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Research Article

Finite-Time Output Feedback Control for a Rigid Hydraulic Manipulator System

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The position tracking control problem of a hydraulic manipulator system is investigated. By utilizing homogeneity theory, a finite-time output feedback controller is designed. Firstly, a finite-time state feedback controller is developed based on homogeneity theory. Secondly, a nonlinear state observer is designed to estimate the manipulator's velocity. A rigorous analysis process is presented to demonstrate the observer's finite-time stability. Finally, the corresponding output feedback tracking controller is derived, which stabilizes the tracking error system in finite time. Simulations demonstrate the effectiveness of the designed finite-time output feedback controller.

1. Introduction

In the past years, robotic system related control problem has been investigated more and more widely for robots application value [1, 2]. In addition to rapid responses and high power-to-weight ratios [3–10], a great many robots are driven by hydraulic actuators. To improve the performance for industrial hydraulic machines, it is essential to enhance the control performance of manipulators actuated by hydraulic actuators [3, 4]. However, hydraulic manipulators control problem is more challenging than their electrical counterparts, due to the nonlinear dynamics and the nonlinear mechanical linkage dynamics. On one hand, mechanical linkage dynamics are composed of strong coupling effects among various joints and significant nonlinearities in the hydraulic actuators [4]. On the other hand, with complex properties of servo valves, frictions, compressibility of hydraulic fluid and volume changes [5, 6, 11], and hydraulic actuators exhibit highly nonlinear characteristics. To realize more accurate control, the hydraulic actuators dynamics must be considered in the hydraulic manipulators control design [4, 12].

For the hydraulic manipulators position tracking control problem, researchers have proposed a great deal of meaningful results in recent years [3, 4, 12–16]. In [13], by using singular perturbation technique, a nonlinear control method was developed to solve the trajectory tracking control problem and the stabilization problem for 6-axis hydraulically actuated robots. Decentralized adaptive controllers were proposed for the hydraulic manipulators [14, 15]. For the Stewart-type hydraulic manipulator [16], a pressure feedback controller was presented, which allow very high proportional position error gains. However, these results have not paid much attention to stability analysis. In [3], based on Lyapunov stability theory, an adaptive robust control algorithm was proposed. With sliding mode control method, a robust control algorithm was presented in [4] to realize accurate position tracking for hydraulic manipulators. By combining pole placement technique, backstepping control, and sliding mode control together, a composite controller was designed in [12] to regulate both flexural vibrations and flexible arm motion.

In the above-mentioned literatures, the proposed methods just realize asymptotical stability for position control problems of hydraulic systems, which indicates that such systems are asymptotically stable with infinite settling time. Thus it is necessary to design more efficient controllers to offer faster convergence rates. To this end, finite-time control is a good choice. To realize finite-time control, nonsmooth

control is a feasible choice. Besides faster convergence rates, nonsmooth control systems usually also possess some other superiorities, such as better disturbance rejection abilities and robustness [17]. Because of such nice features, nonsmooth control has been widely investigated from the perspectives of state feedback control [18], output feedback control [19–21], individual systems [22, 23], and multiagent systems [24–26].

Note that, in the above references, almost all the control designs are based on full-state information. However, in practice, in the case without velocity sensors, the manipulator velocity can not be measured directly. It is impractical to obtain the velocity information by differentiating the measured position, because derivation operation usually results in very noisy velocity data. To this end, an effective way is estimating the manipulator angular velocity via velocity observers.

In this paper, the position tracking control problem of the rigid hydraulic manipulator system is studied. Firstly, a finite-time state feedback controller is developed based on homogeneity theory. Secondly, in the case without angular velocity sensors, by utilizing homogeneous theory, a finite-time convergent observer is developed to obtain the angular velocity information for feedback. A rigorous stability analysis process is presented to demonstrate finite-time stability of the designed state observer. Thirdly, with the designed finite-time state feedback controller and the estimated angular velocity, a finite-time output feedback controller is given.

The remainder of this paper is arranged as follows. In Section 2, some preliminary knowledge is given. In Section 3, the model of the rigid hydraulic manipulator system is presented. The main results are presented in Section 4. Simulations on the proposed finite-time tracking controller are performed in Section 5. Finally, conclusions are drawn in Section 6.

2. Preliminaries

Denote $\mathrm{sig}^{\alpha}(x) = \mathrm{sign}(x)|x|^{\alpha}$, where $\alpha \geq 0, x \in R$. $\mathrm{sign}(\cdot)$ is the sign function. For a vector $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$, the notation $\mathrm{sig}^{\alpha}(x)$ represents the vector $[\mathrm{sig}^{\alpha}(x_1), \ldots, \mathrm{sig}^{\alpha}(x_n)]^{\mathrm{T}}$, where $\alpha \geq 0$. The notation x^{β} represents the vector $[x_1^{\beta}, \ldots, x_n^{\beta}]^{\mathrm{T}}$, where $\beta \in R$. $\mathrm{diag}(x_1, \ldots, x_n)$ represents a diagonal matrix with the diagonal elements $x_1, \ldots, x_n$.

Consider the following nonlinear autonomous system:

$$\dot{x} = f(x), \quad x \subseteq \mathbb{R}^n, \ f(0) = 0, \tag{1}$$

where $f : D \longrightarrow \mathbb{R}^n$ satisfies the locally Lipschitz continuous condition. Under this condition, the definition of finite-time stability can be described as follows.

Definition 1 (finite-time stability, [17]). The equilibrium $x = 0$ of system (1) is finite-time convergent if there are an open neighbourhood U of the origin and a function $T_x : U \setminus \{0\} \longrightarrow (0, \infty)$, such that every solution trajectory $x(t, x_0)$ of system (1) starting from the initial point $x_0 \in U \setminus \{0\}$ is well-defined and unique in forward time for $t \in [0, T_x(x_0))$, and $\lim_{t \longrightarrow T_x(x_0)} x(t, x_0) = 0$. Here $T_x(x_0)$ is called the convergence time (with respect to the initial state x_0). The equilibrium of system (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. If $U = D = \mathbb{R}^n$, the origin is a globally finite-time stable equilibrium.

Lemma 2 (see [27]). *Consider the following system:*

$$\begin{aligned} \dot{x}_1 &= x_2, \ldots, \dot{x}_{n-1} = x_n, \\ \dot{x}_n &= u, \\ y &= x_1. \end{aligned} \tag{2}$$

There exists $\epsilon \in (0, 1)$ such that, for every $\alpha \in (1 - \epsilon, 1)$, the origin is a globally finite-time-stable equilibrium for system (2) under the feedback control law

$$u = -k_1 \mathrm{sig}^{\alpha_1}(x_1) - \cdots - k_n \mathrm{sig}^{\alpha_n}(x_n), \tag{3}$$

where $k_1, \ldots, k_n > 0$ are constants such that the polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ is Hurwitz and $\alpha_1, \ldots, \alpha_n$ satisfy $\alpha_{i-1} = \alpha_i \alpha_{i+1} / (2\alpha_{i+1} - \alpha_i)$, $i = 2, \ldots, n$, with $\alpha_{n+1} = 1$ and $\alpha_n = \alpha$.

Lemma 3 (see [19]). *For system (2), there exists proper constants $a_i > 0$ $(i = 1, \ldots, n)$, so that the observer shown below is finite-time convergent.*

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + a_1 \mathrm{sig}^{\alpha_1}(x_1 - \hat{x}_1), \\ &\vdots \\ \dot{\hat{x}}_{n-1} &= \hat{x}_n + a_{n-1} \mathrm{sig}^{\alpha_{n-1}}(x_1 - \hat{x}_1), \\ \dot{\hat{x}}_n &= u + a_n \mathrm{sig}^{\alpha_n}(x_1 - \hat{x}_1), \end{aligned} \tag{4}$$

where $\alpha_i = 1 + i\alpha, i = 1, \ldots, n, \alpha \in (-1/n, 0)$.

Lemma 4 (see [20]). *Consider the following system:*

$$\dot{x} = f(x) + \hat{f}(x), \quad f(0) = 0, \ x \in \mathbb{R}^n, \tag{5}$$

where $f(x)$ is a continuously homogeneous function of degree $k < 0$ with respect to $[r_1, \ldots, r_n]^T$ and $\hat{f}(x)$ satisfies $\hat{f}(0) = 0$. If system $\dot{x} = f(x)$ is asymptotically stable, then system (5) is locally finite-time stable, when $\lim_{\varepsilon \longrightarrow 0} (\hat{f}_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) / \varepsilon^{k + r_i}) = 0, i = 1, \ldots, n, \forall x \neq 0$.

3. System Models and Problem Formulation

In this section, the hydraulic actuator and the model of the n-link rigid manipulator are presented firstly. Then the integrated model of them is constructed.

3.1. Dynamic of the Hydraulic Actuator. The schematic of the rigid manipulator driven by single-rod hydraulic servo system is depicted in Figure 1. P_1 and P_2 represent the forward and return pressures of the cylinder, respectively. Q_1 is the supply flow rate to the forward chamber and Q_2 is the return flow rate to the return chamber. A_1 is the piston area facing the extending chamber and A_2 is the piston area facing the

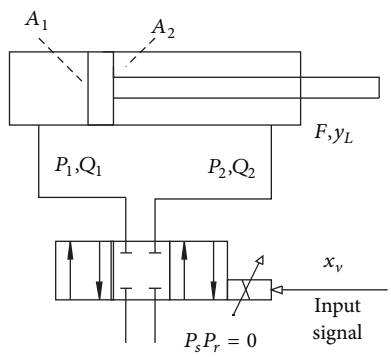

Figure 1: Hydraulic servo system.

retract chamber. P_s is the supply pump pressure and P_r is the return pressure.

From [28], the dynamics of the cylinder pressures P_1 and P_2 can be written as

$$\dot{P}_1 = \frac{\beta_e}{V_1}\left(Q_1 - A_1\dot{y}_L - C_{tm}\left(P_1 - P_2\right)\right), \tag{6a}$$

$$\dot{P}_2 = \frac{\beta_e}{V_2}\left(A_2\dot{y}_L + C_{tm}\left(P_1 - P_2\right) - Q_2\right), \tag{6b}$$

where $Q_1 = kq_1x_s\phi_1(P_1, \mathrm{sign}(x_s))$, $Q_2 = kq_2x_s\phi_2(P_2, \mathrm{sign}(x_s))$, $\phi_1(P_1, \mathrm{sign}(x_s))$, and $\phi_2(P_2, \mathrm{sign}(x_s))$ are defined as (7a) and (7b), β_e is the effective bulk modulus of the hydraulic fluid, V_1 and V_2 are the extend chamber volume and the retract chamber volume, respectively, y_L represents the piston displacement, and C_{tm} is the coefficient of the cylinder internal leakage. It is noted that $V_1 = V_{10} + A_1y_L$ and $V_2 = V_{20} - A_2y_L$, where V_{10} and V_{20} are the two chamber volumes when $y_L = 0$. It is assumed that $\phi_1(P_1, \mathrm{sign}(x_s)) > 0$ and $\phi_2(P_2, \mathrm{sign}(x_s)) > 0$ are given as follows.

$$\phi_1\left(P_1, \mathrm{sign}\left(x_s\right)\right) = \begin{cases} \sqrt{P_s - P_1}, & \text{if } x_s \geq 0 \\ \sqrt{P_1 - P_r}, & \text{if } x_s < 0, \end{cases} \tag{7a}$$

$$\phi_2\left(P_2, \mathrm{sign}\left(x_s\right)\right) = \begin{cases} \sqrt{P_2 - P_r}, & \text{if } x_s \geq 0 \\ \sqrt{P_s - P_2}, & \text{if } x_s < 0. \end{cases} \tag{7b}$$

The dynamic of the servo valve is

$$\tau_s\dot{x}_s = -x_s + k_su, \tag{8}$$

where x_s is the servo-valve displacement, τ_s is the time constant, u is the input signal, and k_s is the servo-valve gain. By neglecting the time constant of the servo valve, the servo-valve dynamics can be simplified as $x_s = k_su$. Hence, the spool displacement x_s can be seen as the control input.

Note that piston displacement vector $\overline{y}_L = [y_{L_1}, \ldots, y_{L_n}]^{\mathrm{T}}$ is the function of joint angular position vector q. By letting $\overline{y}_L = f(q)$, it yields that $\dot{\overline{y}}_L = J(q)\dot{q}$, where $J(q)$ is a $n \times n$ matrix. Then the virtual work of the rigid hydraulic manipulator is $W = F^{\mathrm{T}}\delta\overline{y}_L = F^{\mathrm{T}}J(q)\delta q$, where $\delta\overline{y}_L$ is the actuators piston displacement vector and δq is the virtual variation vector on q. The work of the generalized force is

$$W = \tau^{\mathrm{T}}\delta q. \tag{9}$$

The net force is expressed as $F = P_1A_1 - P_2A_2$. It follows from (9) that

$$\tau = J^{\mathrm{T}}\left(q\right)F, \tag{10}$$

where $J(q)$ is Jacobian Matrix.

3.2. Dynamics of the Rigid Manipulator. By utilizing Lagrangian method, the dynamics of a n-link rigid manipulator are described as follows:

$$M\left(q\right)\ddot{q} + C\left(q, \dot{q}\right)\dot{q} + G\left(q\right) = \tau, \tag{11}$$

where $M(q)$ denotes the robot manipulator inertia matrix, $C(q, \dot{q})\dot{q}$ represents the coriolis and centrifugal torques, $G(q)$ is the vector of gravitational torques, $q, \dot{q}, \ddot{q} \in R^n$ are the vectors of joint angular position, velocity, and acceleration, respectively, and $\tau \in R^n$ is the joint torques vector.

With the manipulator subsystem dynamic and the hydraulic subsystem model, the complete dynamic of a n-link rigid hydraulic manipulator will be constructed.

3.3. Dynamic of the Rigid Hydraulic Manipulator. Before proceeding, the following assumption on the desired position is presented.

Assumption 5. The desired position $q_{id}(t)$ $(i = 1, \ldots, n)$ is third-order differentiable.

Assume that all the hydraulic actuators have the same structure. Let $q_d(t) = [q_{1d}(t), \ldots, q_{nd}(t)]^{\mathrm{T}}$ and $\dot{q}_d(t) = [\dot{q}_{1d}(t), \ldots, \dot{q}_{nd}(t)]^{\mathrm{T}}$ signify the desired position vector and velocity vector of the joint angular. The tracking errors are defined as $e_1 = [e_{11}, \ldots, e_{1n}]^{\mathrm{T}} = q - q_d$, $e_2 = [e_{21}, \ldots, e_{2n}]^{\mathrm{T}} = \dot{q} - \dot{q}_d$.

The complete model of the hydraulic manipulator system is written as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= f_2\left(e\right) + g_2\left(e\right)\left(A_1e_3 - A_2e_4\right), \\ \dot{e}_3 &= f_3\left(e\right) + g_3\left(e, u\right)u, \\ \dot{e}_4 &= f_4\left(e\right) + g_4\left(e, u\right)u, \\ y &= e_1 + q_d, \end{aligned} \tag{12}$$

where $e = [e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, e_3^{\mathrm{T}}, e_4^{\mathrm{T}}]^{\mathrm{T}}$ is the system state vector, $f_2(e) = -M^{-1}(e_1 + q_d)(C(e_1 + q_d, e_2 + \dot{q}_d)(e_2 + \dot{q}_d) + G(e_1 + q_d)) - \ddot{q}_d(t)$, $g_2(e) = M^{-1}(e_1 + q_d)J^{\mathrm{T}}(e_1 + q_d)$, $f_3(e) = -\beta_e\mathrm{diag}(1/V_{11}, \ldots, 1/V_{1n})(A_1J(e_1 + q_d)(e_2 + \dot{q}_d) + C_{tm}(e_3 - e_4))$, $g_3(e, u) = \beta_ekq_1 \times \mathrm{diag}((1/V_{11})\phi_1(e_{31}, \mathrm{sign}(u_1)), \ldots, (1/V_{1n})\phi_1(e_{3n}, \mathrm{sign}(u_n)))$, $f_4(e) = \beta_e\mathrm{diag}(1/V_{21}, \ldots, 1/V_{2n})(A_2J(e_1+q_d)(e_2+\dot{q}_d)+C_{tm}(e_3-e_4))$, $g_4(e, u) =$

$-\beta_e k q_2 \times \operatorname{diag}((1/V_{21})\phi_2(e_{41}, \operatorname{sign}(u_1)), \ldots, (1/V_{2n})\phi_2(e_{4n}, \operatorname{sign}(u_n)))$.

System (12) can be rewritten as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= \xi, \\ \dot{\xi} &= \varphi_1(e) + \varphi_2(e, u)\, u, \end{aligned} \tag{13}$$

where $\xi(e) = (\xi_1(e), \ldots, \xi_n(e))^{\mathrm{T}} = f_2(e) + g_2(e_1)(A_1 e_3 - A_2 e_4)$, $\varphi_1(e) = \dot{f}_2(e) + \dot{g}_2(e)(A_1 x_3 - A_2 e_4) + g_2(e)(A_1 f_3(e) - A_2 f_4(e))$, $\varphi_2(e, u) = g_2(e)(A_1 g_3(e, u) - A_2 g_4(e, u))$.

In the following section, the finite-time position tracking controller will be designed for the rigid hydraulic manipulator system.

4. Output Feedback Control Design

Main results are given in this section. Firstly, a state feedback controller will be developed for tracking error system (13). Then a finite-time state observer and the corresponding output feedback controller are given.

4.1. State Feedback Control Design

Theorem 6. *For tracking error system (13), if the controller is designed as*

$$\begin{aligned} \varphi_2(e, u)\, u = {} & -\varphi_1(e) - k_1 \operatorname{sig}^{\alpha_1}(e_1) - k_2 \operatorname{sig}^{\alpha_2}(e_2) \\ & - k_3 \operatorname{sig}^{\alpha_3}(\xi), \end{aligned} \tag{14}$$

where k_1, k_2, k_3, $\alpha_1, \alpha_2, \alpha_3$ satisfy the constraint conditions in Lemma 2, the closed-loop system (13) and (14) is finite-time stable; i.e., the manipulator position will track the desired position accurately in finite time.

Proof. Closed-loop system (13) and (14) can be written as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= \xi, \\ \dot{\xi} &= -k_1 \operatorname{sig}^{\alpha_1}(e_1) - k_2 \operatorname{sig}^{\alpha_2}(e_2) - k_3 \operatorname{sig}^{\alpha_3}(\xi). \end{aligned} \tag{15}$$

From Lemma 2, it is concluded that system (15) is finite-time stable. This completes the proof. □

4.2. State Observer and Output Feedback Control Design. In the case without an angular velocity sensor, the angular velocity of the manipulator cannot be measured directly. To reconstruct the angular velocity information, a state observer will be designed.

Let $x_1 = [x_{11}, \ldots, x_{1n}]^{\mathrm{T}} = q$, $x_2 = [x_{21}, \ldots, x_{2n}]^{\mathrm{T}} = \dot{q}$. The rigid manipulator system is written as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= M^{-1}(x_1)(-C(x_1, x_2)\, x_2 - G(x_1) + \tau). \end{aligned} \tag{16}$$

Theorem 7. *For system (16), the system state x_2 can be accurately estimated in finite time by the following observer:*

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - l_1 \operatorname{sig}^{\sigma_1}(\hat{x}_1 - x_1), \\ \dot{\hat{x}}_2 &= M^{-1}(x_1)(-C(x_1, \hat{x}_2)\, \hat{x}_2 - G(x_1) + \tau) \\ & \quad - l_2 \operatorname{sig}^{\sigma_2}(\hat{x}_1 - x_1), \end{aligned} \tag{17}$$

where $l_1 > 0, l_2 > 0$, $\sigma_1 = \sigma, \sigma_2 = 2\sigma - 1$, $\sigma \in (1/2, 1)$.

Proof. Let $w_1 = \hat{x}_1 - x_1, w_2 = \hat{x}_2 - x_2$. The observation error system is written as follows:

$$\begin{aligned} \dot{w}_1 &= w_2 - l_1 \operatorname{sig}^{\sigma_1}(w_1), \\ \dot{w}_2 &= M^{-1}(x_1) \\ & \quad \cdot (C(x_1, x_2)\, x_2 - C(x_1, w_2 + x_2)(w_2 + x_2)) \\ & \quad - l_2 \operatorname{sig}^{\sigma_2}(w_1). \end{aligned} \tag{18}$$

Consider the following reduced system of (18):

$$\begin{aligned} \dot{w}_1 &= w_2 - l_1 \operatorname{sig}^{\sigma_1}(w_1), \\ \dot{w}_2 &= -l_2 \operatorname{sig}^{\sigma_2}(w_1). \end{aligned} \tag{19}$$

System (19) is homogeneous of degree $k = \alpha - 1 < 0$ with respect to the dilation $r = [r_1, r_2]^T = [1, \alpha]^T$. From Lemma 3, system (19) is finite-time stable. Let $h(w_1, w_2) = M^{-1}(x_1)(C(x_1, x_2)x_2 - C(x_1, w_2 + x_2)(w_2 + x_2))$. It follows that $\lim_{\varepsilon \to 0}(h(\varepsilon^{r_1} w_1, \varepsilon^{r_2} w_2)/\varepsilon^{k+r_2}) = \lim_{\varepsilon \to 0}(1/\varepsilon^{k+r_2})[M^{-1}(x_1)(C(x_1, x_2)x_2 - C(x_1, \varepsilon^{r_2} w_2 + x_2)(\varepsilon^{r_2} w_2 + x_2))] = \lim_{\varepsilon \to 0} -M^{-1}(x_1) C(x_1, \varepsilon^{r_2} w_2 + x_2) w_2 \varepsilon^{-k} = 0$. From Lemma 4, it is concluded that observation error system (18) is finite-time stable; i.e., the states of system (16) will be accurately estimated in finite time. This completes the proof. □

Based on Theorems 6 and 7, for the position tracking control problem, the finite-time output feedback controller is designed as

$$\begin{aligned} \varphi_2(\hat{e}, u)\, u = {} & -\varphi_1(\hat{e}) - k_1 \operatorname{sig}^{\alpha_1}(\hat{e}_1) - k_2 \operatorname{sig}^{\alpha_2}(\hat{e}_2) \\ & - k_3 \operatorname{sig}^{\alpha_3}(\xi), \end{aligned} \tag{20}$$

where $\hat{e} = [\hat{e}_1, \hat{e}_2]^T$, $\hat{e}_1 = x_1 - q_d$, $\hat{e}_2 = \hat{x}_2 - \dot{q}_d$.

Remark 8. Since $\varphi_2(\hat{e}, u)$ is a function of u, thus controller (20) is coupled. However, from the point view of practice, the coupled controller should be decoupled. Note that $\phi_1(\cdot) > 0$, $\phi_2(\cdot) > 0$, $A_1 > 0, A_2 > 0$, $V_{1i} > 0, V_{2i} > 0$. Let $Z = [z_1, \ldots, z_n]^T = g_2^{-1}(\hat{e})(-\varphi_1(\hat{e}) - k_1 \operatorname{sig}^{\alpha_1}(\hat{e}_1) - k_2 \operatorname{sig}^{\alpha_2}(\hat{e}_2) - k_3 \operatorname{sig}^{\alpha_3}(\xi))$. Then, the coupled controller (20) can be decoupled as

$$u_i = \begin{cases} \dfrac{Z_i}{A_1 \beta_e k_{q_1} \sqrt{P_s - e_{3i}}/V_{1i} + A_2 \beta_e k_{q_2} \sqrt{e_{4i} - P_r}/V_{2i}}, & Z_i \geq 0 \\ \dfrac{Z_i}{A_1 \beta_e k_{q_1} \sqrt{e_{3i} - P_r}/V_{1i} + A_2 \beta_e k_{q_2} \sqrt{P_s - e_{4i}}/V_{2i}}, & Z_i < 0 \end{cases} \tag{21}$$

where u_i is the i-th component of u.

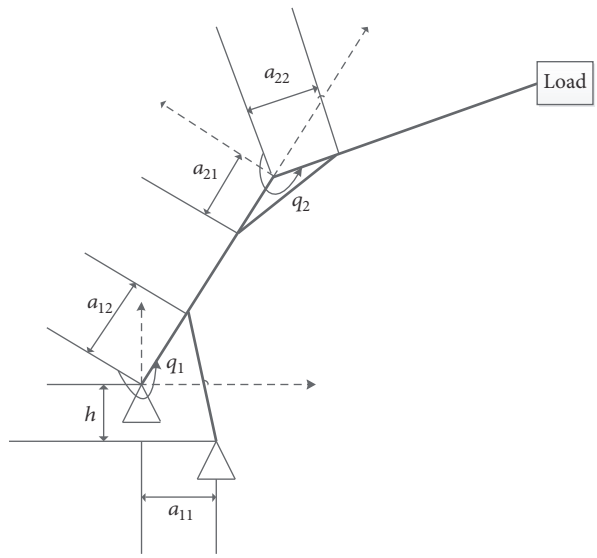

Figure 2: Two-link hydraulic manipulator model.

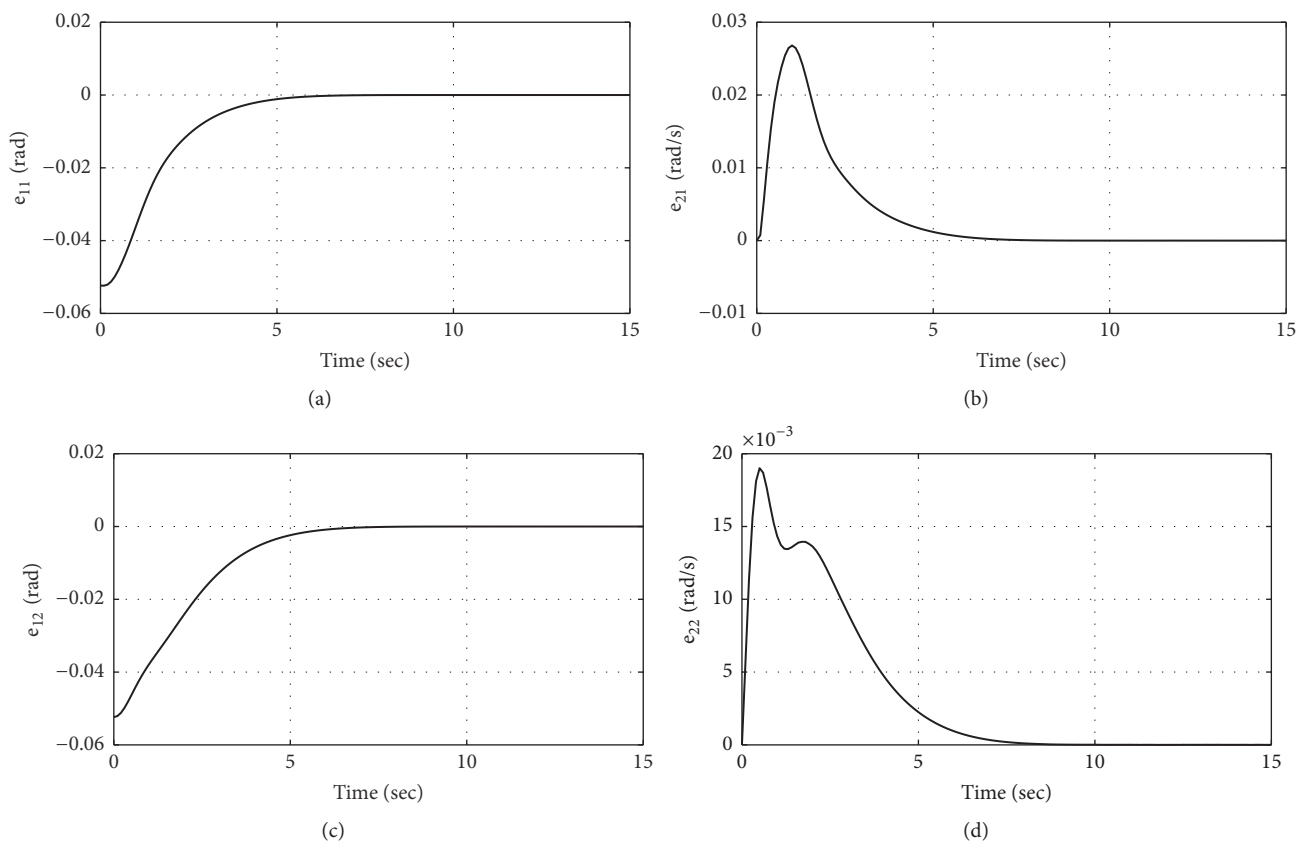

Figure 3: Tracking errors under controller (20). (a) Position tracking error of the first link. (b) Velocity tracking error of the first link. (c) Position tracking error of the second link. (d) Velocity tracking error of the second link.

5. Numerical Simulations

In this section, simulations are performed on a two-link rigid hydraulic manipulator to illustrate the effectiveness of the proposed control method. The model of a two-link rigid manipulator is presented in Figure 2. The inertia matrix, coriolis, and centrifugal torques and gravitational torques are described as $M(q) = [M_{11}, M_{12}; M_{12}, M_{22}]$, where $M_{11} = m_1 l_{c_1}^2 + I_1 + m_2(l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2}\cos(q_2)) + I_2 + m_p(l_1^2 + l_2^2 + 2l_1 l_2 \cos(q_2))$, $M_{12} = m_2(l_1 l_{c_2}\cos(q_2) + l_{c_2}^2) + I_2 + m_p(l_2^2 + l_1 l_2 \cos(q_2))$, $M_{22} = m_2 l_{c_2}^2 + I_2 + m_p l_2^2$. $C(q) =$

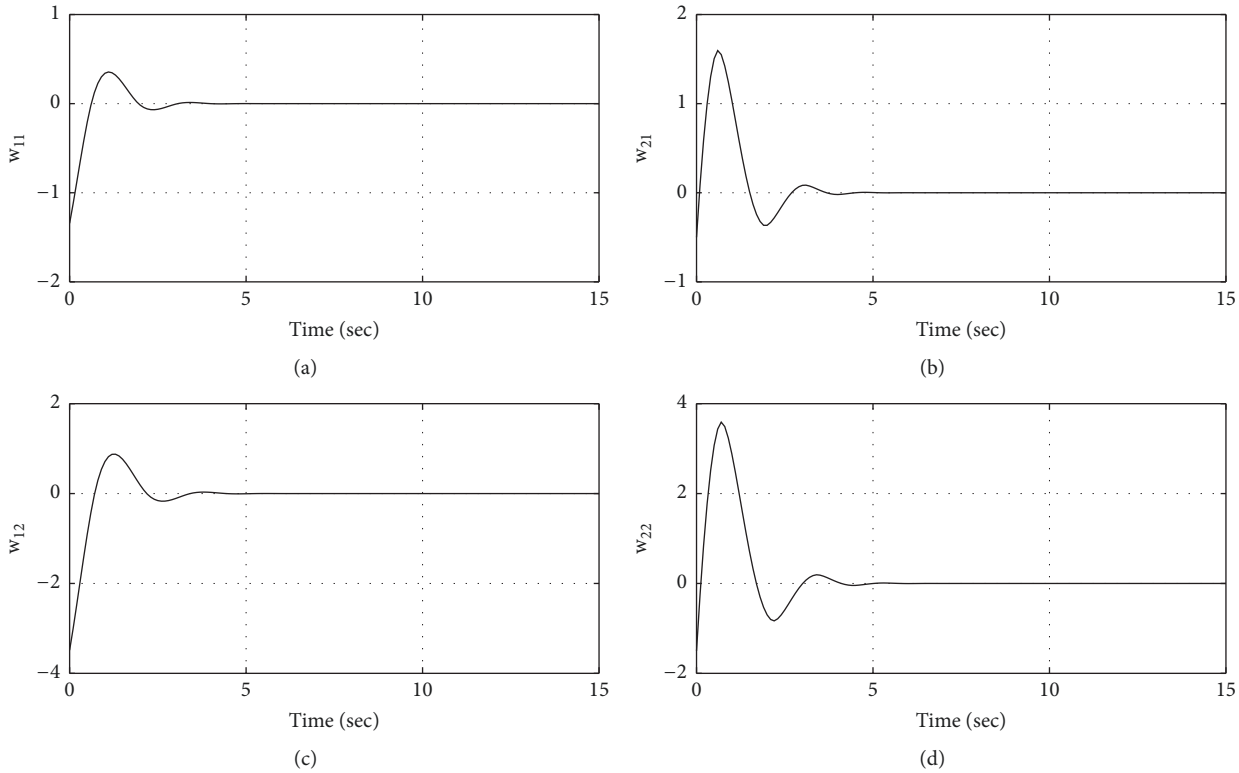

Figure 4: Observation errors. (a) Observation error of x_{11}. (b) Observation error of x_{21}. (c) Observation error of x_{12}. (d) Observation error of x_{22}.

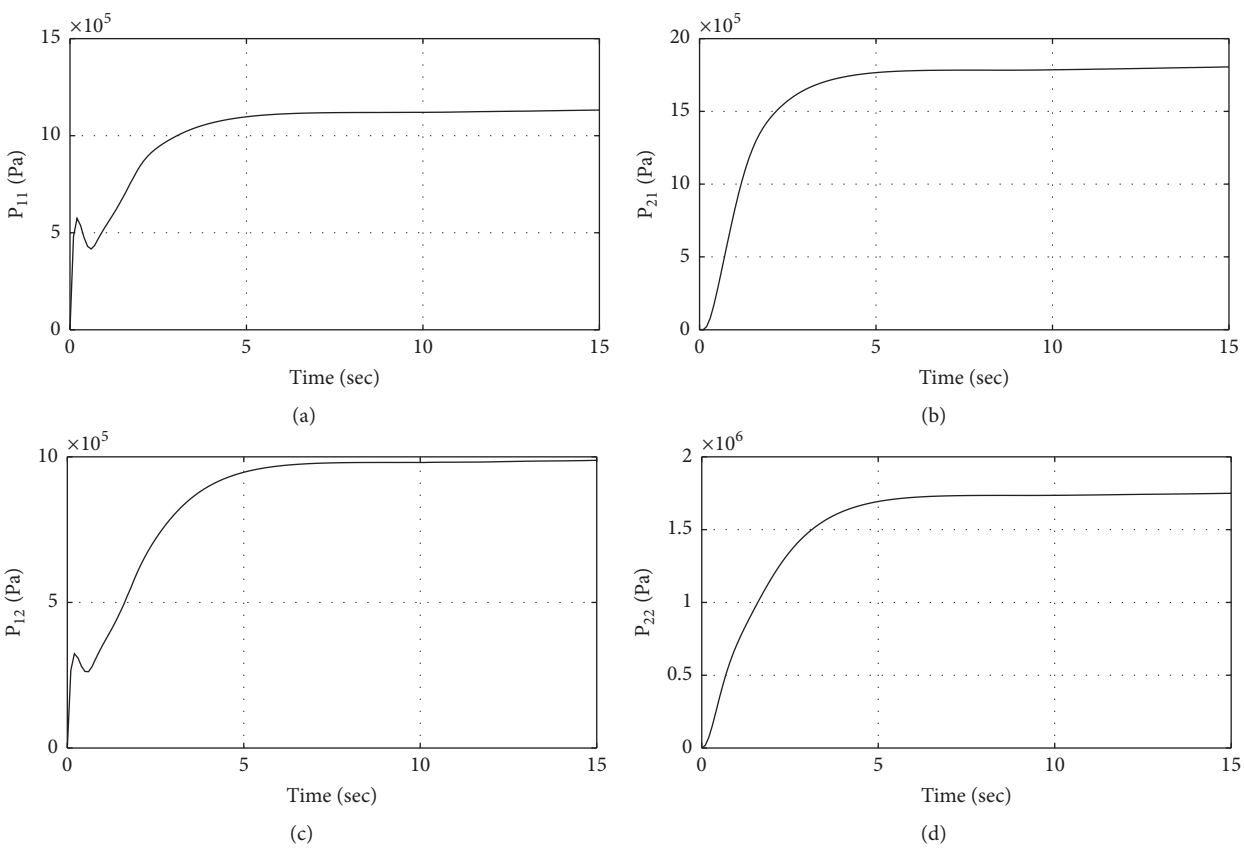

Figure 5: Pressures in the cylinders. (a) Forward pressure in the first actuator. (b) Forward pressure in the second actuator. (c) Return pressure in the first actuator. (d) Return pressure in the second actuator.

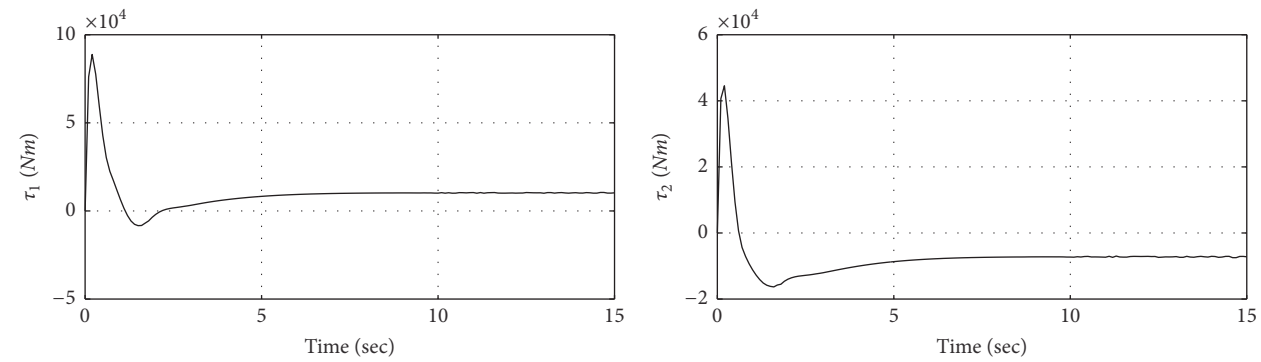

Figure 6: The joint torques. (a) Joint torque of the first link. (b) Joint torque of the second link.

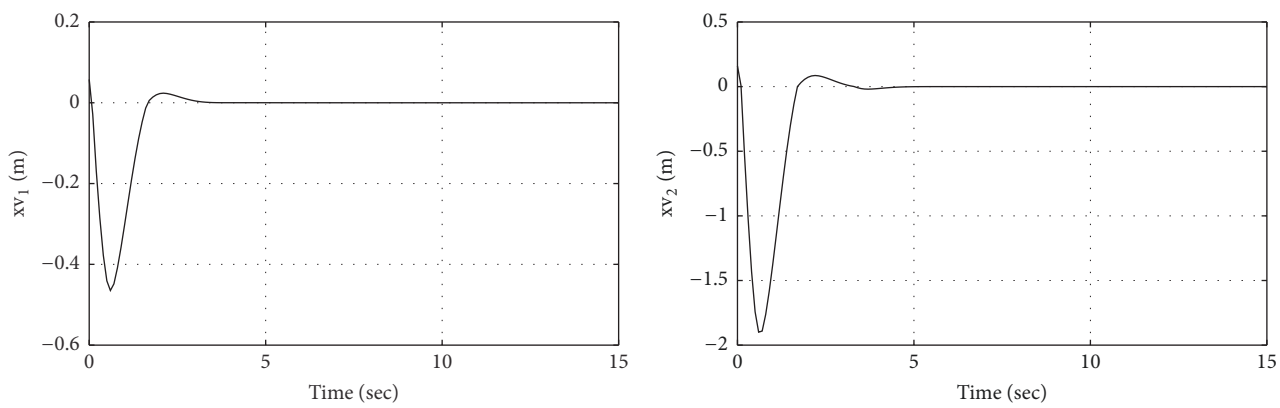

Figure 7: Servo-valve displacements. (a) Servo-valve displacement of the first link. (b) Servo-valve displacement of the second link.

$[C_{11}, C_{12}; C_{12}, C_{22}]$, where $C_{11} = -h_1\dot{q}_2$, $C_{12} = -h_1\dot{q}_2 - h_1\dot{q}_1$, $C_{21} = h_2\dot{q}_1 - h_3\dot{q}_2$, $C_{22} = 0$, where $h_1 = (m_2 l_1 l_{c_2} + m_p l_1 l_2)\sin(q_2)$, $h_2 = m_2 l_1 l_{c_2}\sin(q_2)$, $h_3 = m_p l_1 l_2 \sin(q_2)$. $G(q) = [G_1(q), G_2(q)]^T$, where $G_1(q) = (m_1 l_{c_1} + m_2 l_1 + m_p l_1) g\cos(q_1) + (m_2 l_{c_2} + m_p l_2) g\cos(q_1 + q_2)$, $G_2(q) = (m_2 l_{c_2} + m_p l_2) g\cos(q_1 + q_2)$. The servo-valve displacements are $y_{L_1} = (a_{11}^2 + a_{12}^2 - 2a_{11}a_{12}\cos(q_1) + h^2 + 2ha_{12}\sin(q_1))^{1/2}$, $y_{L_2} = [a_{21}^2 + a_{22}^2 + 2a_{21}a_{22}\cos(q_2)]^{1/2}$, where h is the horizontal distance between the bottom of the lower base and the top of the higher base, which is shown at the bottom of Figure 2. The Jacobian Matrix is $J = [J_1, 0; 0, J_2]$, where $J_1 = (a_{11}a_{12}\sin(q_1) + ha_{12}\cos(q_1))/y_{L_1}$, $J_2 = -a_{21}a_{22}\sin(q_2)/y_{L_2}$.

Model parameters of the hydraulic actuators and the manipulator, borrowed from [29], are given as follows. Parameters of the manipulator are $l_1 = 3.2\text{m}$, $l_{c_1} = 1.6\text{m}$, $l_2 = 2.6\text{m}$, $l_{c_2} = 1.3\text{m}$, $I_1 = 516.3\text{kgm}^2$, $I_2 = 323.2\text{kgm}^2$, $m_1 = 300\text{kg}$, $m_2 = 220\text{kg}$, $m_p = 200\text{kg}$, $a_{11} = 0.4\text{m}$, $a_{21} = 0.4\text{m}$, $a_{12} = 0.4\text{m}$, $a_{22} = 0.4\text{m}$, $h = 0.3\text{m}$, $g = 9.8\text{m/s}^2$. Parameters of the hydraulic manipulator are $C_{tm} = 3.0 \times 10^{-12}\text{m}^5/\text{Ns}$, $\beta_e = 6.013 \times 10^8\text{Pa}$, $P_s = 1.034 \times 10^7\text{Pa}$, $P_r = 0\text{Pa}$, $C_d = 0.61$, $w = 0.023\text{m}$, $\rho = 842\text{kg/m}^3$, $kq_i = 4.835 \times 10^{-4}\text{m}^{5/2}/\sqrt{\text{kg}}$, $A_1 = 2.463 \times 10^{-3}\text{m}^2$, $A_2 = 1.455 \times 10^{-3}\ \text{m}^2$, $V_{10} = 7.239 \times 10^{-4}\text{m}^3$, $V_{20} = 5.021 \times 10^{-4}\text{m}^3$.

The parameters for observer (17) are chosen as $l_1 = 2, l_2 = 5, \sigma_1 = 8/9, \sigma_2 = 7/9$. The parameters of controller (20) are chosen as $k_1 = 4, k_2 = 9, k_3 = 4.5, \alpha_1 = 4/7, \alpha_2 = \frac{2}{3}, \alpha_3 = 4/5$.

Simulation results are presented in Figures 3–7. Figure 3 shows that the manipulator tracks the desired position and velocity accurately in finite time. Figure 4 presents the observation errors of observer (17). The observation errors converge to zero in finite time. The pressure in the cylinders is shown in Figure 5. Figure 6 presents the joint torques of the manipulators. Figure 7 gives the displacement of the servo valve.

6. Conclusions

This paper has studied the finite-time output feedback position tracking control problem for a rigid hydraulic manipulator system. Based on homogeneity theory, in the case without velocity sensors, the finite-time state observer is proposed to estimate the manipulator velocity. With the velocity estimates, the finite-time output feedback controller is developed by utilizing homogeneity theory. Numerical simulations have shown the effectiveness of the proposed output feedback controller.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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