Research Article

Reliable Sideslip Angle Estimation of Four-Wheel Independent Drive Electric Vehicle by Information Iteration and Fusion

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Accurate estimation of longitudinal force and sideslip angle is significant to stability control of four-wheel independent driven electric vehicle. The observer design problem for the longitudinal force and sideslip angle estimation is investigated in this work. The electric driving wheel model is introduced into the longitudinal force estimation, considering the longitudinal force is the unknown input of the system, the proportional integral observer is applied to restructure the differential equation of longitudinal force, and the extended Kalman filter is utilized to estimate the unbiased longitudinal force. Using the estimated longitudinal force, considering the unknown disturbances and uncertainties of vehicle model, the robust sideslip angle estimator is proposed based on vehicle dynamics model. Moreover, the recursive least squares algorithm with forgetting factor is applied to vehicle state estimation based on the vehicle kinematics model. In order to integrate the advantages of the dynamics-model-based observer and kinematics-model-based observer and improve adaptability of observer system in complex working conditions, a vehicle sideslip angle fusion estimation strategy is proposed. The simulations and experiments are implemented and the performance of proposed estimation method is validated.

1. Introduction

With the development of automotive technology, drive mode of electric vehicles (EVs) is also experiencing many changes and innovation [1–4]. Emerging four-wheel independent drive electric vehicles (4WID-EVs), as a potential means of future transportation with the controllability of accurate and independent torque, have proven fruitful in energy-saving and advanced vehicle motion control and attracted the attention of both industrial and academic communities [5–7]. 4WID-EVs possess lots of advantages in vehicle dynamics control, such as DYC, in which the accurate estimation of sideslip angle is significant to control performance [8, 9]. Recently, the increasing requirements of safety and stability in automotive technology promote the rapid growth and all the attendant publicity of intelligent transportation [10–12]. Autonomous ground vehicles and unmanned driving have the great advantages containing higher security, better road

utilization, and greatly reduced mobility costs. With regard to the path-following control, the real time and reliable vehicle states estimation contribute to better control performance. Therefore, owing to the consideration that some vehicle states are hard and costly to obtain, the design of model-based vehicle state estimator is essential to estimate the vehicle state via low-cost sensor measurements.

During the past decade, there have been many researches engaged in the field of longitudinal force and sideslip angle estimation. The common algorithm applied in estimation can be summarized as Kalman filter [13–17], nonlinear observer [18–22], optimal estimation method [23–27], information fusion estimation method [15, 17, 28–31], and robust estimation method [32, 33], and so on. The core technology in most previous approaches concentrates on solving the estimation problem in complex and limited vehicle running conditions, such as high speed [14, 15], low adhesion [16, 20, 34], vehicle parameters time-varying [29, 32], and unknown or nonlinear

disturbances [22–24]. In most prior studies, the longitudinal force observer is designed for traditional internal combustion engine vehicle, and the longitudinal force estimation for EVs especially for 4WID-EVs is still relatively hard to see. In [35], the longitudinal force was calculated by means of integrating the rotational dynamics differential equation of driving wheel, but the influence of noise was accumulated. In 4WID-EVs, the electric driving wheel composed of in-wheel motor and tire is an independent information unit; if we make full use of the measurements of low-cost sensors, such as the current, speed, and bus voltage, the longitudinal force will be achieved cheaply and precisely. Seldom studies have introduced this concept into longitudinal force estimation. For vehicle lateral stability and path-following control, the sideslip angle is an indispensable parameter, and there exists a lot of fruitful progress for sideslip angle estimation. Li et al. presented an adaptive sideslip angle estimator using low-cost sensors in which the multiple model-based Kalman filter was designed to improve the accuracy of estimation [31]. Yoon and Peng estimated the sideslip angle via the combination of the measurements of a global positioning system, an inertial measurement unit, and a magnetometer; the dual Kalman filter was designed to improve the accuracy of estimation via the redundancy of measurements [33].

In this paper, a novel and reliable method of longitudinal force and sideslip angle estimation is proposed for 4WID-EVs. Considering the electric driving wheel model (EDWM) is an uncertain system with unknown input, we utilize the proportional integral observer (PIO) to establish the differential equation of longitudinal force, and the extend Kalman filter (EKF) is introduced to achieve the unbiased estimation of longitudinal force; this design provides a novel thinking of the longitudinal force estimation in 4WID-EVs. In the sideslip angle estimation, the longitudinal forces estimated by longitudinal force observers (LFOs) are regarded and used as the pseudo measurements. The unknown disturbances and uncertainties of vehicle model are considered; the robust sideslip angle estimator is devised on the basis of vehicle dynamics model in which the theoretical robust performance is verified. Furthermore, a recursive least squares method with forgetting factor is used for vehicle state estimation based on the vehicle kinematics model. Then, a vehicle sideslip angle fusion estimation strategy is proposed, in which the accuracy, reliability, and adaptability for the complex working conditions of sideslip angle estimation are improved by the information iteration and fusion between the multimodel-based observers.

The rest of this paper is organized as follows. The vehicle model is presented in Section 2. The vehicle sideslip angle fusion estimation strategy is designed in Section 3. The simulation results are provided in Section 4. The experimental verifications are presented in Section 5, followed by the conclusive remarks.

2. Vehicle Model

2.1. Vehicle Dynamics Model. In this section, a two-degree-of-freedom (2-DOF) vehicle model in longitudinal and lateral

FIGURE 1: Vehicle dynamics model.

directions is introduced to characterize the vehicle motion; the schematic diagram of vehicle dynamic model is shown in Figure 1. The origin of dynamic coordinate system xoy fixed on the vehicle coincides with the vehicle gravity center; the x -axis is the longitudinal axis of the vehicle (the forward direction is positive); the y -axis is the lateral axis of the vehicle (the right-to-left direction is positive). The pitch, roll, and vertical motions and the suspension system of the vehicle are ignored. It is assumed that the mechanical properties of each tire are the same. The serial numbers 1, 2, 3, and 4 of the wheels are, respectively, corresponding to the front-left, the front-right, the rear-left, and the rear-right wheel. Assuming that the front wheel steering angle is small, the lateral dynamics equation of the four-wheel vehicle model can be expressed as

$$\begin{aligned} m v_x \dot{\beta} &= F_{yf} + F_{yr} - m v_x \gamma, \\ I_z \dot{\gamma} &= l_f F_{yf} - l_r F_{yr} + \Delta M_z, \end{aligned} \quad (1)$$

where v_x and v_y represent the longitudinal and lateral vehicle speed at the center of gravity (COG), γ is the yaw rate which represents the angular velocity of vehicle at the COG, β denotes the sideslip of vehicle at the COG, m represents the vehicle mass, and I_z stands for the moment of inertia. l_f and l_r are the distances from vehicle gravity center to the front and rear axle, respectively. F_{yf} and F_{yr} are the generalized front and rear lateral forces, respectively; that is, $F_{yf} = F_{y1} + F_{y2}$, $F_{yr} = F_{y3} + F_{y4}$. ΔM_z is the external yaw moment generated by four in-wheel motors; it can be expressed as

$$\begin{aligned} \Delta M_z &= (F_{x2} - F_{x1}) b_f \cos \delta + (F_{x1} + F_{x2}) l_f \sin \delta \\ &\quad + (F_{x4} - F_{x3}) b_r, \end{aligned} \quad (2)$$

where $b_{f,r}$ is the half tread of wheel base, δ is the steering angle of the front wheels, and F_{xj} ($j = 1, 2, 3, 4$) represents the longitudinal force of the i th tire. The lateral tire force F_{yf} and force F_{yr} can be written as

$$\begin{aligned} F_{yf} &= C_f \alpha_f, \\ F_{yr} &= C_r \alpha_r, \end{aligned} \quad (3)$$

where C_f and C_r are the generalized cornering stiffness of the front and rear tires, respectively. The tire slip angles of front and rear tires can be given by

$$\begin{aligned}\alpha_f &= \delta_f - \frac{l_f \gamma}{v_x} - \beta \\ \alpha_r &= \frac{l_r \gamma}{v_x} - \beta.\end{aligned}\quad (4)$$

The vehicle dynamics model can be derived as

$$\dot{x} = Ax + Bu, \quad (5)$$

where

$$\begin{aligned}x &= [\beta \ \gamma]^T, \\ u &= [\delta \ \Delta M_z]^T, \\ A &= \begin{bmatrix} -\frac{C_f + C_r}{mv_x} & -1 - \frac{l_f C_f - l_r C_r}{mv_x^2} \\ \frac{l_r C_r - l_f C_f}{I_z} & -\frac{l_f^2 C_f + l_r^2 C_r}{v_x I_z} \end{bmatrix}, \\ B &= \begin{bmatrix} \frac{C_f}{mv_x} & 0 \\ \frac{l_f C_f}{I_z} & \frac{1}{I_z} \end{bmatrix}.\end{aligned}\quad (6)$$

2.2. EDWM. Each wheel of the FWID-EV is driven by an in-wheel motor. And the driving wheel consisting of in-wheel motor and tire is an independent driving and informative unit. As shown in Figure 2, the concept of the EDMW is proposed in this study. The rotational dynamic equation of each wheel can be written as

$$J_1 \dot{\omega}_j = T_{Lj} - F_{xj} r, \quad (7)$$

where ω_j is the wheel speed of the j th wheel, J_1 is the wheel moment of inertia, r is the wheel effective rolling radius, and T_{Lj} is the load torque of in-wheel motor. The torque balance equation of the output shaft in in-wheel motor can be given by

$$J_2 \dot{\omega}_j + b \omega_j = K_t i_j - T_{Lj}, \quad (8)$$

where J_2 is the rotational inertia of in-wheel motor rotor, b is the damping coefficient, K_t is the motor torque constant, and i_j is the bus current. The dynamic voltage balance equation of equivalent circuit in in-wheel motor can be modeled as

$$u_j = R i_j + L \dot{i}_j + K_a \omega_j, \quad (9)$$

where u_j is the bus voltage, R is the equivalent resistance of winding, L is the equivalent inductance of winding, and K_a is the inverse electromotive force coefficient.

FIGURE 2: Electric driving wheel model.

2.3. Wheel Speed Coupling Relationship. The rotational speed of four wheels can be expressed as

$$\begin{aligned}\omega_1 &= \frac{[(v_x + \gamma b_f) \cos \delta + (v_y + \gamma l_f) \sin \delta]}{r}, \\ \omega_2 &= \frac{[(v_x - \gamma b_f) \cos \delta + (v_y + \gamma l_f) \sin \delta]}{r}, \\ \omega_3 &= \frac{v_x + \gamma b_r}{r}, \\ \omega_4 &= \frac{v_x - \gamma b_r}{r},\end{aligned}\quad (10)$$

where n_1, n_2, n_3 , and n_4 represent the corresponding wheel speeds, respectively. r is the effective wheel radius.

3. Vehicle Sideslip Angle Fusion Estimation Strategy

3.1. Dynamics-Based Sideslip Angle Robust Estimation

3.1.1. Design of LFO. By substituting (7) into (8) and combining it with (9), we have

$$\begin{aligned}\dot{i}_j &= -\frac{R}{L} i_j - \frac{K_a}{L} \omega_j + \frac{1}{L} u_j, \\ \dot{\omega}_j &= \frac{K_t}{J} i_j - \frac{b}{J} \omega_j - \frac{r}{J} F_{xj},\end{aligned}\quad (11)$$

where $J = J_1 + J_2$. The electric driving wheel model (EDWM) is obtained as

$$\begin{aligned}\dot{x}_w &= A_w x_w + B_w u_w + D_w d + E_w w_1, \\ y_w &= C_w x_w + F_w w_2,\end{aligned}\quad (12)$$

where x_w, u_w, d , and y_w are the state vector, the known input vector, and the unknown input vector, and measurement vector, respectively. w_1 and w_2 are uncorrelated zero mean white noise sequences. The known input and unknown input represent the voltage and longitudinal force, respectively. And

$$\begin{aligned}x_w &= [i_j \ \omega_j]^T, \\ A_w &= \begin{bmatrix} -\frac{R}{L} & -\frac{K_a}{L} \\ \frac{K_t}{J} & -\frac{b}{J} \end{bmatrix},\end{aligned}$$

$$\begin{aligned}
B_w &= \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \\
D_w &= \begin{bmatrix} 0 \\ -\frac{r}{J} \end{bmatrix}, \\
C_w &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
E_w = F_w &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\end{aligned} \tag{13}$$

Owing to the existence of unknown input in system (12), the Luenberger observer is not appropriate for the precise state estimation. Consequently, the proportional integral observer (PIO) is utilized to estimate the longitudinal force:

$$\begin{aligned}
\dot{\hat{x}}_w &= A_w \hat{x}_w + B_w u_w + K_p (y - \hat{y}) + D_w \hat{d} + E_w w_1, \\
\dot{\hat{d}} &= K_i (y_w - \hat{y}_w), \\
\hat{y}_w &= C_w \hat{x}_w + F_w w_2,
\end{aligned} \tag{14}$$

where $\hat{x}_w$ is the state estimation, $\hat{d}$ is the unknown input estimation, K_p is the gain matrix of PIO, and K_i is the integral matrix of the unknown input estimation. The further form of equation (14) can be derived as

$$\begin{aligned}
\dot{\hat{x}}_w &= (A_w - K_p C_w) \hat{x}_w + B_w u_w + K_p y_w + D_w \hat{d} \\
&\quad + E_w w_1 - K_p F_w w_2, \\
\dot{\hat{d}} &= -K_i C_w \hat{x}_w + K_i y_w - K_i F_w w_2.
\end{aligned} \tag{15}$$

Defining the state estimation error as $e_w = \hat{x}_w - x_w$, we get

$$\begin{aligned}
\dot{e}_w &= \dot{\hat{x}}_w - \dot{x}_w \\
&= (A_w - K_p C_w) e_w + D_w (\hat{d} - d) - K_p F_w w_2.
\end{aligned} \tag{16}$$

It can be deduced as

$$\begin{bmatrix} \dot{e}_w \\ \dot{\hat{d}} \end{bmatrix} = A_{we} \begin{bmatrix} e_w \\ \hat{d} \end{bmatrix} + \begin{bmatrix} D_w \\ 0 \end{bmatrix} d - \begin{bmatrix} K_p \\ K_i \end{bmatrix} F_w w_2, \tag{17}$$

where

$$A_{we} = \begin{bmatrix} A_w - K_p C_w & D_w \\ -K_i C_w & 0 \end{bmatrix}. \tag{18}$$

Lemma 1 (see [36]). *If $-\infty < \lim_{t \rightarrow \infty} d(t) < +\infty$, there exists a PIO for system (11) to guarantee $\lim_{t \rightarrow \infty} (\hat{x} - x) = 0$ for any initial states $x_w(0)$, $\hat{x}_w(0)$, and $\hat{d}(0)$ when (A_w, C_w) is observable and $\text{rank} \left\{ \begin{bmatrix} A_w & D_w \\ C_w & 0 \end{bmatrix} \right\} = 3$.*

Remark 2. Notice that the pole assignment of matrix A_{we} can be implemented to the specified positions to obtain the gain matrixes K_p and K_i . With the suitable choice of K_p and K_i , the eigenvalues of A_{we} will locate on the left-half complex plane.

In system (15), the estimation of unknown input is solved by the design of PIO, but the existence of noise will also influence the estimation accuracy. In order to suppress the estimation error caused by noise, the Kalman filter is introduced and combined with PIO for longitudinal force estimation. The logogram of system (15) for Kalman filter design is expressed as

$$\begin{aligned}
\dot{x}_k(t) &= f_k(x_k(t), u_k(t)) + w_{k1}(t), \\
y_k(t) &= h_k(x_k(t), u_k(t)) + w_{k2}(t),
\end{aligned} \tag{19}$$

where $x_k = [\hat{x}_w \quad \hat{d}]^T$, $u_k = [u_w \quad y_w]^T$, $y_k = [i_j \quad w_j]^T$.

The steps of EKF are given by the following.

(1) *Forecasting Process.* Calculate the forecast value

$$\hat{x}_{k+1/k} = \hat{x}_{k/k} + f(\hat{x}_{k/k})T + \frac{A(\hat{x}_{k/k})f(\hat{x}_{k/k})T^2}{2}. \tag{20}$$

Calculate the variance of prediction error

$$P_{k+1/k} = \varphi_k P_{k/k} \varphi_k^T + Q_k, \tag{21}$$

where $A(x_{k/k}) = (\partial f(x)/\partial x)|_{x=\hat{x}_{k/k}}$, $\varphi_k = I + A(\hat{x}_{k/k})T$.

(2) *Trimming Process.* Calculate the matrix of Kalman gain

$$K_{k+1} = P_{k+1/k} H_{k+1}^T (H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1})^{-1}. \tag{22}$$

Update the state estimation

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1} [y_{k+1} - h(\hat{x}_{k+1/k})]. \tag{23}$$

Update the error covariance

$$P_{k+1/k+1} = (I - K_{k+1} H_{k+1}) P_{k+1/k}, \tag{24}$$

where $H = \partial h(x)/\partial x|_{x=\hat{x}(k/k)}$.

3.1.2. Robust Estimation of Sideslip Angle. As we all know, along with the variation of driving manoeuvre and degree of vehicle stability, the estimation deviations will be influenced by unknown disturbances and uncertainties of vehicle model. For (3), considering the uncertainty of tire cornering stiffness, it can be transformed as $F_{yf} = (C_f + \Delta C_f)\alpha_f$, $F_{yr} = (C_r + \Delta C_r)\alpha_r$, where ΔC_f and ΔC_r express the additional nonlinear factors and are considered to be bounded. In order to weaken the influences caused by the above factors, the H_∞ -theory-based estimation method is devised in this work for robust sideslip angle estimation.

The generalized dynamic equation of vehicle model (5) for estimation is written as

$$\begin{aligned}
\dot{x} &= (A + \Delta A)x + Bu + D_1 w_1, \\
y &= Cx + D_2 w_2,
\end{aligned} \tag{25}$$

where $C = [0 \ 1]^T$, D_1 , D_2 are corresponding dimension regular matrixes; w_1 and w_2 represent the unknown disturbances and uncertainties. The estimation of LFO is used as the pseudo measurement of vehicle model to calculate ΔM_z .

Remark 3. Assume that

- (A1) (A, D_1) is controllable and (A, C) is observable;
 (A2) $D_2 D_1^T = 0$, $D_2 D_2^T = I$.

Thus, hypothesis (A1) is the existence condition of the robust observer; hypothesis (A2) is equivalent to the performance target of the two-type optimal control. Define the form of robust observer as $\hat{x} = \phi(y)$, where $\hat{x}$ is the estimation of x . The unbiasedness of estimation means, if there exists arbitrary ε that satisfies the condition $\varepsilon \leq t$, we have $\hat{x}(t) = 0$.

Remark 4. With bounded w_1 and w_2 in (25), the design of robust observer should meet the following condition:

$$J = \sup \left\{ \frac{\|x - \hat{x}\|_2}{\|w_{1,2}\|} \mid w_{1,2} \in L_2(0, \infty), \|w_{1,2}\| \neq 0 \right\} < \lambda, \quad (26)$$

where λ ($\lambda > 0$) is a given constant. This condition means the design of robust observer makes the energy gain of estimation error $e = x - \hat{x}$ less than λ times of w_1 and w_2 to ensure the filter robustness properties.

Remark 5. The robust observer for sideslip angle estimation is proposed as

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}), \quad (27)$$

where K is the feedback gain matrix. Combining (25) with (27), we have

$$\dot{e} = (A - KC)e + (D_1 - KD_2)w_{1,2}. \quad (28)$$

Notice that the transfer function between $w_{1,2}$ and e can be given by

$$G_{we}(s) = \begin{bmatrix} A - KC & D_1 - KD_2 \\ I & 0 \end{bmatrix}. \quad (29)$$

Therefore, performance target of robust estimation can be written as

$$J = \|G_{we}\|_{\infty} < \lambda. \quad (30)$$

Lemma 6. Defining the strict regular rational transfer functional matrix as

$$G(s) = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \quad (31)$$

the corresponding Hamiltonian matrix of $G(s)$ is expressed as

$$H_G \stackrel{\text{def}}{=} \begin{bmatrix} A & \lambda^{-2} B B^T \\ -C^T C & -A A^T \end{bmatrix}. \quad (32)$$

With regard to system (31), if matrix A is stable, thus the necessary and sufficient condition for $\|G\|_{\infty} < \lambda$ can be expressed as follows: $H_G \in \text{dom}(\text{Ric})$, $H = \text{Ric}(H_G) \geq 0$, where H is the solution of matrix Riccati equation (introduced in following manuscript).

Lemma 7. With regard to the two Riccati equations

$$A^T H + HA - HRH + N_1 = 0, \quad (33a)$$

$$A^T H + HA - HRH + N_2 = 0, \quad (33b)$$

where $R = R^T$, $Q_1 = Q_1^T \geq Q_2 = Q_2^T \geq 0$, and $(A, Q_2^{1/2})$ is observable, if (A, R) is steady, that is to say, there exists the solution $H_1 > 0$ of (33a), then there must exist a solution H_2 of (33b) to ensure $H_1 \geq H_2 > 0$, $\text{Re } \lambda(A - RH_2) < 0$.

Theorem 8. With regard to the system with disturbances in (25), assuming conditions (A1) and (A2) are satisfied, there exists unbiased estimator to make the H_{∞} norm of transfer function matrix less than λ . The necessary and sufficient condition of previous description can be stated as follows: if there exists a matrix $H = H^T > 0$, the establishment-condition of following Riccati equation is satisfied:

$$AH + HA^T + H(\lambda^{-2} - C^T C)H + D_1 D_1^T = 0, \quad (34)$$

where the feedback gain matrix is expressed as $K = HC^T$.

Proof. Defining $Q = H^{-1} > 0$, the premultiplication and postmultiplication to (33a) and (33b) are implemented by matrix Q . It can be derived as

$$QA + A^T Q + (\lambda^{-2} - C^T C) + QD_1 D_1^T Q = 0. \quad (35)$$

Equation (35) can be converted as

$$\begin{aligned} & Q(A - Q^{-1}C^T C) + (A - Q^{-1}C^T C)^T Q + \lambda^{-2} \\ & + Q(D_1 - Q^{-1}C^T D_2)(D_1 - Q^{-1}C^T D_2)^T Q \\ & = 0. \end{aligned} \quad (36)$$

The Hamiltonian matrix of (36) is expressed as

$$H_1 = \begin{bmatrix} A - Q^{-1}C^T C & (D_1 - Q^{-1}C^T D_2)(D_1 - Q^{-1}C^T D_2)^T \\ -\lambda^{-2} & -(A - Q^{-1}C^T C)^T \end{bmatrix}. \quad (37)$$

There exists the solution of the Riccati equation; according to Lemma 6, it can be noticed that

$$\left\| \begin{bmatrix} A - Q^{-1}C^T C & D_1 - Q^{-1}C^T D_2 \\ I & 0 \end{bmatrix} \right\|_{\infty} < \lambda, \quad (38)$$

where $A - Q^{-1}C^T C$ is stable. Inequality (37) can be rewritten as

$$\|G_{we}\|_{\infty} = \left\| \begin{bmatrix} A - KC & D_1 - KD_2 \\ I & 0 \end{bmatrix} \right\|_{\infty} < \lambda. \quad (39)$$

The sufficiency of H_{∞} robust performance is proved. $\square$

Owing to the existence of observer gain in (27), the $H\infty$ performance target of robust observer is guaranteed. Referring to Lemma 6, it can be affirmed that the following matrix Riccati equation is tenable.

$$H(A - KC) + (A - KC)^T H + \lambda^{-2} H(D_1 - KD_2)(D_1 - KD_2)^T H + I = 0. \quad (40)$$

On the basis of assumption (A1) and Lemma 1, one can know that $H = H^T > 0$. Defining $Q = H^{-1} > 0$, the premultiplication and postmultiplication to (40) are implemented by matrix Q . It can be derived as

$$Q(A - KC)^T + (A - KC)Q + \lambda^{-2} (D_1 - KD_2)(D_1 - KD_2)^T + QQ = 0. \quad (41)$$

Equation (41) can be converted as

$$\lambda^2 QA^T + \lambda^2 AQ + \lambda^2 Q(I - \lambda^2 C^T C)Q + D_1 D_1^T + (K - \lambda^2 QC^T)(K - \lambda^2 QC^T)^T = 0. \quad (42)$$

On the basis of assumption (A1) and Lemma 7, one can know that there must exist a solution H ($\lambda^2 Q \geq H = H^T > 0$) to ensure the following matrix Riccati equation is tenable:

$$HA^T + AH + H(\lambda^{-2} - C^T C)H + D_1 D_1^T = 0. \quad (43)$$

The necessity of $H\infty$ robust performance is proved. The estimated sideslip angle is denoted as β_H .

3.2. Kinematics-Based Vehicle State Observer Design. The observation equation of wheel speed coupling relationship system in (10) is written as

$$Y = H\xi + V, \quad (44)$$

where Y is the observation vector, ξ is the state to be estimated, H is the measurement matrix, and V is the zero mean white noise sequence. The corresponding vector and matrix are represented as

$$Y = [\omega_1 \ \omega_2 \ \omega_3 \ \omega_4]^T, \quad (45)$$

$$H = \begin{bmatrix} \frac{\cos \delta}{r} & \frac{\sin \delta}{r} & \frac{b_f \cos \delta + l_f \sin \delta}{r} \\ \frac{\cos \delta}{r} & \frac{\sin \delta}{r} & \frac{-b_f \cos \delta + l_f \sin \delta}{r} \\ \frac{1}{r} & 0 & \frac{b_r}{r} \\ \frac{1}{r} & 0 & \frac{-b_r}{r} \end{bmatrix},$$

$$\xi = [v_x \ v_y \ \gamma]^T.$$

According to the characteristics of formula (12), it is known that the least square method (RLS) can be used to estimate the vehicle state like longitudinal speed, lateral

speed, and yaw rate. However, in the iteration process, with the increase of data, the least square method will show the phenomenon of data saturation; that is to say, as time goes on, more and more data are collected, and the information provided by the new data is submerged in the ocean of old data. If the estimation algorithm gives the same degree of trust to the old and new data, it will lose the ability of correction. To overcome the phenomenon of data saturation and to track the change of time-varying parameters, the least square method with forgetting factor is applied to vehicle state estimation, and the recursive algorithm of least square method is given by

$$\begin{aligned} \hat{\xi}(k) &= \hat{\xi}(k-1) + K(k)(Y(k) - H(k)\hat{\xi}(k-1)), \\ K(k) &= \frac{P(k-1)H(k)}{\rho + H^T(k)P(k-1)H(k)}, \end{aligned} \quad (46)$$

$$P(k) = \frac{1}{\rho} (I - K(k)H^T(k))P(k-1),$$

where $K(k)$ is the gain matrix, $P(k)$ is the covariance matrix, and ρ ($0 < \rho \leq 1$) is the forgetting factor used to balance the fast-tracking ability and resisting-disturbance capacity of the estimation results. The estimated vehicle longitudinal speed, lateral speed, and yaw rate are denoted as v_{xR} , v_{yR} , and γ_R , respectively.

3.3. Fusion Estimation Strategy. The robust sideslip angle observer in Section 3.1.2 is designed on the basis of linear tire model; it can get accurate estimations in conventional working condition. Although the uncertainty is considered in the observer design process, in some complex conditions, such as when the tire works in strong nonlinear region, the estimation results will deviate to a certain extent. The least square method with forgetting factor in Section 3.2 improves the reliability of vehicle state estimation results in addition to inheriting the advantages of the least square algorithm. But this method relies on accurate wheel speed measurement, and the estimation effect will be affected when the longitudinal slip rate of tire is too large. Consequently, we can improve the accuracy and reliability of vehicle state estimation by the error iteration and information fusion between the multimodel-based observers. The proposed vehicle state fusion estimation strategy is shown in Figure 3.

In (10), theoretically, ω_1 , ω_2 , ω_3 , and ω_4 show the ideal wheel speed, and it is impossible for ideal wheel speed to totally equal the actual wheel speed. Moreover, if matrix H is an ill-conditioned matrix, little variation may cause the deviation to the solution of RLS. Therefore, considering the deviation caused by the uncertainty of the model in (10), an error compensation mechanism for longitudinal and lateral vehicle speed estimation is developed by PID controller. The estimation error of yaw rate between the real yaw rate and γ_R is defined as $\Delta\gamma = \gamma_R - \gamma$ and used as the input of the PID1 and PID2, the controllers PID1 and PID2 output the corresponding longitudinal and lateral vehicle speed by

FIGURE 3: Vehicle sideslip angle fusion estimation strategy.

FIGURE 4: Degree of membership functions of λ_0 , $|\beta|$, and k_m .

error compensation, respectively, and they can be expressed as follows (denoted as v_{xC} and v_{yC}):

$$\begin{aligned} v_{xC} &= v_{xR} + k_{PID1} \Delta \gamma, \\ v_{yC} &= v_{yR} + k_{PID2} \Delta \gamma, \end{aligned} \quad (47)$$

where k_{PID1} and k_{PID2} are the compensation coefficient of controllers PID1 and PID2. Then the kinematics-based sideslip angle estimation is obtained as $\beta_k = \arctan(v_{xC}/v_{yC})$.

The longitudinal slip rate of tire can be calculated by

$$\lambda_j = \frac{\omega_j r - v_x}{v_x} \quad (j = 1, 2, 3, 4). \quad (48)$$

In this section, the fuzzy controller is designed to fuse the results of kinematics-based sideslip angle estimation and robust sideslip angle estimation. According to the above analysis, the absolute values of sideslip angle $|\beta|$ and $\lambda_0 = |\lambda_1| + |\lambda_2| + |\lambda_3| + |\lambda_4|$ are chosen as the inputs of fuzzy controller, in which $|\beta|$ and λ_0 are used to embody the confidence level of the estimation results of β_k and β_H . When $|\beta|$ is excessively large, the lateral offset of the tire increases and the nonlinear characteristic is enhanced; then it is necessary to reduce the weight of dynamics-based sideslip angle estimation result. When λ_0 is excessively large, it shows that the longitudinal slip is more severe; then it is

TABLE 1: Fuzzy control rules.

k_m		$ \beta $			
		S	M	L	H
λ_0	S	S	M	M	L
	M	M	L	L	H
	L	L	H	H	H
	H	L	H	H	H

necessary to reduce the weight of kinematics-based sideslip angle estimation result. The fuzzy output k_m is calculated by fuzzy controller and used as the fusion weight coefficient between β_k and β_H , and the fusion estimation results of sideslip angle are expressed as

$$\beta = k_m \beta_k + (1 - k_m) \beta_H. \quad (49)$$

The membership functions of the input and output in fuzzy controller are shown in Figure 4. The fuzzy control rules are listed in Table 1.

Synchronously, the fusion estimation result of sideslip angle β is regarded as the pseudo measurement value and used as the input of fuzzy controller; the compensation estimation result of longitudinal vehicle speed v_{xC} is used as the input of slip rate observer. Thus, both β and v_{xC} provide

TABLE 2: Parameters of vehicle and in-wheel motors.

Symbol	Parameters	Value and units
m	Vehicle mass	710 kg
r	Effective radius of wheel	0.245 m
l_f	Distances from vehicle gravity center to the front axle	0.795 m
l_r	Distances from vehicle gravity center to the rear axle	0.975 m
b_f, b_r	Half treads of the front (rear) wheels	0.775 m
C_f	Equivalent cornering stiffness of front wheel	60000 N/rad
C_r	Equivalent cornering stiffness of rear wheel	40000 N/rad
I_z	Moment of inertia	1000 kg·m ²
R	Equivalent resistance of winding	0.688 Ω
K_a	Inverse electromotive force coefficient	0.06 Nm/A
K_t	Motor torque constant	11.43 Nm/A
J	Sum of inertia moment of wheel and motor	7.143 kg·m ²
b	Damping coefficient	0.643 Nm·sec/rad
L	Equivalent inductance of winding	0.125 H

the judgement datum for next-step iterative estimation; then the reliability, anti-interference ability, and self-adaptability in multiple working conditions of presented observer system are improved by the error iteration compensation between the multimodel-based observers.

4. Simulation Results

To validate the effectiveness of the designed LFO and proposed vehicle sideslip angle fusion estimation strategy in this work, the simulations are carried out on a high-fidelity CarSim-Simulink joint simulation platform. CarSim is used to provide the whole vehicle model; the estimation of longitudinal forces and sideslip angle is achieved in Matlab/Simulink. The parameters of vehicle and in-wheel motors are listed in Table 2. In simulation and following road test, the uncertainty of tire stiffness is assumed as $-0.3C_f \leq \Delta C_f \leq 0.3C_f$, $-0.3C_r \leq \Delta C_r \leq 0.3C_r$. In the robust sideslip angle estimator design, in order to adjust to the constraint conditions of (A1) and (A2), system (25) is expanded in the form of

$$\begin{aligned} \dot{x} &= Ax + Bu + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \end{aligned} \quad (50)$$

4.1. Double Lane Changes (DLC) Manoeuvre. In this case, the DLC manoeuvre, as shown in Figure 5, is carried out. In simulation, the road friction coefficient is set to be 0.6. The vehicle speed maintains a constant of 20 m/s.

The designs of LFO for wheels 1, 2, 3, and 4 are implemented, respectively, and the estimation results of longitudinal forces are shown in Figure 6. It can be found that the designed LFOs can estimate the longitudinal force accurately with the measurements of currents, speeds, and voltages. The estimation effectiveness of LFOs integrated with PIO and

FIGURE 5: DLC manoeuvre.

EKF is better than the one only based on PIO; it means that the design of PIO makes it possible to estimate the unknown input of system (longitudinal forces), and the coalition of EKF reduces the influences of noise and guarantees the estimation accuracy. That is to say, the estimated values of LFOs are reliable to substitute the measurements of sensors as the inputs of robust sideslip angle estimator. In DLC manoeuvre, the vehicle speed is set to be relatively larger, and the vehicle actuating manoeuvre changes violently at the beginning; accordingly there was a certain degree tremble in longitudinal forces, but the proposed LFO still can track the real longitudinal force in real time.

The sideslip angle estimation is shown in Figure 7. It is known that both the robust observer and the RLS with compensation can estimate the vehicle sideslip angle in real time. In the DLC manoeuvre with high speed and low road adhesion, the tire longitudinal slip has a certain influence on the RLS estimation result, and the estimation accuracy of RLS is lower than that of robust observer. And the estimation accuracy of sideslip angle can be effectively improved by using the weighted iterative fusion estimation method. In order to further show the accuracy of proposed estimation method, the root-mean-square (RMS) error E_{RMS} between real vehicle state and estimated value is used for the

FIGURE 6: Estimation of longitudinal forces.

FIGURE 7: Estimation of sideslip angle.

quantitative evaluation and can be computed by the following equation:

$$E_{\text{RMS}} = \sqrt{\frac{1}{N_s} \sum_{i=1}^{N_s} (\hat{x}_i - x_i)^2}, \quad (51)$$

where N_s is the number of samples; x_i and $\hat{x}_i$ denote the measured and estimated vehicle state at the i th sample. The comparison of E_{RMS} of vehicle state estimation in DLC manoeuvre simulation is shown in Table 3. In table, the sideslip angle estimation method of robust observer, RLS with compensation, and fusion observer are abbreviated as RO,

RWC and FO. It can be found that E_{RMS} of proposed longitudinal force estimation method and fusion method are less than that of other estimation methods, which indicates that the proposed method improves the accuracy and reliability of longitudinal force and sideslip angle estimation.

4.2. J-Turn Manoeuvre. In the simulation of J-turn manoeuvre, the road friction coefficient is set to be 1.0; in addition, considering the time-varying characteristic of vehicle speed, a sine wave with the amplitude of 1 and frequency of 1 rad/s is superimposed on the base speed of 10 m/s. The corresponding vehicle speed and steering wheel angle in simulation are shown in Figure 8.

The estimation results of longitudinal forces are shown in Figure 9. Similar to the estimation results in DLC manoeuvre, it can be found that the estimation results of longitudinal force based on PIO-EKF show better estimation ability.

As shown in Figure 10, under the rapid steering conditions, the proposed fusion method can still maintain good estimation performance. In the J-turn manoeuvre, owing to the fast-changing steering wheel angle and the time-varying vehicle speed, the vehicle sideslip angle is relatively larger; in this case, the estimation accuracy of the robust observer is lower than RLS with compensation. The presented fusion estimation strategy can integrate the advantages of the two observers effectively, ensure the estimation accuracy, and simultaneously improve the anti-interference performance of estimation system. Moreover, it can be found that the fusion estimation strategy enhances the adaptability and reliability of the estimation system in multiple conditions. Similarly,

TABLE 3: Comparison of E_{RMS} in DLC manoeuvre simulation.

E_{RMS}	Observer				
	Longitudinal force observer		Sideslip angle observer		
	PIO	PIO-EKF	RO	RWC	FO
Longitudinal force F_{x1}	0.4736	0.3284	×	×	×
Longitudinal force F_{x2}	0.4654	0.3948	×	×	×
Longitudinal force F_{x3}	0.4945	0.3166	×	×	×
Longitudinal force F_{x4}	0.4835	0.3154	×	×	×
Sideslip angle	×	×	0.2091	0.2485	0.1714

FIGURE 8: Vehicle speed and steering wheel angle.

FIGURE 9: Estimation of longitudinal forces.

TABLE 4: Comparison of E_{RMS} in J-turn manoeuvre simulation.

E_{RMS}	Observer				
	Longitudinal force observer		Sideslip angle observer		
	PIO	PIO-EKF	RO	RWC	FO
Longitudinal force F_{x1}	0.7678	0.3346	×	×	×
Longitudinal force F_{x2}	0.7144	0.3703	×	×	×
Longitudinal force F_{x3}	0.7513	0.3111	×	×	×
Longitudinal force F_{x4}	0.7098	0.3217	×	×	×
Sideslip angle	×	×	0.4136	0.2877	0.2309

FIGURE 10: Estimation of sideslip angle.

as listed in Table 4, E_{RMS} in J-turn manoeuvre simulation was computed for further verification of the superiority of the proposed method in improving the estimation accuracy of vehicle state. And the presented longitudinal force and sideslip angle estimation method still maintain better estimation performance.

5. Experimental Results

In this section, the experiment is executed for further validation about the estimation performance of proposed estimation method. At present, the sensor for direct longitudinal force measurement is still unavailable on our four-wheel independent drive electric vehicle, so the verification of LFO is implemented on the chassis dynamometer bench. The robust estimator is validated by the experimental data from road test, in which the estimated longitudinal forces by LFOs are regarded and used as the credible pseudo measurements of the fusion estimation system.

5.1. Test on Chassis Dynamometer Bench. The vehicle test on chassis dynamometer bench is shown in Figure 11. The experimental vehicle is a four-wheel independent drive electric vehicle which is refitted from a single-motor drive electric vehicle and actuated by four in-wheel motors. The whole vehicle control system is achieved on a rapid prototyping platform. The currents, speeds, and voltages of in-wheel motors were measured with corresponding sensors and recorded by the host computer through CAN bus. The longitudinal

FIGURE 11: Vehicle test on chassis dynamometer bench.

FIGURE 12: Estimation of longitudinal forces.

forces were recorded by the data acquisition system of chassis dynamometer.

In this work, the experimental data of front-right electric driving wheel is selected to testify the proposed LFO; the experimental result is shown in Figure 12. It can be found that there exists some estimation error during the process of acceleration, but the error is finite. The LFO can track the longitudinal force with high accuracy after 4 s when the driving condition approaches the manoeuvre of uniform speed.

5.2. Road Test. Figure 13(a) shows the photo of vehicle road test. As shown in Figure 13(b), the road test is implemented in the form of serpentine trajectory, 10 traffic piles are placed as

FIGURE 13: Road test.

FIGURE 14: Estimation of sideslip angle.

the obstacle, and the vehicle speed cruising is regulated well by a speed controller. The corresponding steering wheel angle and vehicle speed measured in road test are shown in Figures 13(c) and 13(d), respectively. Figure 14 shows the estimation results of sideslip angle. From Figure 14, it can be seen that forgetting-factor-based RLS algorithm with compensation can track the changing trend of sideslip angle in real time; in addition, by contrasting the sideslip angle estimation results, there exists some fluctuation in the estimation of RLS. The reason is that the magnitude of sideslip angle is far less than

wheel speed, and small disturbances in wheel speeds may lead to relatively large estimation fluctuations. Even so, the forgetting-factor-based RLS algorithm with compensation is still able to maintain better estimation results, and the range of volatility is relatively acceptable. The robust observer has good estimation accuracy and stability, but the robust observer is designed on the basis of dynamic model, so the real time tracking ability of which is weaker than RLS. The fusion estimation strategy can synthesize the estimated results of the two kinds of observers and obtain optimal sideslip angle estimation. In order to further validate the accuracy of the proposed estimation method in practice, the comparisons of E_{RMS} in chassis dynamometer bench test and road test are shown in Table 5. Notice that the effect of the estimation method is satisfactory in actual vehicle application.

6. Conclusion

This work presented a novel longitudinal force and sideslip angle fusion estimation strategy for 4WID-EVs. The EDWM was established and introduced into the longitudinal force estimation. Considering the longitudinal force is the unknown input of EDWM, the PIO was used to restructure the differential equation of longitudinal force, and the EKF was applied to restrain the influence of noise and improve the accuracy of longitudinal force estimation. Based on

TABLE 5: Comparison of E_{RMS} in experiment.

E_{RMS}	Observer				
	Longitudinal force observer		Sideslip angle observer		
	PIO	PIO-EKF	RO	RWC	FO
Longitudinal force	1.9374	1.1007	×	×	×
Sideslip angle	×	×	0.1966	0.1379	0.0743

the longitudinal force estimation, we proposed a robust sideslip angle estimation method with the consideration of the unknown disturbances and uncertainties of vehicle model, and the theoretical proof of robust estimator was provided. Moreover, a kinematics-based vehicle state observer was developed in which the forgetting-factor-based RLS algorithm with error compensation was applied for sideslip angle estimation. Then fusion estimation strategy was presented for more accurate estimation of sideslip angle by the error compensation and iteration between the multimodel-based observers. The simulations of DLC manoeuvre and J-turn manoeuvre were executed, and the results show that the presented LFO can track the longitudinal force in real time, and the fusion estimation strategy has the satisfactory accuracy in estimation. For further validation, the test on chassis dynamometer bench and road test were implemented; the effectiveness of proposed estimation method was verified.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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