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Research Article

Multiperiod Production and Ordering Policies for a Retailer-Led Supply Chain through Option Contracts

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This paper formulates two groups of multiperiod production and ordering models with call and bidirectional option contracts for a two-party supply chain consisting of one followed supplier and one dominant retailer, respectively. Based on dynamic programming theory, we characterize the optimal policy structures for two partners in each period. We also provide an approximation for the corresponding policy parameters evaluation in two cases. Then, we investigate the impacts of different option contracts and the demand risk on the decisions and performances of two members. Our results suggest that, whether concerning call or bidirectional option contracts, the optimal policies for two members always follow a base stock type. When the price parameters are the same for different option contracts, the service levels of both the system and the retailer are higher with call option contracts than with bidirectional ones, whereas the retailer's inventory risk is lower with bidirectional option contracts than with call ones. Under the same conditions stated above, call option contracts can always benefit the supplier, but not the retailer. Owing to the retailer's dominant position, call option contracts are better choice for the supply chain if the option (exercise) price is low (high), while bidirectional option contracts are more suitable choice for the supply chain if the option (exercise) price is high (low). In addition, an increase in the demand risk would prompt the supplier to increase his production quantity and the retailer to reduce the initial firm order quantity, either with call or bidirectional option contracts.

1. Introduction

The past several decades saw the emergence of the dominant retailers, who turn into the major and largest distributors of the suppliers. For example, in the retailing industry, a double-digit percentage of total sales for large-scale producers including P&G and Tandy are conducted through Walmart, one of the largest supermarkets [1, 2]. In the Chinese consumer electronics market, Gome and Suning, two leading companies, have accounted for a growing proportion of sales volume after experiencing an unprecedented development [3]. This phenomenon suggests that the bargaining power is shifting into the downstream. For this reason, the vulnerable suppliers are often coerced to make concessions such as the price dropping and the delivery time reduction to the dominant retailers [4]. The transfer of the leadership deeply affects the decision-making of the players, which differs from the situation with dominant suppliers [5, 6]. Thereby, it has become a significant research field against the background of increasing practical and academic concerns about the retailer-led supply chain.

In many practical settings, there is a usual phenomenon that the sales season experiences multiple periods, even for the perishable products [7]. This phenomenon indicates that the decision-making problems are often characterized by a multiperiodic structure. Different from the single-period model, the correlation between two successive decisions cannot be neglected in the multiperiod model. As the planning horizon is divided into larger numbers of conjoint periods, the optimization process is more and more sophisticated. In recent years, the multiperiod inventory problem for one single firm has gradually become a hot topic, thereby attracting extensive interest of researchers [8–10]. However, very few papers (e.g., [11]) have discussed the related problem from the

perspective of the chain. So far, the retailer-led supply chain management has often been discussed in the single-period setting. However, the multiperiod situation has never been considered. A proper balance between realism and model is struck by this setting.

The implementation of the contract mechanism is a crucial measure to strongly support the efficient operation of the retailer-led supply chain. Owing to the flexibility and risk-sharing attribute, option contracts, originated from financial derivatives, have attracted considerable attention in supply chain management research. In practice, option purchasing practice has been widely applied in various industries such as fresh food, fashion apparel, and electronics. For example, option contracts have been aggressively applied by Hewlett-Packard for 35% of procurement value [12]. In China, flower options have been launched by flower operation firms to the global customers [13]. In generally, there is a variety of different categories in option contracts. One of the most popular types is call option contracts and the other is bidirectional option contracts. For the dominant retailer, the flexibility of adjusting the inventory level upwards is provided by call option contracts while the two-way flexibility is provided by bidirectional option contracts. For the followed supplier, whether concerning call or bidirectional option contracts, partial incomes can be obtained in advance through the sales of the options so that he can arrange the production more rationally. Interestingly, there is an apparent difference relative to the impact of different option contracts on the decisions and performances of the members in the retailer-led supply chain. However, so far, the single-period problem has been unaddressed and let alone discussing the multiperiod problem.

This research is very interesting in that it deals with the multiperiod production and ordering problem for a retailer-led supply chain with call and bidirectional option contracts, respectively. As shown in Section 2, a fair amount of studies on the multiperiod inventory problem for one single firm and the single-period retailer-led supply chain have been reported over the past decade. It is also well known that option contracts indeed contribute significant benefit to the efficient operation of the single-period supplier-led supply chain. However, so far, few studies have developed the multiperiod decision-making models for the supply chain with a dominant retailer and option contracts. Less frequently, the researchers have looked at the impact of different option contracts on the multiperiod retailer-led supply chain. To fulfill this gap, this paper deals with several crucial questions as follows:

(1) What is the optimal production policy for the followed supplier in the multiperiod setting, either with call or bidirectional option contracts?

(2) What is the optimal ordering policy for the dominant supplier in the multiperiod setting, either with call or bidirectional option contracts?

(3) How to develop an approximate algorithm to evaluate the policy parameters, either with call or bidirectional option contracts?

(4) What effect do different option contracts have on decisions and performances of the members in the retailer-led supply chain?

(5) How does the demand risk influences the decisions and performances of the members in the retailer-led supply chain, either with call or bidirectional option contracts?

To answer these above questions, given the dynamic game between the members in the retailer-led supply chain, we formulate two groups of optimality equation for the followed supplier and the dominant retailer with call and bidirectional option contracts, respectively. Based on stochastic dynamic programming, we characterize the optimal policy structures for two members in each period. Then, we provide an approximate algorithm to evaluate the corresponding policy parameters in two cases. Specifically, we explore the impact of different option contracts and the impact of demand risk on the decisions and performances of two members.

The contribution of our research is twofold. The first part formulates two groups of the multiperiod production-inventory models for a two-stage supply chain under two different option contracts, where the retailer is the leader and the supplier is the follower. It is distinctly different from most current research relative to the multiperiod inventory problem for a single company by analyzing the operational strategies of two members in the supply chain. Besides that, our research extends the existing literature on the single-period supply chain either with a dominant retailer or option contracts by looking at the multiperiod situation. In the second part, a fair comparison is conducted to obtain more meaningful findings relative to the role of different option contracts in the decisions and performances of the members in the retailer-led supply chain under the multiperiod situation. This complements the current research that often takes only one type of option contracts into account. A more comprehensive research system has been developed in our paper.

We organize the remainder of this paper as follows: After reviewing the related literature in Section 2, problem description is provided in Section 3. Two groups of multiperiod models for the production and ordering with call and bidirectional option contracts are formulated in Sections 4 and 5, respectively. We explore the role of different option contracts in and the impact of demand risk on the decisions and performances of both partners in Section 6. Finally, we conclude the findings and provide some future work in Section 7.

2. Literature Review

The literature reviewed here is primarily presented in three important streams: (1) multiperiod inventory model, (2) the retailer-Stackelberg game model, and (3) option contracts in supply chain management.

Recently, the multiperiod inventory problem has successfully drawn the attention of more and more researchers. Chao et al. [14] analyze the joint ordering and pricing policies for a multiperiod inventory that faces price-sensitive demand and stochastic supply capacity. Chen et al. [9] characterize the optimal policy structure for a period-review inventory with

and without fixed order costs, where demand is dependent on the inventory level and the shortage is satisfied by the emergency replenishment. By using $L^{\#}$-convexity approach, Chen et al. [15] consider the combined pricing and inventory problem for a multiperiod inventory with linear ordering cost and price-sensitive demand. Yang et al. [16] formulate the multiperiod model for a periodic-review inventory, where the additive demand function, fixed ordering cost, and price adjustment cost are considered. By incorporating customer behavior and reference price, Song et al. [17] identify optimal pricing and inventory decisions for a multiperiod system. All these above papers mainly focus on the multiperiod decision-making from the perspective of one single firm. So far, very few studies (e.g., [11]) have been carried out from the perspective of the chain. By using game theoretic approach, they analyze the multiperiod production and ordering problem with a dominant supplier and without option contracts.

In practice, the members in the supply chain usually have different bargaining powers. Power structure plays an important role in the decision-making process, consequently affecting the participant's profit margin [18]. Game theory is the most applicable approach to model the relationships between the members in the supply chain [19, 20]. When the supplier is more powerful than the retailer, the resulting game is called the supplier-Stackelberg game [21]. Sometimes, the supplier and the retailer have the same power and they play a vertical Nash game [22]. When the retailer is in a stronger competitive position, the retailer needs the best response of the supplier and the members play the retailer-Stackelberg game [23]. Because our focal point is on the case with the dominant retailer, this paper focuses on the retailer-Stackelberg game.

In the past two decades, there were a great number of papers on the retailer-Stackelberg game model. For example, Raju and Zhang [24] demonstrate that quantity discount scheme and two-part tariff scheme cannot be served as effective mechanisms to coordinate the retailer-led supply chain with considering fixed advertising service costs. Geylani et al. [25] investigate how a manufacturer sets a higher wholesale price for the weak retailer in order to respond strategically to the dominant retailer. Almehdawe and Mantin [26] consider the profit maximization problems for a VMI supply chain consisting of one manufacturer and multiple retailers. They consider two different systems: one is controlled by the manufacturer and the other is dominated by one of the retailers. Li et al. [27] design revenue-sharing contracts to coordinate a supply chain controlled by a dominant retailer and explore how the coordination mechanism is influenced after demand disruption. By applying Stackelberg game theoretic approach, Taleizadeh and Noori-Daryan [28] explore the optimal pricing, manufacturing, and inventory policies for a three-level supply chain including a supplier, a producer, and some retailers, where the retailers and the producer act as the leaders in the first and second stages, the demand is sensitive to the price, and the shortage is not permitted. Noori-daryan et al. [20] analyze the combined pricing and replenishment problem for a supply chain consisting of one manufacturer and multiple retailers under blend usage of quantity and freight discount contracts and free shipping contracts, in which the retailers act as the leader, different transportation modes are considered, and the transportation capacity is limited. More related studies can be found in Taleizadeh et al. [29] and so on. From the above literature, we realize that the problem on the retailer-led supply chain can be analyzed from different perspectives. However, all of these above papers focus mainly on the single-period situation. To the best of our knowledge, there are no relevant studies involving the multiperiod retailer-led supply chain management.

Over the past decade, there were plenty of literature sources involving option contracts in supply chain management. The paper written by Barnes-Schuster et al. [30] has been recognized as the first research that introduces option contracts into the supply chain field. They analyze the ordering, production, and coordination problem for a two-period supply chain, where demands across two periods are correlated. Since then, this topic is increasingly popular and attracts substantial attention. Early related research mainly focuses on call option contracts, which offer the right of reserving some units of the extra goods. A great number of typical studies explore the role of call option contracts on the supply chain from different perspectives such as demand information updating [31], service level constraint [12], capital constraint [32], decision preference [33], and random yield [34, 35]. Another type is known as put option contracts, which offer the right of returning some units of the ordered goods. So far, only a few papers have involved this research field. Chen and Parlar [36] consider the role of put option contracts in the replenishment policy of a risk-aversion newsvendor. Xue et al. [37] apply the CVaR downside risk measure to explore the optimal ordering policy for a newsvendor with put option contracts. Wang and Chen [38] analyze the joint ordering and price problem for a newsvendor that faces the additive demand in the presence of put option contracts. Recently, several studies have started paying close attention to bidirectional option contracts, which allow for adjusting the retailer's inventory in two directions. Wang and Tsao [39] study the buyer's optimal ordering policy with bidirectional option contracts, where the demand is assumed to be uniformly distributed. As an extension of Wang and Tsao [39], Zhao et al. [40] consider the ordering and coordination problem of a supply chain that faces stochastic demand in the presence of bidirectional option contracts. Chen et al. [42] study the impact of bidirectional option contracts and service level constraints on the decisions and performances of a supplier-retailer supply chain.

In recent years, there are a few existing literature sources relative to different option contracts simultaneously. Liu et al. [43] analyze the optimal solution for the carrier with call, put, and bidirectional option contracts and then discuss the application strategies of different option contract. Nosoohi and Nookabadi [44] consider the outsourcing decisions with call, put, and bidirectional option contracts for the manufacturer who faces stochastic finial processing cost. They highlight the advantage of option contracts over wholesale price contracts. At the same time, they also analyze the impact of different option contracts on the manufacturer. Wan and

Chen [45] explore the role of call, put, and bidirectional option contracts in the decisions and performances of a supply chain in an inflationary setting. Li et al. [46] develop the production and ordering models for seasonal product supply chain with call and bidirectional option contracts when the demand and production yield are stochastic. However, most of the above papers relative to option contracts, whether concerning one single firm or the whole chain, only consider the single-period situation while neglecting the multiperiod situation.

Wang and Liu [4] have conducted the research that is the closest to our work. In their paper, the single-period production and ordering decisions for two members are analyzed under blend usage of wholesale price contracts and call option contracts, also based on the assumption that the market dominant position has been occupied by the retailer. There exist several noticeable distinctions between their paper and ours. First, their study only considers the single-period situation. However, our work directly extends theirs by considering the multiperiod situation. Moreover, their study focuses on the case with call option contracts. However, we formulate two groups of multiperiod game theoretic models with call and bidirectional option contracts for the followed supplier and the dominant retailer, respectively. In addition, by conducting the mutual comparisons, we investigate the role of different option contracts in the decisions and performances of both partners. Finally, their paper provides the conditions on which the supply chain is coordinated. However, due to the complexity of the multiperiod optimization process, we do not discuss the Stackelberg equilibrium solution.

The brief review of the related literature is shown in Table 1.

3. Problem Description

Consider a two-party supply chain where one supplier and one retailer are the members. The supplier manufactures the goods and distributes them via the retailer to the end customers. The retailer is more powerful and acts as the leader of the market. The whole time horizon is divided into n different and connected periods. The supplier needs to decide his production quantity for each period to obtain the highest profitability over multiple periods. The retailer needs to decide her order quantity for each period to maximize her expected total discounted profit across different periods.

In this research, two contract mechanisms, namely, call and bidirectional option contracts, are applied between the supply chain members, respectively. Call option contracts provide the retailer with the capacity of adjusting the inventory upwards. However, bidirectional option contracts provide the retailer with the capacity of adjusting the inventory either upwards or downwards. Both of them include two parameters. The option price is an upfront allowance that is paid by the retailer for purchasing one unit of option. However, the exercise price is regarded as the main difference in the price parameters of these two contracts. For call option contracts, the exercise price is the payment that is paid to the supplier for exercising one unit of option. For bidirectional option contracts, the exercise price is the payment that is paid by the retailer if the bidirectional option is exercised as the call one; otherwise, the exercise price is the refund that is claimed from the supplier if the bidirectional option is exercised as the put one.

The timeline of the decisions for the multiperiod retailer-Stackelberg game models with option contracts is described as follows. Before stochastic demand in each period t has been observed, the retailer offers the supplier blend usage of wholesale price contracts and option contracts. Based on the negotiated parameters, an initial firm order is placed by the retailer. Upon receipt of the contract mechanism, the supplier decides the production quantity. The supplier's reservation inventory, which equals the production quantity minus the initial firm order quantity, will be purchased by the retailer as her options order. When the sales in each period t begin, the initial firm order is distributed to the retailer immediately. After stochastic demand in each period t has been observed, the options are exercised by the retailer based on the actual realized demand. When the sales in each period t end, unsatisfied demand is backlogged and surplus goods are used for the next period. When the last period ends, the leftover owned by either the supplier or the retailer can be disposed at a unit salvaged value.

The main goal of this paper is to determine the production and ordering decisions for the followed supplier and the dominate retailer such that their objective functions are optimized. The target inventory levels after production and replenishment are the decision variables of the supplier and the retailer for the corresponding multiperiod models, respectively. Though mathematical transformations, we can derive the decisions of two members. The parameters and variables for model development are shown in Notations at the end of the paper.

In addition, the proposed model is formulated under the following assumptions:

(1) The demands in different periods are stochastic and independent.

(2) Similar to Li et al. [46], the exercise price is assumed to be the same for the call and put option under bidirectional option contracts.

(3) $c_t - h_t > o_t$. This condition avoids the possibility that the supplier may arbitrage with the options.

(4) $o_t + e_t > w_t > o_t$. This condition avoids the irrationality of call option contracts.

(5) $w_t + o_t > e_t > w_t - o_t$. This condition ensures the feasibility of bidirectional option contracts.

(6) $e_t + h_t > \alpha c_{t+1}$. This condition avoids the likelihood that the supplier will deliberately manufacture more goods than necessary.

(7) $e_t + h_t > \alpha w_{t+1}$. This condition ensures that more options than necessary are never purchased by the retailer.

Throughout the paper, the subscripts "s" and "r" represent "supplier" and "retailer," while the subscript "t" denotes one particular period over the finite time horizon. In addition, the superscripts "C" and "B" stand for "call" and "bidirectional option contracts." Notation $x^+ = \max(0, x)$.

Table 1: Brief literature review.

Ref.	Period		Perspective			Contracts			
				Supply chain		Option contracts			
	Single period	Multiple periods	Single firms	Supplier-led	Retailer-led	Call	Put	Bidirectional	Other contracts
Chao et al. [14]		√	√						
Chen et al. [9]		√	√						
Chen et al. [15]		√	√						
Yang et al. [16]		√	√						
Song et al. [17]		√	√						
Li et al. [11]		√		√					√
Raju and Zhang [24]	√				√				√
Geylani et al. [25]	√				√				√
Lau et al. [47]	√				√				√
Almehdawe and Mantin [26]	√			√	√				√
Li et al. [27]	√				√				√
Taleizadeh et al. [29]	√				√				√
Taleizadeh and Noori-Daryan [28]	√				√				√
Noori-daryan et al. [20]	√				√				√
Barnes-Schuster et al. [30]		√		√		√			
Wang and Liu [4]	√				√	√			
Buzacott et al. [31]	√			√		√			
Chen and Shen [12]	√			√		√			
Feng et al. [32]	√			√		√			
Liu et al. [33]	√			√		√			
Luo and Chen [34]	√			√		√			
Cai et al. [35]	√			√		√			
Chen and Parlar [36]	√			√			√		
Xue et al. [37]	√			√			√		
Wang and Chen [38]	√			√			√		
Wang and Tsao [39]	√			√				√	
Zhao et al. [40]	√			√				√	
Chen et al. [41]	√			√				√	
Liu et al. [43]	√			√		√	√	√	
Nosoohi and Nookabadi [44]	√			√		√	√	√	
Wan and Chen [45]	√			√		√	√	√	
Li et al. [46]	√			√		√		√	

4. Model with Call Option Contracts

In this section, we deal with the multiperiod production and ordering problems for a followed supplier and a dominant retailer in the presence of call option contracts.

4.1. Supplier's Multiperiod Production Problem. With call option contracts, the expected total discounted profit of the supplier over the entire time horizon, denoted by Π_s^C, is

$$\begin{aligned}\Pi_s^C = \sum_{t=1}^{n}\alpha^{t-1}\Big\{&w_t\left(y_{rt}^C - x_{rt}^C\right) + o_t^C\left(y_{st}^C + x_{rt}^C - y_{rt}^C\right)\\&+ e_t^C E\left(\min\left[y_{st}^C + x_{rt}^C - y_{rt}^C, \left(D_t - y_{rt}^C\right)^+\right]\right)\\&- c_t\left(y_{st}^C - x_{st}^C\right)\\&- h_t E\left(\left[\left(y_{st}^C + x_{rt}^C - y_{rt}^C\right) - \left(D_t - y_{rt}^C\right)^+\right]^+\right)\Big\}\\&+ \alpha^n c_{n+1} x_{sn+1}^C.\end{aligned} \tag{1}$$

In (1), $y_{st}^C - x_{st}^C$ denotes the supplier's production quantity in period t, $y_{rt}^C - x_{rt}^C$ denotes the retailer's initial firm order quantity in period t, and $y_{st}^C + x_{rt}^C - y_{rt}^C$ denotes the supplier's reservation inventory in period t.

$V_{st}^C(x_s^C)$ denotes the supplier's profit-to-go function with call option contracts, providing his original inventory is x_s^C at the beginning of period t. The subscript t is skipped here to avoid the possible confusion. The dynamic program for optimizing (1) can be developed as follows:

$$V_{st}^C\left(x_s^C\right) = c_t x_s^C + \max_{y_s^C \geq \max\{x_s^C, y_{rt}^C - x_{rt}^C\}}\left\{P_{st}^C\left(y_s^C\right)\right\}, \tag{2}$$

where $V_{sn+1}^C(x_s^C) = c_{n+1}x_s^C$ $(x_s^C \geq 0)$ and

$$\begin{aligned}P_{st}^C\left(y_s^C\right) &= H_{st}^C\left(y_s^C\right)\\&\quad + \alpha E V_{st+1}^C\left\{\left[\left(y_s^C + x_{rt}^C - y_{rt}^C\right) - \left(D_t - y_{rt}^C\right)^+\right]^+\right\}\\H_{st}^C\left(y_s^C\right) &= w_t\left(y_{rt}^C - x_{rt}^C\right) + o_t^C\left(y_s^C + x_{rt}^C - y_{rt}^C\right)\\&\quad + e_t^C E\left\{\min\left[y_s^C + x_{rt}^C - y_{rt}^C, \left(D_t - y_{rt}^C\right)^+\right]\right\}\\&\quad - c_t y_s^C\\&\quad - h_t E\left\{\left[\left(y_s^C + x_{rt}^C - y_{rt}^C\right) - \left(D_t - y_{rt}^C\right)^+\right]^+\right\}.\end{aligned} \tag{3}$$

Here, $P_{st}^C(y_s^C)$ is the supplier's expected total discounted profit over periods $t, \ldots, n$, when his initial inventory is x_s^C at the beginning of period t. $H_{st}^C(y_s^C)$ is the supplier's expected profit with initial inventory x_s^C in period t.

Denote $Y_s^C = y_s^C + x_{rt}^C$ as a new decision variable. Let $\overline{V}_{st}^C(x_s^C) = V_{st}^C(x_s^C) - c_t x_s^C$; the recursion equations (2) and (3) are equivalently reformulated as follows:

$$\overline{V}_{st}^C\left(x_s^C\right) = \max_{Y_s^C \geq \max\{x_s^C + x_{rt}^C, y_{rt}^C\}}\left\{\overline{P}_{st}^C\left(Y_s^C\right)\right\}, \tag{4}$$

where $\overline{V}_{sn+1}^C(x_s^C) = 0$ $(x_s^C \geq 0)$ and

$$\begin{aligned}\overline{P}_{st}^C\left(Y_s^C\right) &= \overline{H}_{st}^C\left(Y_s^C\right)\\&\quad + \alpha E \overline{V}_{st+1}^C\left\{\left[\left(Y_s^C - y_{rt}^C\right) - \left(D_t - y_{rt}^C\right)^+\right]^+\right\}\end{aligned} \tag{5}$$

$$\begin{aligned}\overline{H}_{st}^C\left(Y_s^C\right) &= \left(w_t - c_t\right)\left(y_{rt}^C - x_{rt}^C\right) + o_t^C\left(Y_s^C - y_{rt}^C\right)\\&\quad + e_t^C E\left\{\min\left[Y_s^C - y_{rt}^C, \left(D_t - y_{rt}^C\right)^+\right]\right\}\\&\quad - c_t\left(Y_s^C - y_{rt}^C\right)\\&\quad - \left(h_t - \alpha c_{t+1}\right) E\left\{\left[\left(Y_s^C - y_{rt}^C\right) - \left(D_t - y_{rt}^C\right)^+\right]^+\right\}.\end{aligned} \tag{6}$$

Here, $\overline{P}_{st}^C(Y_s^C)$ is the supplier's expected total discounted profit over periods $t, \ldots, n$, when his initial inventory is zero at the beginning of period t. $\overline{H}_{st}^C(Y_s^C)$ is the supplier's expected profit with zero initial inventory in period t.

Based on the above analysis, Proposition 1 follows.

Proposition 1. *With call option contracts, the supplier's optimal production quantity in period t, denoted by Q_{st}^{C*}, is*

$$Q_{st}^{C*} = \begin{cases} S_t^{C*} - x_{rt}^C - x_{st}^C & \text{if } \max\left\{x_{st}^C + x_{rt}^C, y_{rt}^C\right\} \leq S_t^{C*}\\ \left(y_{rt}^C - x_{rt}^C - x_{st}^C\right)^+ & \text{if } \max\left\{x_{st}^C + x_{rt}^C, y_{rt}^C\right\} > S_t^{C*},\end{cases} \tag{7}$$

where $S_t^{C*} \in \arg\max_{Y_s^C}\{\overline{P}_{st}^C(Y_s^C)\}$.

Proof. Equation (6) means that $\overline{H}_{st}^C(Y_s^C)$ is always concave in Y_s^C for all periods. In period n, as $\overline{V}_{sn+1}^C(x_s^C) = 0$, $\overline{P}_{sn}^C(Y_s^C)$ equals $\overline{H}_{sn}^C(Y_s^C)$ and thus is concave. In period $t = 1, \ldots, n-1$, as $\overline{V}_{st+1}^C\{[(Y_s^C - y_{rt}^C) - (D_t - y_{rt}^C)^+]^+\}$ is concave, $\overline{P}_{st}^C(Y_s^C)$ comprises two concave functions and thus is also concave. Thus, for all $t = 1, \ldots, n$, $\overline{V}_{st}^C(x_s^C)$ is a concave function of x_s^C. For this reason, with call option contracts the optimal solution for the decision variable Y_{st}^C in period t is

$$Y_{st}^{C*} = \begin{cases} S_t^{C*} & \text{if } \max\left\{x_{st}^C + x_{rt}^C, y_{rt}^C\right\} \leq S_t^{C*}\\ \max\left\{x_{st}^C + x_{rt}^C, y_{rt}^C\right\} & \text{if } \max\left\{x_{st}^C + x_{rt}^C, y_{rt}^C\right\} > S_t^{C*},\end{cases} \tag{8}$$

where S_t^{C*} is the optimum of Y_s^C that maximizes $\overline{P}_{st}^C(Y_s^C)$. Since $Q_{st}^{C*} = Y_{st}^{C*} - x_{rt}^C - x_{st}^C$, we can derive the above result directly. □

Proposition 1 shows that a base stock policy is optimal for the supplier's production in the framework of call option contracts. It is important to note that S_t^{C*}, namely, the optimal base stock level, stands for the system's inventory level to be raised after the goods have been manufactured in period t; meanwhile, it also represents the retailer's inventory level to be raised after all the call options have been exercised in period t. In addition, this proposition states that the supplier's decision in one particular period can be determined for the retailer's decision in that corresponding period.

As shown in Proposition 1, with call option contracts the supplier operates his optimal production strategy as follows. In each period t, before stochastic demand has been observed, if the larger one between the system's initial inventory $(x_{st}^C + x_{rt}^C)$ and the retailer's target inventory (y_{rt}^C) is at or below S_t^{C*}, the supplier should guarantee the system's inventory level raised up to S_t^{C*} after the goods have been produced; otherwise, more goods than the retailer's initial firm commitment cannot be produced by the supplier. The property of the threshold S_t^{C*} is given as follows.

Corollary 2. *For all $t = 1, \ldots, n$, (1) S_t^{C*} is increasing in o_t^C and e_t^C; (2) S_t^{C*} is increasing in y_{rt}^C.*

Proof. In part (1), (5) means that $\overline{P}_{st}^C(Y_s^C)$ is supermodular with respect to o_t^C and Y_s^C. Thus, S_t^{C*} is an increasing function of o_t^C. In addition, $\overline{P}_{st}^C(Y_s^C)$ is also supermodular with respect to e_t^C and Y_s^C. Thus, S_t^{C*} is an increasing function of e_t^C. Part (2) can be proved in the same approach. Here, we omit it for simplicity. □

Corollary 2 shows that a higher option and/or exercise price in period t will prompt the system to improve the service level in that period, as well as the retailer under the call option contract arrangement. In addition, this corollary states that the optimal base stock level S_t^{C*} increases as the order-up-to level y_{rt}^C is raised. This result, compared with Proposition 1, provides more detailed illustration on the correlation between the decisions of the partners in one specific period.

As pointed out in Li et al. [11] and Inderfurth et al. [48], the policy parameters are too difficult to be calculated accurately. Subsequently, a tractable approximation is provided for the solution evaluation. Denote S_t^{C+} as the near-optimal base stock level for the supplier as he maximizes his current expected profit while ignoring his future expected profit under the call option contract arrangement. Note that $S_t^{C*} < S_t^{C+}$. We get the following proposition.

Proposition 3. *For all $t = 1, \ldots, n$, $S_t^{C+} = F_t^{-1}((o_t^C + e_t^C - c_t)/(e_t^C + h_t - \alpha c_{t+1}))$.*

Proof. Rearranging (6) yields $\overline{H}_{st}^C(Y_s^C) = (w_t - c_t)(y_{rt}^C - x_{rt}^C) - (o_t^C + e_t^C - c_t)y_{rt}^C + (e_t^C + h_t - \alpha c_{t+1})E[(y_{rt}^C - D_t)^+] + (o_t^C + e_t^C - c_t)Y_s^C - (e_t^C + h_t - \alpha c_{t+1})E[(Y_s^C - D_t)^+]$. Solving the first and second order derivatives of $\overline{H}_{st}^C(Y_s^C)$ yields $d\overline{H}_{st}^C(Y_s^C)/dY_s^C = (o_t^C + e_t^C - c_t) - (e_t^C + h_t - \alpha c_{t+1})F_t(Y_s^C)$ and $d^2\overline{H}_{st}^C(Y_s^C)/d(Y_s^C)^2 = -(e_t^C + h_t - \alpha c_{t+1})f_t(Y_s^C) < 0$. Thus, $\overline{H}_{st}^C(Y_s^C)$ is concave in Y_s^C. Then, the near-optimal threshold S_t^{C+} is calculated from $d\overline{H}_{st}^C(Y_s^C)/dY_s^C = 0$. That is, $S_t^{C+} = F_t^{-1}((o_t^C + e_t^C - c_t)/(e_t^C + h_t - \alpha c_{t+1}))$. □

In addition to confirming the validity of Corollary 2, Proposition 3 shows the positive effect of the subsequent production cost on the current near-optimal base stock level in the presence of call option contracts, indicating the correlation between two successive periods in the multi-period model from another perspective. In addition, more interestingly, if $x_{st}^C + x_{rt}^C \leq y_{rt}^C$, owing to the separative property of the dynamic program, $\overline{P}_{st}^C(Y_s^C)$ can be treated as the summation of $\overline{H}_{st}^C(Y_s^C)$ and the optimal solution of $\overline{P}_{st}^C(Y_s^C)$ can be derived by maximizing $\overline{H}_{st}^C(Y_s^C)$. In this case, the supplier's near-optimal base stock policy can remain optimal as the period goes by. The result is in accordance with the relative findings in Wang and Liu [4], which focuses on the analogical problem under a single-period situation.

4.2. Retailer's Multiperiod Ordering Problem. With call option contracts, the expected total discounted profit of the retailer over the finite time horizon, denoted by Π_r^C, is

$$\begin{aligned}\Pi_r^C = \sum_{t=1}^{n} \alpha^{t-1} \Big\{ & p_t E\left[\min\left(Y_{st}^{C*}, D_t\right)\right] - w_t\left(y_{rt}^C - x_{rt}^C\right) \\ & - o_t^C\left(Y_{st}^{C*} - y_{rt}^C\right) \\ & - e_t^C E\left(\min\left[Y_{st}^{C*} - y_{rt}^C, \left(D_t - y_{rt}^C\right)^+\right]\right) \\ & - h_t E\left[\left(y_{rt}^C - D_t\right)^+\right] - g_t E\left[\left(D_t - Y_{st}^{C*}\right)^+\right]\Big\} \\ & + \alpha^n w_{n+1} x_{rn+1}^C.\end{aligned} \tag{9}$$

In (9), $y_{rt}^C - x_{rt}^C$ denotes the retailer's initial firm order quantity in period t and $Y_{st}^{C*} - y_{rt}^C$ denotes the supplier's reservation inventory, as the retailer's options order quantity, in period t.

Let $V_{rt}^C(x_r^C)$ be denoted as the retailer's profit-to-go function with call option contracts, providing her original inventory is x_r^C at the beginning of period t. The dynamic program for optimizing (9) can be developed as follows:

$$V_{rt}^C\left(x_r^C\right) = w_t x_r^C + \max_{y_r^C \geq x_r^C} \left\{P_{rt}^C\left(y_r^C\right)\right\}, \tag{10}$$

where $V_{rn+1}^C(x_r^C) = w_{n+1}x_r^C$ $(x_r^C \geq 0)$ and

$$P_{rt}^C\left(y_r^C\right) = H_{rt}^C\left(y_r^C\right) + \alpha E V_{rt+1}^C\left[\left(y_r^C - D_t\right)^+\right]$$

$$H_{rt}^C\left(y_r^C\right) = p_t E\left[\min\left(Y_{st}^{C*}, D_t\right)\right] - w_t y_r^C$$

$$
\begin{aligned}
&- o_t^C \left(Y_{st}^{C*} - y_r^C\right) \\
&- e_t^C E\left\{\min\left[Y_{st}^{C*} - y_r^C, \left(D_t - y_r^C\right)^+\right]\right\} \\
&- h_t E\left[\left(y_r^C - D_t\right)^+\right] \\
&- g_t E\left[\left(D_t - Y_{st}^{C*}\right)^+\right].
\end{aligned} \tag{11}
$$

Here, $P_{rt}^C(y_r^C)$ is the retailer's expected total discounted profit over periods $t, \ldots, n$, when her initial inventory is x_r^C at the beginning of period t. $H_{rt}^C(y_r^C)$ is the retailer's expected profit with initial inventory x_r^C in period t.

Let $\overline{V}_{rt}^C(x_r^C) = V_{rt}^C(x_r^C) - w_t x_r^C$, the following recursion equals (10) and (11):

$$
\overline{V}_{rt}^C\left(x_r^C\right) = \max_{y_r^C \geq x_r^C}\left\{\overline{P}_{rt}^C\left(y_r^C\right)\right\}, \tag{12}
$$

where $\overline{V}_{rn+1}^C(x_r^C) = 0$ $(x_r^C \geq 0)$ and

$$
\overline{P}_{rt}^C\left(y_r^C\right) = \overline{H}_{rt}^C\left(y_r^C\right) + \alpha E \overline{V}_{rt+1}^C\left[\left(y_r^C - D_t\right)^+\right] \tag{13}
$$

$$
\begin{aligned}
\overline{H}_{rt}^C\left(y_r^C\right) = {} & p_t E\left[\min\left(Y_{st}^{C*}, D_t\right)\right] - w_t y_r^C \\
& - o_t^C\left(Y_{st}^{C*} - y_r^C\right) \\
& - e_t^C E\left\{\min\left[Y_{st}^{C*} - y_r^C, \left(D_t - y_r^C\right)^+\right]\right\} \\
& - \left(h_t - \alpha w_{t+1}\right) E\left[\left(y_r^C - D_t\right)^+\right] \\
& - g_t E\left[\left(D_t - Y_{st}^{C*}\right)^+\right].
\end{aligned} \tag{14}
$$

Here, $\overline{P}_{rt}^C(y_r^C)$ is the retailer's expected total discounted profit over periods $t, \ldots, n$, when her initial inventory is zero at the beginning of period t. $\overline{H}_{st}^C(Y_s^C)$ is the retailer's expected profit with zero initial inventory in period t.

Based on the above analysis, Proposition 4 follows.

Proposition 4. *With call option contracts, the retailer's optimal initial firm order quantity in period t, denoted by Q_{rt}^{C*}, is*

$$
Q_{rt}^{C*} = \begin{cases} s_t^{C*} - x_{rt}^C & \text{if } x_{rt}^C \leq s_t^{C*} \\ 0 & \text{if } x_{rt}^C > s_t^{C*}, \end{cases} \tag{15}
$$

where $s_t^{C*} \in \arg\max_{y_r^C}\{\overline{P}_{rt}^C(y_r^C)\}$.

Proof. Equation (14) means that $\overline{H}_{rt}^C(y_r^C)$ is always concave in y_r^C for all periods. In period n, as $\overline{V}_{rn+1}^C(x_r^C) = 0$, $\overline{P}_{rn}^C(y_r^C)$ equals $\overline{H}_{rn}^C(y_r^C)$ and thus is concave. In period $t = 1, \ldots, n-1$, as $\overline{V}_{rt+1}^C[(y_r^C - D_t)^+]$ is concave, $\overline{P}_{rt}^C(y_r^C)$ comprises two concave functions and thus is also concave. Therefore, for all $t = 1, \ldots, n$, $\overline{V}_{rt}^C(x_r^C)$ is a concave function of x_r^C. For this reason, with call option contracts the optimal solution for the decision variable y_{rt}^C in period t is

$$
y_{rt}^{C*} = \begin{cases} s_t^{C*} & \text{if } x_{rt}^C \leq s_t^{C*} \\ x_{rt}^C & \text{if } x_{rt}^C \leq s_t^{C*}, \end{cases} \tag{16}
$$

where s_t^{C*} is the optimum of y_r^C that maximizes $\overline{P}_{rt}^C(y_r^C)$. Since $Q_{rt}^{C*} = y_{rt}^{C*} - x_{rt}^C$, the above result can be obtained immediately. □

Proposition 4 shows that the retailer's optimal ordering policy is of base stock type in the framework of call option contracts. It is significant to highlight that the optimal base stock level s_t^{C*} stands for the retailer's inventory level to be raised after the initial firm order has been received in period t.

As shown in Proposition 4, with call option contracts the retailer operates her optimal ordering strategy as follows. In each period t, before stochastic demand has been observed, if the retailer's starting inventory is at or below s_t^{C*}, the retailer should guarantee her inventory level raised up to s_t^{C*} when the initial firm order has been received; otherwise, the retailer's inventory level is still maintained at her original state. The property of the threshold s_t^{C*} is given as follows.

Corollary 5. *For all $t = 1, \ldots, n$, (1) s_t^{C*} is increasing in o_t^C and e_t^C; (2) s_t^{C*} is increasing in Y_{st}^{C*}.*

Proof. In part (1), according to (13), $\overline{P}_{rt}^C(y_r^C)$ is supermodular with respect to o_t^C and y_r^C. Thus, s_t^{C*} is an increasing function of o_t^C. In addition, $\overline{P}_{rt}^C(y_r^C)$ is also supermodular with respect to e_t^C and y_r^C. Thus, s_t^{C*} is an increasing function of e_t^C. Part (2) can be proved in the same approach. Here, we omit it for simplicity. □

Corollary 5 shows that a higher option and/or exercise price in period t will induce the retailer to bear higher inventory risk in that period under the call option contract arrangement. This result, combined with (1) in Corollary 2, indicates that s_t^{C*} represents the lower bound of the retailer's minimum committed inventory in period t whereas S_t^{C*} denotes the lower bound of the retailer's maximum committed inventory in period t under the call option contract arrangement. If there is a change in the option and/or exercise price, both thresholds will move towards the same direction. Moreover, this corollary states that the optimal base stock level s_t^{C*} increases as the order-up-to level Y_{st}^{C*} is raised. This result is not embodied in Proposition 4. However, it indicates that the retailer's purchasing decision in each period is indeed sensitive to the supplier's production decision in that corresponding period.

Denote s_t^{C+} as the near-optimal base stock level for the retailer as she maximizes her current expected profit while ignoring her future expected profit under the call option contract arrangement. Note that $s_t^{C*} < s_t^{C+}$. We get the following proposition.

Proposition 6. *For all* $t = 1, \ldots, n$, $s_t^{C+} = F_t^{-1}((o_t^C + e_t^C - w_t)/(e_t^C + h_t - \alpha w_{t+1}))$.

Proof. Rearranging (14) yields $\overline{H}_{rt}^C(y_r^C) = (p_t + g_t - o_t^C - e_t^C)Y_s^{C*} - g_t\mu_t - (p_t + g_t - e_t^C)E[(Y_s^{C*} - D_t)^+] + (o_t^C + e_t^C - w_t)y_{rt}^C - (e_t^C + h_t - \alpha w_{t+1})E[(y_{rt}^C - D_t)^+]$. Solving the first and second order derivatives of $\overline{H}_{rt}^C(y_r^C)$ yields $d\overline{H}_{rt}^C(y_r^C)/dy_r^C = (o_t^C + e_t^C - w_t) - (e_t^C + h_{rt} - \alpha w_{t+1})F_t(y_r^C)$ and $d^2\overline{H}_{rt}^C(y_r^C)/d(y_r^C)^2 = -(e_t^C + h_{rt} - \alpha w_{t+1})f_t(y_r^C) < 0$. Thus, $\overline{H}_{rt}^C(y_r^C)$ is concave in y_r^C. Then, the near-optimal threshold s_t^{C+} is calculated from $d\overline{H}_{rt}^C(y_r^C)/dy_r^C = 0$. That is, $s_t^{C+} = F_t^{-1}((o_t^C + e_t^C - w_t)/(e_t^C + h_t - \alpha w_{t+1}))$. □

Note that $S_t^{C+} > s_t^{C+}$ is equivalent to $o_t^C < ((w_t - c_t)(e_t^C + h_t - \alpha w_{t+1}) - \alpha(w_{t+1} - c_{t+1})(e_t^C - w_t))/\alpha(w_{t+1} - c_{t+1})$. This inequality indicates that if the option price is too high in period t, the call options order will be not placed by the retailer in that period. The cost parameters are assumed to satisfy this condition throughout this paper.

5. Model with Bidirectional Option Contracts

In this section, we address the multiperiod production and ordering problems for a followed supplier and a dominant retailer in the presence of bidirectional option contracts.

5.1. Supplier's Multiperiod Production Problem. With bidirectional option contracts, the expected total discounted profit of the supplier over the entire time horizon, denoted by Π_s^B, is

$$\begin{aligned}\Pi_s^B = \sum_{t=1}^{n}\alpha^{t-1}\Big\{&w_t\left(y_{rt}^B - x_{rt}^B\right) + o_t^B\left(y_{st}^B + x_{rt}^B - y_{rt}^B\right)\\ &+ e_t^B E\left\{\min\left[y_{st}^B + x_{rt}^B - y_{rt}^B, \left(D_t - y_{rt}^B\right)^+\right]\right\}\\ &- e_t^B E\left(\min\left[y_{st}^B + x_{rt}^B - y_{rt}^B, \left(y_{rt}^B - D_t\right)^+\right]\right) - c_t\left(y_{st}^B - x_{st}^B\right)\\ &- h_t E\left(\left[2\left(y_{st}^B + x_{rt}^B - y_{rt}^B\right) - \left(D_t - \left(2y_{rt}^B - y_{st}^B - x_{rt}^B\right)\right)^+\right]^+\right)\Big\}\\ &+ \alpha^n c_{n+1}x_{sn+1}^B.\end{aligned} \tag{17}$$

In (17), $y_{st}^B - x_{st}^B$ denotes the supplier's production quantity in period t, $y_{rt}^B - x_{rt}^B$ denotes the retailer's initial firm order quantity in period t, and $y_{st}^B + x_{rt}^B - y_{rt}^B$ denotes the supplier's reservation inventory in period t.

Let $V_{st}^B(x_s^B)$ be denoted as the supplier's profit-to-go function with bidirectional option contracts, providing his original inventory is x_s^B at the beginning of period t. The dynamic program for optimizing (17) can be developed as follows:

$$V_{st}^B\left(x_s^B\right) = c_t x_s^B + \max_{y_s^B \geq \max\{x_s^B, y_{rt}^B - x_{rt}^B\}}\left\{P_{st}^B\left(y_s^B\right)\right\}, \tag{18}$$

where $V_{sn+1}^B(x_s^B) = c_{n+1}x_s^B$ $(x_s^B \geq 0)$ and

$$\begin{aligned}P_{st}^B\left(y_s^B\right) = H_{st}^B\left(y_s^B\right) + \alpha E V_{st+1}^B\Big\{&\Big[2\left(y_s^B + x_{rt}^B - y_{rt}^B\right)\\ &- \left(D_t - \left(2y_{rt}^B - y_s^B - x_{rt}^B\right)\right)^+\Big]^+\Big\}\end{aligned}$$

$$\begin{aligned}H_{st}^B\left(y_s^B\right) = {}& w_t\left(y_{rt}^B - x_{rt}^B\right) + o_t^B\left(y_s^B + x_{rt}^B - y_{rt}^B\right)\\ &+ e_t^B E\left\{\min\left[y_s^B + x_{rt}^B - y_{rt}^B, \left(D_t - y_{rt}^B\right)^+\right]\right\}\\ &- e_t^B E\left\{\min\left[y_s^B + x_{rt}^B - y_{rt}^B, \left(y_{rt}^B - D_t\right)^+\right]\right\} - c_t y_s^B\\ &- h_t E\Big\{\Big[2\left(y_s^B + x_{rt}^B - y_{rt}^B\right)\\ &- \left(D_t - \left(2y_{rt}^B - y_s^B - x_{rt}^B\right)\right)^+\Big]^+\Big\}.\end{aligned} \tag{19}$$

Here, $P_{st}^B(y_s^B)$ is the supplier's expected total discounted profit over periods $t, \ldots, n$, when his initial inventory is x_s^B at the beginning of period t. $H_{st}^B(y_s^B)$ is the supplier's expected profit with initial inventory x_s^B in period t.

Define $Y_s^B = y_s^B + x_{rt}^B$ and $Y_{rt}^B = 2y_{rt}^B - y_s^B - x_{rt}^B$ as the new decision variables. Let $\overline{V}_{st}^B(x_s^B) = V_{st}^B(x_s^B) - c_t x_s^B$; the recursion equations (18) and (19) are equivalently reformulated as follows:

$$\overline{V}_{st}^B\left(x_s^B\right) = \max_{Y_s^B \geq \max\{x_s^B + x_{rt}^B, Y_{rt}^B\}}\left\{\overline{P}_{st}^B\left(Y_s^B\right)\right\}, \tag{20}$$

where $\overline{V}_{sn+1}^B(x_s^B) = 0$ $(x_s^B \geq 0)$ and

$$\begin{aligned}\overline{P}_{st}^B\left(Y_s^B\right) = {}& \overline{H}_{st}^B\left(Y_s^B\right) + \alpha E\overline{V}_{st+1}^B\Big\{\Big[\left(Y_s^B - Y_{rt}^B\right)\\ &- \left(D_t - Y_{rt}^B\right)^+\Big]^+\Big\}\end{aligned} \tag{21}$$

$$\begin{aligned}\overline{H}_{st}^B\left(Y_s^B\right) = {}& \left(w_t - c_t\right)\left(Y_{rt}^B - x_{rt}^B\right)\\ &+ \frac{\left(w_t - 2c_t\right)\left(Y_s^B - Y_{rt}^B\right)}{2} + \frac{o_t^B\left(Y_s^B - Y_{rt}^B\right)}{2}\\ &+ e_t^B E\left\{\min\left[\frac{\left(Y_s^B - Y_{rt}^B\right)}{2}, \left(D_t - \frac{\left(Y_s^B + Y_{rt}^B\right)}{2}\right)^+\right]\right\}\\ &- e_t^B E\left\{\min\left[\frac{\left(Y_s^B - Y_{rt}^B\right)}{2}, \left(\frac{\left(Y_s^B + Y_{rt}^B\right)}{2} - D_t\right)^+\right]\right\}\\ &- \left(h_t - \alpha c_{t+1}\right)E\left\{\left[\left(Y_s^B - Y_{rt}^B\right) - \left(D_t - Y_{rt}^B\right)^+\right]^+\right\}.\end{aligned} \tag{22}$$

Here, $\overline{P}_{st}^B(Y_s^B)$ is the supplier's expected total discounted profit over periods $t, \ldots, n$, when his initial inventory is zero at the beginning of period t. $\overline{H}_{st}^B(Y_s^B)$ is the supplier's expected profit with zero initial inventory in period t.

Based on the above analysis, Proposition 7 follows.

Proposition 7. *With bidirectional option contracts, the supplier's optimal production quantity in period t, denoted by* Q_{st}^{B*}, *is*

$$Q_{st}^{B*} = \begin{cases} S_t^{B*} - x_{rt}^B - x_{st}^B & \text{if } \max\left\{x_{st}^B + x_{rt}^B, Y_{rt}^B\right\} \leq S_t^{B*}\\ \left(Y_{rt}^B - x_{rt}^B - x_{st}^B\right)^+ & \text{if } \max\left\{x_{st}^B + x_{rt}^B, Y_{rt}^B\right\} > S_t^{B*},\end{cases} \tag{23}$$

where $S_t^{B*} \in \arg\max_{Y_s^B}\{\overline{P}_{st}^B(Y_s^B)\}$.

Proof. Equation (22) means that $\overline{H}^B_{st}(Y^B_s)$ is always concave in Y^B_s for all periods. In period n, as $\overline{V}^B_{sn+1}(x^B_s) = 0$, $\overline{P}^B_{sn}(Y^B_s)$ equals $\overline{H}^B_{sn}(Y^B_s)$ and thus is concave. In period $t = 1, \ldots, n-1$, as $\overline{V}^B_{st+1}\{[(Y^B_s - Y^B_{rt}) - (D_t - Y^B_{rt})^+]^+\}$ is concave, $\overline{P}^B_{st}(Y^B_s)$ comprises two concave functions and thus is also concave. Thus, for all $t = 1, \ldots, n$, $\overline{V}^B_{st}(x^B_s)$ is a concave function of x^B_s. For this reason, with bidirectional option contracts the optimal solution for the decision variable Y^B_{st} in period t is

$$Y^{B*}_{st} = \begin{cases} S^{B*}_t & \text{if } \max\left\{x^B_{st} + x^B_{rt}, Y^B_{rt}\right\} \le S^{B*}_t \\ \max\left\{x^B_{st} + x^B_{rt}, Y^B_{rt}\right\} & \text{if } \max\left\{x^B_{st} + x^B_{rt}, Y^B_{rt}\right\} \le S^{B*}_t, \end{cases} \quad (24)$$

where S^{B*}_t is the optimum of Y^B_s that maximizes $\overline{P}^B_{st}(Y^B_s)$. Since $Q^{B*}_{st} = Y^{B*}_{st} - x^B_{rt} - x^B_{st}$, we can derive the above result directly. □

Proposition 7 shows that a base stock policy is optimal for the supplier's production in the framework of bidirectional option contracts. It is worth noting that S^{B*}_t, namely, the optimal base stock level, denotes the system's inventory level to be raised after the goods have been manufactured in period t; at the same time, it also represents the retailer's inventory level to be raised after all the bidirectional options have been fully exercised as the call ones in period t. In addition, this proposition states that the supplier can determine his decision in a particular period for the retailer's decision in that corresponding period.

As shown in Proposition 7, the supplier should guarantee the system's inventory level in period t to be not less than S^{B*}_t when all the bidirectional options take effect just as the call ones in that period under the bidirectional option contract arrangement. This result, compared with Proposition 1, states that the supplier's optimal production strategy with bidirectional option contracts is similar to those with call ones. This result also suggests that there exists very slight distinction between these two different cases. The unique difference stems from the lower bound constraint of the decision variable Y^B_s. The property of the threshold S^{B*}_t is given as follows.

Corollary 8. *For all $t = 1, \ldots, n$, (1) S^{B*}_t is increasing in o^B_t and w_t; (2) S^{B*}_t is increasing in Y^B_{rt}.*

Proof. In part (1), (21) means that $\overline{P}^B_{st}(Y^B_s)$ is supermodular with respect to o^B_t and Y^B_s. Thus, S^{B*}_t is an increasing function of o^B_t. In addition, $\overline{P}^B_{st}(Y^B_s)$ is also supermodular with respect to w_t and Y^B_s. Thus, S^{B*}_t is an increasing function of w_t. Part (2) can be proved in the same approach. Here, we omit it for simplicity. □

Corollary 8 shows that a higher option price and/or wholesale price in period t will motivate both the system and the retailer to raise the service level in that period under the bidirectional option contract arrangement. In addition, this corollary states that the optimal base stock level S^{B*}_t increases as the order-up-to level Y^B_{rt} is raised. This result, compared with Proposition 7, states that the sensitivity of the supplier's decision in each period is dependent on the retailer's decision in the corresponding period from another perspective.

Denote S^{B+}_t as the near-optimal base stock level for the supplier as he maximizes his current expected profit while ignoring his future expected profit under the bidirectional option contract arrangement. Note that $S^{B*}_t < S^{B+}_t$. Then, we get the following proposition.

Proposition 9. *For all $t = 1, \ldots, n$, $S^{B+}_t = F^{-1}_t[(o^B_t + e^B_t + w_t - 2c_t)/2(e^B_t + h_t - \alpha c_{t+1})]$.*

Proof. Rearranging (22) yields $\overline{H}^B_{st}(Y^B_s) = (w_t - c_t)(Y^B_{rt} - x^B_{rt}) - (o^B_t + e^B_t + w_t - 2c_t)Y^B_{rt}/2 + (e^B_t + h_t - \alpha c_{t+1})E[(Y^B_{rt} - D_t)^+] + (o^B_t + e^B_t + w_t - 2c_t)Y^B_s/2 - (e^B_t + h_t - \alpha c_{t+1})E[(Y^B_s - D_t)^+]$. Solving the first and second order derivatives of $\overline{H}^B_{st}(Y^B_s)$ yields $d\overline{H}^B_{st}(Y^B_s)/dY^B_s = (o^B_t + e^B_t + w_t - 2c_t)/2 - (e^B_t + h_t - \alpha c_{t+1})F_t(Y^B_s)$ and $d^2\overline{H}^B_{st}/d(Y^B_s)^2 = -(e^B_t + h_t - \alpha c_{t+1})f_t(Y^B_s) < 0$. Thus, $\overline{H}^B_{st}(Y^B_s)$ is concave in Y^B_s. Then, the near-optimal threshold S^{B+}_t is calculated from $d\overline{H}^B_{st}(Y^B_s)/dY^B_s = 0$. That is, $S^{B+}_t = F^{-1}_t[(o^B_t + e^B_t + w_t - 2c_t)/2(e^B_t + h_t - \alpha c_{t+1})]$. □

In addition to confirming the validity of Corollary 2, Proposition 9 shows that as the subsequent production cost increases, the supplier's current near-optimal base stock level will increase in the presence of bidirectional option contracts. In addition, this proposition indicates that as the period goes by, the supplier's near-optimal base stock policy can remain optimal under the condition of $x^B_{st} + x^B_{rt} \le y^B_{rt}$. Finally, this result, compared with Proposition 3, states that the analytic expressions for the supplier's near-optimal production policy is more complex with bidirectional option contracts than with call ones.

5.2. Retailer's Multiperiod Ordering Problem. With bidirectional option contracts, the expected total discounted profit of the retailer over the finite time horizon, denoted by Π^B_s, is

$$\begin{aligned} \Pi^B_r = \sum_{t=1}^{n} \alpha^{t-1} \Bigg\{ & p_t E\left[\min\left(Y^{B*}_{st}, D_t\right)\right] \\ & - w_t \left[\frac{\left(Y^{B*}_{st} + Y^B_{rt}\right)}{2} - x^B_{rt}\right] - \frac{o^B_t\left(Y^{B*}_{st} - Y^B_{rt}\right)}{2} \\ & - e^B_t E\left(\min\left[\frac{\left(Y^{B*}_{st} - Y^B_{rt}\right)}{2}, \left(D_t - \frac{\left(Y^{B*}_{st} + Y^B_{rt}\right)}{2}\right)^+\right]\right) \\ & + e^B_t E\left(\min\left[\frac{\left(Y^{B*}_{st} - Y^B_{rt}\right)}{2}, \left(\frac{\left(Y^{B*}_{st} + Y^B_{rt}\right)}{2} - D_t\right)^+\right]\right) \\ & - h_t E\left[\left(Y^B_{rt} - D_t\right)^+\right] - g_t E\left[\left(D_t - Y^{B*}_{st}\right)^+\right] \Bigg\} \\ & + \alpha^n w_{n+1} x^B_{rn+1}. \end{aligned} \quad (25)$$

In (25), $(Y_{st}^{B*} + Y_{rt}^{B})/2 - x_{rt}^{B}$ denotes the retailer's initial firm order quantity in period t and $(Y_{st}^{B*} - Y_{rt}^{B})/2$ denotes the supplier's reservation inventory, as the retailer's options order quantity, in period t.

Let $V_{rt}^{B}(x_r^B)$ be denoted as the retailer's profit-to-go function with bidirectional option contracts, providing her original inventory is x_r^B at the beginning of period t. The dynamic program for optimizing (25) can be developed as follows:

$$V_{rt}^{B}\left(x_r^B\right) = w_t x_r^B + \max_{Y_r^B \geq x_r^B} \left\{P_{rt}^{B}\left(Y_r^B\right)\right\}, \tag{26}$$

where $V_{rn+1}^{B}(x_r^B) = w_{n+1}x_r^B$ $(x_r^B \geq 0)$ and

$$\begin{aligned}
P_{rt}^{B}\left(Y_r^B\right) &= H_{rt}^{B}\left(Y_r^B\right) + \alpha E V_{rt+1}^{B}\left[\left(Y_r^B - D_t\right)^+\right] \\
H_{rt}^{B}\left(Y_r^B\right) &= p_t E\left[\min\left(Y_{st}^{B*}, D_t\right)\right] - \frac{w_t\left(Y_{st}^{B*} + Y_r^B\right)}{2} \\
&\quad - \frac{o_t^B\left(Y_{st}^{B*} - Y_r^B\right)}{2} \\
&\quad - e_t^B E\left\{\min\left[\frac{\left(Y_{st}^{B*} - Y_r^B\right)}{2}, \left(D_t - \frac{\left(Y_{st}^{B*} + Y_r^B\right)}{2}\right)^+\right]\right\} \\
&\quad + e_t^B E\left\{\min\left[\frac{\left(Y_{st}^{B*} - Y_r^B\right)}{2}, \left(\frac{\left(Y_{st}^{B*} + Y_r^B\right)}{2} - D_t\right)^+\right]\right\} \\
&\quad - h_t E\left[\left(Y_r^B - D_t\right)^+\right] - g_t E\left[\left(D_t - Y_{st}^{B*}\right)^+\right].
\end{aligned} \tag{27}$$

Here, $P_{rt}^{B}(Y_r^B)$ is the retailer's expected total discounted profit over periods $t, \ldots, n$, when her initial inventory is x_r^B at the beginning of period t. $H_{rt}^{B}(Y_r^B)$ is the retailer's expected profit with initial inventory x_r^B in period t.

Let $\overline{V}_{rt}^{B}(x_r^B) = V_{rt}^{B}(x_r^B) - w_t x_r^B$; the following recursion equals (26) and (27):

$$\overline{V}_{rt}^{B}\left(x_r^B\right) = \max_{Y_r^B \geq x_r^B} \left\{\overline{P}_{rt}^{B}\left(Y_r^B\right)\right\}, \tag{28}$$

where $\overline{V}_{rn+1}^{B}(x_r^B) = 0$ $(x_r^B \geq 0)$ and

$$\overline{P}_{rt}^{B}\left(Y_r^B\right) = \overline{H}_{rt}^{B}\left(Y_r^B\right) + \alpha E \overline{V}_{rt+1}^{B}\left[\left(Y_r^B - D_t\right)^+\right] \tag{29}$$

$$\begin{aligned}
\overline{H}_{rt}^{B}\left(Y_r^B\right) &= p_t E\left[\min\left(Y_{st}^{B*}, D_t\right)\right] - \frac{w_t\left(Y_{st}^{B*} + Y_r^B\right)}{2} \\
&\quad - \frac{o_t^B\left(Y_{st}^{B*} - Y_r^B\right)}{2} \\
&\quad - e_t^B E\left\{\min\left[\frac{\left(Y_{st}^{B*} - Y_r^B\right)}{2}, \left(D_t - \frac{\left(Y_{st}^{B*} + Y_r^B\right)}{2}\right)^+\right]\right\} \\
&\quad + e_t^B E\left\{\min\left[\frac{\left(Y_{st}^{B*} - Y_r^B\right)}{2}, \left(\frac{\left(Y_{st}^{B*} + Y_r^B\right)}{2} - D_t\right)^+\right]\right\} \\
&\quad - \left(h_t - \alpha w_{t+1}\right) E\left[\left(Y_r^B - D_t\right)^+\right] - g_t E\left[\left(D_t - Y_{st}^{B*}\right)^+\right].
\end{aligned} \tag{30}$$

Here, $\overline{P}_{rt}^{B}(Y_r^B)$ is the retailer's expected total discounted profit over periods $t, \ldots, n$, when her initial inventory is zero at the beginning of period t. $\overline{H}_{rt}^{B}(Y_r^B)$ is the retailer's expected profit with zero initial inventory in period t.

Based on the above analysis, Proposition 10 follows.

Proposition 10. *With bidirectional option contracts, the retailer's optimal initial firm order quantity in period t, denoted by Q_{rt}^{B*}, is*

$$Q_{rt}^{B*} = \begin{cases} \dfrac{\left(Y_{st}^{B*} + s_t^{B*}\right)}{2} - x_{rt}^{B} & \text{if } x_{rt}^{B} \leq s_t^{B*} \\ \dfrac{\left(Y_{st}^{B*} - x_{rt}^{B}\right)}{2} & \text{if } x_{rt}^{B} > s_t^{B*}, \end{cases} \tag{31}$$

where $s_t^{B*} \in \arg\max_{Y_r^B}\{\overline{P}_{rt}^{B}(Y_r^B)\}$.

Proof. Equation (30) means that $\overline{H}_{rt}^{B}(Y_r^B)$ is always concave in Y_r^B for all periods. In period n, as $\overline{V}_{rn+1}^{B}(x_r^B) = 0$, $\overline{P}_{rn}^{B}(Y_r^B)$ equals $\overline{H}_{rn}^{B}(Y_r^B)$ and thus is concave. In period $t = 1, \ldots, n-1$, as $\overline{V}_{rt+1}^{B}[(Y_r^B - D_t)^+]$ is concave, $\overline{P}_{rt}^{B}(Y_r^B)$ comprises two concave functions and thus is also concave. Thus, for all $t = 1, \ldots, n$, $\overline{V}_{rt}^{B}(x_r^B)$ is a concave function of x_r^B. For this reason, the optimal solution for the decision variable Y_{rt}^B in period t is

$$Y_{rt}^{B*} = \begin{cases} s_t^{B*} & \text{if } x_{rt}^{B} \leq s_t^{B*} \\ x_{rt}^{B} & \text{if } x_{rt}^{B} \leq s_t^{B*}, \end{cases} \tag{32}$$

where s_t^{B*} is the optimum of Y_r^B that maximizes $\overline{P}_{rt}^{B}(Y_r^B)$. Since $Q_{rt}^{B*} = (Y_{st}^{B*} + Y_{rt}^{B*})/2 - x_{rt}^{B}$, this above result can be obtained immediately. □

Proposition 10 shows that the retailer's optimal ordering policy is of base stock type in the framework of bidirectional option contracts. It is important to highlight that the optimal base stock level s_t^{B*} denotes the retailer's inventory level to be raised after all the bidirectional options have been fully exercised as the put ones in period t. In addition, this proposition states that the retailer can characterize her purchasing decision in period t based on the specific supplier's production decision in that period.

As shown in Proposition 10, the retailer should guarantee the retailer's inventory level in period t to be not less than s_t^{B*} when all the bidirectional options take effect just as the put ones in that period under the bidirectional option contract arrangement. This result, compared with Proposition 4, suggests that the retailer's optimal ordering strategies with bidirectional option contracts are similar to those with call ones. This result also suggests that there exists very slight distinction between these two different cases. The unique difference stems from the lower bound constraint of the decision variable Y_r^B. The property of the threshold s_t^{B*} is given as follows.

Corollary 11. *For all $t = 1, \ldots, n$, (1) s_t^{B*} is increasing in o_t^B and decreasing in w_t; (2) s_t^{B*} is increasing in Y_{st}^{B*}.*

Proof. In part (1), (29) means that $\overline{P}_{rt}^B(Y_r^B)$ is supermodular with respect to o_t^B and Y_r^B. Thus, s_t^{U*} is an increasing function of o_t^B. In addition, $\overline{P}_{rt}^B(Y_r^B)$ is submodular with respect to w_t and Y_r^B. Thus, s_t^{U*} is an decreasing function of w_t. Part (2) can be proved in the same approach. Here, we omit it for simplicity. □

Corollary 11 shows that a higher option price in period t will cause the retailer to undertake higher inventory risk in that period under the bidirectional option contract arrangement. However, as the wholesale price increases in period t, the retailer will undertake lower inventory risk in that period under the bidirectional option contract arrangement. This result, combined with (1) in Corollary 8, indicates that s_t^{B*} denotes the lower bound of the retailer's minimum committed inventory in period t whereas S_t^{B*} represents that of the retailer's maximum committed inventory in period t under the bidirectional contract arrangement. Apparently, if there is a change in the option price, both thresholds will move towards the same direction; if there is a change in the wholesale price, they move towards an opposite direction. Moreover, this corollary states that the optimal base stock level s_t^{B*} increases as the order-up-to level Y_{st}^{B*} is raised. It indicates that the sensitivity of the retailer's purchasing decision in each period depends on the supplier's production decision in the corresponding period.

Denote s_t^{B+} as the near-optimal base stock level for the retailer as she maximizes her current expected profit while ignoring her future expected profit under the bidirectional option contract arrangement. Note that $s_t^{B*} < s_t^{B+}$. We get the following proposition.

Proposition 12. *For all $t = 1, \ldots, n$, $s_t^{B+} = F_t^{-1}[(o_t^B + e_t^B - w_t)/2(e_t^B + h_t - \alpha w_{t+1})]$.*

Proof. Rearranging (30) yields $\overline{H}_{rt}^B(Y_r^B) = (2p_t + 2g_t - w_t - o_t^B - e_t^B)Y_{st}^{B*}/2 - g_t\mu_t - (p_t + g_t - e_t^B)E[(Y_{st}^{B*} - D_t)^+] + (o_t^B + e_t^B - w_t)Y_r^B/2 - (e_t^B + h_t - \alpha w_{t+1})E[(Y_r^B - D_t)^+]$. Solving the first and second order derivatives of $\overline{H}_{rt}^B(Y_r^B)$ yields $d\overline{H}_{rt}^B(Y_r^B)/dY_r^B = (o_t^B + e_t^B - w_t) - (e_t^B + h_t - \alpha w_{t+1})F_t(Y_r^B)$ and $d^2\overline{H}_{rt}^B(Y_r^B)/d(Y_r^B)^2 = -(e_t^B + h_{rt} - \alpha w_{t+1})f_t(Y_r^B) < 0$. Thus, $\overline{H}_{rt}^B(Y_r^B)$ is concave in Y_r^B. Then, the near-optimal threshold s_t^{B+} is calculated from $d\overline{H}_{rt}^B(Y_r^B)/dY_r^B = 0$. That is, $s_t^{B+} = F_t^{-1}[(o_t^B + e_t^B - w_t)/2(e_t^B + h_t - \alpha w_{t+1})]$. □

Note that $S_t^{B+} > s_t^{B+}$ is equivalent to $o_t^B < (2(w_t - c_t)(e_t^B + h_t - \alpha w_{t+1}) - \alpha(w_{t+1} - c_{t+1})(e_t^B - w_t))/\alpha(w_{t+1} - c_{t+1})$. This inequality indicates that if the option price is too high in period t, the bidirectional options order will not be placed by the retailer in that period. The cost parameters throughout this section are assumed to satisfy this condition.

6. Discussion

In this section, we illustrate the impact of different option contracts and the impact of demand risk on the decisions and performances of both partners.

6.1. Comparisons on Both Partners' Decisions. We first explore the impact of different option contracts on both partners' optimal policies. Considering the relationships between the order-up-to levels with call and bidirectional option contracts, we get the following proposition.

Proposition 13. *For all $t = 1, \ldots, n$, if $o_t^C = o_t^B$ and $e_t^C = e_t^B$, then $S_t^{B+} < S_t^{C+}$ and $s_t^{B+} < s_t^{C+}$.*

Proof. From Propositions 3 and 9, if $o_t^C = o_t^B$ and $e_t^C = e_t^B$, we get $F(S_t^{C+}) = ((o_t^C + e_t^C + w_t - 2c_t)/2(e_t^C + h_t - \alpha c_{t+1}))((o_t^C + e_t^C - w_t)/2(e_t^C + h_t - \alpha c_{t+1})) > (o_t^B + e_t^B + w_t - 2c_t)/2(e_t^B + h_t - \alpha c_{t+1}) = F(S_t^{B+})$. It follows that $S_t^{B+} < S_t^{C+}$. From Propositions 6 and 12, if $o_t^C = o_t^B$ and $e_t^C = e_t^B$, we get $F(s_t^{C+}) = (o_t^C + e_t^C - w_t)/(e_t^C + h_t - \alpha w_{t+1}) > (o_t^B + e_t^B - w_t)/2(e_t^B + h_t - \alpha w_{t+1}) = F(s_t^{B+})$. It follows that $s_t^{B+} < s_t^{C+}$. □

Proposition 13 shows the impact of different options contracts on the upper near-optimal base stock level and the lower one. If there exist no differences on the price parameters of different option contracts in period t, the service levels of both the system and the retailer are greater with call option contracts than with bidirectional ones in that period; at the same time, the retailer's inventory risk is lower with bidirectional option contracts than with call ones in that period. Interestingly, if all the parameters are stationary over all periods, the near-optimal policy is optimal for the optimization problem of an arbitrary partner. In this case, the optimal base stock level for an arbitrary partner is always higher with call option contracts than with bidirectional ones.

6.2. Comparisons on Both Partners' Performances. We conduct sensitivity analysis to illustrate the impact of different option contracts on both partners' performances. We focus on the stationary equilibrium solutions and assume that $p_t = p, w_t = w, c_t = c, o_t = o, e_t = e$, and so on, and D_t is generated by the same Normal distribution; that is, $D_t \sim N(\mu, \sigma^2)$. In addition, the starting inventory levels of both partners are zero at the beginning of the first period. Throughout this numerical study, partial parameters are kept constant and we set $n = 50$, $\mu = 100$, $\alpha = 0.75$, $p = 10$, $g = 12$, $w = 6$, $c = 3$, and $h = 1.2$. The specific values of other parameters are changed to describe the objective problem.

We first analyze the effect of the option price on the maximum expected total discounted profits of both partners. We set $e^C = e^B = 5.5$ and $\sigma = 15$. By changing the value of the option price, we get the results shown in Figure 1.

Several interesting observations can be obtained from Figure 1. (1) Whether concerning call or bidirectional option contracts, an increase in the option price will improve the supplier's performance. In addition, the supplier's performance has a greater magnitude of change with call option

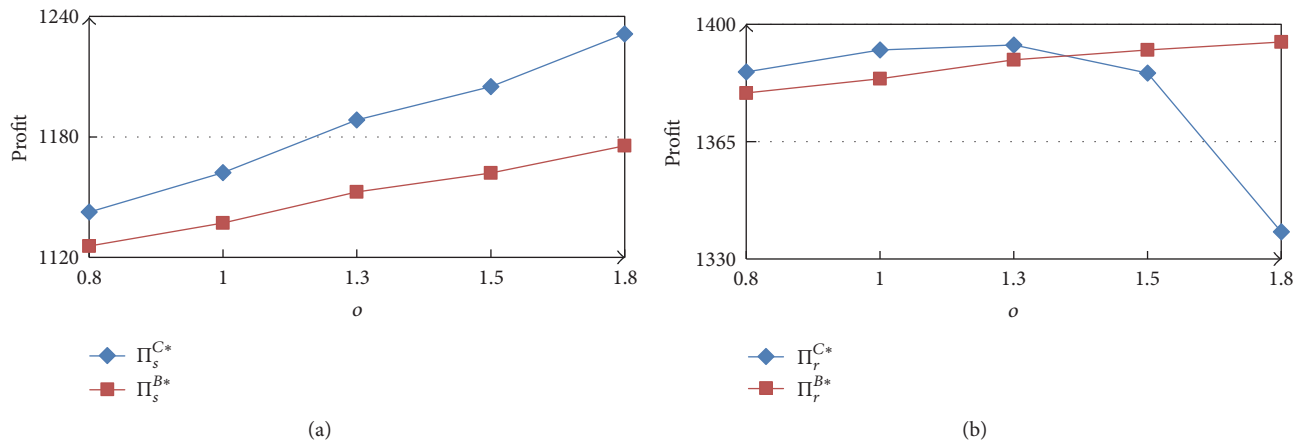

Figure 1: The impact of the option price.

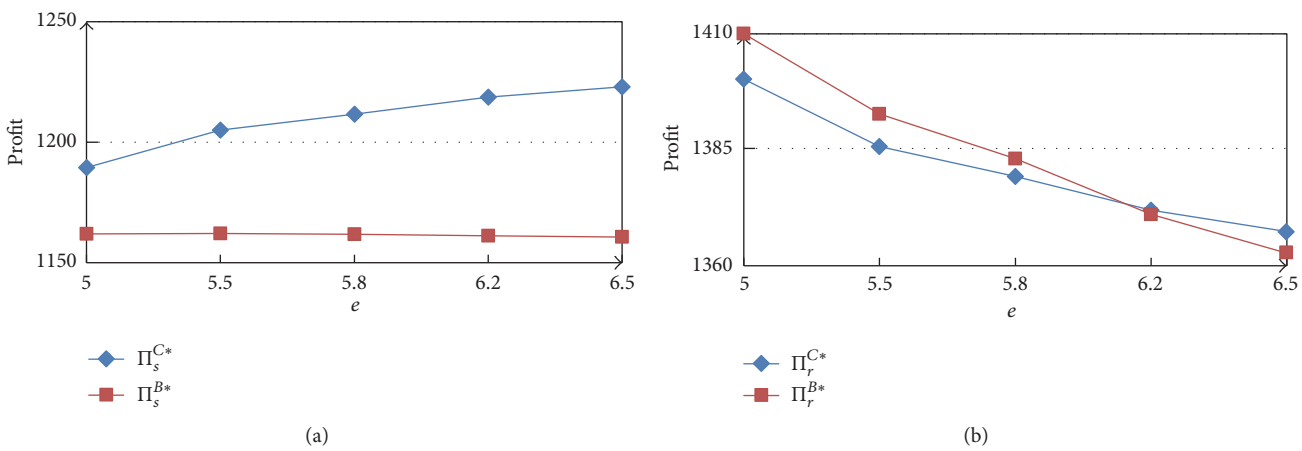

Figure 2: The impact of the exercise price.

contracts than with bidirectional ones. (2) When the price parameters are the same for different option contracts, the supplier always prefers call option contracts rather than bidirectional ones. (3) An increase in the option price brings different outcomes on the retailer's performance under the different contracts arrangements. With call option contracts, the retailer's performance is first increasing and then decreasing in the option price. However, with bidirectional option contracts, the retailer's performance is always increasing in the option price. (4) When the price parameters are the same for different option contracts, call option contracts are better for the retailer facing a low option price, while bidirectional option contracts are better for the retailer facing a high option price. (5) Due to the retailer's dominant position, call option contracts are adopted by the supply chain if the option price is low, while bidirectional option contracts are adopted by the supply chain if the option price is high. Obviously, the option price plays a crucial role for the retailer-led supply chain to select the appropriate option contract types.

We subsequently explore the effect of the exercise price on the maximum expected total discounted profits of both members. We set $o^C = o^B = 1.5$ and $\sigma = 15$. By changing the value of the exercise price, we get the results shown in Figure 2.

Several interesting results can be obtained from Figure 2. (1) An increase in the exercise price brings different outcomes to the supplier's performances under the different contracts arrangements. With call option contracts, the supplier's performance is always increasing in the exercise price. However, with bidirectional option contracts, the supplier's performance is first increasing and then decreasing slightly in the exercise price. In addition, the supplier's performance has a greater magnitude of change with call option contracts than with bidirectional ones. (2) When the price parameters are the same for different option contracts, the supplier always prefers call option contracts rather than bidirectional ones. (3) An increase in the exercise price will weaken the retailer's performance, either with call or bidirectional option contracts. (4) When the price parameters are the same for different option contracts, call option contracts are better for the retailer facing a high exercise price, while bidirectional option contracts are better for the retailer facing low exercise price. (5) Due to the retailer's dominant position, bidirectional option contracts are used by the supply chain

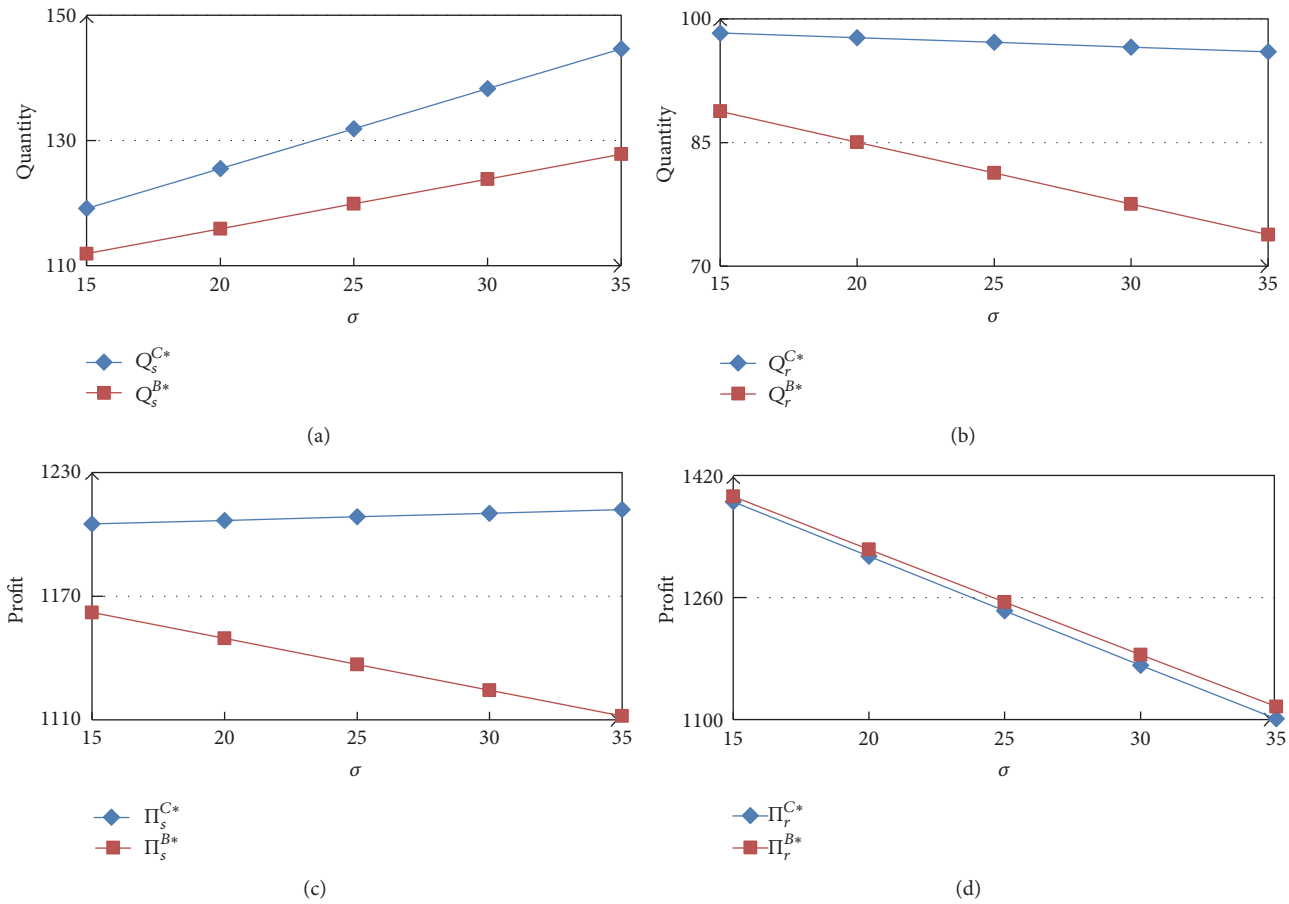

Figure 3: The effect of demand risk.

if the exercise price is low, while call option contracts are adopted by the supply chain if the exercise price is high. Obviously, the exercise price plays a crucial role for the retailer-led supply chain to select the appropriate option contract format.

Combining the above analysis, the following remark can be obtained.

Remark 1. If the parameters are the same for different option contracts, the retailer-led supply chain prefers call option contracts when the option (exercise) price is low (high), while enjoying bidirectional option contracts when the exercise (option) price is low (high).

The supplier, as the follower, always prefers call option contracts. However, call option contracts cannot always benefit the retailer, as the leader. Remark 1 suggests that the option contract parameters are the crucial factors based on which the retailer-led supply chain ultimately decides whether to adopt call or bidirectional option contracts.

6.3. The Effect of Demand Risk. We conduct a numerical example to investigate the impact of demand risk (valued by the standard deviation of the demand). Similar to Section 6.2, we also focus on the stationary equilibrium solutions with zero initial inventory in the first period. We set $n = 50$, $\mu = 100$, $\alpha = 0.75$, $p = 10$, $g = 12$, $w = 6$, $o^C = o^B = 1.5$, $e^C = e^B = 5.5$, $c = 3$, and $h = 1.2$. By changing the size of σ, we get the results shown in Figure 3.

Several interesting findings can be obtained from Figure 3. (1) Whether concerning call or bidirectional option contracts, the supplier's optimal production quantity is always increasing in the demand risk. In addition, the optimal production quantity has a greater magnitude of change with call option contracts than with bidirectional ones. (2) Whether concerning call or bidirectional option contracts, the retailer's optimal initial firm order quantity is always decreasing in the demand risk. In addition, the optimal initial firm order quantity has a greater magnitude of change with bidirectional option contracts than with call ones. (3) An increase in the demand risk brings different outcomes to the supplier's performances under different contracts arrangements. With call option contract, the demand risk has a positive effect on the supplier's performance. With bidirectional option contracts, the demand risk has a negative effect on the supplier's performance. (4) An increase in the demand risk will weaken the retailer's performance, either with call or bidirectional option contracts. In addition, the retailer's performance has a slightly greater magnitude of change with call option contracts than with bidirectional ones.

Combining the above analysis, the following remark can be obtained.

Remark 2. As the demand risk increases, the supplier will enhance his production quantity while the retailer will place a smaller initial firm order, either with call or bidirectional option contracts.

Remark 2 suggests that the decisions and performances of two members in the retailer-led supply chain are influenced by the demand risk, either with call or bidirectional option contracts. In addition, the different level of the demand risk brings different outcomes. Thus, it is urgent to observe whether the demand risk is high or low. Establishing a risk detection mechanism is an important thing for the supply chain members.

7. Conclusions and Further Research

Due to high variation in the market demand, along with the underlying multiperiod structure, it is a big challenge for the retailer-led supply chain to keep a proper balance between supply and demand. This situation is bound to influence the daily operation of the members in the retailer-led supply chain. For the dominant retailer, she should respond flexibly to fluctuating demand even after the order has been placed. For the followed supplier, he should find relief from the great pressure associated with over- and underproduction. Option contracts, as efficient risk hedging, provide the dominant retailer with a certain degree of ordering flexibility; at the same time, they offer tangible benefits to the followed supplier in advance. It is high necessary to introduce this contract mechanism to improve the collaboration between the members in the retailer-led supply chain. This research develops the multiperiod retailer-Stackelberg game models for a retailer-led supply chain with call and bidirectional option contracts, respectively. Based on stochastic dynamic programming, we analyze the optimal policy structures for two members in each period. Meanwhile, we evaluate the corresponding policy parameters via an approximation algorithm. Then, the impacts of different option contracts and demand risk are discussed though a numerical study. The main results are as follows:

(1) A base stock policy is optimal for the followed supplier's production in each period, either with call or bidirectional option contracts. In addition, the dominant retailer's ordering is of base stock type in each period, either with call or bidirectional option contracts.

(2) If the parameters are the same for different option contracts, the service levels of both the system and the retailer are higher with call option contracts than with bidirectional ones, while the retailer's inventory risk is lower with bidirectional option contracts than with call ones.

(3) If the parameters are the same for different option contracts, call option contracts can always benefit the supplier, but not the retailer. Thus, the retailer-led supply chain prefers call option contracts when the option (exercise) price is low (high), while enjoying bidirectional option contracts when the exercise (option) price is low (high).

(4) Whether concerning call or bidirectional option contracts, an increase in the demand risk would prompt the supplier to enhance his production quantity while inducing the retailer to place a smaller initial firm order.

This paper provides several important managerial insights as follows: First, we derive the followed supplier's optimal production policy and the dominant retailer's optimal ordering policy in a multiperiod setting. These conclusions can help the supply chain firms make the rational decisions to maximize their expected total discounted profits over multiple periods. Moreover, we investigate the impact of call and bidirectional option contracts on the decisions and performances of two members in a multiperiod setting. These conclusions can help the supply chain firms rationally choose the appropriate option contract format to improve their operational performance and scientifically design the option contract parameters in a multiperiod setting. Finally, we explore the impact of demand risk on the decisions and performances of two members in a multiperiod setting. These conclusions can help the supply chain firms establish a regularized mechanism to observe and analyze the market environment.

Some unaddressed questions have been left for future research. A common supply chain structure consisting of one supplier and one retailer is considered in our paper. In real practice, a supply chain comprises multiple suppliers or multiple retailers, where the supply chain firms at the same level are often in coopetition. Thus, more sophisticated structure can be considered in the future. In addition, a supply chain has usually subjected to multiple source risks in the rapidly changing market environment. Our study only considers the variation in demand and neglects the impacts of other factors. Therefore, more complex scenario can be considered in the future. Our study considers the retailer-led supply chain; other market power supply chains, such as manufacturer-led supply chain [41, 42] or vertical Nash supply chain [49], can be taken into consideration in the future. Finally, there is only one decision variable in our model. As the dimensions of the problem increase, more intricate multiperiod models are formulated. Thereby, more efficient algorithms need to be provided in the future.

Notations

Parameters

D_t: Stochastic demand in period t, whose cumulative distribution function, probability density function, and expected value are $F_t(\cdot)$, $f_t(\cdot)$, and μ_t
p_t: Unit retail price in period t
w_t: Unit wholesale price in period t
c_t: Unit production cost in period t
o_t: Unit option price in period t
e_t: Unit exercise price in period t
g_t: Unit backlogging cost in period t
h_t: Unit holding cost in period t
w_{n+1}: Unit salvage value for the retailer in the last period
c_{n+1}: Unit salvage value for the supplier in the last period
α: Discount factor; note: $0 < \alpha \leq 1$

x_{st}: Supplier's original inventory level before production in period t
x_{rt}: Retailer's original inventory level before replenishment in period t.

Decision Variables

y_{st}: Supplier's target inventory level after production in period t
y_{rt}: Retailer's target inventory level after replenishment in period t
Q_{st}: Supplier's production quantity in period t; note: $Q_{st} = y_{st} - x_{st}$
Q_{rt}: Retailer's initial firm order quantity in period t; note: $Q_{rt} = y_{rt} - x_{rt}$.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

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References

[1] K. Pan, K. K. Lai, L. Liang, and S. C. H. Leung, "Two-period pricing and ordering policy for the dominant retailer in a two-echelon supply chain with demand uncertainty," *Omega* , vol. 37, no. 4, pp. 919–929, 2009.

[2] X. Chen, X. Wang, and X. Jiang, "The impact of power structure on the retail service supply chain with an O2O mixed channel," *Journal of the Operational Research Society*, vol. 67, no. 2, pp. 294–301, 2015.

[3] J. Li, Z. Sheng, and H. Liu, "Multi-agent simulation for the dominant players' behavior in supply chains," *Simulation Modelling Practice and Theory*, vol. 18, no. 6, pp. 850–859, 2010.

[4] X. Wang and L. Liu, "Coordination in a retailer-led supply chain through option contract," *International Journal of Production Economics*, vol. 110, no. 1-2, pp. 115–127, 2007.

[5] G. Ertek and P. M. Griffin, "Supplier- and buyer-driven channels in a two-stage supply chain," *Institute of Industrial Engineers (IIE). IIE Transactions*, vol. 34, no. 8, pp. 691–700, 2002.

[6] J.-C. Wang, A.-M. Wang, and Y.-Y. Wang, "Markup pricing strategies between a dominant retailer and competitive manufacturers," *Computers & Industrial Engineering*, vol. 64, no. 1, pp. 235–246, 2013.

[7] A. Cheaitou, C. van Delft, Y. Dallery, and Z. Jemai, "Two-period production planning and inventory control," *International Journal of Production Economics*, vol. 118, no. 1, pp. 118–130, 2009.

[8] A. Federgruen and A. Heching, "Combined pricing and inventory control under uncertainty," *Operations Research*, vol. 47, no. 3, pp. 454–475, 1999.

[9] Y. Chen, Y. Lu, and M. Xu, "Optimal inventory control policy for periodic-review inventory systems with inventory-level-dependent demand," *Naval Research Logistics (NRL)*, vol. 59, no. 6, pp. 430–440, 2012.

[10] X. Yan and Y. Wang, "Optimality of two-sided (σ, Σ) policy in capacity expansion problem," *Operations Research Letters*, vol. 42, no. 8, pp. 545–548, 2014.

[11] X. Li, Y. Li, and X. Cai, "On a multi-period supply chain system with supplementary order opportunity," *European Journal of Operational Research*, vol. 209, no. 3, pp. 273–284, 2011.

[12] X. Chen and Z.-J. Shen, "An analysis of a supply chain with options contracts and service requirements," *IIE Transactions*, vol. 44, no. 10, pp. 805–819, 2012.

[13] C. Wang and X. Chen, "Option pricing and coordination in the fresh produce supply chain with portfolio contracts," *Annals of Operations Research*, vol. 248, no. 1-2, pp. 471–491, 2017.

[14] X. Chao, H. Chen, and S. Zheng, "Joint replenishment and pricing decisions in inventory systems with stochastically dependent supply capacity," *European Journal of Operational Research*, vol. 191, no. 1, pp. 142–155, 2008.

[15] X. Chen, G. Hao, and L. Li, "Channel coordination with a loss-averse retailer and option contracts," *International Journal of Production Economics*, vol. 150, pp. 52–57, 2014.

[16] B. Yang, C. Gao, N. Liu, and L. Xu, "Dynamic Inventory and Pricing Policy in a Periodic-Review Inventory System with Finite Ordering Capacity and Price Adjustment Cost," *Mathematical Problems in Engineering*, vol. 2015, Article ID 269695, 8 pages, 2015.

[17] H. Song, L. Ran, and J. Shang, "Multi-period optimization with loss-averse customer behavior: Joint pricing and inventory decisions with stochastic demand," *Expert Systems with Applications*, vol. 72, pp. 421–429, 2017.

[18] Z. Luo, X. Chen, J. Chen, and X. Wang, "Optimal pricing policies for differentiated brands under different supply chain power structures," *European Journal of Operational Research*, vol. 259, no. 2, pp. 437–451, 2017.

[19] A. A. Taleizadeh, M. Noori-Daryan, and R. Tavakkoli-Moghaddam, "Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items," *International Journal of Production Research*, vol. 53, no. 15, pp. 4553–4582, 2015.

[20] M. Noori-daryan, A. A. Taleizadeh, and K. Govindan, "Joint replenishment and pricing decisions with different freight modes considerations for a supply chain under a composite incentive contract," *Journal of the Operational Research Society*, pp. 1–20, 2017.

[21] Y. Yugang, L. Liang, and G. Q. Huang, "Leader–follower game in vendor-managed inventory system with limited production capacity considering wholesale and retail prices," *International Journal of Logistics Research and Applications*, vol. 9, no. 4, pp. 335–350, 2006.

[22] Y. Dai, S. X. Zhou, and Y. Xu, "Competitive and collaborative quality and warranty management in supply chains," *Production Engineering Research and Development*, vol. 21, no. 1, pp. 129–144, 2012.

[23] B. Yan, T. Wang, Y. Liu, and Y. Liu, "Decision analysis of retailer-dominated dual-channel supply chain considering cost misreporting," *International Journal of Production Economics*, vol. 178, pp. 34–41, 2016.

[24] J. Raju and Z. J. Zhang, "Channel coordination in the presence of a dominant retailer," *Marketing Science*, vol. 24, no. 2, pp. 254–262, 2005.

[25] T. Geylani, A. J. Dukes, and K. Srinivasan, "Strategic manufacturer response to a dominant retailer," *Marketing Science*, vol. 26, no. 2, pp. 164–178, 2007.

[26] E. Almehdawe and B. Mantin, "Vendor managed inventory with a capacitated manufacturer and multiple retailers: Retailer versus manufacturer leadership," *International Journal of Production Economics*, vol. 128, no. 1, pp. 292–302, 2010.

[27] J. Li, X. Liu, J. Wu, and F. Yang, "Coordination of supply chain with a dominant retailer under demand disruptions," *Mathematical Problems in Engineering*, vol. 2014, Article ID 854681, 10 pages, 2014.

[28] A. A. Taleizadeh and M. Noori-Daryan, "Pricing, manufacturing and inventory policies for raw material in a three-level supply chain," *International Journal of Systems Science*, vol. 47, no. 4, pp. 919–931, 2016.

[29] A. A. Taleizadeh, M. Noori-Daryan, and K. Govindan, "Pricing and ordering decisions of two competing supply chains with different composite policies: A Stackelberg game-theoretic approach," *International Journal of Production Research*, vol. 54, no. 9, pp. 2807–2836, 2016.

[30] D. Barnes-Schuster, Y. Bassok, and R. Anupindi, "Coordination and flexibility in supply contracts with options," *Manufacturing & Service Operations Management*, vol. 4, no. 3, pp. 171–207, 2002.

[31] J. Buzacott, H. Yan, and H. Zhang, "Risk analysis of commitment-option contracts with forecast updates," *IIE Transactions*, vol. 43, no. 6, pp. 415–431, 2011.

[32] Y. Feng, Y. Mu, B. Hu, and A. Kumar, "Commodity options purchasing and credit financing under capital constraint," *International Journal of Production Economics*, vol. 153, no. 1, pp. 230–237, 2014.

[33] Z. Liu, L. Chen, L. Li, and X. Zhai, "Risk hedging in a supply chain: Option vs. price discount," *International Journal of Production Economics*, vol. 151, pp. 112–120, 2014.

[34] J. Luo and X. Chen, "Risk hedging via option contracts in a random yield supply chain," *Annals of Operations Research*, vol. 257, no. 1-2, pp. 697–719, 2017.

[35] J. Cai, M. Zhong, J. Shang, and W. Huang, "Coordinating VMI supply chain under yield uncertainty: Option contract, subsidy contract, and replenishment tactic," *International Journal of Production Economics*, vol. 185, pp. 196–210, 2017.

[36] F. Chen and M. Parlar, "Value of a put option to the risk-averse newsvendor," *Institute of Industrial Engineers (IIE). IIE Transactions*, vol. 39, no. 5, pp. 481–500, 2007.

[37] W. Xue, L. Ma, and H. Shen, "Optimal inventory and hedging decisions with CVaR consideration," *International Journal of Production Economics*, vol. 162, pp. 70–82, 2015.

[38] C. Wang and X. Chen, "Joint order and pricing decisions for fresh produce with put option contracts," *Journal of the Operational Research Society*, pp. 1–11, 2017.

[39] Q. Wang and D.-B. Tsao, "Supply contract with bidirectional options: the buyer's perspective," *International Journal of Production Economics*, vol. 101, no. 1, pp. 30–52, 2006.

[40] Y. Zhao, L. Ma, G. Xie, and T. C. E. Cheng, "Coordination of supply chains with bidirectional option contracts," *European Journal of Operational Research*, vol. 229, no. 2, pp. 375–381, 2013.

[41] X. Chen, X. Wang, and H. K. Chan, "Manufacturer and retailer coordination for environmental and economic competitiveness: A power perspective," *Transportation Research Part E: Logistics and Transportation Review*, vol. 97, pp. 268–281, 2017.

[42] X. Chen, N. Wan, and X. Wang, "Flexibility and coordination in a supply chain with bidirectional option contracts and service requirement," *International Journal of Production Economics*, vol. 193, pp. 183–192, 2017.

[43] C. Liu, Z. Jiang, L. Liu, and N. Geng, "Solutions for flexible container leasing contracts with options under capacity and order constraints," *International Journal of Production Economics*, vol. 141, no. 1, pp. 403–413, 2013.

[44] I. Nosoohi and A. S. Nookabadi, "Outsource planning through option contracts with demand and cost uncertainty," *European Journal of Operational Research*, vol. 250, no. 1, pp. 131–142, 2016.

[45] N. Wan and X. Chen, "Contracts choice for supply chain under inflation," *International Transactions in Operational Research*, 2016.

[46] J.-C. Li, Y.-W. Zhou, and W. Huang, "Production and procurement strategies for seasonal product supply chain under yield uncertainty with commitment-option contracts," *International Journal of Production Economics*, vol. 183, pp. 208–222, 2017.

[47] A. H. L. Lau, H.-S. Lau, and J.-C. Wang, "Pricing and volume discounting for a dominant retailer with uncertain manufacturing cost information," *European Journal of Operational Research*, vol. 183, no. 2, pp. 848–870, 2007.

[48] K. Inderfurth, P. Kelle, and R. Kleber, "Dual sourcing using capacity reservation and spot market: Optimal procurement policy and heuristic parameter determination," *European Journal of Operational Research*, vol. 225, no. 2, pp. 298–309, 2013.

[49] X. Chen and X. Wang, "Free or bundled: channel selection decisions under different power structures," *Omega* , vol. 53, pp. 11–20, 2015.