Research Article

Reliability Modeling and Estimation of the Gear System

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Traditional reliability models of the gear system either simplify the gear system unreasonably or are too complicated to be applicable for some gear systems. Therefore, a reliability model of the gear system considering statistically dependent failure based on the theory of the order statistics is developed in this paper. Firstly, the gear bending fatigue test of small sample is implemented to obtain the P-S-N curves that are fitted by using the least square method and the linear regression method of the statistical parameters under the test stress levels. Then, according to stress-strength interference theory and the theory of the order statistics, the reliability models considering the number of load cycles from a gear to gear system are built, which indicates clear modeling process of the gear system. Finally, the proposed reliability model of gear system is validated by using the Monte Carlo Simulation (MCS) and the system reliability under special load history is analyzed qualitatively.

1. Introduction

Gear is an important component of the mechanical transmission equipment. Its reliability has the great influence on the system service life. Therefore, scholars have done lots of research works to estimate the gear system reliability. For instance, the simplification that a tooth denotes a gear is widely used for reliability estimation [1–5]. Based on the linear damage theory, a long-term fatigue damage analysis method for the gear bending fatigue of the wind turbine is developed [6, 7]. By the introduction of correlation coefficient, the reliability models of the gear system considering failure-dependence are developed [8–10]. Recently, Xie [11] considered the gear system as multiconfiguration system and built the reliability models considering the tooth-pair meshing sequence based on the stress-strength interference theory.

Though lots of reliability methods of the gear system have been proposed, they either considered the gear as a component or built too complicated models. The methods introducing the correlation coefficient that is calculated experientially may cause the great error. Therefore, building an efficient and applicable reliability model of the gear system is necessary.

The life distribution under the special stress level can be determined by the test. The load-weighted average model for

the fatigue reliability calculation is developed based on the life distribution [12]. Thus, for the random constant amplitude load history, the reliability can be calculated using this model.

Based on this idea, the reliability model of gear system considering statistically dependent failure is developed according to the theory of the order statistics in this paper.

2. Test of Gear Bending Fatigue Life

The gear bending fatigue life is tested by the gear test rig of closed power flow (Figure 1(a)). In the equipment, the failure of gear bending fatigue is monitored by the vibration signal detection sensor. The tested gear is lubricated by the spraying oil below 60°C. The oil is cooled by the water cooling system. The error of cycle counting is between -0.1% and 0.1% [13]. The revolution speed is 2000 rpm. The gear parameters are listed in Table 1. The tested gear and failure mode are shown in Figures 1(b) and 1(c).

The permissible bending stress is calculated in accordance with ISO 6336-3:2006 (5) as follows [14].

$$\sigma_{FP} = \frac{\sigma_{Flim} \sigma_{ST} \sigma_{\delta relT} \sigma_{RrelT} \sigma_X}{S_{Fmin}} = \frac{520 \text{ N/mm}^2 \times 2}{1.5} \quad (1)$$

$$= 693.33 \text{ MPa}$$

FIGURE 1: Test of gear bending fatigue life. (a) Test rig. (b) Tested gear. (c) Tooth root failure.

TABLE 1: Parameters of gear.

Number of teeth	25
Module (mm)	4
Pressure angle (degree)	20
Tooth surface width (mm)	25
Radius of tooth root fillet (mm)	1.52
Precision grade	6
Material	20CrMnTi
Tooth surface hardness	HRC 58-64
Gear core hardness	HRC 32-45

where the factors can be found in ISO gear standards, $\sigma_{Fmin}=520$ MPa is estimated by the GB/T 3840-97 [15], and $S_{Fmin}=1.5$ is recommended by GB/T 3840-93 for the reliability purpose, whose failure probability is less than $1E-4$.

Therefore, 700 MPa is used for the maximum stress level of the gear bending test. Since $\sigma_{Fmin}=520$ MPa, 550 MPa is used for the minimum stress level. 40% of total stress range [13] (i.e., 60 MPa for this test) is recommended for the stress interval between the maximum stress level and secondary stress level. Then, 50 MPa and 40 MPa are used for the secondary and third stress intervals, respectively. So,

700 MPa, 640 MPa, 590 MPa, and 550 MPa are used for the four stress levels.

Not less than 5 samples are recommended for each stress level in GB/T 14230. Thus, the testing method of small sample is used for the test of gear bending fatigue and 5 samples are used for each stress level. The gear bending fatigue lives are shown in Figure 2.

3. Statistical Analysis of Test Data

The distribution function of test data is unknown. In the test of gear bending fatigue, the normal distribution, Log-normal distribution, and two-parameter Weibull distribution are recommended for the test of the life distribution [13] as follows.

$$\begin{aligned}
 P(N) &= \Phi\left(\frac{N - \lambda_N}{\eta_N}\right) \\
 P(N) &= \Phi\left(\frac{\ln N - \lambda_{\ln N}}{\eta_{\ln N}}\right) \\
 P(N) &= 1 - \exp\left[\left(-\frac{N}{\lambda}\right)^\eta\right]
 \end{aligned} \tag{2}$$

where N denotes the tested data; P denotes the fatigue life distribution; λ_N and η_N denote the mean and standard deviation

FIGURE 2: Test data of gear bending fatigue.

of tested data, respectively; $\lambda_{\ln N}$ and $\eta_{\ln N}$ denote the mean and standard deviation of logarithmic data, respectively; λ denotes the scale parameter; η denotes the shape parameter.

For the test of fit goodness, the weighted least square method is used. The linear regression for the three distributions can be expressed uniformly as follows.

$$Y = a + bX \quad (3)$$

where a and b denote, respectively, the intercept and slope of the linear regression function, which are calculated by the weighted least square method (WLSM) [16, 17].

$$\begin{aligned} \omega_i &= \frac{\partial \hat{Y}_i}{\partial \hat{P}_i} \\ b &= \frac{\sum_{i=1}^u \omega_i^2 X_i \hat{Y}_i \cdot \sum_{i=1}^u \omega_i^2 - \sum_{i=1}^u \omega_i^2 \hat{Y}_i \cdot \sum_{i=1}^u \omega_i^2 X_i}{\sum_{i=1}^u \omega_i^2 X_i^2 \cdot \sum_{i=1}^u \omega_i^2 - (\sum_{i=1}^u \omega_i^2 X_i)^2} \\ a &= \frac{\sum_{i=1}^u \omega_i^2 X_i \hat{Y}_i - b \sum_{i=1}^u \omega_i^2 X_i^2}{\sum_{i=1}^u \omega_i^2 X_i} \end{aligned} \quad (4)$$

where i is the ordinal of tested data from small to large; ω_i^2 denotes the weight factor; ω_i^2 , X_i , and Y_i for the three distributions are given in Table 2, whose linear functions are given in Appendix A.

The failure probability $\hat{P}$ is estimated by the empirical failure probability as follows [18].

$$\hat{P}(N) = \frac{i - 0.3}{n + 0.4} \quad (5)$$

where n denotes the number of tested lives under each stress level.

FIGURE 3: Statistical parameters of the log-normal distribution.

The correlation coefficient R^2 is used for the comparisons of fit goodness between the three distributions, which is expressed as follows.

$$R^2 = \frac{\sum_{i=1}^u [\hat{P}(N_i) - P(N_i)]^2}{\sum_{i=1}^u [\hat{P}(N_i) - 1/u \cdot \sum_{i=1}^u \hat{P}(N_i)]^2} \quad (6)$$

By virtue of the least square method, the statistical parameters of the fatigue lives under each stress level are shown in Table 3.

From Table 3, the correlation coefficient R^2 associated with the log-normal distribution is better than other two distributions. When the minimum value of linear correlation coefficient satisfies the three distributions ($R^2 > 0.8$ represents the high correlation), the log-normal distribution with maximum correlation coefficient has the priority to be selected [13].

The studies [19–21] illustrate that the linear relationship exists between the stress level and the mean/std of log-normal distribution with parameters $\hat{\lambda}_{\ln N}$ and $\hat{\eta}_{\ln N}$. Then, the statistical parameters of log-normal distribution can be fitted with 95% confidence level by the least square method, which are shown in Figure 3. $E(s_j)$ and $D(s_j)$ denote, respectively, the mean and variance of log-normal distribution, which can be expressed as

$$\begin{aligned} \ln[E(s_j)] &= 25.1088 - 0.0198s_j \\ \ln\{[D(s_j)]^{-0.5}\} &= 24.8890 - 0.0198s_j \\ E(s_j) &= e^{\lambda(s_j) + \eta(s_j)^2/2} \\ D(s_j) &= (e^{\eta(s_j)^2} - 1)e^{2\lambda(s_j) + \eta(s_j)^2} \end{aligned} \quad (7)$$

TABLE 2: Fitting parameters of the three distributions.

Distribution	X	Y	a	b	ω^2
Normal	N	$\Phi^{-1}[\hat{P}(N)]$	$-\lambda_N/\eta_N$	$1/\eta_N$	1
Lognormal	$\ln N$	$\Phi^{-1}[\hat{P}(N)]$	$-\lambda_{\ln N}/\eta_{\ln N}$	$1/\eta_{\ln N}$	1
Weibull	$\ln N$	$\ln \ln[1-\hat{P}(N)]^{-1}$	$-\eta \ln \lambda$	η	$\{[1-\hat{P}(N)] \ln[1-\hat{P}(N)]\}^{-1}^{-2}$

TABLE 3: Statistical parameters of each stress level.

Stress level	Distribution	$\hat{\lambda}$	$\hat{\eta}$	R^2
s_1	Normal	7.8639e4	5.0025e4	0.9442
	Lognormal	11.1490	0.6680	0.9692
	Weibull	9.1207e4	1.8990	0.9466
s_2	Normal	3.4904e5	1.8556e4	0.9090
	Lognormal	12.6872	0.5061	0.9466
	Weibull	4.0993e5	2.6560	0.8561
s_3	Normal	6.2751e5	3.4650e5	0.9562
	Lognormal	13.2488	0.6671	0.9078
	Weibull	7.4114e5	1.7708	0.9376
s_4	Normal	1.5802e6	9.7182e5	0.9369
	Lognormal	14.1547	0.6502	0.9582
	Weibull	1.8278e6	1.9524	0.9387

Then, the parameters $\lambda_{\ln N}$, $\eta_{\ln N}$ and probability density function (pdf) can be expressed as follows.

$$\begin{aligned}
 \lambda_{\ln N}(s_j) &= \ln[E(s_j)] - \frac{1}{2} \\
 &\quad \cdot \ln \left[1 + \frac{D(s_j)}{D(s_j) + [E(s_j)]^2} \right] \\
 \eta_{\ln N}(s_j) &= \sqrt{\ln \left[1 + \frac{D(s_j)}{D(s_j) + [E(s_j)]^2} \right]} \\
 f(N | s_j) &= \frac{1}{N \cdot \eta_{\ln N}(s_j) \cdot \sqrt{2\pi}} \\
 &\quad \cdot \exp \left[-\frac{1}{2} \left[\frac{\ln N - \lambda_{\ln N}(s_j)}{\eta_{\ln N}(s_j)} \right]^2 \right]
 \end{aligned} \quad (8)$$

The parameters $\lambda_{\ln N}$ and $\eta_{\ln N}$ of each stress level calculated by using (7) and (8) are listed in Table 4.

The logarithmic fatigue life $\ln N_{P,s_j}$ under each stress level with the failure probability P can be calculated by (9) [22]. Then, P-S-N curves can be fitted by the least square method, which are shown in Figure 4.

$$\ln N_{P,s_j} = \lambda_{s_j} - k_{(P,1-\alpha,v)} \eta_{s_j} \quad (9)$$

where $k_{(P,1-\alpha,v)}$ is the one-sided tolerance coefficient with a failure probability P , a confidence level of $1-\alpha$, and degrees of freedom v [22].

Shown in Figure 4 are the median S-N curve and the P-S-N curves of gear bending fatigue with the confidence level

FIGURE 4: P-S-N curves of gear bending fatigue.

of 95%, failure probability 10% that is fitted by the proposed regression method, and Ref. [23] method that shares the fatigue lives under the different stress levels and fits the P-S-N curves of small sample based on the principle of sample polymerization (detailed method is shown in Appendix B). Both methods provide the different life dispersions under different stress levels, which is different from ISO 12107:2012 method that has the same slope. The relative error of the slopes for the P-S-N curves of C95P10 obtained by using the proposed method and Ref. [23] method is 2.08%. This illustrates that the proposed method is applicable to the fitting of P-S-N curves.

TABLE 4: Statistical regression parameters of the four stress levels.

Stress level	s_1	s_2	s_3	s_4
$\lambda_{\ln N}$	11.3286	12.4957	13.4683	14.2463
$\eta_{\ln N}$	0.5103	0.5159	0.5207	0.5244

4. Gear System Reliability Model

4.1. Reliability of a Gear Tooth. During the operation of the aircraft engine or wind turbine, the gear is subjected to the torque load, which can be described using a variable amplitude load history. For any meshing tooth-pair, the load can be considered as random variable.

For a simple case, under constant amplitude load history T_γ , the reliability of a gear tooth, whose bending fatigue life is greater than the special life N^* , can be expressed as

$$R(N^*) = \int_{N^*}^{+\infty} f[N | s(T_\gamma)] dN \quad (10)$$

where $f[N | s(T_\gamma)]$ denotes the pdf of gear bending fatigue life under the stress level s , which is the function of torque load T_γ .

If the occurrence probability of load T_γ is p_γ ($\gamma=1,2,\dots,m$, $\sum_{\gamma=1}^m p_\gamma = 1$), then, the reliability of a gear tooth bending fatigue life can be expressed as

$$R(N^*) = \sum_{\gamma=1}^m p_\gamma \int_{N^*}^{+\infty} f[N | s(T_\gamma)] dN \quad (11)$$

If the torque load T_γ is a random variable and can be described as the pdf: $f(T_\gamma)$, then p_γ can be expressed as

$$p_\gamma \approx f(T_\gamma) \Delta T_\gamma \quad (12)$$

where ΔT_γ is the load discrete interval.

Then, (11) can be expressed as

$$R(N^*) = \sum_{\gamma=1}^m f(T_\gamma) \Delta T_\gamma \int_{N^*}^{+\infty} f[N | s(T_\gamma)] dN \quad (13)$$

Let ΔT_γ tend to zero and m tend to infinity; (13) can be expressed as

$$R(N^*) = \int_0^{+\infty} f(T_\gamma) \int_{N^*}^{+\infty} f[N | s(T_\gamma)] dN dT \quad (14)$$

4.2. Reliability of the Gear System. For the gear with z teeth, the strengths of its teeth are independent and identically distributed (iid) random variables [11]. With respect to a gear, a tooth failure can cause the failure of the gear. Therefore, a gear can be considered as a series system composed of its teeth. Then, the reliability of a gear with z teeth using the traditional method can be expressed as

$$R_{\text{gear}}(N^*) = \left\{ \int_0^{+\infty} f(T_\gamma) \int_{s(T_\gamma)}^{+\infty} f[S | N^*] dS dT \right\}^z \quad (15)$$

where $f(S | N^*)$ denotes the pdf of strength S under the special life N^* .

The scholars [24, 25] have proved that the probability that the fatigue life N is less than the special life N^* under the stress s^* equal to the probability that the strength S is less than s^* under the special life N^* (i.e., $P(N < N^* | s^*) = P(S < s^* | N^*)$). Then, (15) can be expressed as

$$R_{\text{gear-1}}(N^*) = \left\{ \int_0^{+\infty} f(T_\gamma) \int_{N^*}^{+\infty} f[N | s(T_\gamma)] dN dT \right\}^z \quad (16)$$

It can be seen that (16) is the z square of (14). It considers that the failure of the gear tooth is statistically independent. Then, the reliability of gear system with δ gears can be expressed as

$$R_{\text{sys-1}}(N_e^*) = \prod_{e=1}^{\delta} R_{\text{gear-1}}(N^*) = \prod_{e=1}^{\delta} \left\{ \int_0^{+\infty} f(T_\gamma) \int_{N_e^*}^{+\infty} f[N | s_e(T_\gamma)] dN dT \right\}^{z_e} \quad (17)$$

where $e=1, 2, 3, \dots, \delta$ denotes the Ids of gears.

Eq. (17) is established from the viewpoint of statistically independent failure. Actually, a gear failure depends on the weakest component (tooth) in the viewpoint of probability. Then, for the gear with z teeth, according to the definition of order statistics, the distribution of the fatigue life equals the distribution of the minimum order statistics of its teeth lives. Therefore, for the z samples (teeth), the pdf and reliability of the gear fatigue life can be expressed as (18) and (19), respectively.

$$g[N | s(T_\gamma)] = z \cdot \{1 - F[N | s(T_\gamma)]\}^{z-1} \cdot f[N | s(T_\gamma)] \quad (18)$$

$$R_{\text{gear-2}}(N^*) = \int_0^{+\infty} f(T_\gamma) \int_{N^*}^{+\infty} g[N | s(T_\gamma)] dN dT \quad (19)$$

Due to the gear system subjected to the common input torque load, the failures of these gears are statistically dependent. Then, the reliability of the gear system with δ gears can be expressed as

$$R_{\text{sys-2}}(N_e^*) = \int_0^{+\infty} f(T_\gamma) \prod_{e=1}^{\delta} \int_{N_e^*}^{+\infty} z_e$$

TABLE 5: Parameters of gears.

	Sun	Planetary	Ring	Gear-1	Gear-2	Gear-3	Gear-4
Tooth Numbers	30	57	144	81	25	46	29
Normal module (mm)	12	12	12	14	14	22	22
Pressure angle (degree)	20	20	20	20	20	20	20
Helix angle (degree)	15	15	15	15	15	15	15
Face width (mm)	220	220	220	110	110	50	50
Profile shift coefficient	-0.33	0.38	-0.43	0.52	0.44	0.36	-0.23

$$\begin{aligned}
& \bullet \left\{ 1 - F \left[N \mid s_e(T_\gamma) \right] \right\}^{Z_e - 1} \\
& \bullet f \left[N \mid s_e(T_\gamma) \right] dN dT
\end{aligned} \quad (20)$$

Eqs. (17) and (20) are two kinds of gear reliability models which are developed from two viewpoints. The proposed model (i.e., (20)) considers the statistically dependent failure between gears.

5. Reliability Analysis of a Planetary Gear System under Variable Amplitude Load History

The topology of a planetary gear system is shown in Figure 5, which includes three planetary gears. The parameters of all gears are listed in Table 5.

The stress of tooth root is calculated as [14]

$$s = \frac{F_t}{b \cdot m_n} Y_F Y_S Y_\beta Y_D T K_A K_V K_{F\beta} K_{Fa} K_\gamma \quad (21)$$

where the variables and factors can be found in the ISO gear standards [14, 15].

The cycles of each gear are different. The relationship between them can be described using the gear ratio. Thus, cycles of each gear can be determined as

$$\begin{aligned}
n_s &= n_1 = (1 + i_{z_r:z_s}) n_c - i_{z_r:z_s} n_r \\
n_2 &= n_3 = i_{z_1:z_2} n_1 \\
n_4 &= i_{z_3:z_4} n_3
\end{aligned} \quad (22)$$

where n_s , n_r , n_c , n_1 , n_2 , n_3 , and n_4 denote, respectively, the cycles of the sun gear, ring gear, planetary carrier, gear-1, gear-2, gear-3, and gear-4. $i_{z_r:z_s}$ denotes the ratio of the tooth numbers between the ring gear and sun gear. $i_{z_1:z_2}$ denotes the ratio of the tooth numbers between the gear-1 and gear-2. $i_{z_3:z_4}$ denotes the ratio of the tooth numbers between the gear-3 and gear-4.

In order to validate the reliability model, MCS (Monte Carlo Simulation) is used for the reliability analysis. In the MCS, each tooth is given randomly an S-N curve from the P-S-N curves of gear fatigue life. The MCS procedure is shown in Figure 6. It is assumed that the torque history of the input shaft follows the normal distribution, $T \sim N(800, 50^2)$. Two kinds of methods of (17) and (20) are used for the reliability

FIGURE 5: Planetary gear system.

TABLE 6: Statistical parameters of the torque loads.

Torque loads	Mean	Std
Load 1	750	50
Load 2	800	50
Load 3	850	50

estimation. Eq. (17) is denoted by the traditional method that denotes the reliability model of independent failure between the gears and (20) is denoted by the proposed method. The system reliability is shown in Figure 7.

From Figure 7, the reliability obtained by the proposed method is in good agreement with that of the MCS. The traditional method underestimates the reliability of the gear system. It considers that the failures of each tooth and gear are statistically independent, which may cause the incorrect results.

The proposed dynamic reliability model (i.e., (20)) is built based on the P-S-N curves and load distribution. Because the P-S-N curves are determined by the test, the load dispersions also have the influences on the system reliability, which will be analyzed in the next section. Two cases are used for the influence analyses.

Case 1. The torque load distributions with different means are listed in Table 6. The estimation results with different means are shown in Figure 8.

Case 2. The torque distributions with different standard deviations are listed in Table 7. The estimation results with different standard deviations are shown in Figure 9.

FIGURE 6: Procedure of the Monte Carlo Simulation.

FIGURE 7: Comparison between three methods.

From Figure 8, the fatigue life decreases with the increase of the mean of the torque distribution, which reflects the correct fatigue failure law. From Figure 9, at the early stage of the fatigue life, the reliability of the gear system with a large dispersion of the torque load is low. At the later stage of the fatigue life, a large dispersion of the torque load leads to a high reliability of the gear system. When the mean value is constant, a large dispersion of the torque load leads to a

FIGURE 8: Reliability under different means of load.

FIGURE 9: Reliability under different Stds of load.

TABLE 7: Statistical parameters of the torque loads.

Torque loads	Mean	Std
Load 1	800	50
Load 2	800	70
Load 3	800	90

large dispersion of the fatigue life, which increases the system reliability at the later stage.

6. Conclusions

The proposed dynamic reliability model is established based on the P-S-N curves rather than the initial strength of component, which avoids the difficulties of determining the initial strength and strength degradation function [26]. The failure relationship between the teeth of a gear is expressed by the

order statistics, which simplifies the complicated modeling process considering the statistically dependent failure [11] based on the stress intensity interference theory. The main conclusions are as follows.

(1) To obtain the accurate P-N-S curves, the fitting method of P-S-N curves based on the LSM and regression functions of the statistical parameters is proposed. The fitting results obtained by the proposed method and Ref. [23] method show the good agreement.

(2) The modeling method from a component (tooth) to the system (gear system) is used for the reliability modeling of the gear system, which indicates clear modeling process of the gear system. According to the principle of the system failure starting from the weakest component and the common cause failure, the reliability model of the gear system is built based on the theory of the order statistics.

(3) Through comparison with MCS, the reliability estimated by the proposed model is in good agreement with that of the MCS. By using the proposed model, the influences of the load dispersion on the system reliability at different service life stages are analyzed.

Appendix

A.

Linear regression functions can be expressed as follows:

(1) Normal distribution:

$$\Phi [P(N)]^{-1} = \frac{1}{\eta_N} (N - \lambda_N) \quad (\text{A.1})$$

(2) Log-normal distribution:

$$\Phi [P(N)]^{-1} = \frac{1}{\eta_{\ln N}} (\ln N - \lambda_{\ln N}) \quad (\text{A.2})$$

(3) Weibull distribution:

$$\ln \ln \frac{1}{1 - P(N)} = \eta (\ln N - \ln \lambda) \quad (\text{A.3})$$

B.

Taking the tested fatigue lives with 5-5-5-5 samples, for example (4 stress levels with 5 samples of each stress level are used for the test), the procedures of estimating η_{s_j} and k are as follows.

- (1) The fatigue lives tested under the four stress levels should range from 10^4 to 10^7 .
- (2) The logarithmic life mean under each stress level can be estimated by (B.1) and the median S-N curve can be fitted by the least square method. Then, the mean value of the fatigue lives under each stress level can be calculated by the median S-N curve: $\lambda_{s_1}, \lambda_{s_2}, \lambda_{s_3}, \lambda_{s_4}$.

$$\lambda_{s_j} = \frac{1}{m_{s_j}} \sum_{i=1}^{m_{s_j}} \ln N_{i,s_j} \quad j = 1, 2, 3, 4 \quad (\text{B.1})$$

- (3) Let standard deviation of s_1 stress level $\tilde{\eta}_{s_1} = \alpha \lambda_{s_1}$ ($\alpha=0.001, 0.002, \dots$) and let $k_{u+1}=k_u+\Delta k$ (Δk is the step length, such as $1e-6$). According to each $\tilde{\eta}_{s_1}$, select all possible k values for the estimation of the standard deviation of other stress level by (B.2). Then, convert the fatigue lives of the other several stress levels into the equivalent fatigue life N_{i,s_j}^e under the s_1 stress level by (B.3). Thus, all the equivalent fatigue lives can be obtained.

$$\eta_{s_j} = \tilde{\eta}_{s_1} + k (s_1 - s_i) \quad (\text{B.2})$$

$$\ln N_{i,s_1}^e = \frac{\ln N_{i,s_j} - \lambda_{s_j}}{\eta_{s_j}} \quad j = 2, 3, 4 \quad (\text{B.3})$$

- (4) Calculate the equivalent $\eta_{s_1}^e$ of all fatigue lives under s_1 stress level. Calculate the relative error between the equivalent $\eta_{s_1}^e$ and $\tilde{\eta}_{s_1}$ by (B.4). If k_u ($u=0,1,2,3,\dots$) satisfies (B.4), the right k and η_{s_j} can be determined. Otherwise, $k_{u+1}=k_u+\Delta k$ (Δk is the step length, such as $1e-6$), return to the step (3) and recalculate η_{s_j} , until the right k and η_{s_j} are obtained. Then, the P-S-N curves can be fitted by the least square method based on the obtained parameters.

$$\Delta = \frac{\eta_{s_1}^e - \tilde{\eta}_{s_1}}{\tilde{\eta}_{s_1}} \leq 0.001 \quad (\text{B.4})$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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