensional plane strain finite difference analysis was performed to predict the behavior of a two-bench retained excavation. Half of the excavation was modeled and the finite difference model is shown in Figure 9. The first and second excavation depths (\( H_1 \) and \( H_2 \)) were both 4 m. The bench width \( B \) was 8 m. The retaining structures were assumed to be 1 m thick retaining walls. The lengths of the 1RS and 2RS, \( L_1 \) and \( L_2 \), were 24 m and 12 m, respectively. The left and right boundaries of the numerical model were fixed with roller supports, and the bottom boundary was fixed with pin supports.

As Speswhite China kaolin was widely used in geotechnical research and its properties were well known; the soil layer was modeled using Speswhite China kaolin [21]. The soil
was overconsolidated by applying 200 kPa vertical loading on top of the soil surface. According to critical state soil mechanics [22], the void ratio of the overconsolidated soil can be calculated, as shown in Figure 10.

The Modified Cam-clay (MCC) model was adopted to simulate the soil. The material parameters of kaolin clay are shown in Table 1.

### Table 1: Material parameters of kaolin clay [23].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk weight (kN/m$^3$)</td>
<td>17.6</td>
</tr>
<tr>
<td>Stress ratio, $M$</td>
<td>0.84</td>
</tr>
<tr>
<td>Slope of one-dimensional compression line in $v$--$\ln p'$ space, $\lambda$</td>
<td>0.25</td>
</tr>
<tr>
<td>Slope of unload-reload line in $v$--$\ln p'$ space, $\kappa$</td>
<td>0.05</td>
</tr>
<tr>
<td>Specific volume of the critical state line at unit pressure, $\Gamma$</td>
<td>3.48</td>
</tr>
<tr>
<td>Poisson's ratio for effective stress analysis, $v'$</td>
<td>0.33</td>
</tr>
<tr>
<td>Coefficient of earth pressure at rest, $K_0$</td>
<td>0.63</td>
</tr>
<tr>
<td>Saturated horizontal permeability, $k_h$ (m/s)</td>
<td>$4.43 \times 10^{-9}$</td>
</tr>
<tr>
<td>Saturated vertical permeability, $k_v$ (m/s)</td>
<td>$2.89 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

The retaining structures were made of reinforced concrete and simulated by liner elements, which have interfaces on both sides. The retaining structures were assumed to be elastic. Young's modulus and the equivalent thickness of the liner were determined to be 30 GPa and 1 m. The friction angle of the interface of the soil and retaining structures was 16.7$^\circ$.

Water table was located at the ground surface. As the excavation process was assumed to be conducted in a very short period, undrained condition was applied and the seepage flow was not allowed during the excavation.

### 5. Comparison between MSD and FD

The undrained shear strength in MCC model can be calculated and is shown in Figure 11. The representative point was located at the middle height of each retaining structure. The representative stress $(q)$-strain $(\varepsilon_q)$ curve is shown in Figure 12.
According to (5), (8), (11), and (12), the weighted average superimposed shear strain can be given by \( \delta \varepsilon^d = \left(9/4\right)\delta \theta_2 - \left(47/12\right)\delta \theta_1 \). The calculated result of the mobilized shear strength of the IRS \( c_{m1} \) is 19 kPa and \( R_1 \) is 1.1 m. The mobilized deviatoric stress is given by \( q_{m1} = 2c_{m1} = 38 \) kPa. From the stress-strain curve, the corresponding triaxial shear strain \( \varepsilon_{q1} \) is 0.0159. The engineering shear strain \( \varepsilon_{q1} \) is equal to 1.5 times the triaxial shear strain \( \varepsilon_{q1} \) [20], and thus \( \varepsilon_{q1} = 1.5\varepsilon_{q1} = 0.02385 \). The rotation angle of the IRS is given by \( \delta \theta_1 = \varepsilon_{q1}/2 = 0.01193 \). The height of the wall above the pivot point is calculated by \( L_1' = L_1 - R_1 = 23.1 \) m. Thus, the crest horizontal displacement of the IRS is \( L_1'\delta \theta_1 = 27.56 \) cm. The crest displacement of the 2RS can be calculated through the same procedure above.

The results predicted by the MSD and FD method are listed in Table 2. The calculated crest displacements of the IRS and 2RS using the MSD method were close to those calculated using the FD analysis.

Figure 13 shows the lateral earth pressures acting on the IRS and 2RS predicted by the MSD and FD method. The predicted results are close to each other.

The bending moment distributions of the IRS and 2RS are shown in Figure 14. As the retaining structures were assumed to be rigid, the MSD calculated results are larger than the FD results. When the retaining structures are more rigid, the FD results will be closer to the MSD results, as shown in Figure 14.

Osman and Bolton [20] discussed the influence of flexibility of the retaining wall. If a ductile retaining structure is designed to resist MSD earth pressure, it would attract smaller moment than the calculated value and cannot collapse, which is on the safe side. If it is so stiff that it would develop bending moments in excess of the MSD values, extra plastic displacements would be caused to relieve the earth pressures and reduce the bending moments [20].

6. Conclusions

The analysis method for the multibench retaining system based on the MSD theory was introduced in this paper. The kinematically admissible deformation mechanism of cantilever retaining wall is adopted to represent the deformation characteristics of the IRS and 2RS in a two-bench retaining system. The weighted average shear strain of the superimposed strain field and the lateral earth pressure distributions acting on each retaining structure were deduced.
based on the MSD concept. Following the MSD analysis procedure proposed in this paper, the crest displacements, earth pressures, and bending moment in a two-bench retained excavation can be calculated using this method. Basically, the MSD calculated results matched the finite difference results.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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