Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 8410360, 11 pages
https://doi.org/10.1155/2018/8410360

Research Article

Adaptive Constrained Control for Uncertain Nonlinear Time-Delay System with Application to Unmanned Helicopter

Rong Li,[1] **Qingxian Wu**,[1] **Qingyun Yang**,[2] **and Hui Ye**[3]

[1] *College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
[2] *School of Mechanical and Electrical Engineering, Zaozhuang University, Zaozhuang 277160, China*
[3] *School of Electronics and Information, Jiangsu University of Science and Technology, Zhenjiang 212003, China*

Correspondence should be addressed to Qingxian Wu; wuqingxian@nuaa.edu.cn

Received 30 June 2017; Revised 16 September 2017; Accepted 26 September 2017; Published 13 February 2018

Academic Editor: Libor Pekař

This paper investigates a class of nonlinear time-delayed systems with output prescribed performance constraint. The neural network and DOB (disturbance observer) are designed to tackle the uncertainties and external disturbance, and prescribed performance function is constructed for the output prescribed performance constrained problem. Then the robust controller is designed by using adaptive backstepping method, and the stability analysis is considered by using Lyapunov-Krasovskii. Furthermore, the proposed method is employed into the unmanned helicopter system with time-delay aerodynamic uncertainty. Finally, the simulation results illustrate that the proposed robust prescribed performance control system achieved a good control performance.

1. Introduction

Time-delay systems have drawn considerable attention in the past decade [1, 2]. The adaptive backstepping technology was employed into the uncertain nonlinear time-delay system in [3]. The dynamic surface method was presented for the nonlinear time-delay system in [4]. In [5], the nonlinear stochastic system with time delay was studied. The finite-time control method was proposed for a class of time-delay systems in [6, 7]. In the previous studies on time-delay system, the uncertain nonlinear systems consisting of both constraint and external disturbances were not considered. In this paper, we will study a class of uncertain nonlinear time-delay systems subject to constraint.

It is well known that the uncertainty and external disturbance have an effect on the tracking performance of closed systems. Neural network is popular for its ability to cope with uncertainty [8]. In [9], the neural network was introduced into a class of nonlinear systems with unknown coefficient matrices. Combining RBFNN (radial basis function neural network) and disturbance observer, fault tolerant control method was presented to deal with input saturated system with actuator faults in [10]. Moreover, the disturbance observer is a valid method to deal with external disturbance [11]. In [12], the disturbance observer was proposed for permanent-magnet synchronous motor drivers. In [13], the sliding mode disturbance observer was presented to deal with mismatched disturbance. In [14], the disturbance observer was employed into a transport aircraft control system subject to continuous heavy cargo airdrop. In this paper, the neural network and disturbance observer will be utilized to tackle uncertainties, time delay, and external disturbance.

Another challenging problem in controller design lies in the constrained condition of the nonlinear systems [15]. The existence of constraint condition may degrade the performance or cause the instability of the closed control systems [16]. Using the Barrier Lyapunov function and adaptive backstepping technology, a robust constrained controller for a class of nonlinear strict systems was presented in

[17]. In [18], the Barrier Lyapunov function was employed into the switched systems subject to output constraints. In [19], the Barrier Lyapunov function and high-gain observer were introduced to deal with the constrained trajectory tracking problem of the marine surface vessel. Additionally, prescribed performance is another method to cope with output constraints, by defining the appropriate prescribed performance. In [20], the prescribed performance-based feedback linearization method was proposed to deal with output tracking error constraints for the MIMO (multiple-input multiple-output) nonlinear systems. In [21], the prescribed performance and adaptive fuzzy logic were employed into the nonlinear adaptive controller design. To the best of the authors' knowledge, there is still no research about uncertain nonlinear time-delay system considering uncertainties, external disturbance, and output constraints. Thus, in this paper, we will present a prescribed performance-based adaptive constrained control method for the time-delay nonlinear systems.

Nowadays, the unmanned helicopter system has received an increasing attention, and there is an amount of studies about the flight control approaches [22, 23]. In [24], chattering-free sliding mode was proposed for the miniature helicopter system. To solve the tracking problem with nonlinearity, the model predictive control method for unmanned helicopters was presented in [25]. In [26], a trajectory tracking control method was proposed for unmanned helicopter system with constraint conditions. However, with the increasing demands for real time and accuracy, the aerodynamic disturbance caused by transmission delay for unmanned helicopter control system cannot be ignored. In this paper, we will apply the prescribed performance-based robust adaptive control approach for the uncertain unmanned helicopter systems with external disturbance, time delay, and output constraints.

This paper is organized as follows. In Section 2, problem statement and preliminaries of time-delay system and prescribed performance are introduced. Section 3 presents the entire adaptive controller design and stability analysis. In Section 4, the prescribed performance-based control method is employed into the unmanned helicopter system. Finally, simulation and conclusion are given in Sections 5 and 6, respectively.

2. Problem Statement and Preliminaries

In this subsection, we will review some preliminary knowledge about nonlinear time-delay system, prescribed performance, and neural network, which are necessary in the following controller design. Firstly, consider a class of uncertain MIMO nonlinear time-delay systems in the form of

$$\begin{aligned}
\dot{x}_i(t) &= f_i(\overline{x}_i(t)) + g_i(\overline{x}_i(t))\, x_{i+1}(t) + h_i(\overline{x}_i(t-\tau_i)) \\
&\quad + \Delta f_i(\overline{x}_i(t)) + d_i(t), \quad 1 \le i \le n-1 \\
\dot{x}_n(t) &= f_n(\overline{x}_n(t)) + g_n(\overline{x}_n(t))\, u(t) + h_n(\overline{x}_n(t-\tau_n)) \\
&\quad + \Delta f_n(\overline{x}_n(t)) + d_n(t) \\
y(t) &= x_1(t),
\end{aligned} \tag{1}$$

where $x_i(t) \in R^m$, $i = 1, 2, \ldots, n$, are the system state vectors which are assumed to be measurable, $\overline{x}_i(t) = [x_1^T(t), x_2^T(t), \ldots, x_i^T(t)]^T \in R^{m\cdot i}$, $y \in R^m$ is the output vector, and $u(t) \in R^m$ is the control input vector. $f_i(\overline{x}_i(t)) \in R^m$, $i = 1, 2, \ldots, n$, are the known smooth nonlinear function, $g_i(\overline{x}_i) \in R^{m\times m}$, $i = 1, 2, \ldots, n$, represent the known control coefficient matrices, and $h_i(\overline{x}_i(t-\tau_i)) \in R^{m\times m}$, $i = 1, 2, \ldots, n$, indicate the known time-delay functions. $\Delta f_i(\overline{x}_i(t)) \in R^m$, $i = 1, 2, \ldots, n$, denote the unknown nonlinear functions which contain both parametric and nonparametric uncertainties. $\tau_i \in R$, $i = 1, 2, \ldots, n$, stand for constant unknown time delays. $d_i(t) \in R^m$, $i = 1, 2, \ldots, n$, mean the unknown external disturbance. In this paper, we impose the following assumptions and lemmas.

Assumption 1 (see [5]). The ideal tracking signals $y_r(t)$ and their derivatives $\dot{y}_r(t)$ are known and continuous.

Assumption 2 (see [11]). The external disturbance $d_i(t)$ means the slow varying signal, while it is restricted in the bound of $\|\dot{d}_i(t)\| \le \overline{d}_i$, $\overline{d}_i > 0$.

Assumption 3 (see [10]). For $i = 1, \ldots, n$, the known continuous function $g_i(\overline{x}_i(t))$ satisfies $g_i(\overline{x}_i(t)) \neq 0$.

Lemma 4 (see [9]). *Consider a class of nonlinear systems $\dot{x} = f(x)$. For any initial conditions $x(0) \in R^n$, if there exists a C^1 continuous and positive definite Lyapunov function satisfying $\varpi_1(x) \le V(x) \le \varpi_2(x)$, $\varpi_i(\|x\|) : R^n \to R$, $i = 1, 2$, mean the K class functions, such that $\dot{V} \le -c_1 V(x) + c_2$, $c_1 > 0$, $c_2 > 0$, then one can conclude that the solution $x(t)$ is uniformly bounded.*

Lemma 5 (see [3]). *For $v > 0$, if there exists one set Z defined by $Z := \{z \mid |z| \le 0.8814v\}$, then one can conclude that, for any $z \notin Z$, the inequality $1 - 2\tanh^2(z/v) < 0$ is satisfied.*

Lemma 6 (see [10]). *For any continuous function $f(z)$, there exists a valid linear neural network to approximate it by choosing enough nodes on the compact set $z \in N_z$. The basic function can be selected as Gaussian function $s_i(z) = e^{-\|z-\mu_i\|^2/c_i}$, where $c_i > 0$ is the width of function and $\mu_i \in R$ represent the center of function. The neural network approximator consists of the weight estimate vector $\widehat{W}(z)$ and Gaussian function $S(z) = [s_1(z), \ldots, s_n(z)]^T$, which can be written as $f_p(z) = \widehat{W}^T(z)S(z) + \varepsilon$. Assume that $W^*(z)$ means the optimal approximating weight, such that*

$$W^*(z) := \arg\min_{\widehat{W}\in R^n} \left\{ \sup_{z\in\Omega_z} \left\| f(z) - f_p(z) \right\| \right\}. \tag{2}$$

In addition, the optimal approximator of continuous function $f(z)$ can be written as

$$f(z) = W^{*T}(z) S(z) + \varepsilon^*, \tag{3}$$

where ε^* indicates the optimal approximate error.

3. Controller Design and Stability Analysis

In this section, the objective is to propose a robust prescribed performance control law for uncertain nonlinear systems such that the closed-loop errors converge to a small neighborhood of the origin.

3.1. Prescribed Performance Controller Design

Step 1. Define the tracking errors $\tilde{z}_1(t) \in R^m$ and $z_2(t) \in R^m$ as follows:

$$\begin{aligned} \tilde{z}_1(t) &= x_1(t) - y_d(t) \\ z_2(t) &= x_2(t) - x_2^*(t), \end{aligned} \tag{4}$$

where $y_d(t) \in R^m$ is the ideal tracking signal and $x_2^*(t) \in R^m$ is the immediate control. The output error transformation can be defined in the form of [18]

$$\begin{aligned} z_{1j}(t) &= Q^{-1}\left(\frac{\tilde{z}_{1j}(t)}{\rho_j(t)}, \overline{\alpha}_j(t), \underline{\alpha}_j(t)\right) \\ &= \frac{1}{2}\ln\frac{\underline{\alpha}_j + \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)}{\overline{\alpha}_j - \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)}, \quad j = 1, \ldots, m, \end{aligned} \tag{5}$$

where $\overline{\alpha}_j$ and $\underline{\alpha}_j$ are the positive constants and $\rho_j(t)$ indicates the performance function, which can be chosen as [26]

$$\rho_j(t) = \left(\rho_{j0} - \rho_{j\infty}\right) e^{-lt} + \rho_{j\infty}. \tag{6}$$

The constant $\rho_{j\infty} > 0$ is the maximum amplitude of the tracking error at the steady state. The decreasing rate $e^{-l_j t}$ of $\rho_j(t)$ represents the desired convergence speed of the tracking error. Therefore, the appropriate choice of the performance function $\rho_j(t)$ and the design constant imposes bounds on the system output trajectory.

Define $f_1 = [f_{11}, \ldots, f_{1m}]^T \in R^m$ and $g_1 = [g_{11}^T, \ldots, g_{1m}^T]^T \in R^{m\times m}$. Thus, the time derivative of $z_{1j}(t)$ becomes

$$\begin{aligned} \dot{z}_{1j}(t) &= \frac{\partial z_{1j}(t)}{\partial\left(\tilde{z}_{1j}(t)/\rho_j(t)\right)}\frac{1}{\rho_j(t)}\dot{\tilde{z}}_{1j}(t) - \frac{\partial z_{1j}(t)}{\partial\left(\tilde{z}_{1j}(t)/\rho_j(t)\right)}\frac{\tilde{z}_{1j}(t)}{\rho_j^2(t)}\dot{\rho}_j(t) \\ &= \frac{1}{2}\frac{\overline{\alpha}_j + \underline{\alpha}_j}{\left(\underline{\alpha}_j + \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)\left(\overline{\alpha}_j - \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)}\left(\frac{1}{\rho_j(t)}\dot{\tilde{z}}_{1j}(t) - \frac{\tilde{z}_{1j}(t)}{\rho_j^2(t)}\dot{\rho}_j(t)\right) \\ &= \frac{1}{2}\frac{\overline{\alpha}_j + \underline{\alpha}_j}{\left(\underline{\alpha}_j + \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)\left(\overline{\alpha}_j - \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)} \\ &\quad \cdot\left(-\frac{\tilde{z}_{1j}(t)}{\rho_j^2(t)}\dot{\rho}_j(t) + \frac{f_{1j}(x_1(t)) + g_{1j}(x_1(t))x_2(t) + h_{1j}(x_1(t-\tau_1)) + \Delta f_{1j}(x_1(t)) + d_{1j}(t) - \dot{y}_{dj}(t)}{\rho_j(t)}\right) \end{aligned} \tag{7}$$

In order to simplify the analysis, we define M_{1j} and N_{1j} as follows:

$$\begin{aligned} M_{1j}(t) &= \frac{\overline{\alpha}_j + \underline{\alpha}_j}{2\rho_j\left(\underline{\alpha}_j + \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)\left(\overline{\alpha}_j - \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)} > 0. \\ N_{1j}(t) &= \frac{\overline{\alpha}_j + \underline{\alpha}_j}{2\rho_j^2(t)\left(\underline{\alpha}_j + \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)\left(\overline{\alpha}_j - \left(\tilde{z}_{1j}(t)/\rho_j(t)\right)\right)} \cdot \tilde{z}_{1j}(t)\dot{\rho}_j(t) \end{aligned} \tag{8}$$

Furthermore, we can define $M_1(t) = \mathrm{diag}\{M_{11}(t), \ldots, M_{1m}(t)\}$ and $N_1(t) = [N_{11}(t), \ldots, N_{1m}(t)]^T$, then we have

$$\begin{aligned} \dot{z}_1(t) &= M_1(t)\dot{\tilde{z}}_1(t) - N_1(t) = M_1(t)\left(f_1(x_1(t))\right. \\ &\quad + g_1(x_1(t))x_2(t) - \dot{y}_d(t) + \Delta f_1(x_1(t)) \\ &\quad \left. + h_1(x_1(t-\tau_1)) + d_1(t)\right) - N_1(t). \end{aligned} \tag{9}$$

Since $\Delta f_1(x_1(t))$ is unknown, using the RBFNN to approximate it, we obtain

$$M_1(t)\Delta f_1(x_1(t)) + \gamma_1(t) = \eta_1^{-1}W_1^{*T}S_1 + \varepsilon_1^*, \tag{10}$$

where $\gamma_1(t) = [\gamma_{1,1}(t), \ldots, \gamma_{1,m}(t)]^T$, and

$$\gamma_{1,j}(t) = \frac{16}{\tilde{z}_{1j}(t)}\tanh^2\left(\frac{\tilde{z}_{1j}(t)}{v_1}\right)h_{1j}^2(x_1(t)), \quad j = 1, \ldots, m. \tag{11}$$

Substituting (10) into (1) results in

$$\begin{aligned}\dot{x}_1(t) &= f_1(x_1(t)) + g_1(x_1(t))x_2(t)\\ &\quad + h_1(x_1(t-\tau_1)) - M_1^{-1}(t)\gamma_1(t)\\ &\quad + \eta_1^{-1}M_1^{-1}(t)W_1^{*T}S_1 + \chi_1(t).\end{aligned} \tag{12}$$

Invoking (4), (9), and (10), the time derivative of $z_1(t)$ can be rewritten as

$$\begin{aligned}\dot{z}_1(t) &= M_1(t)\\ &\quad \cdot (f_1(x_1(t)) + g_1(x_1(t))z_2(t) + g_1(x_1(t))x_2^*(t)\\ &\quad + h_1(x_1(t-\tau_1)) - \dot{y}_d(t)) - \gamma_1(t) + \eta_1^{-1}W_1^{*T}S_1\\ &\quad + M_1(t)\chi_1(t) - N_1(t).\end{aligned} \tag{13}$$

Construct updating law of the RBFNN

$$\dot{\widehat{W}}_1 = \Gamma_1\left(\eta_1^{-1}z_1(t)S_1 - \sigma_1\Gamma_1^{-1}\widehat{W}_1\right), \tag{14}$$

where $\sigma_1 > 0$ is a design parameter. Furthermore, the DOB can be chosen as

$$\begin{aligned}\widehat{\chi}_1(t) &= w_1(t) + \eta_1 x_1(t)\\ \dot{w}_1(t) &= -\eta_1 w_1(t) - \eta_1\left(f_1(x_1(t)) + g_1(x_1(t))x_2(t)\right.\\ &\quad \left. + \eta_1^{-1}\widehat{W}_1^T S_1 + \eta_1 x_1(t)\right) + M_1^T(t)z_1(t).\end{aligned} \tag{15}$$

According to (15), we obtain

$$\dot{\widehat{\chi}}_1(t) = \eta_1\widetilde{\chi}_1(t) + M_1^T(t)z_1(t) + \widetilde{W}_1^T S_1. \tag{16}$$

According to the neural network updating law (14) and DOB (15), the immediate control is chosen

$$\begin{aligned}x_2^*(t) &= g_1^{-1}(x_1(t))\left(-M_1^{-1}(t)K_1 z_1(t) - f_1(x_1(t))\right.\\ &\quad + \dot{y}_d(t) + M_1^{-1}(t)N_1(t) - M_1^{-1}(t)\gamma_1(t)\\ &\quad \left. - \eta_1^{-1}\widehat{W}_1^T S_1 - \widehat{\chi}_1(t)\right),\end{aligned} \tag{17}$$

where $M_1(t) = \mathrm{diag}\{M_{11}(t), \ldots, M_{1m}(t)\}$. $\dot{y}_d(t)$ is the time derivative of reference trajectory, and $K_1 \in R^{m\times m}$ is the constant positive definite matrices. $\widehat{W}_1$ is the estimated values of W_1^*, $\widetilde{W}_1$ represents the estimated error, and $\widetilde{W}_1 = W_1^* - \widehat{W}_1$. Substituting (17) into (13), the time derivative of $z_1(t)$ becomes

$$\begin{aligned}\dot{z}_1(t) &= -K_1 z_1(t) + M_1(t)g_1(x_1(t))z_2(t) - \gamma(t)\\ &\quad + M_1(t)h_1(x_1(t-\tau_1)) + \eta_1^{-1}\widetilde{W}_1^T S_1\\ &\quad + M_1(t)\widetilde{\chi}_1(t).\end{aligned} \tag{18}$$

Choose the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V_1 &= \frac{z_1^T(t)z_1(t)}{2} + \int_{t-\tau_1}^{t} P_1(x_1(\tau))\,d\tau\\ &\quad + tr\left(\widetilde{W}_1^T\Gamma_1^{-1}\widetilde{W}_1\right) + \frac{1}{2}\widetilde{\chi}_1^T(t)\widetilde{\chi}_1(t),\end{aligned} \tag{19}$$

where the positive function $P_1(x_1(t))$ can be designed as follows:

$$P_1(x_1(t)) = \frac{h_1^T(x_1(t))h_1(x_1(t))}{2}. \tag{20}$$

Then the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1^T(t)\left(-K_1 z_1(t) + M_1(t)g_1(x_1(t))z_2(t)\right.\\ &\quad + M_1(t)h_1(x_1(t-\tau_1)) - \gamma_1(t) + \eta_1^{-1}\widetilde{W}_1^T S_1\\ &\quad \left. + M_1(t)\chi_1(t)\right) + P_1(x_1(t)) - P_1(x_1(t-\tau_1))\\ &\quad - tr\left(\widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1\right) - \widetilde{\chi}_1^T(t)\dot{\widehat{\chi}}_1(t) + \widetilde{\chi}_1^T(t)\dot{\chi}_1(t)\\ &= -z_1^T(t)K_1 z_1(t) + z_1^T(t)\widetilde{W}_1^T S_1 + z_1^T(t)\\ &\quad \cdot h_1(x_1(t-\tau_1)) - z_1^T(t)\gamma_1(t) + z_1^T(t)M_1(t)\\ &\quad \cdot g_1(x_1(t))z_2(t) + \eta_1^{-1}z_1^T(t)\widetilde{W}_1^T S_1 + z_1^T(t)M_1(t)\\ &\quad \cdot \widetilde{\chi}_1(t) + P_1(x_1(t)) - P_1(x_1(t-\tau_1))\\ &\quad - tr\left(\widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1\right) - \widetilde{\chi}_1^T(t)\dot{\widehat{\chi}}_1(t) + \widetilde{\chi}_1^T(t)\dot{\chi}_1(t).\end{aligned} \tag{21}$$

Substituting (14) into (21), we obtain

$$\begin{aligned}\dot{V}_1 &= -z_1^T(t)K_1 z_1(t) + z_1^T(t)\widetilde{W}_1^T S_1\\ &\quad + z_1^T(t)h_1(x_1(t-\tau_1))\\ &\quad + z_1^T(t)M_1(t)g_1(x_1(t))z_2(t) - z_1^T(t)\gamma_1(t)\\ &\quad + z_1^T(t)M_1(t)\widetilde{\chi}_1 + P_1(x_1(t))\\ &\quad - P_1(x_1(t-\tau_1)) + \sigma_1 tr\left(\widetilde{W}_1^T\Gamma_1^{-1}\widehat{W}_1\right)\\ &\quad - \widetilde{\chi}_1^T(t)\dot{\widehat{\chi}}_1(t) + \widetilde{\chi}_1^T(t)\dot{\chi}_1(t).\end{aligned} \tag{22}$$

Substituting (16) into (22) yields

$$\begin{aligned}\dot{V}_1 &= -z_1^T(t)K_1 z_1(t) + z_1^T(t)\widetilde{W}_1^T S_1\\ &\quad + z_1^T(t)h_1(x_1(t-\tau_1)) - z_1^T(t)\gamma_1(t)\\ &\quad + z_1^T(t)M_1(t)g_1(x_1(t))z_2(t) + P_1(x_1(t))\\ &\quad - P_1(x_1(t-\tau_1)) + \sigma_1 tr\left(\widetilde{W}_1^T\Gamma_1^{-1}\widehat{W}_1\right)\\ &\quad - \eta_1\widetilde{\chi}_1^T(t)\widetilde{\chi}_1(t) - \widetilde{\chi}_1^T(t)\widetilde{W}_1^T S_1 + \widetilde{\chi}_1^T(t)\dot{\chi}_1(t).\end{aligned} \tag{23}$$

Step i. Define the error variables $z_i(t) \in R^m$ and $z_{i+1}(t) \in R^m$

$$\begin{aligned}z_i(t) &= x_i(t) - x_i^*(t)\\ z_{i+1}(t) &= x_{i+1}(t) - x_{i+1}^*(t),\end{aligned} \tag{24}$$

where $x_i^*(t)$ is a virtual control law. Combining (1) and (24) and differentiating $z_i(t)$ with respect to time, we have

$$\begin{aligned}\dot{z}_i(t) &= f_i(\overline{x}_i(t)) + g_i(\overline{x}_i(t))x_{i+1}(t) + h_i(\overline{x}_i(t-\tau_i))\\ &\quad + \Delta f_i(\overline{x}_i(t)) + d_i(t) - \dot{x}_i^*(t).\end{aligned} \tag{25}$$

Since $\Delta f_i(\overline{x}_i(t))$ is unknown, we use RBFNN to approximate uncertain function. $\widehat{W}_i$ indicates the estimate of W_i^*, and ε_i^* is the approximate error. $\widetilde{W}_i$ represents the estimated error, which is defined as $\widetilde{W}_i = W_i^* - \widehat{W}_i$. Define the variable $\chi_i(t) = \varepsilon_i^* + d_i(t)$.

$$\Delta f_i(\overline{x}_i(t)) + \gamma_i(t) = \eta_i^{-1} W_i^{*T} S_i + \varepsilon_i^*, \tag{26}$$

where $\gamma_i(t) = [\gamma_{i,1}(t), \ldots, \gamma_{i,m}(t)]^T$, and

$$\gamma_{i,j}(t) = \frac{16}{z_{ij}(t)} \tanh^2\left(\frac{z_{ij}(t)}{\nu_i}\right) h_{ij}^2(\overline{x}_i(t)), \quad j = 1, \ldots, m. \tag{27}$$

Substituting (26) into (1) yields

$$\begin{aligned}\dot{x}_i(t) = {} & f_i(\overline{x}_i(t)) + g_i(\overline{x}_i(t)) x_{i+1}(t) \\ & + h_i(\overline{x}_i(t-\tau_i)) - \gamma_i(t) + \eta_i^{-1} W_i^{*T} S_i + \chi_i(t).\end{aligned} \tag{28}$$

Moreover, substituting (26) into (25), we have

$$\begin{aligned}\dot{z}_i(t) = {} & f_i(\overline{x}_i(t)) + g_i(\overline{x}_i(t)) x_{i+1}(t) \\ & + h_i(\overline{x}_i(t-\tau_i)) - \gamma_i(t) + \eta_i^{-1} W_i^{*T} S_i + \chi_i(t) \\ & - \dot{x}_i^*(t).\end{aligned} \tag{29}$$

Construct updating law of the RBFNN

$$\dot{\widehat{W}}_i = \Gamma_i\left(\eta_1^{-1} z_i(t) S_i - \sigma_i \Gamma_i^{-1} \widehat{W}_i\right), \tag{30}$$

where $\sigma_i > 0$ is a design parameter. Furthermore, the DOB can be chosen as

$$\begin{aligned}\widehat{\chi}_i(t) &= w_i(t) + \eta_i x_i(t) \\ \dot{w}_i(t) &= -\eta_i w_i(t) - \eta_i\left(f_i(\overline{x}_i(t)) + g_i(\overline{x}_i(t)) x_{i+1}\right. \\ &\quad \left. + \eta_i^{-1} \widehat{W}_i^T S_i + \eta_i x_i(t)\right) + z_i(t).\end{aligned} \tag{31}$$

According to (31), we obtain

$$\dot{\widetilde{\chi}}_i(t) = \eta_i \widetilde{\chi}_i(t) + z_i(t) + \widetilde{W}_i^T S_i. \tag{32}$$

Hence, the virtual control law $x_{i+1}^*(t)$ is proposed as

$$\begin{aligned}x_{i+1}^*(t) = {} & g_i^{-1}(\overline{x}_i(t))\left(-K_i z_i(t)\right. \\ & - g_{i-1}^T(\overline{x}_{i-1}(t)) z_{i-1}(t) - f_i(\overline{x}_i(t)) + \dot{x}_i^*(t) \\ & \left. - \eta_i^{-1} \widehat{W}_i^T S_i - \widehat{\chi}_i(t)\right).\end{aligned} \tag{33}$$

Choose the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V_i = {} & \frac{1}{2} z_i^T(t) z_i(t) + \int_{t-\tau_i}^{t} P_i(\overline{x}_i(\tau)) d\tau \\ & + tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right) + \frac{1}{2} \widetilde{\chi}_i^T(t) \widetilde{\chi}_i(t),\end{aligned} \tag{34}$$

where the positive function $P_i(\overline{x}_i(\tau))$ is given in the form of

$$P_i(\overline{x}_i(\tau)) = \frac{1}{2} h_i^T(\overline{x}_i(\tau)) h_i(\overline{x}_i(\tau)). \tag{35}$$

Similar to Step 1, the time derivative of $\dot{V}_i$ can be rewritten as

$$\begin{aligned}\dot{V}_i = {} & -z_i^T(t) K_i z_i(t) + z_i^T(t) g_i(\overline{x}_i(t)) z_{i+1}(t) \\ & - z_i^T(t) g_{i-1}^T(\overline{x}_{i-1}(t)) z_{i-1}(t) - z_i^T(t) \gamma_i(t) \\ & + z_i^T(t) h_i(\overline{x}_i(t-\tau_i)) + P_i(\overline{x}_i(t)) \\ & - P_i(\overline{x}_i(t-\tau_i)) + \sigma_i tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widehat{W}_i\right) \\ & - \eta_i \widetilde{\chi}_i^T(t) \widetilde{\chi}_i(t) - \widetilde{\chi}_i^T(t) \widetilde{W}_i^T S_i + \widetilde{\chi}_i^T(t) \dot{\chi}_i(t).\end{aligned} \tag{36}$$

Step n. Define the error variable $z_n(t) \in R^m$:

$$z_n(t) = x_n(t) - x_n^*(t) \tag{37}$$

Invoking (1) and (37), differentiating $z_n(t)$ with respect to time yields

$$\begin{aligned}\dot{z}_n(t) = {} & f_n(\overline{x}_n(t)) + g_n(\overline{x}_n(t)) u(t) + h_n(\overline{x}_n(t-\tau)) \\ & + \Delta f_n(\overline{x}_n(t)) - \dot{x}_n^*(t).\end{aligned} \tag{38}$$

Since $\Delta f_n(\overline{x}_n(t))$ is unknown, the RBFNN is used to approximate $\Delta f_n(\overline{x}_n(t))$. $\widehat{W}_n$ indicates the estimate of W_n^*, and ε_n^* is the approximate error. $\widetilde{W}_n$ represents the estimated error, which is defined as $\widetilde{W}_n = W_n^* - \widehat{W}_n$. Define the variable $\chi_n = \varepsilon_n^* + d_n(t)$.

$$\Delta f_n(\overline{x}_n(t)) + \gamma_n(t) = \eta_n^{-1} W_n^{*T} S_n + \varepsilon_n^*, \tag{39}$$

where $\gamma_n(t) = [\gamma_{n,1}(t), \ldots, \gamma_{n,m}(t)]^T$, and

$$\gamma_{n,j}(t) = \frac{16}{z_{nj}(t)} \tanh^2\left(\frac{z_{nj}(t)}{\nu_n}\right) h_{nj}^2(\overline{x}_n(t)), \quad j = 1, \ldots, m. \tag{40}$$

Substituting (39) into (1) yields

$$\begin{aligned}\dot{x}_n(t) = {} & f_n(\overline{x}_n(t)) + g_n(\overline{x}_n(t)) u(t) \\ & + h_n(\overline{x}_n(t-\tau_n)) - \gamma_n(t) + \eta_n^{-1} W_n^{*T} S_n \\ & + \chi_n(t).\end{aligned} \tag{41}$$

Invoking (38) and (39), one obtains

$$\begin{aligned}\dot{z}_n(t) = {} & f_n(\overline{x}_n(t)) + g_n(\overline{x}_n(t)) u(t) \\ & + h_n(\overline{x}_n(t-\tau_n)) - \gamma_n(t) + \eta_n^{-1} W_n^{*T} S_n \\ & + \chi_n(t) - \dot{x}_n^*(t).\end{aligned} \tag{42}$$

Construct updating law of the RBFNN

$$\dot{\widehat{W}}_n(t) = \Gamma_n\left(\eta_n^{-1} z_n(t) S_n - \sigma_n \Gamma_n^{-1} \widehat{W}_n(t)\right), \tag{43}$$

where $\sigma_n \in R$, $\sigma_n > 0$, is a design parameter. Furthermore, the DOB can be chosen as

$$\begin{aligned}\hat{\chi}_n(t) &= w_n(t) + \eta_n x_n(t)\\ \dot{w}_n(t) &= -\eta_n w_n(t) - \eta_n\left(f_n(\bar{x}_n(t)) + g_n(\bar{x}_n(t))u(t)\right.\\ &\quad\left.+\eta_n^{-1}\widehat{W}_n^T S_n + \eta_n x_n(t)\right) + z_n(t).\end{aligned} \tag{44}$$

According to (44), we obtain

$$\dot{\tilde{\chi}}_n(t) = \eta_n\tilde{\chi}_n(t) + z_n(t) + \widetilde{W}_n^T S_n. \tag{45}$$

Therefore, design the control law $u(t)$ as

$$\begin{aligned}u(t) &= g_n^{-1}(\bar{x}_n(t))\left(-K_n z_n(t)\right.\\ &\quad - g_{n-1}^T(\bar{x}_{n-1}(t))z_{n-1}(t) - f_n(\bar{x}_n(t)) + \dot{x}_n^*(t)\\ &\quad\left.-\eta_n^{-1}\widehat{W}_n^T S_n - \hat{\chi}_n(t)\right),\end{aligned} \tag{46}$$

where $K_n \in R^{m\times m}$, $K_n > 0$ is design matrix. Choose the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V_n &= \frac{1}{2}z_n^T(t)z_n(t) + \int_{t-\tau_n}^{t} P_n(\bar{x}_n(\tau))\,d\tau\\ &\quad + tr\left(\widetilde{W}_n^T\Gamma_n^{-1}\widetilde{W}_n\right) + \frac{1}{2}\tilde{\chi}_n^T(t)\tilde{\chi}_n(t)\end{aligned} \tag{47}$$

and the positive function $P_n(\bar{x}_n(\tau))$ can be designed as follows:

$$P_n(\bar{x}_n(\tau)) = \frac{1}{2}h_n^T(\bar{x}_n(\tau))h_n(\bar{x}_n(\tau)). \tag{48}$$

According to the derivatives in Step 1 and Step i, we have

$$\begin{aligned}\dot{V}_n &= -z_n^T(t)K_n z_n(t) - z_n^T(t)g_{n-1}^T(\bar{x}_{n-1}(t))z_{n-1}(t)\\ &\quad + z_n^T(t)h_n(\bar{x}_n(t-\tau_n)) - z_n^T(t)\gamma_n(t)\\ &\quad + P_n(\bar{x}_n(t)) - P_n(\bar{x}_n(t-\tau_n))\\ &\quad - \sigma_n tr\left(\widetilde{W}_n^T\Gamma_n^{-1}\widehat{W}_n\right) - \eta_n\tilde{\chi}_n^T(t)\tilde{\chi}_n(t)\\ &\quad - \tilde{\chi}_n^T(t)\widetilde{W}_n^T S_n + \tilde{\chi}_n^T(t)\dot{\chi}_n(t).\end{aligned} \tag{49}$$

From the above inductive design procedure, moreover, we can conclude the following theorem.

3.2. Stability Analysis

Theorem 7. *Considering the system dynamics described by (1), under the adaptive neural updated laws (14), (30), and (43), the disturbance observers (15), (31), and (45), and the prescribed performance-based control laws (17), (33), and (46), we can conclude that the trajectories of the closed-loop system are semiglobally uniformly bounded, while the tracking error $\tilde{z}_1(t)$ converges to a compact set Q_z asymptotically, where C and κ are defined in (50).*

$$\begin{aligned}\kappa &= \min\left\{\left(\lambda_{\min}(K_i) - \frac{\vartheta_{\xi i}+1}{2}\right),\right.\\ &\quad\left.\left(\eta_i - \frac{1}{2\vartheta_{\xi i}} - \frac{\delta_S\varphi_i}{2} - \frac{\pi_i}{2}\right), \frac{\sigma_i}{2}\right\}\\ C &= \sum_{i=1}^{n}\frac{\sigma_i}{2}\lambda_{\min}\left(\Gamma_i^{-1}\right)\left\|W_i^*\right\|^2 + \sum_{i=1}^{n}\frac{1}{2\pi_i}\left\|\dot{\chi}_i(t)\right\|^2,\end{aligned} \tag{50}$$

where Q_z can be made as small as desired by appropriately choosing design parameters. $\lambda_{\min}$ represents the minimum eigenvalue of the matrix. $\lambda_{\min}$ represents the minimum eigenvalue of the matrix.

Proof. For analytical purposes, define the total Lyapunov function

$$V_\Xi = \sum_{i=1}^{n} V_i, \tag{51}$$

where definition of V_i can be referred to (19), (34), and (47). According to (23), (36), and (49), we obtain

$$\begin{aligned}\dot{V}_\Xi &= -\sum_{i=1}^{n} z_i^T(t)K_i z_i(t) - \sum_{i=1}^{n}\eta_i\tilde{\chi}_i^T(t)\tilde{\chi}_i(t)\\ &\quad + \sum_{i=1}^{n} z_i^T(t)h_i(\bar{x}_i(t-\tau_i)) + \sum_{i=1}^{n} z_i^T(t)\tilde{\chi}_i(t)\\ &\quad - \sum_{i=1}^{n} z_i^T(t)\gamma_i(t) - \sum_{i=1}^{n}\tilde{\chi}_i^T(t)\widetilde{W}_i^T S_i\\ &\quad + \sum_{i=1}^{n}\tilde{\chi}_i^T(t)\dot{\chi}_i(t)\\ &\quad + \sum_{i=1}^{n}\left(P_i(\bar{x}_i(t)) - P_i(\bar{x}_i(t-\tau_i))\right)\\ &\quad - \sum_{i=1}^{n}\sigma_i tr\left(\widetilde{W}_i^T\Gamma_i^{-1}\widehat{W}_i\right).\end{aligned} \tag{52}$$

Considering the fact that

$$\begin{aligned}&z_i^T(t)h_i(\bar{x}_i(t-\tau_i))\\ &\quad\le \frac{1}{2}z_i^T(t)z_i(t) + \frac{1}{2}h_i^T(\bar{x}_i(t-\tau_i))h_i(\bar{x}_i(t-\tau_i))\end{aligned} \tag{53}$$

and $P_i(\overline{x}_i(t-\tau_i)) = (1/2)h_i^T(\overline{x}_i(t-\tau_i))h_i(\overline{x}_i(t-\tau_i))$, we have

$$\begin{aligned}\dot{V}_{\Xi} \le & -\sum_{i=1}^{n} z_i^T(t) K_i z_i(t) - \sum_{i=1}^{n} \eta_i \widetilde{\chi}_i^T(t) \widetilde{\chi}_i(t) \\ & + \sum_{i=1}^{n} z_i^T(t) \widetilde{\chi}_i(t) - \sum_{i=1}^{n} z_i^T(t) \gamma_i(t) \\ & - \sum_{i=1}^{n} \widetilde{\chi}_i^T(t) \widetilde{W}_i^T S_i + \sum_{i=1}^{n} \widetilde{\chi}_i^T(t) \dot{\chi}_i(t) \\ & + \frac{1}{2}\sum_{i=1}^{n} z_i^T(t) z_i(t) + \sum_{i=1}^{n} P_i(\overline{x}_i(t)) \\ & - \sum_{i=1}^{n} \sigma_i tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right).\end{aligned} \tag{54}$$

Then we have the following results:

$$\begin{aligned}\dot{V}_{\Xi} \le & -\sum_{i=1}^{n} z_i^T(t)\left(K_i - \frac{1}{2} I^{m\times m}\right) z_i(t) \\ & - \sum_{i=1}^{n} \eta_i \widetilde{\chi}_i^T(t) \widetilde{\chi}_i(t) + \sum_{i=1}^{n} z_i^T(t) \widetilde{\chi}_i(t) \\ & - \sum_{i=1}^{n} z_i^T(t) \gamma_i(t) - \sum_{i=1}^{n} \widetilde{\chi}_i^T(t) \widetilde{W}_i^T S_i \\ & + \sum_{i=1}^{n} \widetilde{\chi}_i^T(t) \dot{\chi}_i(t) + \sum_{i=1}^{n} P_i(\overline{x}_i(t)) \\ & - \sum_{i=1}^{n} \sigma_i tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right).\end{aligned} \tag{55}$$

In addition, it is clear that there exists $\|S_i\| \le \delta_{Si}$, $\delta_{Si} > 0$; we have the following facts:

$$\begin{aligned}\sigma_i tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right) &\le -\frac{\sigma_i}{2} tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right) \\ &\quad + \frac{\sigma_i}{2} tr\left(W_i^{*T} \Gamma_i^{-1} W_i^*\right) \\ z_i^T(t) \widetilde{\chi}_i(t) &\le \frac{\vartheta_{\xi i}}{2} z_i^T(t) z_i(t) + \frac{1}{2\vartheta_{\xi i}} \left\|\widetilde{\chi}_i(t)\right\|^2 \\ \widetilde{\chi}_i^T(t) \widetilde{W}_i^T S_i &\le \frac{\delta_{Si}\varphi_i}{2} \left\|\widetilde{\chi}_i(t)\right\|^2 + \frac{\delta_{Si}}{2\varphi_i} \left\|\widetilde{W}_i\right\|^2 \\ \widetilde{\chi}_i^T(t) \dot{\chi}_i(t) &\le \frac{\pi_i}{2} \left\|\widetilde{\chi}_i(t)\right\|^2 + \frac{1}{2\pi_i} \left\|\dot{\chi}_i(t)\right\|^2,\end{aligned} \tag{56}$$

where $\vartheta_{\xi i} > 0$ and $\varphi_i > 0$ are the constants, and then we have

$$\begin{aligned}\dot{V}_{\Xi} \le & -\sum_{i=1}^{n}\left(\lambda_{\min}(K_i) - \frac{\vartheta_{\xi i}+1}{2}\right)\|z_i(t)\| - \sum_{i=1}^{n}\Bigg(\eta_i \\ & - \frac{1}{2\vartheta_{\xi i}} - \frac{\delta_{Si}\varphi_i}{2} - \frac{\pi_i}{2}\Bigg)\left\|\widetilde{\chi}_i(t)\right\| \\ & - \sum_{i=1}^{n} \frac{\sigma_i}{2} tr\left(\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i\right) + \frac{1}{2} \\ & \cdot \sum_{i=1}^{n}\sum_{j=1}^{m}\left(1 - 16\tanh^2\left(\frac{z_{ij}(t)}{v_i}\right)\right) h_{ij}^T(\overline{x}_i(t)) \\ & \cdot h_{ij}(\overline{x}_i(t)) + \sum_{i=1}^{n} \frac{\sigma_i}{2} tr\left(W_i^{*T} \Gamma_i^{-1} W_i^*\right) \\ & + \sum_{i=1}^{n} \frac{1}{2\pi_i} \left\|\dot{\chi}_i(t)\right\|.\end{aligned} \tag{57}$$

Invoking Lemma 5, we obtain

$$\dot{V}_{\Xi} \le -\kappa V_{\Xi} + C. \tag{58}$$

$\kappa > 0, C > 0$ are defined in (50). Therefore, according to Lemma 4, we can conclude that the solution of the closed-loop system remains within a compact subset. □

4. Application to Unmanned Helicopter System

In this section, we will apply the proposed robust adaptive control strategy to solve the problem of attitude tracking for a class of uncertain unmanned helicopter systems.

The unmanned helicopters rigid-body dynamics consist of two parts, attitude angular dynamics and flapping dynamics. The unsteady aerodynamics bring out the time-delay nonlinear uncertainty; thus in this section we consider the attitude control for attitude control subject to time delay. Firstly, the attitude angular and angular velocity dynamics can be described as [26]

$$\begin{aligned}\dot{\Theta}(t) &= R(\Theta(t))\,\Omega(t) \\ I\dot{\Omega}(t) &= -\Omega(t) \times (I\Omega(t)) + \overline{A}(t)\, v_c(t) \\ &\quad + h(\Theta(t), \Omega(t)) + \overline{B}(t) \\ &\quad + \Delta_{\Omega}(\Theta(t), \Omega(t)) + d_{\Omega}(t),\end{aligned} \tag{59}$$

where $\Theta = [\psi, \theta, \phi]^T \in R^3$ indicates the attitude angle and $\Omega = [p, q, r]^T \in R^3$ represents the attitude angle velocity. $d_{\Omega}(t) \in R^3$ stands for the disturbance, and $\Delta_{\Omega} \in R^3$ stands for the uncertainties; $h(\Theta(t), \Omega(t)) \in R^3$ means the unknown

nonlinear block of time delay. $v_c = [a, b, T_T]^T \in R^3$, $R(\Theta(t))$ denotes

$$R(\Theta(t)) = \begin{bmatrix} 1 & \sin\phi(t)\tan\theta(t) & \cos\phi(t)\tan\theta(t) \\ 0 & \cos\phi(t) & -\sin\phi(t) \\ 0 & \dfrac{\sin\phi(t)}{\cos\theta(t)} & \dfrac{\cos\phi(t)}{\cos\theta(t)} \end{bmatrix}. \tag{60}$$

$\overline{A}$ and $\overline{B}$ denote

$$\overline{A} = \begin{bmatrix} -G_R & -z_R T_R + K_\beta & z_T \\ -z_R T_R + K_\beta & G_R & 0 \\ y_R T_R & x_R T_R & -x_T \end{bmatrix}$$
$$\overline{B} = \begin{bmatrix} -y_R T_R \\ x_R T_R \\ -G_R \end{bmatrix} \tag{61}$$
$$G_R = C^R |T_R|^{1.5} + D^R.$$

In the unmanned helicopter attitude control loop, the main rotor thrusts T_R are fixed. $[x_R, y_R, z_R]^T \in R^3$ indicates the location of the main rotor relative to the center of gravity; $[x_T, y_T, z_T]^T \in R^3$ indicates the location of the tail rotor relative to the center of gravity. $K_\beta \in R$ means the stiffness of the rotor hub. C^R and D^R are constants, which are associated with the antitorque. The control design procedure can be presented as follows.

Step 1. Define the attitude angle tracking errors in the form of $\tilde{z}_\psi(t) = \psi(t) - \psi_r(t)$, $\tilde{z}_\theta(t) = \theta(t) - \theta_r(t)$, $\tilde{z}_\phi(t) = \phi(t) - \phi_r(t)$, and $\tilde{z}_\Theta(t) = [\tilde{z}_\psi(t), \tilde{z}_\theta(t), \tilde{z}_\phi(t)]^T$, where $\psi_r(t)$, $\theta_r(t)$, and $\phi_r(t)$ are the reference trajectories and $\Theta_r = [\psi_r(t), \theta_r(t), \phi_r(t)]^T$. Then the time derivative of $\tilde{z}_\Theta(t)$ is achieved as

$$\tilde{z}_\Theta = R(\Theta(t))\,\Omega(t) - \dot{\Theta}_r(t). \tag{62}$$

According to the analysis in the previous section, the prescribed performance function is chosen as

$$z_{\Theta j}(t) = \frac{1}{2}\ln\left[\frac{\left(\underline{\alpha}_j + \tilde{z}_{\Theta j}(t)/\rho_j(t)\right)}{\left(\overline{\alpha}_j - \tilde{z}_{\Theta j}(t)/\rho_j(t)\right)}\right], \quad j = 1, \ldots, 3, \tag{63}$$

and the definition of $\rho_j(t)$ can be referred to in (6). Choose the virtual control signal as

$$\Omega^*(t) = R^{-1}(\Theta(t)) \cdot \left(-M_\Theta^{-1}(t) K_\Theta z_\Theta(t) + \dot{\Theta}_r(t) + M_\Theta^{-1}(t) N_\Theta(t)\right). \tag{64}$$

Choose the Lyapunov function:

$$V_\Theta = \frac{z_\Theta^T(t) z_\Theta(t)}{2}. \tag{65}$$

Step 2. Define the tracking errors $z_p(t) = p(t) - p_r^*(t)$, $z_q(t) = q(t) - q_r^*(t)$, $z_r(t) = r(t) - r_r^*(t)$, and $\Omega^*(t) = [p_r^*(t), q_r^*(t), r_r^*(t)]^T$. Using the neural network to compensate the uncertainties and time-delay terms, such that $\Delta_\Omega(\Theta(t), \Omega(t)) + \gamma_\Omega(t) = \eta_\Omega^{-1} W_\Omega^{*T} S_\Omega + \varepsilon_\Omega^*$, assume that ε_Ω^* is the optimal approximate error, and define $\chi_\Omega(t) = \varepsilon_\Omega^* + d_\Omega(t)$. Design the neural updated law as

$$\dot{\widehat{W}}_\Omega(t) = \Gamma_\Omega\left(z_\Omega(t) S_\Omega(\Theta(t), \Omega(t)) - \sigma_\Omega \Gamma_\Omega^{-1} \widehat{W}_\Omega(t)\right). \tag{66}$$

The DOB can be constructed in the form of

$$\begin{aligned} \widehat{\chi}_\Omega(t) &= w_\Omega(t) + \eta_\Omega \Omega(t) \\ \dot{w}_\Omega(t) &= -\eta_\Omega w_\Omega(t) - \eta_\Omega\left(-\Omega(t) \times (I\Omega(t)) \right. \\ &\quad \left. + \overline{A}(t) v_c(t) + \overline{B}(t) + \eta_\Omega^{-1} \widehat{W}_\Omega^T S_\Omega + \eta_\Omega \Omega(t)\right) \\ &\quad + z_\Omega(t). \end{aligned} \tag{67}$$

Based on the above design, the adaptive control law is achieved as

$$\begin{aligned} v_c(t) &= \overline{A}^{-1}(t)\left(-K_\Omega z_\Omega(t) - R^T(\Theta) z_\Theta(t) + \dot{\Omega}^*(t) \right. \\ &\quad \left. + \Omega(t) \times (I\Omega(t)) - \overline{B}(t) - \eta_\Omega^{-1} \widehat{W}_\Omega^T S_\Omega - \widehat{\chi}_\Omega(t)\right). \end{aligned} \tag{68}$$

Choose the Lyapunov function:

$$\begin{aligned} V_\Omega &= \frac{1}{2}\Omega^T(t)\Omega(t) + \int_{t-\tau}^{t} P\left(\Theta(t_\tau), \Omega(t_\tau)\right) dt_\tau \\ &\quad + tr\left(\widetilde{W}_\Omega^T \Gamma_\Omega^{-1} \widetilde{W}_\Omega\right) + \frac{1}{2}\widetilde{\chi}_\Omega^T(t)\widetilde{\chi}_\Omega(t). \end{aligned} \tag{69}$$

Similar to Theorem 7, we can conclude the following theorem.

Theorem 8. *For the unmanned helicopter system (59), combining the DOB (67) and neural network update law (66), the control procedure can be achieved as (64) and (68); thus the closed-loop signals are bounded, and the output trajectories are satisfying the prescribed performance conditions.*

For Theorem 8, the detailed proof can be referred to in Theorem 7 in the previous section.

5. Simulation Results

In this section, the numerical simulation about unmanned helicopter system is utilized for illustrating the effectiveness of the previous control method in this paper. Consider the unmanned helicopter system with the following parameters (Table 1) [24, 26].

Now let us look at the design of adaptive controller design, the DOB design parameters are chosen as $\eta_\Omega = 6$. The neural update law has been given by 70, and its parameters are chosen as $\Gamma_\Omega = 0.1 \cdot \mathrm{diag}\{1, 1, 1, 1, 1, 1, 1\}$, $\sigma_\Omega = 10$, $\mu = [-3, -2, -1, 0, 1, 2, 3]$, and $c_\Omega = [1, 1, 1, 1, 1, 2, 3]$, and the controller design parameters are chosen as $K_\Theta = \{5, 5, 5\}$ and

Table 1: Helicopter parameters.

Symbol	Unit	Number
I	kg·m^2	diag (0.18, 0.34, 0.28)
m	kg	8.2
g	m/sec^2	9.81
x_T	m	−0.91
z_T	m	−0.08
z_R	m	−0.235
x_R	m	0
y_R	m	0
z_R	m	0
K_β	N·m/rad	52
C^R	m/$\sqrt{\text{N}}$	0.004452
D^R	N · m	0.6304

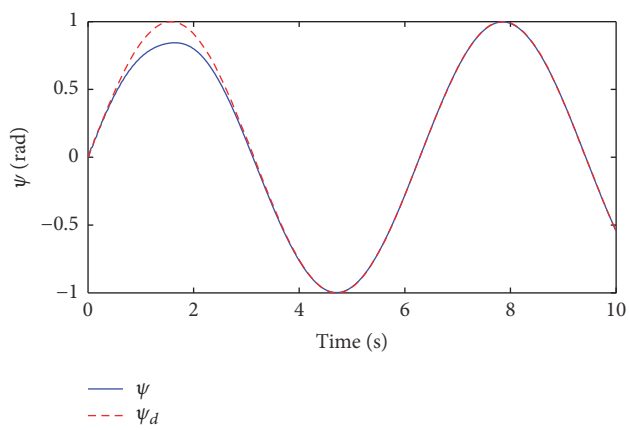

Figure 1: Yaw angle.

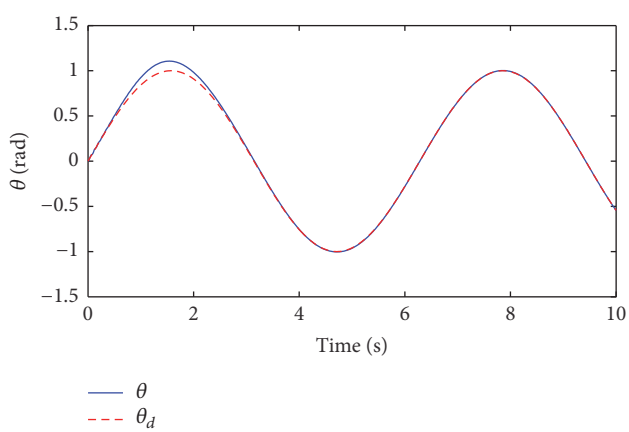

Figure 2: Pitch angle.

$K_\Omega = \{10, 10, 10\}$. The simulation environment is built under Matlab, and the simulation step is chosen as $\tau = 0.02$. The initial conditions are chosen as follows: $\psi(0) = 1$ rad, $\psi(0) = 0$ rad, and $\psi(0) = 0$ rad. The time-delay block is chosen in the form of $h(\Theta(t), \Omega(t)) = \Omega(t-\tau)^2 + \sin(0.1 \cdot \Omega(t-3\tau))$. The external disturbance is chosen as $d_\Omega(t) = 2 \cdot \sin(2 \cdot t + 0.6)$.

From the curves in Figures 1–3, it can be seen that the tracking trajectories soon reach the target trajectories, and

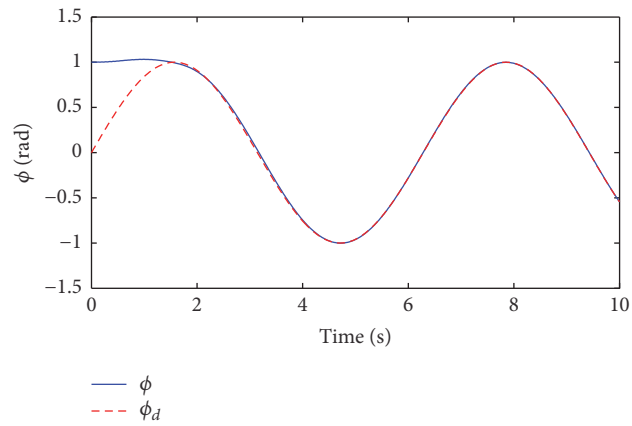

Figure 3: Roll angle.

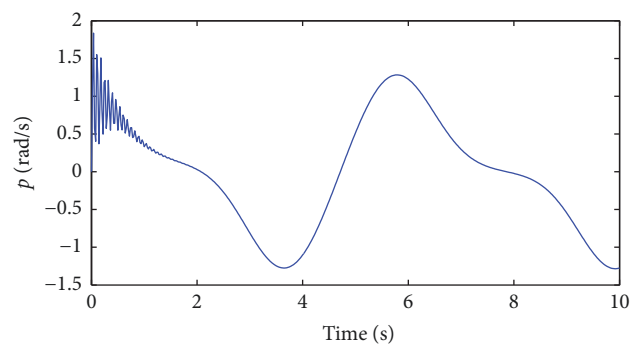

Figure 4: Yaw angular velocity.

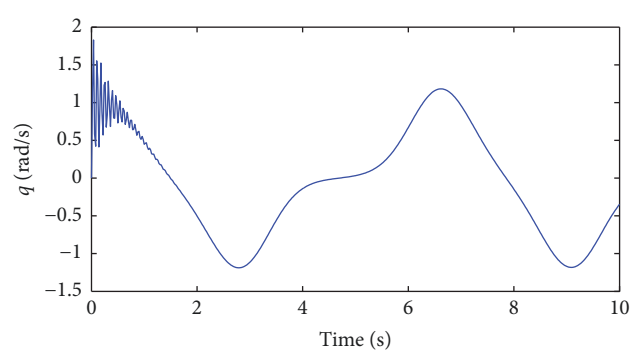

Figure 5: Pitch angular velocity.

the tracking errors are bounded in the appointed region. Figures 4–6 show the attitude angular velocities. Figures 7–9 show the control input signals. From the numerical simulation, we can conclude that the proposed control approach is valid for a class of uncertain unmanned helicopter systems with unsteady aerodynamics. Furthermore, it can be illustrated that the closed-loop system output signals are asymptotically tracking the ideal trajectories, and they are restricted in the region of output constraints.

6. Conclusion

In this paper, an adaptive prescribed performance control procedure has been proposed for a class of nonlinear time-delay systems with uncertainties, external disturbances, and

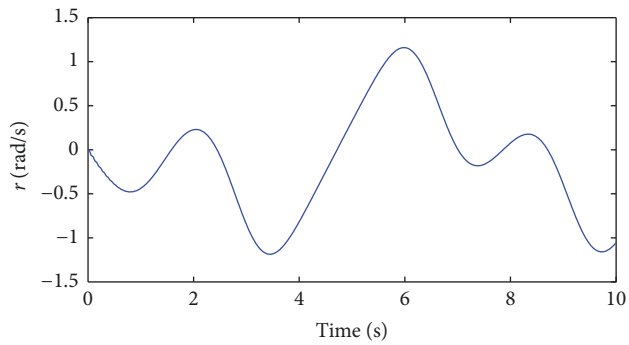

Figure 6: Roll angular velocity.

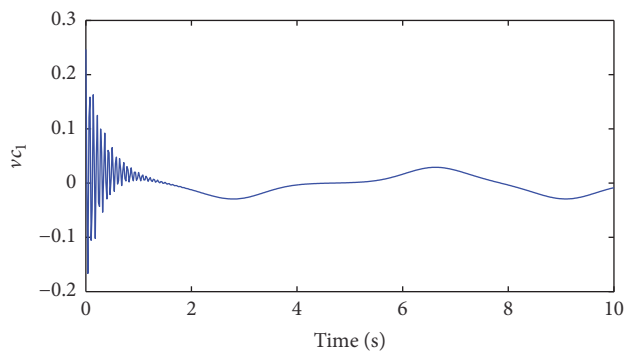

Figure 7: Control input signal $v_{c1}(t)$.

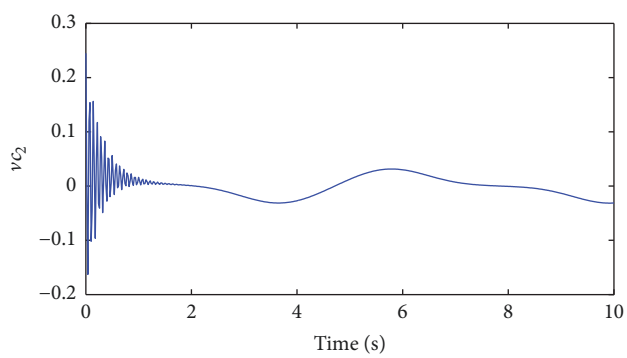

Figure 8: Control input signal $v_{c2}(t)$.

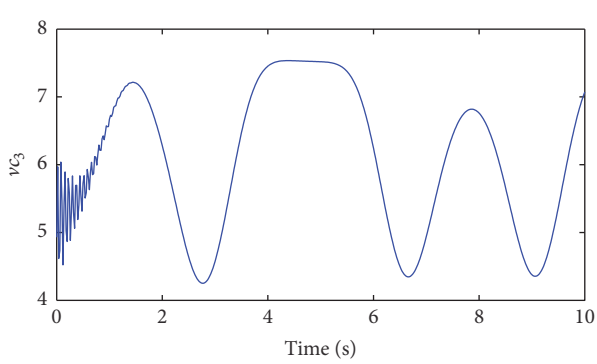

Figure 9: Control input signal $v_{c3}(t)$.

output constraints. Additionally, the robust controller has been applied to unmanned helicopter systems with unsteady aerodynamics. At last, the simulation illustrates that the proposed control approach is valid for the uncertain constrained time-delay system.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This article was supported by the National Natural Science Foundation of China (nos. 61573184, 61374212), the Fundamental Research Funds for the Central Universities (no. NE2016101), and the Aeronautical Science Foundation of China (no. 20165752049).

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