Research Article

Stability Analysis of Solutions for a Kind of Integro-Differential Equations with a Delay

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Stability of zero solution for second-order integro-differential equations with a delay is analyzed and some new results are presented. Through constructing Lyapunov functional, we give the corresponding sufficient conditions on stability of zero solution for two integro-differential equations. Moreover, an illustrative example is considered to support our new results.

1. Introduction

Stability and the existence of solutions for nonlinear differential equations have been studied by many scholars [1–20] due to their many applications to problems in information theory, control theory, mechanics, chemistry, physics, and so on. In [5], the authors considered a second-order functional integro-differential equation with multiple delays

$$\begin{aligned} x''(t) + a(t)f(t, x(t), x'(t))x'(t) \\ + g(t, x(t), x'(t)) + \sum_{i=1}^n h_i(x(t - \tau_i)) \\ = \int_0^t C(t, \xi)x'(\xi)d\xi, \end{aligned} \quad (1)$$

and they gave some new conditions on the continuability and boundedness of solutions. Li [6] studied Ψ -stability of the trivial solutions of the following three nonlinear systems with time delay,

$$x'(t) = f(t, x(t)) + g(t, x(t - \tau(t))), \quad (2)$$

and two Volterra integro-differential equations

$$\begin{aligned} x'(t) = f(t, x(t)) + g(t, x(t - \tau(t))) \\ + p(t, x(t)) \int_0^t q(s, x(s - \tau(s)))ds, \end{aligned} \quad (3)$$

$$\begin{aligned} x'(t) = f(t, x(t)) + g(t, x(t - \tau(t))) \\ + p(t, x(t - \tau(t))) \int_0^t q(s, x(s))ds. \end{aligned} \quad (4)$$

In [1], Afuwape studied the asymptotic stability and the uniformly ultimate boundedness of the solutions for a kind of third-order delay differential equations and gave some sufficient conditions.

This paper investigates second-order integro-differential equations with a delay and obtains some new results on the stability of zero solution. By constructing Lyapunov functional, the corresponding sufficient conditions are present on stability of zero solution for two integro-differential equations. Moreover, an illustrative example is considered to show that our results are effective.

The rest of this paper is organized as follows. Section 2 presents the main results of this paper. In Section 3, an illustrative example is given to support our new results.

2. Main Results

Consider the following integro-differential equation with a delay:

$$\begin{aligned} x'' + f(t, x, x')x' + g(t, x, x') + h(x(t-\tau)) \\ = p(t, x(t)) \int_0^t q(s, x'(s)) ds, \end{aligned} \quad (5)$$

where $\mathbb{R}_+ = [0, +\infty)$, $\tau > 0$ is a constant, $f, g : \mathbb{R}_+ \times \mathbb{R}^2 \rightarrow \mathbb{R}$ and $h : \mathbb{R}_+ \rightarrow \mathbb{R}$ are continuous with $h(0) = 0$, and $p, q : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}$ with $p(t, 0) = 0$ and $q(t, 0) = 0$.

We can rewrite (5) as the system

$$\begin{aligned} x' &= y, \\ y' &= -f(t, x, y)y - g(t, x, y) - h(x) \\ &\quad + \int_{t-\tau}^t h'(x(s))y(s)ds \\ &\quad + p(t, x) \int_0^t q(s, y(s))ds. \end{aligned} \quad (6)$$

Theorem 1. Consider system (6). There exist nonnegative constants K^* , R , b , M , and N , a positive number l , and functions $P(t), Q(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, such that the following conditions hold:

$$(C_1) \quad h(x)x^{-1} \geq l \text{ when } x \neq 0, |h'(x)| \leq K^*.$$

$$(C_2) \quad g(t, x, 0) = 0, g(t, x, y)y^{-1} \geq b \text{ when } y \neq 0.$$

$$(C_3) \quad |p(t, x)| \leq P(t) \leq M, |q(t, y)| \leq Q(t)|y|, Q(t) \leq N.$$

$$(C_4) \quad N \int_0^\infty P(s)ds + M \int_0^\infty Q(s)ds + K^*(\tau + 1) \leq 2f(t, x, y) + 2b.$$

Then the zero solution of system (6) is stable.

Proof. Define a Lyapunov functional $V(t) = V(t, x, y)$ as

$$\begin{aligned} V(t) &= \frac{1}{2}y^2 + \int_0^x h(\eta) d\eta + \lambda \int_{-\tau}^t ds \int_{t+s}^t y^2(\theta) d\theta \\ &\quad + \frac{1}{2} \int_0^t ds \int_t^\infty |p(\theta, x(\theta))| Q(s) y^2(s) d\theta, \end{aligned} \quad (7)$$

where λ is a positive constant to be determined. Using condition (C_1) , we have

$$\int_0^x h(\eta) d\eta = \int_0^x \frac{h(\eta)}{\eta} \eta d\eta \geq \int_0^x l \eta d\eta = \frac{l}{2} x^2 > 0, \quad (8)$$

and $V(t) \geq (l/2)x^2 + (1/2)y^2$.

Suppose that $(x(t), y(t))$ is a solution of (6). Then

$$\begin{aligned} \frac{dV}{dt} &= yy' + h(x)y + \lambda \int_{-\tau}^0 [y^2(t) - y^2(t+s)] ds \\ &\quad + \frac{1}{2} Q(t) y^2(t) \int_t^\infty |p(\theta, x(\theta))| d\theta \\ &\quad - \frac{1}{2} |p(t, x(t))| \int_0^t Q(s) y^2(s) ds \\ &= -f(t, x, y) y^2 - g(t, x, y) y + \lambda \tau y^2 \\ &\quad + y(t) \int_{t-\tau}^t h'(x(s)) y(s) ds \\ &\quad + y(t) p(t, x(t)) \int_0^t q(s, y(s)) ds \\ &\quad - \lambda \int_{t-\tau}^t y^2(s) ds \\ &\quad + \frac{1}{2} Q(t) y^2(t) \int_t^\infty |p(\theta, x(\theta))| d\theta \\ &\quad - \frac{1}{2} |p(t, x(t))| \int_0^t Q(s) y^2(s) ds. \end{aligned} \quad (9)$$

By conditions (C_1) and (C_3) and $2ab \leq a^2 + b^2$, we have

$$\begin{aligned} &y(t) \int_{t-\tau}^t h'(x(s)) y(s) ds \\ &\leq |y(t)| \int_{t-\tau}^t |h'(x(s))| |y(s)| ds \\ &\leq \frac{K^*}{2} \int_{t-\tau}^t [y^2(t) + y^2(s)] ds \\ &= \frac{K^*}{2} \tau y^2 + \frac{K^*}{2} \int_{t-\tau}^t y^2(s) ds, \\ &y(t) p(t, x(t)) \int_0^t q(s, y(s)) ds \\ &\leq |y(t)| |p(t, x(t))| \int_0^t |q(s, y(s))| ds \end{aligned} \quad (10)$$

$$\begin{aligned}
&\leq |p(t, x(t))| |y(t)| \int_0^t Q(s) |y(s)| ds \\
&\leq \frac{1}{2} |p(t, x(t))| \int_0^t Q(s) [y^2(t) + y^2(s)] ds \\
&\leq \frac{My^2}{2} \int_0^\infty Q(s) ds \\
&\quad + \frac{1}{2} |p(t, x(t))| \int_0^t Q(s) y^2(s) ds.
\end{aligned} \tag{11}$$

Then,

$$\begin{aligned}
\frac{dV}{dt} &\leq \left[-f(t, x, y) - b + \lambda\tau + \frac{K^*}{2}\tau \right] y^2 \\
&\quad + \left(\frac{K^*}{2} - \lambda \right) \int_{t-\tau}^t y^2(s) ds \\
&\quad + \frac{N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds}{2} y^2.
\end{aligned} \tag{12}$$

Choosing $\lambda = K^*/2$, we have

$$\frac{dV}{dt} \leq \frac{-2f(t, x, y) - 2b + K^*(\tau + 1) + N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds}{2} y^2 \leq 0. \tag{13}$$

Therefore, the zero solution of system (6) is stable and the proof is completed. $\square$

Theorem 1 can be generalized to the form with a variable delay $\tau(t)$. It only needs a new condition. Then, the following result is obtained.

Corollary 2. Consider system (6) with a variable delay $\tau(t)$. Conditions (1)-(3) of Theorem 1 are satisfied. Moreover,

$$\begin{aligned}
(C_4') & N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds + ((2-\beta)/(1-\beta)) K^* \tau \leq 2f(t, x, y) + 2b, \\
(C_5) & \text{ there are } \tau > 0 \text{ and } 0 < \beta < 1, \text{ such that } 0 \leq \tau(t) \leq \tau \text{ and } \tau'(t) \leq 1.
\end{aligned}$$

Then the zero solution of system (6) with a variable delay $\tau(t)$ is stable.

Proof. The proof is similar to that of Theorem 1. But

$$\begin{aligned}
\frac{dV}{dt} &\leq \left[-f(t, x, y) - b + \lambda\tau(t) + \frac{K^*}{2}\tau(t) \right] y^2 \\
&\quad + \left[\frac{K^*}{2} - \lambda(1-\beta) \right] \int_{t-\tau(t)}^t y^2(s) ds \\
&\quad + \frac{N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds}{2} y^2.
\end{aligned} \tag{14}$$

Thus, choosing $\lambda = K^*/2(1-\beta)$, we have

$$\frac{dV}{dt} \leq \frac{-2f(t, x, y) - 2b + ((2-\beta)/(1-\beta)) K^* \tau + N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds}{2} y^2 \leq 0. \tag{15}$$

Therefore, the zero solution of system (6) with a variable delay $\tau(t)$ is stable and the proof is completed. $\square$

Next, we consider another second-order integro-differential system with time delay

$$\begin{aligned}
&x'' + f(t, x, x') x' + g(t, x, x') + h(x(t)) \\
&= p(x(t-\tau)) \int_0^t q(s, x'(s)) ds,
\end{aligned} \tag{16}$$

System (16) can be rewritten as follows:

$$\begin{aligned}
&x' = y \\
&y' = -f(t, x, y) y - g(t, x, y) - h(x(t)) \\
&\quad - p(x) \int_0^t q(s, y(s)) ds \\
&\quad + \int_{t-\tau}^t p'(x(s)) y(s) ds \int_0^t q(s, y(s)) ds,
\end{aligned} \tag{17}$$

Theorem 3. Consider system (17). There are nonnegative constants K, R, L_1, L_2 , and N , positive numbers l, M , and M_1 , and functions $P(t)$ and $Q(t)$, such that the conditions hold:

$$\begin{aligned}
(D_1) & h(x)x^{-1} \geq l \text{ when } x \neq 0, |p'(x)| \leq K. \\
(D_2) & f(t, x, 0) = 0, g(t, x, 0) = 0, f(t, x, y)y^{-2} \geq L_1, \\
& g(t, x, y)y^{-1} \geq L_2 \text{ when } y \neq 0. \\
(D_3) & |p(x)| \leq P(t), M_1 \leq P(t) \leq M, |q(t, y)| \leq Q(t)y, \\
& Q(t) \leq N. \\
(D_4) & M \int_0^\infty Q(s) ds + ((M + K\tau)N/M_1) \int_0^\infty P(s) ds \leq 2L_2. \\
(D_5) & K\tau \int_0^\infty Q(s) ds \leq 2L_1.
\end{aligned}$$

Then the zero solution of system (17) is stable.

Proof. Define a Lyapunov functional $V(t) = V(t, x, y)$ as

$$V(t) = \frac{1}{2}y^2 + \int_0^x h(\eta) d\eta + \lambda \int_{-\tau}^0 ds \int_{t+s}^t y^4(\theta) d\theta + \mu \int_0^t ds \int_t^\infty |p(x(\theta))| Q(s) y^2(s) d\theta, \quad (18)$$

where λ and μ are two positive constants to be determined. Using condition (D_1) , we have

$$\int_0^x h(\eta) d\eta = \int_0^x \frac{h(\eta)}{\eta} \eta d\eta \geq \int_0^x l\eta d\eta = \frac{l}{2}x^2, \quad (19)$$

and then $V(t) \geq (l/2)x^2 + (l/2)y^2$.

Let $(x(t), y(t))$ be a solution of (17). Then,

$$\begin{aligned} \frac{dV}{dt} &= yy' + h(x)y + \lambda \int_{-\tau}^0 [y^4(t) - y^4(t+s)] ds \\ &\quad + \mu Q(t)y^2(t) \int_t^\infty |p(x(\theta))| d\theta \\ &\quad - \mu |p(x(t))| \int_0^t Q(s)y^2(s) ds \\ &= -f(t, x, y)y^2 - g(t, x, y)y \\ &\quad - yp(x) \int_0^t q(s, y(s)) ds \\ &\quad + y \int_{t-\tau}^t p'(x(s))y(s) ds \int_0^t q(s, y(s)) ds \\ &\quad + \lambda \tau y^4 - \lambda \int_{t-\tau}^t y^4(s) ds \\ &\quad + \mu Q(t)y^2 \int_t^\infty |p(x(\theta))| d\theta \\ &\quad - \mu |p(x(t))| \int_0^t Q(s)y^2(s) ds. \end{aligned} \quad (20)$$

Applying conditions (D_1) and (D_3) and $2ab \leq a^2 + b^2$, we have

$$\begin{aligned} &y \int_{t-\tau}^t p'(x(s))y(s) ds \int_0^t q(s, y(s)) ds \\ &\leq |y| \int_{t-\tau}^t |p'(x(s))| |y(s)| ds \int_0^t |q(s, y(s))| ds \\ &\leq \frac{K}{2} \int_{t-\tau}^t [y^2(t) + y^2(s)] ds \int_0^t Q(\theta) |y(\theta)| d\theta \\ &\leq \frac{K\tau}{2} y^2(t) \int_0^t Q(\theta) |y(\theta)| d\theta \\ &\quad + \frac{K}{2} \int_{t-\tau}^t y^2(s) \int_0^t Q(\theta) |y(\theta)| d\theta ds \\ &\leq \frac{K\tau}{4} \int_0^t Q(\theta) [y^4(t) + y^2(\theta)] d\theta \end{aligned}$$

$$\begin{aligned} &+ \frac{K}{4} \int_{t-\tau}^t \int_0^t Q(\theta) [y^4(s) + y^2(\theta)] d\theta ds \\ &\leq \frac{K\tau}{4} y^4 \int_0^\infty Q(s) ds + \frac{K\tau}{2} \int_0^t Q(s) y^2(s) ds \\ &\quad + \frac{K}{4} \int_0^\infty Q(s) ds \int_{t-\tau}^t y^4(s) ds, \end{aligned} \quad (21)$$

$$\begin{aligned} &- y(t)p(x(t)) \int_0^t q(s, y(s)) ds \\ &\leq |y(t)| |p(t, x(t))| \int_0^t |q(s, y(s))| ds \\ &\leq |p(x(t))| |y(t)| \int_0^t Q(s) |y(s)| ds \\ &\leq \frac{1}{2} |p(x(t))| \int_0^t Q(s) [y^2(t) + y^2(s)] ds \\ &\leq \frac{My^2}{2} \int_0^\infty Q(s) ds + \frac{M}{2} \int_0^t Q(s) y^2(s) ds. \end{aligned} \quad (22)$$

By condition (D_2) , we have

$$\begin{aligned} \frac{dV}{dt} &\leq - \left[L_1 - \frac{K\tau}{4} \int_0^\infty Q(s) ds - \lambda \tau \right] y^4 \\ &\quad - \left[L_2 - \frac{M}{2} \int_0^\infty Q(s) ds - \mu N \int_0^\infty P(s) ds \right] y^2 \\ &\quad + \left[\frac{K}{4} \int_0^\infty Q(s) ds - \lambda \right] \int_{t-\tau}^t y^4(s) ds \\ &\quad + \left[\frac{M + K\tau}{2} - \mu M_1 \right] \int_0^t Q(s) y^2(s) ds. \end{aligned} \quad (23)$$

Thus, we choose $\lambda = (K/4) \int_0^\infty Q(s) ds$ and $\mu = (M + K\tau)/2M_1$. Using conditions (D_4) and (D_5) , we have

$$\begin{aligned} \frac{dV}{dt} &\leq - \left[L_1 - \frac{K\tau}{2} \int_0^\infty Q(s) ds \right] y^4 - \left[L_2 \right. \\ &\quad \left. - \frac{M}{2} \int_0^\infty Q(s) ds - \frac{(M + K\tau)N}{2M_1} \int_0^\infty P(s) ds \right] y^2 \\ &\leq 0. \end{aligned} \quad (24)$$

Therefore, the zero solution of system (17) is stable and the proof is completed. $\square$

Corollary 4. Consider system (17) with a variable delay $\tau(t)$. Conditions (1)-(3) of Theorem 3 are satisfied. Moreover,

$$(D'_5) \quad (2 - \beta)K\tau \int_0^\infty Q(s) ds \leq 4(1 - \beta)L_1,$$

(D'_6) there are $\tau > 0$ and $0 < \beta < 1$, such that $0 \leq \tau(t) \leq \tau$ and $\tau'(t) \leq 1$.

Then the zero solution of system (17) with a variable delay $\tau(t)$ is stable.

Proof. The proof is similar to that of Theorem 3. But

$$\begin{aligned}
 \frac{dV}{dt} &\leq - \left[L_1 - \frac{K\tau(t)}{4} \int_0^\infty Q(s) ds - \lambda\tau(t) \right] y^4 \\
 &\quad - \left[L_2 - \frac{M}{2} \int_0^\infty Q(s) ds - \mu N \int_0^\infty P(s) ds \right] y^2 \\
 &\quad + \left[\frac{K}{4} \int_0^\infty Q(s) ds - \lambda(1 - \tau'(t)) \right] \int_{t-\tau(t)}^t y^4(s) ds \\
 &\quad + \left[\frac{M + K\tau(t)}{2} - \mu P(t) \right] \int_0^t Q(s) y^2(s) ds \\
 &\leq - \left[L_1 - \frac{K\tau}{4} \int_0^\infty Q(s) ds - \lambda\tau \right] y^4 \\
 &\quad - \left[L_2 - \frac{M}{2} \int_0^\infty Q(s) ds - \mu N \int_0^\infty P(s) ds \right] y^2 \\
 &\quad + \left[\frac{K}{4} \int_0^\infty Q(s) ds - \lambda(1 - \beta) \right] \int_{t-\tau(t)}^t y^4(s) ds \\
 &\quad + \left[\frac{M + K\tau}{2} - \mu M_1 \right] \int_0^t Q(s) y^2(s) ds.
 \end{aligned} \tag{25}$$

Thus, choosing $\lambda = (K/4(1 - \beta)) \int_0^\infty Q(s) ds$ and $\mu = (M + K\tau)/2M_1$, we have

$$\begin{aligned}
 \frac{dV}{dt} &\leq - \left[L_1 - \frac{(2 - \beta)K\tau}{4(1 - \beta)} \int_0^\infty Q(s) ds \right] y^4 - \left[L_2 \right. \\
 &\quad \left. - \frac{M}{2} \int_0^\infty Q(s) ds - \frac{(M + K\tau)N}{2M_1} \int_0^\infty P(s) ds \right] y^2 \\
 &\leq 0.
 \end{aligned} \tag{26}$$

Then, the zero solution of system (6) with a variable delay $\tau(t)$ is stable and the proof is completed. $\square$

3. An Illustrative Example

In this section, we give an illustrative example to the effectiveness of results obtained in this paper. Moreover, we list a graph of solutions of an integral-differential equation with a delay to verify the correctness of the conclusion.

Example 1. Consider the following example:

$$\begin{aligned}
 x''(t) + 2e^{t+x^2(t)+x'^2(t)} x'(t) + \frac{1}{4} x'(t) + \frac{1}{2} x(t-2) \\
 = e^{-2t} \int_0^t e^{-s} x'(s) ds.
 \end{aligned} \tag{27}$$

In the example, $f(t, x, y) = 2e^{t+x^2+y^2}$, $g(t, x, y) = (1/4)y$, $h(x) = (1/2)x$, $p(t, x) = e^{-2t}$, $q(t, y) = \int_0^t e^{-s} y(s) ds$, and a delay $\tau = 2$. Obvious, this system is the same form as (5). Moreover,

FIGURE 1: Stability of solutions for system (27).

$$(C_1) \quad h(x)x^{-1} \geq l = 1/2, |h'(x)| = 1/2 \leq K^* = 1/2;$$

$$(C_2) \quad g(t, x, 0) = 0, g(t, x, y)y^{-1} \geq b = 1/4;$$

$$(C_3) \quad |p(t, x)| = P(t) \leq M = 1, |q(t, y)| \leq Q(t)|y|, Q(t) = e^{-t} \leq 1;$$

$$(C_4) \quad N \int_0^\infty P(s) ds + M \int_0^\infty Q(s) ds + K^*(\tau+1) = \int_0^\infty e^{-2t} dt + \int_0^\infty e^{-t} dt + 1/2 \times 2 = 7/2, 2f(t, x, y) + 2b \geq 4 + 2 \times 1/4 \geq 7/2.$$

Thus, all conditions of Theorem 1 are satisfied and the zero solution of system (27) is stable by the obtained result. To show the effectiveness of the result, we carry out a simulation result with the following choices. Initial Condition: $x(t) = x'(t) = 0$ for $t \in [-2, 0)$. The simulation result is shown in Figure 1, which is the stability of solutions for system (27).

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The first author is the main completer. The second author is the corresponding author and provides the thought of this paper and advice in the process of writing.

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