Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 9095014, 9 pages
https://doi.org/10.1155/2018/9095014

Research Article

H_∞ Control for LPV Discrete Systems with Random Time-Varying Network Delay

Kewang Huang[1] **Tao Ma**[2] **and Feng Pan**[2]

[1] *School of Internet of Things Technology, Wuxi Institute of Technology, Wuxi, Jiangsu 214121, China*
[2] *Key Laboratory of Advanced Process Control for Light Industry, Ministry of Education, Jiangnan University, Wuxi, Jiangsu 214122, China*

Correspondence should be addressed to Kewang Huang; huangkw@wxit.edu.cn

Received 6 June 2018; Revised 21 September 2018; Accepted 17 October 2018; Published 29 October 2018

Academic Editor: Xiangyu Meng

In this paper, we study the H_∞ control problem for Linear Parameter Varying (LPV) discrete systems with random time-varying network delay. The state matrices of LPV discrete systems are deterministic functions and changed with parameters; the range of parameters is measurable. Considering the characteristics of networks with random time-varying delay, we proposed a new parameter-dependent H_∞ performance criterion based on the Lyapunov stability theory. The coupling between Lyapunov functions and system matrices could be eliminated by introducing an additional matrix in this criterion, which made it easier for numerical implementation. On this basis, we designed a state feedback controller by virtue of linear matrix inequalities, which transformed the sufficient conditions into existence condition of solution of parametric linear matrix inequalities. The designed controller could keep the closed-loop system asymptotically stable under given time delay and probability and meet predefined performance metric. The validity of the proposed method is verified by numerical simulation.

1. Introduction

In recent years, the gain scheduling control has become one of the most effective methods in nonlinear control domains, and the application of gain scheduling control based on Linear Parameter-Varying (LPV) system is boosting. The LPV system is a type of system with constantly changing parameters [1–4]. The state matrices of LPV systems are deterministic functions with time-varying parameters, and the range of these time-varying parameters associated with the functions can be online measured. However, most of the control research works for physical systems use a normal (or regular) model. In fact, LPV systems are of quite importance for the physical representation of some real systems [5–7]. Nowadays, the LPV system is widely used in aviation, aerospace, robotic, industrial control, and so on. The reports about the LPV systems can be found in much literature [7–10].

The modern networked control systems are developing rapidly by virtue of such advantages as low cost, easy installation, high reliability/flexibility, strong fault tolerance/diagnosis ability, convenient remote manipulation/control, and so on. One of the most important requirements for a control system is the so-called robustness [11]. However, the combination of LPV system with network will cause problems as data quantization, network delay, and data loss, which will deteriorate the performance metric of the whole system and even lead to system instability [12]. Due to the network delay, system analysis will become much more complex and difficult in real closed-loop control system and the designed controller without considering network delay will always cause instability or the degradation of performance metric. Therefore, it is necessary to investigate the LPV system with network delay. To date, there are only a few reports about the networked LPV system. Wang [13] and Mehendale & Grigoriadis [14] investigated H_∞ control of the LPV system with inherent delay. Hencey & Alleyne [15] analyzed the stability of LPV system with time delay. Wu & Su & Shi [16], Xiao & Jia & Matsuno [17], Shao [18], and Deuce & Sipahi [19] analyzed the time delay problems for networked control system. Faiz & Sing [20] and Luan & Shi & Liu [21] researched the stability problems of networked control system with random time delay. They proposed the design of controller with random time delay, which will keep the closed-loop system asymptotically stable. Yuan [22] investigated the stability of LPV system with state time delay and the controller design; he proposed the system model structure and designed an effective controller, which will keep the system asymptotically stable. However, H_∞ control problem of LPV discrete systems with random time-varying network delay is still an open problem. The investigation of these systems has practical reference value and realistic signification. Therefore, we investigate the above-mentioned LPV discrete systems with random time-varying network delay in this paper.

In summary, we investigate the control of LPV discrete system with time-varying network delay while there is very little literature on this aspect. The main contribution of this paper is as follows. First, it has lower conservative property in comparison with constant time delay system. We constructed a suitable Lyapunov function, and the occurrence of network-induced time delay was described by sequence generated from Bernoulli distribution. Secondly, the corresponding controller was designed to keep the quantized closed-loop LPV system asymptotically stable. Finally, the proposed controller design conditions were transformed into an optimization problem. The optimal solutions can be obtained by linear matrix inequalities based on stability theory and system can meet optimal H_∞ performance metric [23–25].

The following sections of this paper are as follows. Section 2 outlines the problem formulation. Section 3 describes the main theorems with their proofs. Section 4 demonstrates numerical simulation results. Section 5 gives conclusion for this paper.

2. Problem Formulation

Consider the following polytypic LPV discrete control system:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}(\boldsymbol{\rho}(k))\mathbf{x}(k) + \mathbf{B}_1(\boldsymbol{\rho}(k))\mathbf{u}(k) \\ &\quad + \mathbf{B}_2(\boldsymbol{\rho}(k))\boldsymbol{\omega}(k) \qquad (1)\\ \mathbf{z}(k) &= \mathbf{C}(\boldsymbol{\rho}(k))\mathbf{x}(k) + \mathbf{D}(\boldsymbol{\rho}(k))\mathbf{u}(k)\end{aligned}$$

$$\mathbf{x}(k) = \boldsymbol{\Phi}(k), \quad k \in [-d, 0] \qquad (2)$$

where $\left[\begin{smallmatrix}\mathbf{A}(\boldsymbol{\rho}(k)) & \mathbf{B}_1(\boldsymbol{\rho}(k)) & \mathbf{B}_2(\boldsymbol{\rho}(k)) \\ \mathbf{C}(\boldsymbol{\rho}(k)) & \mathbf{D}(\boldsymbol{\rho}(k)) & \mathbf{0}\end{smallmatrix}\right] = \left\{\sum_{i=1}^{r}\alpha_i \left[\begin{smallmatrix}\mathbf{A}_i & \mathbf{B}_{1i} & \mathbf{B}_{2i} \\ \mathbf{C}_i & \mathbf{D}_i & \mathbf{0}\end{smallmatrix}\right], \alpha_i \geq 0, \sum_{i=1}^{r}\alpha_i = 1\right\}$;

$\mathbf{x}(k) \in \mathbb{R}^n$ represents the state vector; $\mathbf{z}(k) \in \mathbb{R}^n$ represents the system output; $\mathbf{u}(k) \in \mathbb{R}^p$ represents the system control input; $\boldsymbol{\omega}(k) \in \mathbb{R}^q$ represents the disturbance input. α_i represents the coordinate of polytope. $\mathbf{A}_i$, $\mathbf{B}_{1i}$, $\mathbf{B}_{2i}$, $\mathbf{C}_i$, and $\mathbf{D}_i$ represent each vertex of the polytope for system matrix $\mathbf{A}(\boldsymbol{\rho}(k))$, $\mathbf{B}_1(\boldsymbol{\rho}(k))$, $\mathbf{B}_2(\boldsymbol{\rho}(k))$, $\mathbf{C}(\boldsymbol{\rho}(k))$, and $\mathbf{D}(\boldsymbol{\rho}(k))$, respectively. $\{\boldsymbol{\Phi}(k), k = -d, -d+1, \cdots, 0\}$ represents a sequence with known initial conditions. $\boldsymbol{\rho}(k) = [\rho_1(k)\ \rho_2(k)\ \cdots\ \rho_s(k)]^{\mathrm{T}}$ represents a parameter vector and $\rho_i(k)$ can be online measured and fall into the closed interval of $[\underline{\rho}_i, \overline{\rho}_i]$. For convenience, we use $\boldsymbol{\rho}$ and ρ_i to represent $\boldsymbol{\rho}(k)$ and $\rho_i(k)$, respectively.

Construct the following state feedback controller:

$$\mathbf{u}(k) = (1-\delta_k)\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) + \delta_k\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k)) \qquad (3)$$

where $1 \leq d_m \leq d(k) \leq d_M$ and $\mathbf{K}(\boldsymbol{\rho}) \in \mathbb{R}^{p\times n}$ represents the unknown gain matrix of state feedback controller, which depends on parameter vector $\boldsymbol{\rho}(k)$, $d(k)$ represents the random time-varying network delay. d_m and d_M represent the lower and upper bound of time delay, respectively. δ_k represents whether a delay will occur or not when the signal is transmitted from the sensor to the controller through network channel. The Bernoulli distribution sequence with a value of 0 or 1 is utilized to represent the occurrence of time delay. $\delta_k = 0$ means no random time delay while $\delta_k = 1$ means random time delay.

Suppose the probability of random time-varying delay is

$$\begin{aligned} prob\{\delta_k = 1\} &= E\{\delta_k\} = \alpha \\ prob\{\delta_k = 0\} &= E\{1-\delta_k\} = 1-\alpha \end{aligned} \qquad (4)$$

where $0 \leq \alpha \leq 1$ is a known constant.

From (1) and (3), we can obtain the following closed-loop LPV control system:

$$\begin{aligned}\mathbf{x}(k+1) &= \overline{\mathbf{A}}(\boldsymbol{\rho})\mathbf{x}(k) + \mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k)) \\ &\quad + \mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k) \\ &\quad + (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) \\ &\quad - (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k)) \qquad (5)\\ \mathbf{z}(k) &= \overline{\mathbf{C}}(\boldsymbol{\rho})\mathbf{x}(k) + \overline{\mathbf{D}}(\boldsymbol{\rho})\mathbf{x}(k-d(k)) \\ &\quad + (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) \\ &\quad - (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\end{aligned}$$

where $\overline{\mathbf{A}}(\boldsymbol{\rho}) = \mathbf{A}(\boldsymbol{\rho}) + (1-\alpha)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$, $\mathbf{A}_d(\boldsymbol{\rho}) = \alpha\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$, $\overline{\mathbf{C}}(\boldsymbol{\rho}) = \mathbf{C}(\boldsymbol{\rho}) + (1-\alpha)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$, and $\overline{\mathbf{D}}(\boldsymbol{\rho}) = \alpha\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$.

First, we investigate the H_∞ performance criteria for (1). Secondly, we design the corresponding feedback controller, which should meet the following two metrics.

(a) Equation (5) should be exponentially mean square stable.

(b) Equation (5) has a certain level of disturbance attenuation for H_∞; i.e., the gain from the disturbance input $\boldsymbol{\omega}(k)$ to the system output $\mathbf{z}(k)$ is less than a predefine value. Moreover, (5) should meet the following criterion under zero initial condition (i.e., $\boldsymbol{\Phi}(k) = 0$)

$$E\left\{\|\mathbf{z}\|_2^2\right\} \leq \gamma^2\|\boldsymbol{\omega}\|_2^2 \qquad (6)$$

where performance metric $\gamma > 0$, $\|\boldsymbol{\omega}\|_2^2 = \sum_{k=0}^{\infty}\boldsymbol{\omega}^{\mathrm{T}}(k)\boldsymbol{\omega}(k)$ and $\|\mathbf{z}\|_2^2 = \sum_{k=0}^{\infty}\mathbf{z}^{\mathrm{T}}(k)\mathbf{z}(k)$.

3. Main Theorems

Lemma 1 (see [26] (Schur complement)). *Given real matrices L_1, L_2, L_3 where $L_1 = L_1^T$ and $L_2 = L_2^T < 0$, then $L_1 - L_3^TL_2^{-1}L_3 \leq 0$ if and only if $\left[\begin{smallmatrix}L_1 & L_3^T \\ L_2 & L_2\end{smallmatrix}\right] \leq 0$ or $\left[\begin{smallmatrix}L_2 & L_3 \\ L_3^T & L_1\end{smallmatrix}\right] \leq 0$.*

Theorem 2. *For (5), given the probability of communication time delay occurrence $0 < \alpha \leq 1$ and the performance metric $\gamma > 0$, if there are matrices $\mathbf{P}(\rho) > 0$, $\mathbf{Q}(\rho) > 0$, $\mathbf{Z} > 0$, $\mathbf{Y}_1$, and $\mathbf{Y}_2$ such that the following linear matrix inequality equation holds for all parameters changing trajectories, then (5) meets the above-mentioned two performance metrics.*

$$\begin{bmatrix} \prod_1 & \prod_2 \\ * & \prod_3 \end{bmatrix} < 0 \tag{7}$$

where $*$ represents the symmetric matrices block transpose,

$$\prod_1 = \begin{bmatrix} -\mathbf{P}(\rho(k)) + (d_M - d_m + 1)\mathbf{Q}(\rho(k)) + \mathbf{Y}_1 + \mathbf{Y}_1^T & \mathbf{Y}_2^T - \mathbf{Y}_1 & 0 \\ * & -\mathbf{Q}(\rho(k-d_k)) - \mathbf{Y}_2 - \mathbf{Y}_2^T & 0 \\ * & * & -\gamma^2\mathbf{I} \end{bmatrix}$$

$$\prod_2 = \begin{bmatrix} \overline{\mathbf{A}}^T(\rho)\mathbf{P}(\rho(k+1)) & d_M\left(\overline{\mathbf{A}}^T(\rho) - \mathbf{I}\right)\mathbf{Z} & l\cdot(\mathbf{B}_1(\rho)\mathbf{K}(\rho))^T\mathbf{P}(\rho(k+1)) & -d(k)\mathbf{Y}_1 & \overline{\mathbf{C}}^T(\rho) & \beta\cdot(\mathbf{D}(\rho)\mathbf{K}(\rho))^T \\ \mathbf{A}_d^T(\rho)\mathbf{P}(\rho(k+1)) & d_M\mathbf{A}_d^T(\rho)\mathbf{Z} & -l\cdot(\mathbf{B}_1(\rho)\mathbf{K}(\rho))^T\mathbf{P}(\rho(k+1)) & -d(k)\mathbf{Y}_2 & \overline{\mathbf{D}}^T(\rho) & -\beta\cdot(\mathbf{D}(\rho)\mathbf{K}(\rho))^T \\ \mathbf{B}_2^T(\rho)\mathbf{P}(\rho(k+1)) & d_M\mathbf{B}_2^T(\rho)\mathbf{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\prod_3 = \begin{bmatrix} -\mathbf{P}(\rho(k+1)) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -d_M\mathbf{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{P}(\rho(k+1)) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -d(k)\mathbf{Z} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\mathbf{I} & \mathbf{0} \\ * & * & * & * & * & -\mathbf{I} \end{bmatrix}, \tag{8}$$

$$l = \sqrt{(1+d_M)\alpha(1-\alpha)},\quad \beta = \sqrt{\alpha(1-\alpha)}.$$

Proof. Construct the following parameter-dependent Lyapunov function:

$$\mathbf{V}(k,\rho(k)) = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 + \mathbf{V}_4 \tag{9}$$

where $\mathbf{V}_1 = \mathbf{x}^T(k)\mathbf{P}(\rho(k))\mathbf{x}(k)$, $\mathbf{V}_2 = \sum_{i=k-d(k)}^{k-1}\mathbf{x}^T(i)\mathbf{Q}(\rho(i))\mathbf{x}(i)$, $\mathbf{V}_3 = \sum_{j=-d_M+2}^{-d_m+1}\sum_{i=k+j-1}^{k-1}\mathbf{x}^T(i)\mathbf{Q}(\rho(i))\mathbf{x}(i)$, $\mathbf{V}_4 = \sum_{i=-d_M}^{-1}\sum_{m=k+i}^{k-1}\boldsymbol{\delta}^T(m)\mathbf{Z}\boldsymbol{\delta}(m)$, $\boldsymbol{\delta}(k) = \mathbf{x}(k+1) - \mathbf{x}(k)$.

Note that $E\{(\alpha-\delta_k)^2\} = \beta^2$ and $E\{(\alpha-\delta_k)\} = 0$; therefore we have

$$E\{\mathbf{V}(k+1,\rho(k+1))\} - \mathbf{V}(k,\rho(k)) = E\{\Delta\mathbf{V}_1\} + E\{\Delta\mathbf{V}_2\} + E\{\Delta\mathbf{V}_3\} + E\{\Delta\mathbf{V}_4\}$$

$$\begin{aligned}\Delta\mathbf{V}_1 &= \mathbf{x}^T(k+1)\mathbf{P}(\rho(k+1))\mathbf{x}(k+1) - \mathbf{x}^T(k) \\ &\cdot \mathbf{P}(\rho(k))\mathbf{x}(k) = \left[\overline{\mathbf{A}}(\rho)\mathbf{x}(k) + \mathbf{A}_d(\rho)\mathbf{x}(k \right. \\ &- d(k)) + \mathbf{B}_2(\rho)\mathbf{w}(k) + (\alpha-\delta_k)\mathbf{B}_1(\rho)\mathbf{K}(\rho)\mathbf{x}(k) \\ &\left. - (\alpha-\delta_k)\mathbf{B}_1(\rho)\mathbf{K}(\rho)\mathbf{x}(k-d(k))\right]^T \cdot \mathbf{P}(\rho(k \\ &+ 1)) \cdot \left[\overline{\mathbf{A}}(\rho)\mathbf{x}(k) + \mathbf{A}_d(\rho)\mathbf{x}(k-d(k)) + \mathbf{B}_2(\rho)\right. \\ &\cdot \mathbf{w}(k) + (\alpha-\delta_k)\mathbf{B}_1(\rho)\mathbf{K}(\rho)\mathbf{x}(k) - (\alpha-\delta_k) \\ &\left.\cdot \mathbf{B}_1(\rho)\mathbf{K}(\rho)\mathbf{x}(k-d(k))\right] - \mathbf{x}^T(k)\mathbf{P}(\rho(k))\mathbf{x}(k)\end{aligned}$$

$$\begin{aligned}\Delta\mathbf{V}_2 &= \mathbf{x}^T(k)\mathbf{Q}(\rho(k))\mathbf{x}(k) - \mathbf{x}^T(k-d(k))\mathbf{Q}(\rho(k \\ &- d(k)))\mathbf{x}(k-d(k)) + \sum_{i=k-d_m+1}^{k-1}\mathbf{x}^T(i)\mathbf{Q}(\rho(i))\mathbf{x}(i) \\ &+ \sum_{i=k-d(k+1)+1}^{k-d_m}\mathbf{x}^T(i)\mathbf{Q}(\rho(i))\mathbf{x}(i) - \sum_{i=k-d(k)+1}^{k-1}\mathbf{x}^T(i) \\ &\cdot \mathbf{Q}(\rho(i))\mathbf{x}(i) \leq \mathbf{x}^T(k)\mathbf{Q}(\rho(k))\mathbf{x}(k) - \mathbf{x}^T(k \\ &- d(k))\mathbf{Q}(\rho(k-d(k)))\mathbf{x}(k-d(k)) \\ &+ \sum_{i=k-d_M+1}^{k-d_m}\mathbf{x}^T(i)\mathbf{Q}(\rho(i))\mathbf{x}(i)\end{aligned}$$

$$\begin{aligned}\Delta\mathbf{V}_3 &= \sum_{j=-d_M+2}^{-d_m+1}\left[\mathbf{x}^T(k)\mathbf{Q}(\rho(k))\mathbf{x}(k) \right. \\ &\left. - \mathbf{x}^T(k+j-1)\mathbf{Q}(\rho(k+j-1))\mathbf{x}(k+j-1)\right] \\ &= (d_M - d_m)\mathbf{x}^T(k)\mathbf{Q}(\rho(k))\mathbf{x}(k) - \sum_{i=k-d_M+1}^{k-d_m}\mathbf{x}^T(i)\end{aligned}$$

$$\cdot \mathbf{Q}(\boldsymbol{\rho}(i))\mathbf{x}(i)$$

$$\Delta \mathbf{V}_4 = \sum_{i=-d_M}^{-1}\left[\boldsymbol{\delta}^T(k)\mathbf{Z}\boldsymbol{\delta}(k) - \boldsymbol{\delta}^T(k+i)\mathbf{Z}\boldsymbol{\delta}(k+i)\right]$$
$$\leq d_M[\mathbf{x}(k+1)-\mathbf{x}(k)]^T\mathbf{Z}[\mathbf{x}(k+1)-\mathbf{x}(k)]$$
$$-\sum_{m=k-d(k)}^{k-1}\boldsymbol{\delta}^T(m)\mathbf{Z}\boldsymbol{\delta}(m) = d_M\left[\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)\mathbf{x}(k)\right.$$
$$+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k)) + \mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k) + (\alpha-\delta_k)$$
$$\cdot \mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) - (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k$$
$$\left.-d(k))\right]^T\cdot\mathbf{Z}\cdot\left[\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-I\right)\mathbf{x}(k)+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k\right.$$
$$-d(k)) + \mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k) + (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k)$$
$$\left.-(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]$$
$$-\sum_{m=k-d(k)}^{k-1}\boldsymbol{\delta}^T(m)\mathbf{Z}\boldsymbol{\delta}(m) \tag{10}$$

Then we have

$$\Delta\mathbf{V}(k,\boldsymbol{\rho}(k)) = \Delta\mathbf{V}_1+\Delta\mathbf{V}_2+\Delta\mathbf{V}_3+\Delta\mathbf{V}_4$$
$$\leq \frac{1}{d(k)}\sum_{m=k-d(k)}^{k-1}\Xi(m,\boldsymbol{\rho}(k)) \tag{11}$$

where

$$\Xi(m,\boldsymbol{\rho}(k)) = \left[\overline{\mathbf{A}}(\boldsymbol{\rho})\mathbf{x}(k)+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right.$$
$$+\mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k) + (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k)$$
$$\left.-(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]^T$$
$$\cdot\mathbf{P}(\boldsymbol{\rho}(k+1))\cdot\left[\overline{\mathbf{A}}(\boldsymbol{\rho})\mathbf{x}(k)+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right.$$
$$+\mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k) + (\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k)$$
$$\left.-(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]-\mathbf{x}^T(k)$$
$$\cdot\mathbf{P}(\boldsymbol{\rho}(k))\mathbf{x}(k)+(d_M-d_m+1)\mathbf{x}^T(k)\mathbf{Q}(\boldsymbol{\rho}(k))$$
$$\cdot\mathbf{x}(k)-\mathbf{x}^T(k-d(k))\mathbf{Q}(\boldsymbol{\rho}(k-d(k)))\mathbf{x}(k$$
$$-d(k))+d_M\cdot\left[\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)\mathbf{x}(k)\right.$$
$$+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k))+\mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k)$$
$$+(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k)$$
$$\left.-(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]^T\cdot\mathbf{Z}$$
$$\cdot\left[\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)\mathbf{x}(k)+\mathbf{A}_d(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right.$$
$$+\mathbf{B}_2(\boldsymbol{\rho})\mathbf{w}(k)+(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k)$$
$$\left.-(\alpha-\delta_k)\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]-d(k)$$
$$\cdot\boldsymbol{\delta}^T(m)\mathbf{Z}\boldsymbol{\delta}(m). \tag{12}$$

Based on $\mathbf{x}(k)-\mathbf{x}(k-d(k)) = \sum_{m=k-d(k)}^{k-1}\boldsymbol{\delta}(m)$, for any matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$,

$$\Lambda = \frac{1}{d(k)}\sum_{m=k-d(k)}^{k-1}\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{x}(k-d(k))\end{bmatrix}^T\begin{bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{bmatrix}[\mathbf{x}(k)$$
$$-\mathbf{x}(k-d(k))-d(k)\boldsymbol{\delta}(m)] = 0 \tag{13}$$

Then we have

$$E\{\Delta\mathbf{V}(k,\boldsymbol{\rho}(k))\} = E\{\Delta\mathbf{V}(k,\boldsymbol{\rho}(k))+2\Lambda\}$$
$$\leq\frac{1}{d(k)}\sum_{m=k-d(k)}^{k-1}\boldsymbol{\eta}^T(k,m)\boldsymbol{\psi}\boldsymbol{\eta}(k,m) \tag{14}$$

where

$$\boldsymbol{\eta}(k,m) = \begin{bmatrix}\mathbf{x}(k)\\ \mathbf{x}(k-d(k))\\ \mathbf{w}(k)\\ \boldsymbol{\delta}(m)\end{bmatrix},$$

$$\boldsymbol{\psi} = \begin{bmatrix}\psi_{11} & \psi_{12} & \psi_{13} & -d(k)\mathbf{Y}_1\\ * & \psi_{22} & \psi_{23} & -d(k)\mathbf{Y}_2\\ * & * & \psi_{33} & \mathbf{0}\\ * & * & * & -d(k)\mathbf{Z}\end{bmatrix},$$

$$\psi_{11} = \overline{\mathbf{A}}^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{P}(\boldsymbol{\rho}(k))$$
$$+(d_M-d_m+1)\mathbf{Q}(\boldsymbol{\rho}(k))+\mathbf{Y}_1+\mathbf{Y}_1^T$$
$$+d_M\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)^T\mathbf{Z}\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)$$
$$+l^2(\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$$

$$\psi_{12} = \mathbf{Y}_2^T-\mathbf{Y}_1+\overline{\mathbf{A}}^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{A}_d(\boldsymbol{\rho})$$
$$+d_M\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)^T\mathbf{Z}\mathbf{A}_d(\boldsymbol{\rho})$$
$$-l^2(\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$$

$$\psi_{22} = \mathbf{A}_d^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{A}_d(\boldsymbol{\rho})$$
$$+d_M\mathbf{A}_d^T(\boldsymbol{\rho})\mathbf{Z}\mathbf{A}_d(\boldsymbol{\rho})-\mathbf{Q}(\boldsymbol{\rho}(k-d(k)))-\mathbf{Y}_2$$
$$-\mathbf{Y}_2^T$$
$$+l^2(\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_1(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})$$

$$\begin{aligned}\psi_{13} &= \overline{\mathbf{A}}^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_2(\boldsymbol{\rho}) \\ &\quad + d_M\left(\overline{\mathbf{A}}(\boldsymbol{\rho})-\mathbf{I}\right)^T\mathbf{Z}\mathbf{B}_2(\boldsymbol{\rho}), \\ \psi_{23} &= \mathbf{A}_d^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_2(\boldsymbol{\rho}) + d_M\mathbf{A}_d^T(\boldsymbol{\rho})\mathbf{Z}\mathbf{B}_2(\boldsymbol{\rho}) \\ \psi_{33} &= \mathbf{B}_2^T(\boldsymbol{\rho})\mathbf{P}(\boldsymbol{\rho}(k+1))\mathbf{B}_2(\boldsymbol{\rho}) + d_M\mathbf{B}_2^T(\boldsymbol{\rho})\mathbf{Z}\mathbf{B}_2(\boldsymbol{\rho})\end{aligned} \tag{15}$$

Further, the norm of LPV system output is

$$\begin{aligned}\|\mathbf{z}(k)\|^2 &= \left[\overline{\mathbf{C}}(\boldsymbol{\rho})\mathbf{x}(k) + \overline{\mathbf{D}}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right. \\ &\quad + (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) \\ &\quad \left.- (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]^T \\ &\quad \cdot \left[\overline{\mathbf{C}}(\boldsymbol{\rho})\mathbf{x}(k) + \overline{\mathbf{D}}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right. \\ &\quad + (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k) \\ &\quad \left.- (\alpha-\delta_k)\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho})\mathbf{x}(k-d(k))\right]\end{aligned} \tag{16}$$

Combining (14) and (16), we have

$$\begin{aligned}&E\{\Delta\mathbf{V}(k,\boldsymbol{\rho}(k))\} + E\left\{\|\mathbf{z}(k)\|^2\right\} - \gamma^2\|\mathbf{w}(k)\|^2 \\ &\quad \le \boldsymbol{\eta}^T(k)\boldsymbol{\psi}\boldsymbol{\eta}(k) - \gamma^2\mathbf{w}(k)^T\mathbf{w}(k) + \boldsymbol{\eta}^T(k) \\ &\quad \cdot \begin{bmatrix}\overline{\mathbf{C}}^T(\boldsymbol{\rho}) \\ \overline{\mathbf{D}}^T(\boldsymbol{\rho}) \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{C}}^T(\boldsymbol{\rho}) \\ \overline{\mathbf{D}}^T(\boldsymbol{\rho}) \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix}^T \eta(k) + \boldsymbol{\eta}^T(k) \\ &\quad \cdot \begin{bmatrix}\beta\cdot(\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T \\ -\beta\cdot(\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix}\begin{bmatrix}\beta\cdot(\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T \\ -\beta\cdot(\mathbf{D}(\boldsymbol{\rho})\mathbf{K}(\boldsymbol{\rho}))^T \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix}^T \\ &\quad \cdot \boldsymbol{\eta}(k)\end{aligned} \tag{17}$$

According to Schur complement theory, (7) can keep that $E\{\Delta\mathbf{V}(k,\boldsymbol{\rho}(k))\} + E\{\|\mathbf{z}(k)\|^2\} - \gamma^2\|\mathbf{w}(k)\|^2 < 0$; i.e., the closed-loop LPV system is exponential mean square stable and meets the predefined H_∞ performance metrics. This completes the proof. □

We can conclude from Theorem 2 that there exists a product term concerning with the Lyapunov function matrix and the closed-loop system matrix, which makes (7) change into bilinear matrix inequalities. To solve this problem, an additional matrix is introduced to eliminate the coupling between the system matrix and the parameter-dependent Lyapunov function [27]. A new H_∞ performance metric is obtained for further design of gain scheduling feedback controller.

Theorem 3. *Given the probability of communication time delay occurrence $0 < \alpha \le 1$ and the performance metric $\gamma > 0$, if there are symmetric positive definite matrices $\overline{\mathbf{P}}$, $\overline{\mathbf{Q}}(\boldsymbol{\rho})$, $\mathbf{Z}$, and matrices $\overline{\mathbf{K}}(\boldsymbol{\rho})$, $\overline{\mathbf{Y}}_1$, and $\overline{\mathbf{Y}}_2$, such that the following linear matrix inequality equation holds for all parameters changing trajectories, then (5) is asymptotically stable and meets the predefined H_∞ performance metrics.*

$$\begin{bmatrix}\boldsymbol{\Theta}_{11} & \overline{\mathbf{Y}}_2^T-\overline{\mathbf{Y}}_1 & \mathbf{0} & \boldsymbol{\Theta}_{14} & d_M\boldsymbol{\Theta}_{15} & l\mathbf{M} & -d(k)\overline{\mathbf{Y}}_1 & \boldsymbol{\Theta}_{18} & \beta\mathbf{N} \\ * & \boldsymbol{\Theta}_{22} & \mathbf{0} & \overline{\mathbf{P}}\mathbf{A}_d^T(\boldsymbol{\rho}) & d_M\overline{\mathbf{P}}\mathbf{A}_d^T(\boldsymbol{\rho}) & -l\mathbf{M} & -d(k)\overline{\mathbf{Y}}_2 & \overline{\mathbf{P}}\,\overline{\mathbf{D}}^T(\boldsymbol{\rho}) & -\beta\mathbf{N} \\ * & * & -\gamma^2\mathbf{I} & \mathbf{B}_2^T(\boldsymbol{\rho}) & d_M\mathbf{B}_2^T(\boldsymbol{\rho}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\overline{\mathbf{P}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & d_M\mathbf{Z}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\overline{\mathbf{P}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -d(k)\overline{\mathbf{P}}\mathbf{Z}\overline{\mathbf{P}} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -\mathbf{I} & \mathbf{0} \\ * & * & * & * & * & * & * & * & -\mathbf{I}\end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned}
\Theta_{11} &= -\overline{\mathbf{P}} + (d_M - d_m + 1)\,\overline{\mathbf{Q}}(\rho(k)) + \overline{\mathbf{Y}}_1 + \overline{\mathbf{Y}}_1^T, \\
\Theta_{14} &= \overline{\mathbf{P}}\,\overline{\mathbf{A}}^T(\rho) + (1-\alpha)(\mathbf{B}_1(\rho)\mathbf{K}(\rho))^T, \\
\Theta_{15} &= \overline{\mathbf{P}}\left(\overline{\mathbf{A}}^T(\rho) - \mathbf{I}\right) + (1-\alpha)(\mathbf{B}_1(\rho)\mathbf{K}(\rho))^T, \\
\Theta_{18} &= \overline{\mathbf{P}}\,\overline{\mathbf{C}}^T(\rho) + (1-\alpha)(\mathbf{D}(\rho)\mathbf{K}(\rho))^T, \\
\Theta_{22} &= -\overline{\mathbf{Q}}(\rho(k-d(k))) - \overline{\mathbf{Y}}_2 - \overline{\mathbf{Y}}_2^T, \\
\mathbf{A}_d(\rho) &= \alpha\mathbf{B}_1(\rho)\mathbf{K}(\rho), \\
\overline{\mathbf{D}}(\rho) &= \alpha\mathbf{D}(\rho)\mathbf{K}(\rho), \\
\mathbf{M} &= \overline{\mathbf{P}}(\mathbf{B}_1(\rho)\mathbf{K}(\rho))^T, \\
\mathbf{N} &= \overline{\mathbf{P}}(\mathbf{D}(\rho)\mathbf{K}(\rho))^T,
\end{aligned} \tag{19}$$

If (18) has a solution, then the gain matrix of controller is

$$\mathbf{K}(\rho(k)) = \overline{\mathbf{K}}(\rho(k))\,\overline{\mathbf{P}}^{-1} \tag{20}$$

Proof. Suppose there are symmetric positive definite matrices $\overline{\mathbf{P}} > 0$, $\overline{\mathbf{Q}}(\rho) > 0$, $\mathbf{Z} > 0$, and matrices $\overline{\mathbf{K}}(\rho)$, $\overline{\mathbf{Y}_1}$, and $\overline{\mathbf{Y}_2}$ meet (18). We do the equal transformation on (18) based on reversible matrix $\text{diag}\{\overline{\mathbf{P}}^{-1}, \overline{\mathbf{P}}^{-1}, \mathbf{I}, \overline{\mathbf{P}}^{-1}, \mathbf{Z}, \overline{\mathbf{P}}^{-1}, \overline{\mathbf{P}}^{-1}, \mathbf{I}, \mathbf{I}\}$. Moreover, letting $\mathbf{P} = \overline{\mathbf{P}}^{-1}$, $\mathbf{Q}(\rho(k)) = \overline{\mathbf{P}}^{-1}\overline{\mathbf{Q}}(\rho(k))\overline{\mathbf{P}}^{-1}$, $\mathbf{Y}_1 = \overline{\mathbf{P}}^{-1}\overline{\mathbf{Y}_1}\,\overline{\mathbf{P}}^{-1}$, $\mathbf{Y}_2 = \overline{\mathbf{P}}^{-1}\overline{\mathbf{Y}_2}\,\overline{\mathbf{P}}^{-1}$, $\mathbf{P}(\rho(k)) \equiv \mathbf{P}$, we can draw the conclusion that (18) is equivalent to (7). Therefore, the gain of controller designed by (18) can keep the closed-loop system stable, and system can meet the predefined performance metrics. This completes the proof. □

4. Numerical Simulations

In this section, we will provide numerical simulation to show the effectiveness of the proposed method. The system matrix is originated from literature [22]. We consider the following polytypic LPV control system:

$$\begin{aligned}
\mathbf{x}(k+1) &= \begin{bmatrix} 0.1 & 1+0.2\rho_1 \\ -1.2 & -1+0.1\rho_2 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}\mathbf{u}(k) \\
&\quad + \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}\boldsymbol{\omega}(k) \\
\mathbf{z}(k) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}(k)
\end{aligned} \tag{21}$$

where ρ_1 and ρ_2 are online measurable time-varying parameters, and they are in the intervals $\rho_1 \in [-1, 1]$ and $\rho_2 \in [1, 2]$, respectively.

The vertex of polytypic Θ is $\{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\} = \{[-1, 1], [-1, 2], [1, 1], [1, 2]\}$. Let $\mathbf{A}(\rho) = \sum_{i=1}^{r}\alpha_i\mathbf{A}_i$ and the initial state of the system is $x_0 = [1, -0.5]$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\alpha_3 = 0.3$, $\alpha_4 = 0.4$. The disturbance is defined as follows:

$$\omega(k) = \frac{1}{(0.1 + k^2)}. \tag{22}$$

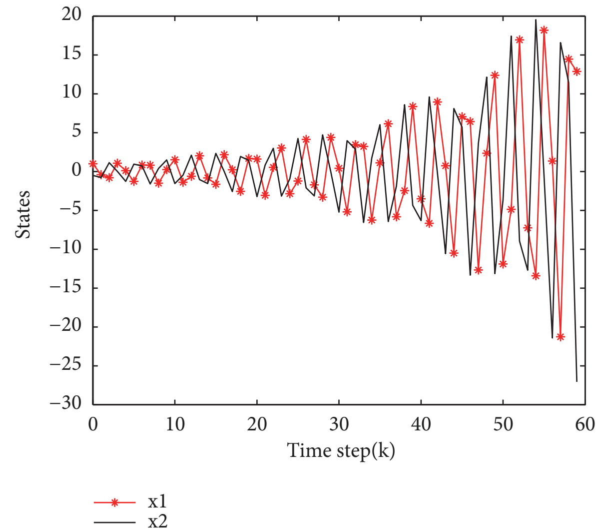


Figure 1: The state response of open-loop system.

The state response of (21) is shown in Figure 1. As can be seen in Figure 1, the state response is divergent within the range of vector $\rho(k)$, and the final state response will approximate infinity. Therefore, the original polytypic LPV system described by (21) is unstable.

Consider the H_∞ control, letting $1 \le d(k) \le 3$, $d_m = 1$, $d_M = 3$, $\mathbf{K}(\rho) = \sum_{i=1}^{r}\alpha_i\mathbf{K}_i$, we substitute Θ into the system matrix. When $d(k) = 1$ and α changes in the interval $[0, 1]$, we can obtain optimal performance metric under different time delay probabilities by solving (18), as shown in Table 1. The corresponding state responses are shown in Figures 2–4.

Table 1 implies that γ will gradually increase with the increase of α. This means the anti-interference ability will gradually deteriorate. As can be seen in Figures 2–4, the system is stable if $d(k) = 1$ is fixed and α is less than or equal to 0.5; the system will become unstable gradually if $d(k) = 1$ is fixed and α is greater than 0.6. In most cases, α is less than or equal to 0.5. Therefore, the controller designed in this paper can keep the closed-loop system stable and make system meet the predefined H_∞ performance metric.

When $\alpha = 0.5$ is fixed, the performance metric of the system will also change with the time delay, as shown in Table 2, and its corresponding state response is shown in Figure 5.

Table 2 implies that γ will gradually increase with the increase of $d(k)$. This means the anti-interference ability will gradually deteriorate. As can be seen in Figures 3 and 5, system stability and performance metric will be affected with the increase of $d(k)$ when $\alpha = 0.5$ is fixed; the system is stable with good performance when $d(k)$ is less than or equal to 2; the system is divergent, and it will gradually change into unstable system when $d(k)$ is greater than or equal to 3.

Table 1: Performance metric γ under different time delay probabilities α.

time delay probability α	performance metric γ
0	0.2079
0.2	0.2151
0.5	0.2271
0.6	0.2317
0.8	0.2429
1.0	0.2587

Table 2: Performance metric γ under different time delay scale $d(k)$.

time delay scale $d(k)$	performance metric γ
1	0.2271
2	0.2551
3	0.2764

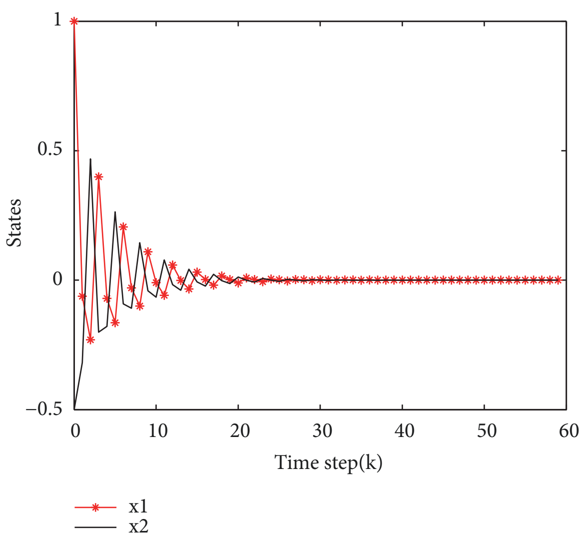


Figure 2: The state response of closed-loop system when $\alpha = 0.2$ and $d(k) = 1$.

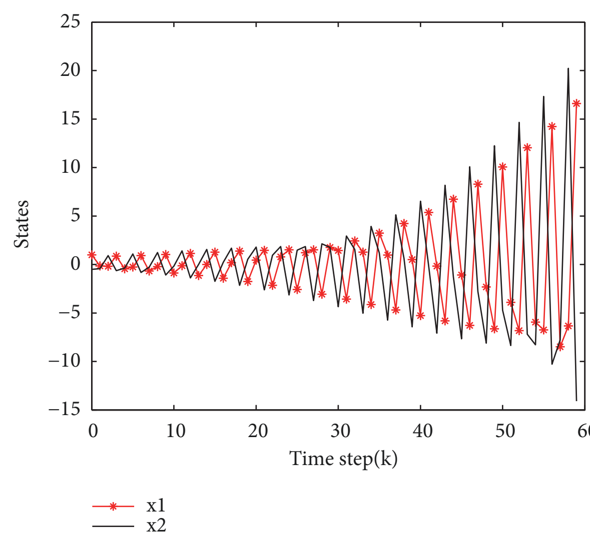


Figure 4: The state response of closed-loop system when $\alpha = 0.6$ and $d(k) = 1$.

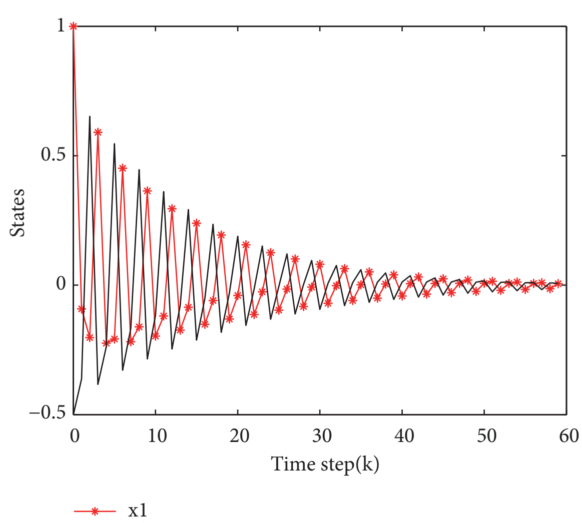


Figure 3: The state response of closed-loop system when $\alpha = 0.5$ and $d(k) = 1$.

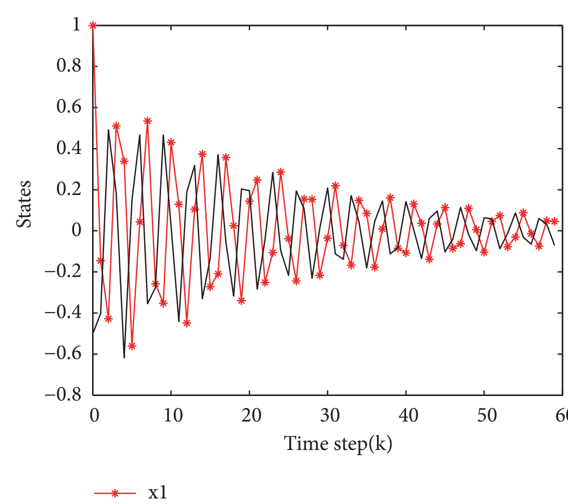


Figure 5: The state response of the closed-loop system when $\alpha = 0.5$ and $d(k) = 2$.

In summary, the stability and performance of the system are both affected by time delay probability and scale. The stability and performance of the system will deteriorate with the increasing of time delay probability α or time delay scale $d(k)$. The controller designed in this paper can keep the closed-loop system asymptotically stable and meet predefined performance metric under given delay and probability.

5. Conclusion

In this paper, we investigate H_∞ control of LPV polytypic system with random time-varying network delay. A method for the robust H_∞ controller of gain scheduling LPV with random time-varying network delay is proposed based on the gain scheduling and H_∞ theory. Utilizing LPV convex decomposition technique, each vertex of the polytypic LPV system is used to design the feedback gain so as to meet the H_∞ performance metric and dynamic characteristics, respectively. Firstly, the feedback controller should be exponentially mean square stable under random delay distribution sequence conditions. Secondly, the designed system should meet predefined performance metric when taking anti-interference ability into account. Compared with the time delay in existing literature, we consider network-induced time delay; it will change stochastically, and it is a sequence under certain distribution. We can give the conclusion from equation derivations and numerical simulations that time delay is a key element for system stability. The stability and performance of the system will deteriorate if the time delay probability or the time delay scale is increasing. The proposed controller in this paper can keep closed-loop system asymptotically stable and meet expected performance under given delay and probability.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

1. We acknowledge financial support for this work from the National Nature Science Foundation of China (no. 61773182).
2. We acknowledge financial support for this work from scientific research project of Wuxi Institute of Technology of china in 2018.

References

[1] W. J. Rugh and J. S. Shamma, "Research on gain scheduling," *Automatica*, vol. 36, no. 10, pp. 1401–1425, 2000.

[2] D. Yang and J. Zhao, "H∞ output tracking control for a class of switched LPV systems and its application to an aero-engine model," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 12, pp. 2102–2120, 2017.

[3] Y. Hu, G. Duan, and F. Tan, "Finite-time control for LPV systems with parameter-varying time delays and exogenous disturbances," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 17, pp. 3841–3861, 2017.

[4] H. Yin, J. Gao, and Z. Liu, "A parameter dependent controller design approach for delayed LPV system," *Asian Journal of Control*, vol. 19, no. 1, pp. 391–398, 2017.

[5] M.-H. Wang, G. Liu, P.-T. Zhao, and S.-H. Yang, "Variable gain state feedback H∞ control for hypersonic vehicle based on LPV," *Yuhang Xuebao/Journal of Astronautics*, vol. 34, no. 4, pp. 488–495, 2013.

[6] J. Chen, Y.-Y. Cao, and W. Zhang, "A fault detection observer design for LPV systems in finite frequency domain," *International Journal of Control*, vol. 88, no. 3, pp. 571–584, 2015.

[7] H. J. Shi, L. F. Qian, Y. D. Xu, and L. M. Chen, "Optimization of manipulator structure with interval uncertainties," *Journal of Nanjing University of Science and Technology*, vol. 38, no. 1, pp. 100–105, 2014.

[8] Y. Zhang, F. Yang, and Q.-L. Han, "H_∞ control of LPV systems with randomly multi-step sensor delays," *International Journal of Control, Automation, and Systems*, vol. 12, no. 6, pp. 1207–1215, 2014.

[9] M. Rodrigues, H. Hamdi, N. B. Braiek, and D. Theilliol, "Observer-based fault tolerant control design for a class of LPV descriptor systems," *Journal of The Franklin Institute*, vol. 351, no. 6, pp. 3104–3125, 2014.

[10] H. Zhang, Y. Shi, and A. S. Mehr, "Parameter-dependent mixed H2/H1filtering for linear parameter-varying systems," *IET Signal Processing*, vol. 6, no. 7, pp. 697–703, 2012.

[11] P. Shi and S.-P. Shue, "Robust H∞ control for linear discrete-time systems with norm-bounded nonlinear uncertainties," *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 108–111, 1999.

[12] S. Wang, X. Meng, and T. Chen, "Wide-area control of power systems through delayed network communication," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 2, pp. 495–503, 2012.

[13] J. L. Wang, *Delay Linear Parameter Varying System Stability Analysis and Gain Scheduling Control*, Science Press, Beijing, China, 2008.

[14] C. S. Mehendale and K. M. Grigoriadis, "Performance of LPV gain-scheduled systems," in *Proceedings of the 2006 American Control Conference*, pp. 2915–2920, USA, June 2006.

[15] B. Hencey and A. Alleyne, "Robust gain-scheduled control," in *Proceedings of the 2010 American Control Conference, ACC 2010*, pp. 3075–3081, USA, July 2010.

[16] L. Wu, X. Su, P. Shi, and J. Qiu, "A new approach to stability analysis and stabilization of discrete-time T-S fuzzy time-varying delay systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 41, no. 1, pp. 273–286, 2011.

[17] N. Xiao, Y. Jia, and F. Matsuno, "Delay distribution dependent stability criteria for discrete-time systems with interval time-varying delay," in *Proceedings of the 2013 1st American Control Conference, ACC 2013*, pp. 1733–1738, USA, June 2013.

[18] H. Y. Shao, "Further improvement on delay-dependent stability results for linear systems with time-varying delays," in *Proceedings of the in Proceedings of the 30th Chinese Control Conference*, Yantai, China, July 2011.

[19] I. I. Delice and R. Sipahi, "Delay-independent stability test for systems with multiple time-delays," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 57, no. 4, pp. 963–972, 2012.

[20] F. Rasool and S. K. Nguang, “Quantized robust H∞ control of discrete time systems with random communication delays,” *International Journal of Systems Science*, vol. 42, no. 1, pp. 129–138, 2011.

[21] X. L. Luan, P. Shi, and F. Liu, “Stabilization of networked control systems with random delays,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 9, pp. 4323–4330, 2011.

[22] W. Yuan, *Research on system control of linear parameter varying with state delay*, Daqing Petroleum Institute, Heilongjiang, 2008.

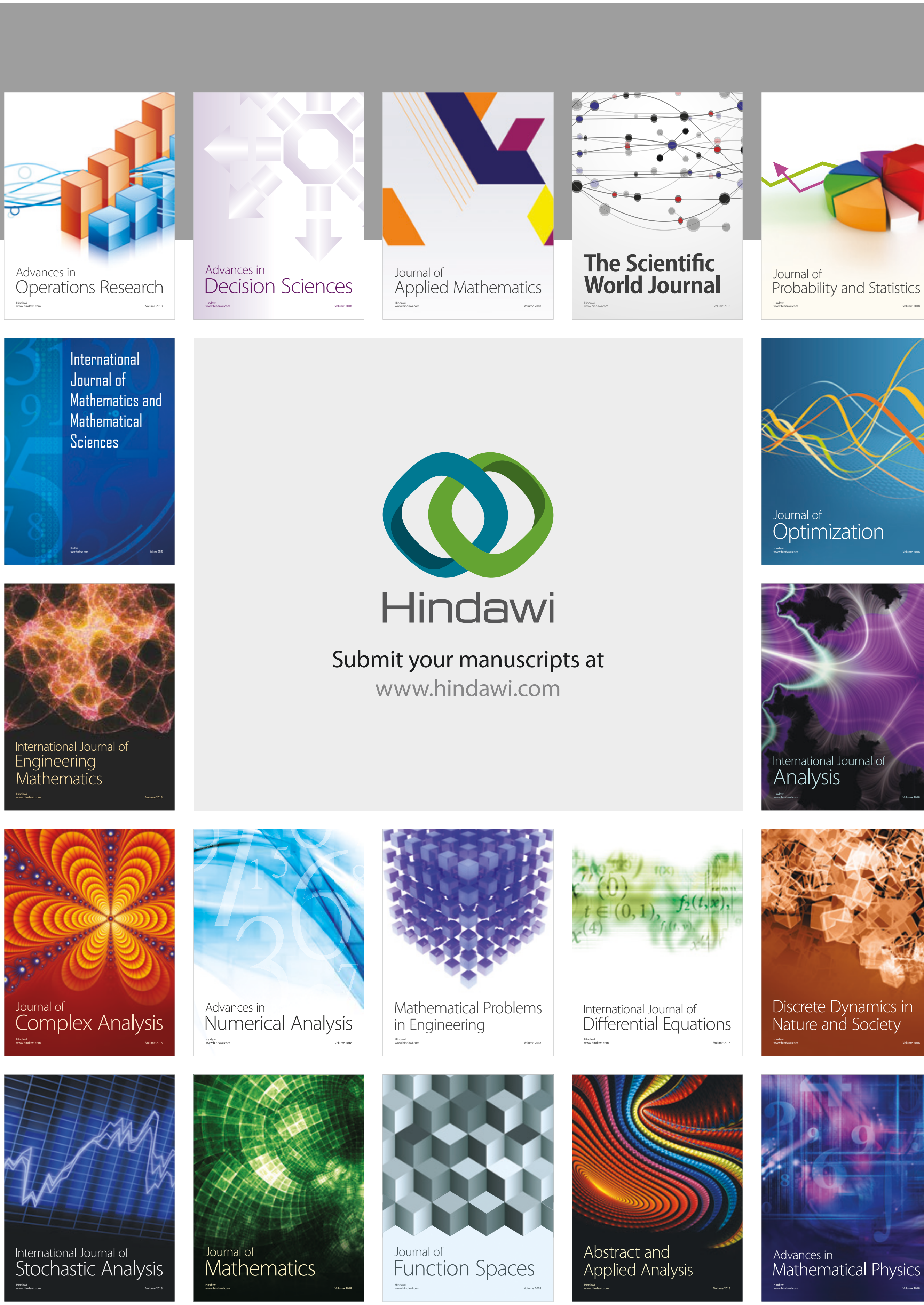
[23] L. Yu, *Robust control-linear matrix inequality processing*, Tsinghua University Press, Beijing, 2002.

[24] Y. Jia, “Alternative proofs for improved LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty: A predictive approach,” *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 48, no. 8, pp. 1413–1416, 2003.

[25] S. K. Nguang, P. Shi, and S. Ding, “Delay-dependent fault estimation for uncertain time-delay nonlinear systems: an LMI approach,” *International Journal of Robust and Nonlinear Control*, vol. 16, no. 18, pp. 913–933, 2006.

[26] F. Yang and Y. Li, “Set-membership filtering for systems with sensor saturation,” *Automatica*, vol. 45, no. 8, pp. 1896–1902, 2009.

[27] Y. Jia, “Robust control with decoupling performance for steering and traction of 4WS vehicles under velocity-varying motion,” *IEEE Transactions on Control Systems Technology*, vol. 8, no. 3, pp. 554–569, 2000.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
The Scientific World Journal
Journal of Probability and Statistics
International Journal of Mathematics and Mathematical Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of Optimization
International Journal of Engineering Mathematics
International Journal of Analysis
Journal of Complex Analysis
Advances in Numerical Analysis
Mathematical Problems in Engineering
International Journal of Differential Equations
Discrete Dynamics in Nature and Society
International Journal of Stochastic Analysis
Journal of Mathematics
Journal of Function Spaces
Abstract and Applied Analysis
Advances in Mathematical Physics