Research Article
Iterative Learning Control for Nonlinear Weighing and Feeding Process

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Due to the nonlinear dynamics in weighing and feeding process, it is difficult to achieve high accuracy with conventional control methods. This paper uses a piecewise linearization method for the nonlinear problem and discusses the application of iterative learning control in weighing and feeding process. First, the nonlinear problem and the repeatability are discussed based on dynamic analysis of weighing and feeding process. Next, a linear state space model is established with a piecewise linearization method. Then, an iterative learning controller is presented by utilizing repetitive characteristics, and the controller parameters are obtained by using a multi-objective optimization method. Finally, simulation results show that the presented control method improves the control performances and the accuracy of feeding.

1. Introduction

A weighing and feeding process is a process in which a certain material is weighted and added to a reaction vessel through an actuator, which repeats many times to get more products. It is widely used in modern industries, such as metallurgical industry, pharmaceutical industry, and food processing industry [1]. Vibratory feeders are the main actuators in weighing and feeding processes, which use vibration and gravity to feed material to a weighing hopper. The weight of the material in the weighing hopper is one of the most important control objectives in a weighing and feeding process. However, it is difficult to achieve high accuracy of the weight because of the existence of a nonlinear process of the vibratory feeder [2].

In the past few years, some good methods were researched to establish accurate mathematical models for vibratory feeders. For example, a model of a resonant linear electromagnetic vibratory feeder was established based on the kinetic and potential energies, the dissipative function of the mechanical system, and Lagrangian formulation [3]. Czubak [4] considered the vibratory feeder as a single degree of freedom system (SDOF). The relations between the structural elements of the mode and the vibratory feeder were analyzed based SDOF [5]. Furthermore, Chandravanshi [6, 7] analyzed the dynamic characteristics of vibratory feeders and examined experimentally the vibration behavior of the particles on the conveying surface. The purposes of these researches are to dynamically analyze the vibratory feeders and the vibration behavior of the particles. These researches reveal the nonlinear relationship between the feed rate and the motor speed (vibratory motors rotate to generate vibration to make the vibratory feeder work). It is obvious that vibratory feeders are nonlinear devices in weighing and feeding processes.

Regarding the control and modeling of weighing and feeding processes, many results were published for one feeding batch, which considered the vibrating feeder as a linear device. For instance, a generalized-predictive-control (GPC)-based PID controller was presented for a linear weighing and feeding process, which considered the relationship between the feed rate and motor speed as a linear model [8]. This method derived PID parameters based on GPC and considered a ramp-type signal as the reference signal. The method presented in [8] was improved in [9], in which the
Table 1: Some notations used in this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( t, k )</td>
<td>The time and batch index, respectively</td>
</tr>
<tr>
<td>( \alpha, \beta )</td>
<td>The time period in a batch and the total number of batches, respectively</td>
</tr>
<tr>
<td>( R )</td>
<td>The set of real number</td>
</tr>
<tr>
<td>( \text{I, } 0 )</td>
<td>The identity matrix and the zero matrix with appropriated dimensions, respectively</td>
</tr>
<tr>
<td>( f, U )</td>
<td>The output frequency and input voltage of the inverter, respectively</td>
</tr>
<tr>
<td>( \omega )</td>
<td>The motor speed</td>
</tr>
<tr>
<td>( v )</td>
<td>The feeding rate</td>
</tr>
<tr>
<td>( w )</td>
<td>The weight of the material in the weighing hopper</td>
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</table>

estimated plant parameters are updated when the prediction error increases. A feedback controller was used in the control of an electromagnetic vibratory feeder, in which the vibratory feeder is considered as a linear device [10]. And an amplitude-frequency control method of the vibratory feeder is presented by using a current-controlled power converter [11]. The previous researches on the weighing and feeding processes achieved good results. However, the achieved control accuracy can be further improved by considering the nonlinear characters and the repetitive nature of weighing and feeding processes.

In summary, previous researches on modeling of vibratory feeders had proven that vibratory feeders are nonlinear devices. However, previous researches on the control and modeling of weighing and feeding processes did not consider the nonlinear characters of the weighing and feeding process. In our previous paper [12], the repetitive nature of a weighing and feeding process was considered, and then the feeding accuracy was improved to some extent. However, this performance improvement is limited because the paper did not consider the nonlinear characters of vibratory feeders.

Many good methods were presented to solve nonlinear problems, such as the fixed point index theory in cones which proved the existence of positive solutions of nonlinear boundary value problem [13, 14], the method of upper and lower solutions and different monotone iterative techniques which analyzed a nonlinear third-order differential equation [15], a weighted norm method which analyzed nonlinear Schrodinger equations [16], and using neural networks to approximate the nonlinear function [17, 18]. These researches provide good references for the nonlinear problem of weighing and feeding processes. Iterative learning control (ILC) [19] can improve the control performance with the increasing number of the batch by taking full advantage of the history control information to correct control input. With the effectiveness of solving the problems with repetitive characteristics, ILC is widely used in the batch processes [20, 21] and repetitive processes [22, 23].

For the repeatability and the nonlinear problem of weighing and feeding processes, this paper presents an ILC and a piecewise linearization method. The rest of this paper is organized as follows: Section 2 analyzes the dynamic characteristics and the nonlinear problem of the process and then establishes an approximate linear model. Section 3 gives a 2D ILC system based on an improved state space model and a controller parametrization method by using a multi-objective optimization method. Section 4 gives the simulation results to illustrate the effectiveness of the proposed method. Finally, Section 5 draws a conclusion.

Some notations used in this paper are described in Table 1.

2. Analysis and Modeling of Weighing and Feeding Process

This section presents the structure of a weighing and feeding process, analyzes the dynamic characteristic of the weighing and feeding process, discusses its nonlinear problem, and presents an approximate linear model by using a piecewise linearization method.

2.1. Weighing and Feeding Process. The schematic diagram of a weighing and feeding process is shown in Figure 1. The vibratory feeder is the main device, which consists of springs for damping, vibration motors for oscillating, and a trough for conveying material. The vibration motors are fixed under the bottom of the trough, which generates an exciting force by driving a rotating unbalanced mass. The trough is inclined at an angle which performs sinusoidal vibration caused by the exciting force. When the trough vibrates due to the exciting force, materials are shaken into the weighing hopper by...
Mathematical Problems in Engineering

Controller

Inverter

Vibratory
motors

Trough

Weighing
hopper

Load cell

Reference input

Input voltage

Frequency

Force

Feed rate

Discharged mass

Vibratory feeder

Figure 2: Weighing and feeding process control system.

Figure 3: The exciting force.

Figure 4: The mechanical model of the vibratory feeder.

the vibrating trough. Weighing hopper measures materials weight by a load cell. The feed rate of the material is adjusted by changing the exciting force and the vibration frequency. In addition, the replenishing hopper replenishes the loss material on the trough. The spring is installed between the trough and the support unit to reduce damage to the support unit caused by the vibration motors.

The block diagram of weighing and feeding process control system is shown in Figure 2, which consists of a controller, an inverter, a vibratory feeder, a weighing hopper, and a load cell. The controller adjusts the speed of the motor by changing the input voltage of the inverter. When the weight of material reaches the set value, the vibratory feeder stops working and prepares for the next batch. In order to get more products, the feeding process like that repeats many times.

The dynamic characteristics of the inverter are approximated as an amplifier [8], which is expressed as

$$f = K_i U,$$  \hspace{1cm} (1)

where $K_i$ is the gain of the inverter. $f$ and $U$ denote the output frequency and input voltage of the inverter, respectively.

The trough of the vibratory feeder vibrates under the drive of double rotating unbalances which produces a periodic force excitation in a definite direction. The electrical model of the motor can be written as a first-order system

$$\dot{\omega} + \frac{1}{T} \omega = \frac{K_m}{T} f,$$  \hspace{1cm} (2)

where $K_m$ is the gain and $T$ is assumed to be the time constant of the motor.

The exciting forces, $F_1$ and $F_2$, are induced by a single rotating unbalance mass (see Figure 3), $m$, respectively. The values of $F_1$ and $F_2$ are both given as

$$F_0 = mE\omega^2,$$  \hspace{1cm} (3)

where $E$ is the eccentricity of the unbalance.

Decomposition of excitation force into horizontal and vertical directions

$$F_x = F_0 \cos \omega t + F_0 \cos (\pi - \omega t) = 0,$$  \hspace{1cm} (4)

$$F_y = F_0 \sin \omega t + F_0 \sin (\pi - \omega t) = 2F_0 \sin \omega t.$$  \hspace{1cm} (5)

The mechanical model of the vibratory feeder is assumed as an SDOF which consists of a mass, $M$, spring with stiffness coefficient, $k_s$, and damping with damping coefficient, $c$, [5] as shown in Figure 4.

The equation of the vibration is expressed as

$$M \frac{d^2 Y}{dt^2} + c \frac{dY}{dt} + k_s Y = F_y,$$  \hspace{1cm} (6)

where $M$ is the equivalent weight which consists of the weight of the vibratory motor, the trough, and the materials in the trough. $Y$ denotes the displacement from the balance position in vertical directions. The damping is the consumption of vibratory energy as heat and sound. $k_s$ is the equivalent stiffness of springs. The exciting force in vertical directions changes sinusoidally. By analyzing (6) through vibration theory, the maximum amplitude, $Y_{\text{max}}$, of the vibratory trough is written as

$$Y_{\text{max}} = \frac{2F_0}{\sqrt{(k_s - \omega^2 M)^2 + (\omega c)^2}}.$$  \hspace{1cm} (7)
The moving distance of the material in the trough is related to the vibration amplitude in one vibration. During a certain period of time, all the materials in the trough move multiple times to form an average feed rate. The average feed rate, \( V \), and the maximum amplitude of vibration are approximately proportion

\[
V = K \frac{\omega}{2\pi} Y_{\text{max}} \tag{8}
\]

where \( K \) is the structure coefficient, which depends on the width of the trough and the angle of inclination of the trough.

Vibratory feeder adds material to the hopper, which can be approximated as an integrator

\[
w = \int v \, dt, \tag{9}
\]

where \( w \) denotes the weight of the material in the weighing hopper.

2.2. Piecewise Linearization Method. By substituting (7) into (8), the relationship between the feeding rate and the motor speed is written as

\[
v = \frac{K m E \omega^3}{\pi \sqrt{(k_s - \omega^2 M)^2 + (\omega c)^2}} \tag{10}
\]

It is obvious that the feed rate increases first and then decreases when the motor speed increases through mathematical analysis of the relationship between \( v \) and \( \omega \). And when the vibration frequency of the vibratory motor is equal to the natural frequency, \( \omega_n \), the vibrating feeder reaches the maximum feeding rate. At this time, the motor speed \( \omega_n \) is

\[
\omega_n = \sqrt{\frac{k_s}{M}}. \tag{11}
\]

Through the analysis of the relationship between the feeding rate and the motor speed, the feeding rate is decreasing when the motor speed is increasing \((\omega > \omega_n)\). It should be avoided to work in this state because increasing the speed of the motor reduces the feeding speed, which is an inefficient production way.

Based on the considerations above, the motor of vibratory feeder should work in motor speed range \(0 < \omega < \omega_n\). Then, the nonlinear relationship between motor speed and feeding rate is piecewise linearized into proportional relation

\[
v = \begin{cases} a_1 \omega + b_1, & 0 \leq \omega < \omega_1 \\ a_2 \omega + b_2, & \omega_1 \leq \omega < \omega_2 \\ a_3 \omega + b_3, & \omega_2 \leq \omega < \omega_3 \end{cases} \tag{12}
\]

which can be written as

\[
v = a_i \omega + b_i, \quad \omega \in \Omega_i, \quad 1 \leq i \leq 3. \tag{13}
\]

The nonlinear character and the piecewise linearization result are shown in Figure 5. In this paper, the motor speed range is divided equally into three intervals. Then the parameters of (13) are obtained by the corresponding motor speed and feeding rate. The accuracy of the piecewise linearized model significantly depends on the number of the intervals and more intervals can increase the accuracy of the piecewise linearized model.

Use the following coordinate transformation

\[
\begin{align*}
\mathbf{V} &= \begin{bmatrix} v \\ 0 \end{bmatrix}, \tag{14} \\
\mathbf{\omega} &= \begin{bmatrix} \omega \\ 1 \end{bmatrix}. \tag{15}
\end{align*}
\]

Then the nonlinear relationship between the feeding rate and the motor speed transforms into an approximate linear model

\[
\mathbf{V} = \begin{bmatrix} a_i & b_i \\ 0 & 0 \end{bmatrix} \mathbf{\omega}, \quad \mathbf{\omega} \in \Omega_i, \quad 1 \leq i \leq 3. \tag{16}
\]

2.3. The State Space Model of the Weighing and Feeding Process. According to the previous analysis, the relationship between motor speed and the input voltage is obtained from (1) and (2), the relationship between motor speed and the feeding rate is obtained from (16), and the relationship between the feeding rate and the weight of the material in the weighing hopper is obtained from (9). They are used as parts of (17) and (18). Then the model of the control object is described as

\[
\dot{x} = \begin{bmatrix} \frac{1}{T} & 0 & 0 \\ 0 & \frac{1}{T} & 0 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} K_s K_i \omega \end{bmatrix} u, \quad 1 \leq i \leq 3, \tag{17}
\]

\[
y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x. \tag{18}
\]
3. Iterative Learning Control System

This section discusses the batch characteristic of weighing and feeding process and gives the structure of iterative learning controller based on an improved state space model. Then the controller parameters are found through a multi-objective optimization method.

3.1 2D System Representation. A discrete state space equation which is obtained by using the zero-order hold to discrete (17) and (18) over \(1 \leq t \leq \alpha, k \geq 1\) is

\[
\begin{align*}
    x_k(t+1) &= A_i x_k(t) + B_i u_k(t), \quad 1 \leq i \leq 3, \\
y_k(t) &= C x_k(t),
\end{align*}
\]

where \(u_k(t) \in R\) is the control input, \(y_k(t) \in R\) is the output, \(x_k(t) \in R^2\) is the state vector, including the motor speed \(\omega\), and the discharged mass \(w\) at time \(t\) in \(k\)th batch.

The discrete dynamic model in the \(k\)th batch is described as above. However, this process will be repeated many times to get more products in practice. Based on the above considerations, define \(x(t, k)\) and \(y(t, k)\) as the state and output of the process at time \(t\) in \(k\)th batch. The following discrete model can be established from (19) and (20) to describe the relationship between batches:

\[
\begin{align*}
x(t+1, k) &= A_i x(t, k) + B_i u(t, k), \quad 1 \leq i \leq 3, \\
y(t, k) &= C x(t, k).
\end{align*}
\]

Let \(y_{ref}(t)\) be a vector-valued reference representing desired output behavior. Then the error in the \(k\)th batch is

\[
e(t, k) = y_{ref}(t) - y(t, k).
\]

In ILC, a general form of the control law is

\[
\Sigma_{ilc}: \begin{cases}
    u(t, k) = u(t, k - 1) + u_e(t, k) \\
    u(t, 0) = 0,
\end{cases}
\]

where \(u(t, k)\) is constructed from the control in previous batch and a correction term \(u_e(t, k)\) from the history control information and \(u(t, 0)\) denotes the initial control. \(u_e(t, k)\) is the updating law to be determined.

Define \(f_e(t, k + 1)\) as the difference of \(f\) between two consecutive batches

\[
f_e(t, k + 1) = f(t, k + 1) - f(t, k),
\]

where \(f\) may be state \(x\) or output \(y\) and thus is in accordance with the ILC updating law shown in (24).

The tracking error integral \(\sum e(t, k + 1)\) is an additional state variable in ILC design, which is defined as

\[
\sum e(t, k + 1) = \sum_{i=0}^{t} e(i, k + 1).
\]

It follows from (21), (22) and (25) that

\[
x_e(t + 1, k + 1) = A_i x_e(t, k + 1) + B_i u_e(t, k + 1), \quad 1 \leq i \leq 3.
\]

Equation (23) can be written as

\[
e(t + 1, k + 1) = y_{ref}(t + 1) - y(t + 1, k + 1) + y(t + 1, k) - y(t + 1, k)
\]

\[
e(t + 1, k) - y_e(t + 1, k + 1).
\]

It follows from (21), (22), (25), and (27) that

\[
y_e(t + 1, k + 1) = CA_i x_e(t, k + 1) + CB_i u_e(t, k + 1), \quad 1 \leq i \leq 3.
\]

According to (29), (28) can be written as

\[
e(t + 1, k + 1) = e(t + 1, k) - CA_i x_e(t, k + 1) - CB_i u_e(t, k + 1).
\]

It follows from (26) and (30) that

\[
\sum e(t + 1, k + 1) = \sum e(t + 1, k) + e(t + 1, k) - CA_i x_e(t, k + 1) - CB_i u_e(t, k + 1), \quad 1 \leq i \leq 3.
\]

Introduce a new state variable as

\[
\bar{x}(t + 1, k + 1) = \begin{bmatrix}
x_e(t + 1, k + 1) \\
e(t + 1, k + 1) \\
\sum e(t + 1, k + 1)
\end{bmatrix}.
\]

Then the corresponding state space model is derived as

\[
\bar{x}(t + 1, k + 1) = A_i \bar{x}(t, k + 1) + A_{i+1} \bar{x}(t + 1, k) + \tilde{B}_1 u_e(t, k + 1), \quad 1 \leq i \leq 3,
\]

where

\[
A_{i,1} = \begin{bmatrix}
    A_i & 0 & 0 \\
    -CA_i & 0 & 0 \\
    -CA_i & 0 & 1
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
    0 & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 1 & 0
\end{bmatrix},
\]

\[
\tilde{B}_1 = \begin{bmatrix}
    B_i \\
    -CB_i \\
    -CB_i
\end{bmatrix}.
\]

3.2 Controller Design. Design an updating law as

\[
u_e(t, k + 1) = K_{i,1} \bar{x}(t, k + 1) + K_{i,2} \bar{x}(t + 1, k), \quad 1 \leq i \leq 3,
\]
Then the improved 2D system of (21) and (22) is given by

\[
\begin{align*}
\dot{x}(t+1,k+1) &= \ddot{A}_{i,1} \ddot{x}(t,k+1) + \ddot{A}_{i,2} \ddot{x}(t+1,k), \\
\end{align*}
\]

where

\[
\ddot{A}_{i,1} = \begin{bmatrix} A_i + B_i K_{i,1} & B_i K_{i,12} & B_i K_{i,13} \\ -C_A K_{i,1} & -C_B K_{i,12} & -C_B K_{i,13} \\ -C_A K_{i,1} & -C_B K_{i,12} & I - C_B K_{i,13} \end{bmatrix},
\ddot{A}_{i,2} = \begin{bmatrix} B_i K_{i,21} & B_i K_{i,22} & B_i K_{i,23} \\ -C_B K_{i,1} & I - C_B K_{i,22} & -C_B K_{i,23} \\ -C_B K_{i,1} & I - C_B K_{i,22} & I - C_B K_{i,23} \end{bmatrix}.
\]

Then an improved 2D system of (21) and (22) is given by

\[
\ddot{x}(t+1,k+1) = \ddot{A}_{i,1} \ddot{x}(t,k+1) + \ddot{A}_{i,2} \ddot{x}(t+1,k),
\]

where

\[
\ddot{A}_{i,1} = \begin{bmatrix} A_i + B_i K_{i,11} & B_i K_{i,12} & B_i K_{i,13} \\ -C_A K_{i,11} & -C_B K_{i,12} & -C_B K_{i,13} \\ -C_A K_{i,11} & -C_B K_{i,12} & I - C_B K_{i,13} \end{bmatrix},
\ddot{A}_{i,2} = \begin{bmatrix} B_i K_{i,21} & B_i K_{i,22} & B_i K_{i,23} \\ -C_B K_{i,11} & I - C_B K_{i,22} & -C_B K_{i,23} \\ -C_B K_{i,11} & I - C_B K_{i,22} & I - C_B K_{i,23} \end{bmatrix}.
\]

More states information of (21) and (22) is considered, and the control law (37) contains control information of the current and historical batches. The appropriate controller parameters, \(K_{i,1}, K_{i,2}\), will give the system a better control performance.

3.3. Stability Analysis

**Theorem 1.** The closed-loop 2D system in (40) is asymptotically stable and if symmetric positive matrices \(P > Q\) exist, the following linear matrix inequality holds

\[
\Psi_i = \begin{bmatrix} -P & PA_{i,1} + Y_1 & PA_{i,2} + Y_2 \\ * & -Q & 0 \\ * & * & -P + Q \end{bmatrix} < 0,
\]

where

\[
Y_1 = P \tilde{B}_i K_{i,1},
Y_2 = P \tilde{B}_i K_{i,2}.
\]

And the controller parameters \(K_{i,1}, K_{i,2}\) can be solved as

\[
K_{i,1} = \tilde{B}_i^{-1} P^{-1} Y_1,
K_{i,2} = \tilde{B}_i^{-1} P^{-1} Y_2,
\]

where \(P\) and \(Q\) are symmetric positive matrices to be determined, and \(P > Q\).

The state energy increment in both time and batch directions is

\[
\Delta V = V_{1,1} - V_{0,1} - V_{1,0}.
\]

It follows from (40) and (51) that

\[
\dot{\Delta V} = \xi^T \Psi \dot{\xi},
\]

where

\[
\xi = \begin{bmatrix} \ddot{x}(t,k+1)^T, \ddot{x}(t+1,k)^T \end{bmatrix}^T,
\Psi_i = \begin{bmatrix} \tilde{A}_{i,1}^T P \tilde{A}_{i,1} - Q & \tilde{A}_{i,1}^T P \tilde{A}_{i,2} \\ \tilde{A}_{i,2}^T P \tilde{A}_{i,1} & \tilde{A}_{i,2}^T P \tilde{A}_{i,2} - P + Q \end{bmatrix}.
\]

Therefore, \(\Delta V < 0\) guarantees asymptotic stability of closed-loop 2D system in (40).

Applying Schur’s complement \(\Delta V < 0\) is equivalent to

\[
\begin{bmatrix} -P & PA_{i,1} & PA_{i,2} \\ * & -Q & 0 \\ * & * & -P + Q \end{bmatrix} < 0.
\]

Replace \(\ddot{A}_{i,1}, \ddot{A}_{i,2}\) with \(A_{i,1} + \tilde{B}_i K_{i,1}, A_{i,2} + \tilde{B}_i K_{i,2}\), and defining \(P \tilde{B}_i K_{i,1} = Y_1, P \tilde{B}_i K_{i,2} = Y_2\, LMI\) constraints in (43) can be obtained.

3.4. Controller Parametrization. Theorem 1 is a sufficient and unnecessary condition for system stability, and there are solutions to the LMI Equation (43). The controller with them as parameters stabilizes the system but has different control performance. An efficient combination of linear matrix inequalities and single-objective optimization algorithms was proposed to develop a control law parametrization that guarantees performance [23]. Based on this research, this paper proposes a method of controller parametrization, which combines LMI and multi-objective optimization algorithms.

For the convenience of writing, \(K_{i,1}, K_{i,2}\) are described as \(K_1, K_2\) in this section.

In order to measure the control performance of the system, the first function is chosen as

\[
f_1(K_1, K_2) = \sum_{k \in [1, \alpha]} \sum_{l \in [1, \beta]} \left( y_l - y_{ref,l} \right)^2,
\]

which is the sum of squares of the output error for all samples in all batches. \(f_i(K_1, K_2)\) contains all the system output error information, and the smaller the \(f_i(K_1, K_2)\) is, the better the system’s dynamic performance and steady-state performance are.

In order to measure the deviation of the output from the reference signal, the second function is chosen as

\[
f_2(K_1, K_2) = \max_{k \in [1, \alpha]} \left\{ \left| y(t,k) - y_{ref,l}(t) \right| \right\}_{l \in [1, \beta]}.
\]
Step 1. Initialize parameters, the initial temperature, $T$, and the lowest temperature, $T_f$

Step 2. Solve the LMI \( \psi_i < 0 \), apply (46) and (47) to calculate \( K_1, K_2 \), set \( K_{1,opt} = K_1, K_{2,opt} = K_2 \), simulate (21) and (22) and the nonlinear system with the control law (37), and calculate and set \( f_{1,new} = f_1(K_1, K_2), f_{2,new} = f_2(K_1, K_2) \).

Step 3. After the controller matrices \( K_{1,opt}, K_{2,opt} \), obtain the new matrices \( K_1, K_2 \), simulate (21) and (22) and the nonlinear system with the control law (37), and calculate the \( f_{1,new} = f_1(K_1, K_2), f_{2,new} = f_2(K_1, K_2) \).

Step 4. If \( f_{1,new} < f_{1,opt} \), and \( f_{2,new} < f_{2,opt} \), set \( K_{1,opt} = K_1, K_{2,opt} = K_2, f_{1,opt} = f_{1,new}, f_{2,opt} = f_{2,new} \). If it does not, calculate \( \mu = (1 - \alpha) \cdot (f_{1,new} - f_{1,opt})/(f_{1,new} + f_{1,opt}) + \alpha \cdot (f_{2,new} - f_{2,opt})/(f_{2,new} + f_{2,opt}) \), and set \( K_{1,opt} = K_1, K_{2,opt} = K_2, f_{1,opt} = f_{1,new}, f_{2,opt} = f_{2,new} \) with the probability \( p_{accept} = \exp(-(1 - \mu)/T) \).

Step 5. Reduce the temperature $T$. If $T > T_f$, go to Step 3. If it does not, go to Step 6.

Step 6. Verify \( K_{1,opt}, K_{2,opt} \) subject to \( \psi_i < 0 \), \( 1 \leq i \leq 3 \), using \( K_{1,opt}, K_{2,opt} \).

Table 2: The plant parameters.

<table>
<thead>
<tr>
<th>Plant parameter</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The coefficient of the inverter</td>
<td>( k_i )</td>
<td>5</td>
</tr>
<tr>
<td>The weight of the unbalanced mass</td>
<td>( m )</td>
<td>3.5</td>
</tr>
<tr>
<td>The eccentricity of weight from the shaft</td>
<td>( E )</td>
<td>0.06</td>
</tr>
<tr>
<td>The structure coefficient</td>
<td>( K )</td>
<td>0.882</td>
</tr>
<tr>
<td>The equivalent stiffness of springs</td>
<td>( k_s )</td>
<td>76520</td>
</tr>
<tr>
<td>Weight of trough</td>
<td>( M )</td>
<td>60</td>
</tr>
<tr>
<td>The coefficient damping</td>
<td>( c )</td>
<td>752</td>
</tr>
<tr>
<td>The gain of the motor</td>
<td>( k_m )</td>
<td>0.6</td>
</tr>
<tr>
<td>The time-constant of the motor</td>
<td>( T )</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: The parameters of the controller.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{1,1} )</td>
<td>([-1.862, -0.050, 0.150, -0.027, -0.605])</td>
</tr>
<tr>
<td>( K_{1,2} )</td>
<td>([0.035, 0.055, 0.020, 0.081, 0.206])</td>
</tr>
<tr>
<td>( K_{2,1} )</td>
<td>([-1.959, 0.054, -0.087, 0.052, -0.192])</td>
</tr>
<tr>
<td>( K_{2,2} )</td>
<td>([0.034, 0.060, -0.072, -0.124, 0.282])</td>
</tr>
<tr>
<td>( K_{3,1} )</td>
<td>([-1.757, 0.005, -0.082, -0.033, 0.015])</td>
</tr>
<tr>
<td>( K_{3,2} )</td>
<td>([0.076, 0.052, 0.020, 0.029, 0.291])</td>
</tr>
</tbody>
</table>

4. Simulation

This section gives numerical simulation results, which show that more accurate feed accuracy is obtained after several feeding batches.

The plant parameters are set in Table 2. The discrete state space model is obtained with a sampling period of \( T_s = 1 \) s. And the weight of material in one batch is set: 20 kg. The duration of a batch is set to 200 s. Then the reference trajectory is obtained

\[
y_{ref} = 0.1t, \quad 1 \leq t \leq 200.
\]

The controller parameters \( K_{1,1} \) and \( K_{1,2} \) are obtained by using the controller parameterization method presented in Section 3.4, which is shown as Table 3.

The updating law is obtained by substituting the controller parameters into (37). Then the input voltage is calculated based on (24) and (37), which is the control input applied to the inverter. Then the output response and the process state are obtained in this sampling time. The output responses in all batches are shown in Figure 6.

Figure 6 shows that the output response curve has large fluctuations in the first batch, which means the feeding performance is not good enough. However, the feeding performance is improved. Figure 6 shows that the fluctuation of the output response curve in the latter batch is smaller than the previous.

Figure 7 shows the output response in the first, fourth, and eighth batches, which shows that the errors between the output responses and the reference signal are decreased with the number of batch increasing.

Two performance indicators, the squared error, and the maximum error are presented to evaluate the control performance of the ILC for the weighing and feeding process.
Table 4: Two performance indicators in first 9 batches.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Squared error</td>
<td>555</td>
<td>276</td>
<td>123</td>
<td>53</td>
<td>40</td>
<td>35</td>
<td>36</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Maximum error</td>
<td>2.34</td>
<td>2.21</td>
<td>1.56</td>
<td>1.38</td>
<td>1.26</td>
<td>1.17</td>
<td>1.09</td>
<td>1.03</td>
</tr>
</tbody>
</table>

The squared error denotes the output error in all sampling time in one batch. The small squared error means good steady-state performance. The maximum error denotes the maximum output error in one batch. The small maximum error means good dynamic state performance. Two performance indicators in the first 9 batches are shown in Table 4. Two values of performance indicators decrease with the number of batch increasing.

Figure 6 shows the change of two performance indicators in the first 9 batches, which demonstrates intuitively that the two performance indicators decrease with the number of batch increasing. It means that the control performances and the accuracy of feeding are improved with the number of batches increasing, and the accuracy of feeding is acceptable after 3-4 batches.

5. Conclusions

This paper presented an ILC method and a piecewise linearization method for a weighing and feeding process with the nonlinear problem. A piecewise linearization method was presented to establish an approximate linear model. Then an iterative learning controller was designed and the controller parametrization method was given. The simulation result showed that ILC and piecewise linearization effectively solved the control problem of weighing and control process.

Our future work will design a new control method considering the changes in the parameters of the weighing and feeding process model, caused by changes in the properties of the material. And we plan to develop a control system to apply the control method in a real-world weighing and feeding process.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
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References
