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Research Article

Observer Design for Delayed Markovian Jump Systems with Output State Saturation

Guoliang Wang and Bo Feng

School of Information and Control Engineering, Liaoning Shihua University, Fushun, Liaoning 113001, China

Correspondence should be addressed to Guoliang Wang; gliangwang@aliyun.com

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This paper considers the observer design problem of continuous-time delayed Markovian jump systems with output state saturation. Different from the traditionally observer-based saturation control methods, a kind of system output state saturation with a partially delay-dependent property is proposed, where both nondelay and delay states exist at the same time but happen asynchronously. By exploiting the Bernoulli variable, the probability distributions of such two states are described and considered in the observer design. Based on an improved equality applied to deal with saturation terms, sufficient conditions for the designed observer with three kinds of output saturations are all provided with LMI forms. Finally, a numerical example is given to indicate the effectiveness of the obtained results.

1. Introduction

As we know, Markovian jump system (MJS) is a special kind of stochastic hybrid dynamical systems. Because of two kinds of mechanisms contained, it is very suitable to model such actual systems whose structures or parameters change [1, 2]. Over the past years, many research topics on MJSs have been extensively studied, like stability analysis [3–6], stabilization [7–11], robust control [12–15], adaptive control [16–19], H_∞ filtering and control [20, 21], state estimation [22–25], synchronization [26–30], and so on.

On the other hand, saturation problem is commonly encountered in many physical systems. It is mainly because the transmitted signal is usually affected by environmental factors, physical factors, and even technical factors. When signal occurs with saturation, the system performance will be reduced. Therefore, the effect of saturation is necessarily studied, as in [31–35]. By investigating the above results, it is found that they mainly focused on the estimate of domain of attraction, in which an equivalent description of saturation actuator is necessary. However, the introduction of this transformation will make the computation complexity very large, especially for an MJS with N operation modes. Recently, one inequality was introduced in [36] and applied to deal with saturation. In the modern control theory, it is obvious that the state feedback control has advantages in solving the problems of system stability, pole placement, stabilization, and optimal control. However, system state usually cannot be measured directly, or the measurement is limited by objective conditions. In this case, it is not easy to achieve the object by state feedback control. The better way is to make full use of system output. When output is saturated, some results were presented in [37, 38]. By investigating these references, it is found that there are still many problems to be considered. For example, there is no time delay in the saturated output, while time delay is usually associated with most system states and could lead to unstable system. Thus, it is necessary to consider them together. To our best knowledge, very few results are available to study the above problems. All observations motivate current research.

In this paper, the observer design problem of continuous-time delayed Markovian jump systems with output state saturation is studied, where the output state saturation is partially delay-dependent. The main contributions of this paper are generalized as follows: (1) A kind of observer based on partially delay-dependent output is proposed. Here, both nondelay and delay states are contained in output saturation simultaneously, but their occurrences are asynchronous. (2)

The probability distributions of such two states are embodied by the Bernoulli variable and fully considered in the observe design. (3) Sufficient conditions of the designed observer are obtained by applying an improved inequality to deal with the saturation terms. In order to make the computation solved easily and directly, the existence conditions are presented within LMI framework by introducing some additional variables and inequalities. (4) Based on the proposed methods and techniques, another two kinds of observers based on different output saturations are proposed, whose conditions are all LMIs.

Notation 1. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{q\times n}$ is the set of all $q \times n$ real matrices. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathbb{P}$ is the probability measure on $\mathcal{F}$. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag$\{\cdots\}$ for a block-diagonal matrix, and $(M)^{\star} \triangleq M + M^T$.

2. Problem Formulation

Consider the following continuous-time delayed Markovian jump systems with output state saturation:

$$\begin{aligned} \dot{x}(t) &= A(r_t)\,x(t), \\ y(t) &= C(r_t)\,\mathrm{sat}\left[\alpha(t)\,x(t) + (1-\alpha(t))\,x(t-d)\right], \\ x(t) &= \phi(t), \quad -d \le t \le 0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^q$ is the output, and $\phi(t) \in \mathbb{R}^n$ is the initial condition. Matrices $A(r_t)$ and $C(r_t)$ are known matrices of compatible dimensions. $\{r_t,\ t \ge 0\}$ is a continuous-time Markov process taking values in a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$ with transition rate matrix (TRM) $\Pi \triangleq (\pi_{ij}) \in \mathbb{R}^{N\times N}$ given by

$$\Pr\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \ne j \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$, $\pi_{ij} \ge 0$, if $i \ne j$, and $\pi_{ii} = -\sum_{j\ne i}\pi_{ij}$ for all $i, j \in \mathbb{S}$. Time delay d satisfies $d > 0$, and $\alpha(t)$ is the Bernoulli stochastic variable and defined as

$$\alpha(t) = \begin{cases} 1, & \text{if } x(t) \text{ is available} \\ 0, & \text{if } x(t-d) \text{ is available.} \end{cases} \tag{3}$$

It is satisfied as

$$\begin{aligned} \Pr\{\alpha(t) = 1\} &= \mathscr{E}\{\alpha(t)\} = \alpha, \\ \Pr\{\alpha(t) = 0\} &= 1 - \alpha. \end{aligned} \tag{4}$$

Function sat$(\cdot)$: $\mathbb{R}^n \to \mathbb{R}^n$ is the standard saturation function and defined as

$$\mathrm{sat}(u_i) = \begin{cases} \mu_i, & u_i > \mu_i \\ u_i, & -\mu_i \le u_i \le \mu_i \quad i \in 1, 2, \ldots, n, \\ -\mu_i, & u_i < -\mu_i, \end{cases} \tag{5}$$

where

$$\begin{aligned} \mathrm{sat}(u) &= \left[\mathrm{sat}(u_1) \ \ \cdots \ \ \mathrm{sat}(u_n)\right]^T, \\ \mu &= \left[\mu_1 \ \ \cdots \ \ \mu_n\right]^T. \end{aligned} \tag{6}$$

Remark 1. It is worth mentioning that saturation in output is more general and has some advantages. Firstly, compared with saturations described in [36, 39–42] where nondelay state should be available online, system state $x(t)$ in output is not necessary and could be replaced by delay state $x(t-d)$. Secondly, compared with the saturated controllers designed for delayed systems without delay state [43, 44], both nondelay and delay states are included in output, whose distribution probabilities are also considered. Because all the delay terms could affect delayed systems, the obtained results are more conservative without considering them. In this sense, it is said that our formulation about output saturation has a larger application scope and less conservatism.

By letting $\eta = x - \mathrm{sat}(x)$ and $\eta_d = x_d - \mathrm{sat}(x_d)$ and considering the definition of $\alpha(t)$, it is known that

$$\begin{aligned} &\mathrm{sat}\left[\alpha(t)\,x(t) + (1-\alpha(t))\,x(t-d)\right] \\ &\quad = \alpha(t)\,x(t) + (1-\alpha(t))\,x(t-d) \\ &\qquad - \left[\alpha(t)\,\eta + (1-\alpha(t))\,\eta_d\right] \\ &\quad = \alpha x(t) + (1-\alpha)\,x(t-d) \\ &\qquad + (\alpha(t)-\alpha)\,(x(t) - x(t-d)) \\ &\qquad - \left[\alpha\eta + (1-\alpha)\,\eta_d + (\alpha(t)-\alpha)\,(\eta - \eta_d)\right]. \end{aligned} \tag{7}$$

Then, system (1) is rewritten to be

$$\begin{aligned} \dot{x}(t) &= A(r_t)\,x(t), \\ y(t) &= C(r_t)\,\{\alpha x(t) + (1-\alpha)\,x(t-d) \\ &\quad + (\alpha(t)-\alpha)\,(x(t) - x(t-d)) \\ &\quad - \left[\alpha\eta + (1-\alpha)\,\eta_d + (\alpha(t)-\alpha)\,(\eta - \eta_d)\right]\}, \\ x(t) &= \phi(t), \quad -d \le t \le 0. \end{aligned} \tag{8}$$

In this paper, the designed state observer system is described by

$$\begin{aligned} \dot{\hat{x}}(t) &= A(r_t)\,\hat{x}(t) + G(r_t)\,(y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C(r_t)\,\hat{x}(t), \end{aligned} \tag{9}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of $x(t)$, and $\hat{y}(t) \in \mathbb{R}^q$ is the corresponding output. Define $e(t) = x(t) - \hat{x}(t)$; the resulting error system is rewritten to be

$$\begin{aligned} \dot{e}(t) &= (A(r_t) - G(r_t)\,C(r_t))\,e(t) \\ &\quad + (1-\alpha)\,G(r_t)\,C(r_t)\,x(t) \end{aligned}$$

$$\begin{aligned}
&- (1-\alpha) G(r_t) C(r_t) x(t-d) \\
&+ \alpha G(r_t) C(r_t) \eta + (1-\alpha) G(r_t) C(r_t) \eta_d \\
&+ (\alpha(t) - \alpha) G(r_t) C(r_t) \\
&\times (x(t-d) - x(t) + \eta - \eta_d).
\end{aligned} \tag{10}$$

Here, it is assumed that stochastic variables $\alpha(t)$ and r_t are independent.

Proposition 2. *Let $u = \alpha(t)Kx(t) + (1-\alpha(t))K_d x(t-d)$ and $\eta = u - sat(u)$. Then, there exist real numbers $\varepsilon \in (0,1)$ and $\varepsilon_d \in (0,1)$ such that*

$$\begin{aligned}
\eta^T \eta \le\ & \alpha(t) \varepsilon x^T(t) K^T K x(t) \\
& + (1-\alpha(t)) \varepsilon_d x^T(t-d) K_d^T K_d x(t-d),
\end{aligned} \tag{11}$$

where $\eta = [\eta_1 \ \cdots \ \eta_n]^T$, and η_i is the dead-zone nonlinearity function, $i = \{1, 2, \ldots, n\}$.

Proof. Based on the formulation of $u(t)$, it is obtained that

$$\begin{aligned}
\eta^T \eta = & \{\alpha(t) Kx(t) + (1-\alpha(t)) K_d x(t-d) \\
& - \operatorname{sat}[\alpha(t) Kx(t) + (1-\alpha(t)) K_d x(t-d)]\}^T \\
& \cdot \{\alpha(t) Kx(t) + (1-\alpha(t)) \times K_d x(t-d) \\
& - \operatorname{sat}[\alpha(t) Kx(t) + (1-\alpha(t)) K_d x(t-d)]\}.
\end{aligned} \tag{12}$$

On the one hand, if $\alpha(t) = 1$, one has $u = Kx(t)$. Similar to [36], we have the following.

(i) If $u_i > \mu_i$, it is obtained that

$$\begin{aligned}
\eta^T \eta &= (Kx(t) - \operatorname{sat}(Kx(t)))^T (Kx(t) - \operatorname{sat}(Kx(t))) \\
&= x^T(t) K^T K x(t) - 2x^T(t) K^T \operatorname{sat}(Kx(t)) \\
&\quad + (\operatorname{sat}(Kx(t)))^T (\operatorname{sat}(Kx(t))) \\
&= x^T(t) K^T K x(t) - 2x^T(t) K^T \mu + \mu^T \mu \\
&< x^T(t) K^T K x(t) - \mu^T \mu \le \varepsilon x^T(t) K^T K x(t),
\end{aligned} \tag{13}$$

where $\varepsilon \in (0,1)$ could be selected.

(ii) If $u_i < -\mu_i$, it is found that

$$\begin{aligned}
\eta^T \eta &= (Kx(t) - \operatorname{sat}(Kx(t)))^T (Kx(t) - \operatorname{sat}(Kx(t))) \\
&= x^T(t) K^T K x(t) - 2x^T(t) K^T \operatorname{sat}(Kx(t)) \\
&\quad + (\operatorname{sat}(Kx(t)))^T (\operatorname{sat}(Kx(t))) \\
&= x^T(t) K^T K x(t) + 2x^T(t) K^T \mu + \mu^T \mu \\
&< x^T(t) K^T K x(t) - \mu^T \mu \le \varepsilon x^T(t) K^T K x(t),
\end{aligned} \tag{14}$$

where $\varepsilon \in (0,1)$ could be obtained too.

(iii) If $-\mu_i \le u_i \le \mu_i$, we have $\eta^T \eta = 0$. Then, there is always a scalar $\varepsilon \in (0,1)$ such that

$$\eta^T \eta \le \varepsilon x^T(t) K^T K x(t). \tag{15}$$

On the other hand, when $\alpha(t) = 0$, it is obtained that $u = K_d x(t-d)$. Similarly, we have the following cases.

(i) If $u_i > \mu_i$, it is obtained that

$$\begin{aligned}
\eta^T \eta = & \{K_d x(t-d) - \operatorname{sat}[K_d x(t-d)]\}^T \\
& \cdot \{K_d x(t-d) - \operatorname{sat}[K_d x(t-d)]\} = x^T(t-d) \\
& \cdot K_d^T K_d x(t-d) - 2x^T(t) K_d^T \operatorname{sat}[K_d x(t-d)] \\
& + (\operatorname{sat}[K_d x(t-d)])^T \operatorname{sat}[K_d x(t-d)] \\
& = x^T(t-d) K_d^T K_d x(t-d) - 2x^T(t-d) K_d^T \mu \\
& + \mu^T \mu < x^T(t-d) K_d^T K_d x(t-d) - \mu^T \mu \\
& \le \varepsilon_d x^T(t-d) K_d^T K x(t-d),
\end{aligned} \tag{16}$$

where $\varepsilon_d \in (0,1)$.

(ii) If $u_i < -\mu_i$, it is found that

$$\begin{aligned}
\eta^T \eta = & \{K_d x(t-d) - \operatorname{sat}[K_d x(t-d)]\}^T \\
& \cdot \{K_d x(t-d) - \operatorname{sat}[K_d x(t-d)]\} = x^T(t-d) \\
& \cdot K_d^T K_d x(t-d) - 2x^T(t) K_d^T \operatorname{sat}[K_d x(t-d)] \\
& + (\operatorname{sat}[K_d x(t-d)])^T \operatorname{sat}[K_d x(t-d)] \\
& = x^T(t-d) K_d^T K_d x(t-d) + 2x^T(t-d) K_d^T \mu \\
& + \mu^T \mu < x^T(t-d) K_d^T K_d x(t-d) - \mu^T \mu \\
& \le \varepsilon_d x^T(t-d) K_d^T K x(t-d),
\end{aligned} \tag{17}$$

where $\varepsilon_d \in (0,1)$.

(iii) If $-\mu_i \le u_i \le \mu_i$, we have $\eta^T \eta = 0$. Then, there is always a scalar $\varepsilon_d \in (0,1)$ such that

$$\eta^T \eta \le \varepsilon_d x^T(t-d) K_d^T K_d x(t-d). \tag{18}$$

This completes the proof. □

3. Main Results

Theorem 3. *Given system (1) with $d > 0$, there exists an observer (9) such that system (1) and error system (12) are asymptotically stable, if for given positive scalars $\alpha \in [0,1]$, $\varepsilon_i \in (0,1)$, and $\varepsilon_{di} \in (0,1)$, there exist $P_i > 0$, $Q > 0$, $Z > 0$, $\gamma_i > 0$, $\lambda_i > 0$, and Y_i such that*

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & Z & 0 & \Theta_{i3} & \Theta_{i4} \\ * & \Theta_{i5} & \Theta_{i6} & Z & 0 & \Theta_{i7} \\ * & * & \Theta_{i8} & 0 & \Theta_{i9} & \Theta_{i10} \\ * & * & * & \Theta_{11} & 0 & 0 \\ * & * & * & * & \Theta_{i12} & 0 \\ * & * & * & * & * & \Theta_{i12} \end{bmatrix} < 0, \quad (19)$$

$$\begin{bmatrix} -\gamma_i I & C_i^T Y_i^T \\ * & -P_i \end{bmatrix} \leq 0, \quad (20)$$

$$\begin{bmatrix} -\lambda_i I & C_i^T Y_i^T \\ * & \Theta_{i12} \end{bmatrix} \leq 0, \quad (21)$$

where

$$\begin{aligned}
\Theta_{i1} &= (P_i A_i)^* + \sum_{j=1}^{N} \pi_{ij} P_j + \alpha \varepsilon_i \gamma_i I + Q + d^2 A_i^T Z A_i \\
&\quad - Z + 2\alpha d^2 \varepsilon_i \lambda_i I + 4\alpha\overline{\alpha} d^2 \varepsilon_i \lambda_i I, \\
\Theta_{i2} &= \overline{\alpha} C_i^T Y_i^T, \\
\Theta_{i3} &= -\sqrt{2\alpha\overline{\alpha}} d C_i^T Y_i^T, \\
\Theta_{i4} &= \sqrt{2}\overline{\alpha} d C_i^T Y_i^T, \\
\Theta_{i5} &= (P_i A_i - Y_i C_i)^* + P_i + \sum_{j=1}^{N} \pi_{ij} P_j + Q - Z, \\
\Theta_{i6} &= -\overline{\alpha} Y_i C_i, \\
\Theta_{i7} &= \sqrt{2} d \left(A_i^T P_i - C_i^T Y_i^T\right), \\
\Theta_{i8} &= \overline{\alpha} \varepsilon_{di} \gamma_i I + 2\overline{\alpha} d^2 \varepsilon_{di} \lambda_i I + 4\alpha\overline{\alpha} d^2 \varepsilon_{di} \lambda_i I - Q - Z, \\
\Theta_{i9} &= \sqrt{2\alpha\overline{\alpha}} d C_i^T Y_i^T, \\
\Theta_{i10} &= -\sqrt{2}\overline{\alpha} d C_i^T Y_i^T, \\
\Theta_{11} &= -Q - Z, \\
\Theta_{i12} &= -2P_i + Z, \\
\overline{\alpha} &= 1 - \alpha.
\end{aligned} \quad (22)$$

Then, the gain of observer (9) is computed by

$$G_i = Y_i P_i^{-1}. \quad (23)$$

Proof. For systems (1) and (12), choose the following Lyapunov function:

$$V(x_t) = V_1(x_t) + V_2(x_t), \quad (24)$$

where

$$\begin{aligned}
V_1(x_t, r_t) &= x^T(t) P(r_t) x(t) + \int_{t-d}^{t} x^T(s) Q x(s)\, ds \\
&\quad + d \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z \dot{x}(s)\, ds\, d\theta, \\
V_2(x_t, r_t) &= e^T(t) P(r_t) e(t) + \int_{t-d}^{t} e^T(s) Q e(s)\, ds \\
&\quad + d \int_{-d}^{0} \int_{t+\theta}^{t} \dot{e}^T(s) Z \dot{e}(s)\, ds\, d\theta.
\end{aligned} \quad (25)$$

Let $\mathscr{L}$ be the weak infinitesimal generator of random process $\{x_t, r_t\}$, for each $r_t = i \in \mathbb{S}$; it is defined as

$$\begin{aligned}
\mathscr{L}V(x_t, t, i) &= \lim_{\Delta t \to 0^+} \\
&\quad \cdot \frac{1}{\Delta t} \{\mathscr{E}[V(x_{t+\Delta t}, r_{t+\Delta t}, t+\Delta t) \mid x_t, r_t = i] \\
&\quad - V(x_t, i, t)\}.
\end{aligned} \quad (26)$$

Then, it is further obtained that

$$\begin{aligned}
&\mathscr{L}V_1(x_t) \\
&\quad = x^T(t) (P_i A_i)^* x(t) + x^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x(t) \\
&\qquad + x^T(t) Q x(t) - x^T(t-d) Q x(t-d) \\
&\qquad + d^2 x^T(t) A_i^T Z A_i x(t) - \int_{t-d}^{t} \dot{x}^T(\theta) Z \dot{x}(\theta)\, d\theta, \\
&\mathscr{L}V_2(x_t) \\
&\quad = e^T(t) [P_i (A_i - G_i C_i)]^* e(t) \\
&\qquad + 2\overline{\alpha} e^T(t) P_i G_i C_i x(t) \\
&\qquad - 2\overline{\alpha} e^T(t) P_i G_i C_i x(t-d) + 2\alpha e^T(t) P_i G_i C_i \eta \\
&\qquad + 2\overline{\alpha} e^T(t) P_i G_i C_i \eta_d + e^T(t) \sum_{j=1}^{N} \pi_{ij} P_j e(t) \\
&\qquad - \int_{t-d}^{t} \dot{e}^T(\theta) Z \dot{e}(\theta)\, d\theta + e^T(t) Q e(t) \\
&\qquad - e^T(t-d) Q e(t-d) \\
&\qquad + d^2 (f_{i1} + f_{i2})^T Z (f_{i1} + f_{i2}) \\
&\qquad + \alpha\overline{\alpha} d^2 [G_i C_i (x(t-d) - x(t) + \eta - \eta_d)]^T \\
&\qquad \times Z [G_i C_i (x(t-d) - x(t) + \eta - \eta_d)],
\end{aligned} \quad (27)$$

where

$$f_{i1} = (A_i - G_iC_i)\,e(t) + \overline{\alpha}G_iC_ix(t) - \overline{\alpha}G_iC_ix(t-d), \quad f_{i2} = \alpha G_iC_i\eta + \overline{\alpha}G_iC_i\eta_d. \tag{28}$$

Based on the Jensen inequality, one has

$$-\int_{t-d}^{t}\dot{x}^T(\theta)\,Z\dot{x}(\theta)\,d\theta \le -x^T(t)\,Zx(t) + 2x^T(t)\,Zx(t-d) - x^T(t-d)\,Zx(t-d), \tag{29}$$

$$-\int_{t-d}^{t}\dot{e}^T(\theta)\,Z\dot{e}(\theta)\,d\theta \le -e^T(t)\,Ze(t) + 2e^T(t)\,Ze(t-d) - e^T(t-d)\,Ze(t-d). \tag{30}$$

Moreover, it is concluded that

$$2\alpha e^T(t)\,P_iG_iC_i\eta \le \alpha e^T(t)\,P_ie(t) + \alpha\eta^TC_i^TG_i^TP_iG_iC_i\eta, \tag{31}$$

$$2\overline{\alpha}e^T(t)\,P_iG_iC_i\eta_d \le \overline{\alpha}e^T(t)\,P_ie(t) + \overline{\alpha}\eta_d^TC_i^TG_i^TP_iG_iC_i\eta_d, \tag{32}$$

$$2d^2(\alpha G_iC_i\eta + \overline{\alpha}G_iC_i\eta_d)^T Z(\alpha G_iC_i\eta + \overline{\alpha}G_iC_i\eta_d) \le 2\alpha d^2\eta^TC_i^TG_i^TZG_iC_i\eta + 2\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d, \tag{33}$$

$$2\alpha\overline{\alpha}d^2(G_iC_i\eta - G_iC_i\eta_d)^T Z(G_iC_i\eta - G_iC_i\eta_d) \le 4\alpha\overline{\alpha}d^2\eta^TC_i^TG_i^TZG_iC_i\eta + 4\alpha\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d, \tag{34}$$

$$d^2(f_{i1}+f_{i2})^T Z(f_{i1}+f_{i2}) \le 2d^2 f_{i1}^TZf_{i1} + 2d^2(\alpha G_iC_i\eta + \overline{\alpha}G_iC_i\eta_d)^T \cdot Z(\alpha G_iC_i\eta + \overline{\alpha}G_iC_i\eta_d), \tag{35}$$

$$\begin{aligned}&\alpha\overline{\alpha}d^2[G_iC_i(x(t-d) - x(t) + \eta - \eta_d)]^T \cdot Z[G_iC_i(x(t-d) - x(t) + \eta - \eta_d)] \\ &\le 2\alpha\overline{\alpha}d^2[G_iC_i(x(t-d) - x(t))]^T \cdot Z[G_iC_i(x(t-d) - x(t))] \\ &\quad + 2\alpha\overline{\alpha}d^2(G_iC_i\eta - G_iC_i\eta_d)^T Z(G_iC_i\eta - G_iC_i\eta_d).\end{aligned} \tag{36}$$

Based on these conditions, the following is computed:

$$\begin{aligned}\mathscr{L}V(x_t) &\le x^T(t)\,(P_iA_i)^*\,x(t) + x^T(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\ &+ e^T(t)\,[P_i(A_i - G_iC_i)]^*\,e(t) + 2\overline{\alpha}e^T(t) \\ &\cdot P_iG_iC_ix(t) - 2\overline{\alpha}e^T(t)\,P_iG_iC_ix(t-d) + e^T(t) \\ &\cdot P_ie(t) + \alpha\eta^TC_i^TG_i^TP_iG_iC_i\eta + \overline{\alpha}\eta_d^TC_i^TG_i^TP_iG_iC_i\eta_d \\ &+ e^T(t)\sum_{j=1}^{N}\pi_{ij}P_je(t) + x^T(t)\,Qx(t) - x^T(t-d) \\ &\cdot Qx(t-d) + e^T(t)\,Qe(t) - e^T(t-d)\,Qe(t-d) \\ &+ d^2x^T(t)\,A_i^TZA_ix(t) - x^T(t)\,Zx(t) + 2x^T(t) \\ &\cdot Zx(t-d) - x^T(t-d)\,Zx(t-d) \\ &+ 2\alpha d^2\eta^TC_i^TG_i^TZG_iC_i\eta + 2\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d \\ &+ 2d^2[(A_i - G_iC_i)\,e(t) + \overline{\alpha}G_iC_ix(t) \\ &- \overline{\alpha}G_iC_ix(t-d)]^T \times Z[(A_i - G_iC_i)\,e(t) \\ &+ \overline{\alpha}G_iC_ix(t) - \overline{\alpha}G_iC_ix(t-d)] \\ &+ 2\alpha\overline{\alpha}d^2[G_iC_i(x(t-d) - x(t))]^T \\ &\cdot Z[G_iC_i(x(t-d) - x(t))] \\ &+ 4\alpha\overline{\alpha}d^2\eta^TC_i^TG_i^TZG_iC_i\eta \\ &+ 4\alpha\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d - e^T(t)\,Ze(t) + 2e^T(t) \\ &\cdot Ze(t-d) - e^T(t-d)\,Ze(t-d).\end{aligned} \tag{37}$$

Moreover, the other conditions are assumed to be

$$C_i^TG_i^TP_iG_iC_i \le \gamma_iI, \tag{38}$$

$$C_i^TG_i^TZG_iC_i \le \lambda_iI. \tag{39}$$

Based on the above conditions and applying Proposition 2, it is obtained that (37) is implied by

$$\mathscr{L}V(x_t) \le \xi^T(t)\left(\begin{bmatrix}\Theta_{i1} & \Theta_{i2} & Z & 0\\ * & \Theta_{i5} & \Theta_{i6} & Z\\ * & * & \Theta_{i8} & 0\\ * & * & * & \Theta_{11}\end{bmatrix}\right. + 2\alpha\overline{\alpha}d^2\begin{bmatrix}-C_i^TG_i^T\\ 0\\ C_i^TG_i^T\\ 0\end{bmatrix} Z \begin{bmatrix}-C_i^TG_i^T\\ 0\\ C_i^TG_i^T\\ 0\end{bmatrix}^T$$

$$+ 2d^2 \begin{bmatrix} \overline{\alpha} C_i^T G_i^T \\ \left(A_i - C_i^T G_i\right)^T \\ -\overline{\alpha} C_i^T G_i^T \\ 0 \end{bmatrix} Z \begin{bmatrix} \overline{\alpha} C_i^T G_i^T \\ \left(A_i - C_i^T G_i\right)^T \\ -\overline{\alpha} C_i^T G_i^T \\ 0 \end{bmatrix}^T \Bigg)$$

$$\cdot \xi(t), \tag{40}$$

where

$$\xi^T(t) = \left[x^T(t) \quad e^T(t) \quad x^T(t-d) \quad e^T(t-d)\right]. \tag{41}$$

By using the Schur complement lemma, it is further obtained that

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & Z & 0 & \overline{\Theta}_{i3} & \overline{\Theta}_{i4} \\ * & \Theta_{i5} & \Theta_{i6} & Z & 0 & \overline{\Theta}_{i7} \\ * & * & \Theta_{i8} & 0 & \overline{\Theta}_{i9} & \overline{\Theta}_{i10} \\ * & * & * & \Theta_{11} & 0 & 0 \\ * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{42}$$

where

$$\begin{aligned} \overline{\Theta}_{i3} &= -\sqrt{2\alpha\overline{\alpha}} d C_i^T G_i^T, \\ \overline{\Theta}_{i4} &= \sqrt{2}\overline{\alpha} d C_i^T G_i^T, \\ \overline{\Theta}_{i9} &= \sqrt{2\alpha\overline{\alpha}} d C_i^T G_i^T, \\ \overline{\Theta}_{i7} &= \sqrt{2} d \left(A_i^T - C_i^T G_i^T\right), \\ \overline{\Theta}_{i10} &= -\sqrt{2}\overline{\alpha} d C_i^T G_i^T. \end{aligned} \tag{43}$$

By pre- and postmultiplying both its sides with $\operatorname{diag}\{I, I, I, I, P_i, P_i\}$ and its transpose, respectively, one gets

$$\begin{bmatrix} \Theta_{i1} & \Theta_{i2} & Z & 0 & \Theta_{i3} & \Theta_{i4} \\ * & \Theta_{i5} & \Theta_{i6} & Z & 0 & \Theta_{i7} \\ * & * & \Theta_{i8} & 0 & \Theta_{i9} & \Theta_{i10} \\ * & * & * & \Theta_{11} & 0 & 0 \\ * & * & * & * & -\overline{Z} & 0 \\ * & * & * & * & * & -\overline{Z} \end{bmatrix} < 0, \tag{44}$$

where $\overline{Z} = P_i Z^{-1} P_i$. It is further obtained that $\overline{Z} > 0$ could be guaranteed by

$$-P_i Z^{-1} P_i \le -2P_i + Z < 0. \tag{45}$$

As for conditions (38) and (39), they are equivalent to

$$\begin{aligned} -\gamma_i I + C_i^T G_i^T P_i G_i C_i &\le 0, \\ -\lambda_i I + C_i^T G_i^T Z G_i C_i &\le 0. \end{aligned} \tag{46}$$

By applying the Schur complement lemma and considering representation (23), it is known that condition (38) is equivalent to condition (20). As for condition (39), by pre- and postmultiplying both its sides with $\operatorname{diag}\{I, P_i\}$ and applying inequality (45), one could easily get condition (21) with representation (23) implying condition (39). This completes the proof. □

Remark 4. Due to α included obviously, it plays important roles in analysis and synthesis of systems with output saturation. With two special cases that only system state with $\alpha = 0$ or delay state with $\alpha = 1$ is contained in output saturation, our results could be viewed as extension results on the saturation output problem from output without time delay to stochastic delay output. On the other hand, from the proof process, it is seen that some inequalities, like (31), (33), (35), and so on, have been used to get the LMI conditions and could lead to more conservatism. In order to reduce its conservatism, some additional variables could be introduced in these inequalities. However, such variables will lead to larger computation complexity. Moreover, the conservatism of inequality (45) could be further reduced by applying similar methods in [45]. Similarly, much larger computation complexity in terms of more variables and inequalities should be needed. Based on these facts, it is said that whether to choose these methods or not should be considered in concrete situations.

Remark 5. It is seen from this theorem that such a probability should be given exactly. It will be impossible or of high cost in some practical applications. Instead, only its estimation is available, even if it is totally unknown. In other words, there will be a uncertainty between the real and estimated values. As we know, such a uncertainty will degrade the system performance and even lead to unstable system. So, it is necessary and meaningful to consider this general case. Moreover, some necessarily additional variables and inequalities are introduced to obtain LMI conditions, where the variables are to be computed instead of being given beforehand. Because of the given results with LMI forms, the above general case could be handled by exploiting the methods in this paper and [46, 47] together.

Next, we consider another state observer system described as

$$\begin{aligned} \dot{\hat{x}}(t) &= A(r_t)\hat{x}(t) + G(r_t)\left(y(t) - \hat{y}(t)\right), \\ \hat{y}(t) &= C(r_t)\left[\alpha(t)\hat{x}(t) + (1-\alpha(t))\hat{x}(t-d)\right]. \end{aligned} \tag{47}$$

Then, the resulting error system is rewritten to be

$$\begin{aligned} \dot{e}(t) &= \left(A(r_t) - \alpha G(r_t) C(r_t)\right) e(t) - (1-\alpha) G(r_t) \\ &\quad \cdot C(r_t) e(t-d) + \alpha G(r_t) C(r_t) \eta + (1-\alpha) \\ &\quad \cdot G(r_t) C(r_t) \eta_d + (\alpha(t) - \alpha) G(r_t) C(r_t) \\ &\quad \cdot \left(x(t-d) - x(t) + \eta - \eta_d\right). \end{aligned} \tag{48}$$

Similarly, we have the following theorem.

Theorem 6. *Given system (1) with $d > 0$, there exists observer (47) such that system (1) and error system (48) are asymptotically stable, if for given positive scalars $\alpha \in [0,1]$, $\varepsilon_i \in (0,1)$, and $\varepsilon_{di} \in (0,1)$, there exist $P_i > 0$, $Q > 0$, $Z > 0$, $\gamma_i > 0$, $\lambda_i > 0$, and Y_i satisfying conditions (20) and (21) and*

$$\begin{bmatrix} \Theta_{i1} & 0 & Z & 0 & 0 & 0 \\ * & \Psi_{i1} & 0 & \Psi_{i2} & \Theta_{i3} & \Psi_{i3} \\ * & * & \Theta_{i8} & 0 & 0 & 0 \\ * & * & * & \Theta_{11} & \Theta_{i9} & \Theta_{i10} \\ * & * & * & * & \Theta_{i12} & 0 \\ * & * & * & * & * & \Theta_{i12} \end{bmatrix} < 0, \quad (49)$$

where

$$\begin{aligned} \Psi_{i1} &= \left(P_i A_i - \alpha Y_i C_i\right)^* + P_i + \sum_{j=1}^{N} \pi_{ij} P_j + Q - Z, \\ \Psi_{i2} &= -\overline{\alpha} Y_i C_i + Z, \\ \Psi_{i3} &= \sqrt{2} d \left(A_i^T P_i - \alpha C_i^T Y_i^T\right). \end{aligned} \quad (50)$$

Then, the gain of observer (47) could be computed by (23).

Proof. For systems (1) and (48), choose the same Lyapunov function (24). Similar to (35) and (36), it is obtained that

$$\begin{aligned} d^2 \left(f_{i3} + f_{i2}\right)^T Z \left(f_{i3} + f_{i2}\right) &\le 2d^2 f_{i3}^T Z f_{i3} \\ &+ 2d^2 \left(\alpha G_i C_i \eta + \overline{\alpha} G_i C_i \eta_d\right)^T \\ &\cdot Z \left(\alpha G_i C_i \eta + \overline{\alpha} G_i C_i \eta_d\right), \end{aligned} \quad (51)$$

where

$$f_{i3} = \left(A_i - \alpha G_i C_i\right) e(t) - \overline{\alpha} G_i C_i e(t-d), \quad (52)$$

$$\begin{aligned} &\alpha \overline{\alpha} d^2 \left[G_i C_i \left(e(t-d) - e(t) + \eta - \eta_d\right)\right]^T \\ &\quad \cdot Z \left[G_i C_i \left(e(t-d) - e(t) + \eta - \eta_d\right)\right] \\ &\quad \le 2\alpha \overline{\alpha} d^2 \left[G_i C_i \left(e(t-d) - e(t)\right)\right]^T \\ &\quad \cdot Z \left[G_i C_i \left(e(t-d) - e(t)\right)\right] \\ &\quad + 2\alpha \overline{\alpha} d^2 \left(G_i C_i \eta - G_i C_i \eta_d\right)^T Z \left(G_i C_i \eta - G_i C_i \eta_d\right). \end{aligned} \quad (53)$$

Then, it is obtained that

$$\begin{aligned} \mathscr{L}V(x_t) &\le x^T(t) \left(P_i A_i\right)^* x(t) + x^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x(t) \\ &+ e^T(t) \left[P_i \left(A_i - G_i C_i\right)\right]^* e(t) - 2\overline{\alpha} e^T(t) \\ &\cdot P_i G_i C_i e(t-d) + e^T(t) P_i e(t) \\ &+ \alpha \eta^T C_i^T G_i^T P_i G_i C_i \eta + \overline{\alpha} \eta_d^T C_i^T G_i^T P_i G_i C_i \eta_d + e^T(t) \\ &\cdot \sum_{j=1}^{N} \pi_{ij} P_j e(t) + x^T(t) Q x(t) - x^T(t-d) \\ &\cdot Q x(t-d) + e^T(t) Q e(t) - e^T(t-d) Q e(t-d) \\ &+ d^2 x^T(t) A_i^T Z A_i x(t) - x^T(t) Z x(t) + 2x^T(t) \\ &\cdot Z x(t-d) - x^T(t-d) Z x(t-d) \\ &+ 2\alpha d^2 \eta^T C_i^T G_i^T Z G_i C_i \eta + 2\overline{\alpha} d^2 \eta_d^T C_i^T G_i^T Z G_i C_i \eta_d \\ &+ 2d^2 \left[\left(A_i - \alpha G_i C_i\right) e(t) - \overline{\alpha} G_i C_i e(t-d)\right]^T \\ &\times Z \left[\left(A_i - \alpha G_i C_i\right) e(t) - \overline{\alpha} G_i C_i e(t-d)\right] \\ &+ 2\alpha \overline{\alpha} d^2 \left[G_i C_i \left(e(t-d) - e(t)\right)\right]^T \\ &\cdot Z \left[G_i C_i \left(e(t-d) - e(t)\right)\right] \\ &+ 4\alpha \overline{\alpha} d^2 \eta^T C_i^T G_i^T Z G_i C_i \eta \\ &+ 4\alpha \overline{\alpha} d^2 \eta_d^T C_i^T G_i^T Z G_i C_i \eta_d - e^T(t) Z e(t) + 2e^T(t) \\ &\cdot Z e(t-d) - e^T(t-d) Z e(t-d). \end{aligned} \quad (54)$$

Based on conditions (20), (21), (51), and (53), it is obtained that (54) is implied by

$$\begin{aligned} \mathscr{L}V(x_t) \le \xi^T(t) &\left(\begin{bmatrix} \Theta_{i1} & 0 & Z & 0 \\ * & \Psi_{i1} & 0 & \Psi_{i2} \\ * & * & \Theta_{i8} & 0 \\ * & * & * & \Theta_{11} \end{bmatrix} \right. \\ &+ 2\alpha \overline{\alpha} d^2 \begin{bmatrix} 0 \\ -C_i^T G_i^T \\ 0 \\ C_i^T G_i^T \end{bmatrix} Z \begin{bmatrix} 0 \\ -C_i^T G_i^T \\ 0 \\ C_i^T G_i^T \end{bmatrix}^T \\ &\left. + 2d^2 \begin{bmatrix} 0 \\ \left(A_i - \alpha C_i^T G_i\right)^T \\ 0 \\ -\overline{\alpha} C_i^T G_i^T \end{bmatrix} Z \begin{bmatrix} 0 \\ \left(A_i - \alpha C_i^T G_i\right)^T \\ 0 \\ -\overline{\alpha} C_i^T G_i^T \end{bmatrix}^T \right) \\ &\cdot \xi(t). \end{aligned} \quad (55)$$

By using similar methods given in Theorem 3, it is known that condition (55) could be guaranteed by inequality (49). This completes the proof. □

Finally, we consider the following state observer system described by

$$\begin{aligned} \dot{\hat{x}}(t) &= A(r_t) \hat{x}(t) + G(r_t) \left(y(t) - \hat{y}(t)\right), \\ \hat{y}(t) &= C(r_t) \operatorname{sat}\left(\alpha(t) \hat{x}(t) + (1 - \alpha(t)) \hat{x}(t-d)\right) \end{aligned} \quad (56)$$

which is equivalent to

$$\begin{aligned}\dot{\hat{x}}(t) &= A(r_t)\hat{x}(t) + G(r_t)(y(t) - \hat{y}(t)),\\ \hat{y}(t) &= C(r_t)\{\alpha\hat{x}(t) + \overline{\alpha}\hat{x}(t-d)\\ &\quad + (\alpha(t) - \alpha)(\hat{x}(t) - \hat{x}(t-d))\\ &\quad - [\alpha\hat{\eta} + \overline{\alpha}\hat{\eta}_d + (\alpha(t) - \alpha)(\hat{\eta} - \hat{\eta}_d)]\}.\end{aligned} \tag{57}$$

Then, the resulting error system is rewritten to be

$$\begin{aligned}\dot{e}(t) &= (A(r_t) - \alpha G(r_t)C(r_t))e(t) - \overline{\alpha}G(r_t)C(r_t)\\ &\quad\cdot e(t-d) + \alpha G(r_t)C(r_t)(\eta - \hat{\eta}) + \overline{\alpha}G(r_t)\\ &\quad\cdot C(r_t)(\eta_d - \hat{\eta}_d) + (\alpha(t) - \alpha)G(r_t)C(r_t)\\ &\quad\cdot (x(t-d) - x(t) + \eta - \hat{\eta} - \eta_d + \hat{\eta}_d).\end{aligned} \tag{58}$$

Theorem 7. *Given system (1) with $d > 0$, there exists observer (56) such that system (1) and error system (58) are asymptotically stable, if for given positive scalars $\alpha \in [0,1]$, $\varepsilon_i \in (0,1)$, and $\varepsilon_{di} \in (0,1)$, there exist $P_i > 0$, $Q > 0$, $Z > 0$, $\gamma_i > 0$, $\lambda_i > 0$, and Y_i satisfying conditions (20) and (21) and*

$$\begin{bmatrix}\Omega_{i1} & \Omega_{i2} & Z & 0 & 0 & 0\\ * & \Omega_{i3} & 0 & \Psi_{i2} & \Theta_{i3} & \Psi_{i3}\\ * & * & \Omega_{i4} & \Omega_{i5} & 0 & 0\\ * & * & * & \Omega_{i6} & \Theta_{i9} & \Theta_{i10}\\ * & * & * & * & \Theta_{i12} & 0\\ * & * & * & * & * & \Theta_{i12}\end{bmatrix} < 0, \tag{59}$$

where

$$\begin{aligned}\Omega_{i1} &= (P_iA_i)^* + \sum_{j=1}^{N}\pi_{ij}P_j + 2\alpha\varepsilon_i\gamma_iI + Q + d^2A_i^TZA_i\\ &\quad - Z + 8\alpha d^2\varepsilon_i\lambda_iI + 16\alpha\overline{\alpha}d^2\varepsilon_i\lambda_iI,\\ \Omega_{i2} &= -\alpha\varepsilon_i\gamma_iI - 4\alpha d^2\varepsilon_i\lambda_iI - 8\alpha\overline{\alpha}d^2\varepsilon_i\lambda_iI,\\ \Omega_{i3} &= (P_iA_i - \alpha Y_iC_i)^* + 2P_i + \sum_{j=1}^{N}\pi_{ij}P_j + Q - Z\\ &\quad + \alpha\varepsilon_i\gamma_iI + 4\alpha d^2\varepsilon_i\lambda_iI + 8\alpha\overline{\alpha}d^2\varepsilon_i\lambda_iI,\\ \Omega_{i4} &= 2\overline{\alpha}\varepsilon_{di}\gamma_iI + 8\overline{\alpha}d^2\varepsilon_{di}\lambda_iI + 16\alpha\overline{\alpha}d^2\varepsilon_{di}\lambda_iI - Q\\ &\quad - Z,\\ \Omega_{i5} &= -\overline{\alpha}\varepsilon_{di}\gamma_iI - 4\overline{\alpha}d^2\varepsilon_{di}\lambda_iI - 8\alpha\overline{\alpha}d^2\varepsilon_{di}\lambda_iI,\\ \Omega_{i6} &= \overline{\alpha}\varepsilon_{di}\gamma_iI + 4\overline{\alpha}d^2\varepsilon_{di}\lambda_iI + 8\alpha\overline{\alpha}d^2\varepsilon_{di}\lambda_iI - Q - Z.\end{aligned} \tag{60}$$

Then, (23) could be used to compute the gain of observer (56).

Proof. Firstly, the same Lyapunov function is also selected for systems (1) and (56). Similar to computation process of Theorems 3 and 6, the corresponding terms are handled as

$$\begin{aligned}&-2\alpha e^T(t)P_iG_iC_i\hat{\eta}\\ &\quad\le \alpha e^T(t)P_ie(t) + \alpha\hat{\eta}^TC_i^TG_i^TP_iG_iC_i\hat{\eta},\\ &-2\overline{\alpha}e^T(t)P_iG_iC_i\hat{\eta}_d\\ &\quad\le \overline{\alpha}e^T(t)P_ie(t) + \overline{\alpha}\hat{\eta}_d^TC_i^TG_i^TP_iG_iC_i\hat{\eta}_d,\\ &d^2(f_{i3} + f_{i4})^TZ(f_{i3} + f_{i4})\\ &\quad\le 2d^2f_{i3}^TZf_{i3} + 2d^2f_{i4}^TZf_{i4},\end{aligned} \tag{61}$$

where

$$\begin{aligned}&f_{i4} = \alpha G_iC_i(\eta - \hat{\eta}) + \overline{\alpha}G_iC_i(\eta_d - \widehat{\eta_d}),\\ &2d^2f_{i4}^TZf_{i4} \le 2\alpha d^2(\eta - \hat{\eta})^TC_i^TG_i^TZG_iC_i(\eta - \hat{\eta})\\ &\quad + 2\overline{\alpha}d^2(\eta_d - \widehat{\eta_d})^TC_i^TG_i^TZG_iC_i(\eta_d - \widehat{\eta_d})\\ &\quad \le 4\alpha d^2\eta^TC_i^TG_i^TZC_i^TG_i^T\eta + 4\alpha d^2\hat{\eta}^TC_i^TG_i^TZG_iC_i\hat{\eta}\\ &\quad + 4\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d + 4\overline{\alpha}d^2\hat{\eta}_d^TC_i^TG_i^TZG_iC_i\hat{\eta}_d,\\ &d^2(f_{i3} + f_{i4})^TZ(f_{i3} + f_{i4}) \le 2d^2f_{i3}^TZf_{i3}\\ &\quad + 2d^2f_{i4}^TZf_{i4},\\ &\alpha\overline{\alpha}d^2\{G_iC_i[e(t-d) - e(t) + (\eta - \hat{\eta}) - (\eta_d - \widehat{\eta_d})]\}^T\\ &\quad \times Z\{G_iC_i[e(t-d) - e(t) + (\eta - \hat{\eta}) - (\eta_d - \widehat{\eta_d})]\}\\ &\quad \le 2\alpha\overline{\alpha}d^2[G_iC_i(e(t-d) - e(t))]^T\\ &\quad \cdot Z[G_iC_i(e(t-d) - e(t))] + 2\alpha\overline{\alpha}d^2[G_iC_i(\eta - \hat{\eta})\\ &\quad - G_iC_i(\eta_d - \widehat{\eta_d})]^TZ \times [G_iC_i(\eta - \hat{\eta}) - G_iC_i(\eta_d\\ &\quad - \widehat{\eta_d})].\end{aligned} \tag{62}$$

Moreover, it is obtained that

$$\begin{aligned}&2\alpha\overline{\alpha}d^2[G_iC_i(\eta - \hat{\eta}) - G_iC_i(\eta_d - \widehat{\eta_d})]^T\\ &\quad\cdot Z[G_iC_i(\eta - \hat{\eta}) - G_iC_i(\eta_d - \widehat{\eta_d})]\\ &\quad\le 8\alpha\overline{\alpha}d^2\eta^TC_i^TG_i^TZC_i^TG_i^T\eta\\ &\quad + 8\alpha\overline{\alpha}d^2\hat{\eta}^TC_i^TG_i^TZG_iC_i\hat{\eta}\\ &\quad + 8\alpha\overline{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d\\ &\quad + 8\alpha\overline{\alpha}d^2\hat{\eta}_d^TC_i^TG_i^TZG_iC_i\hat{\eta}_d.\end{aligned} \tag{63}$$

Based on these conditions, it is obtained that

$$\begin{aligned}
\mathscr{L}V(x_t) \le{}& x^T(t)(P_iA_i)^{*}x(t) + x^T(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\
&+ e^T(t)[P_i(A_i - G_iC_i)]^{*}e(t) - 2\bar{\alpha}e^T(t) \\
&\cdot P_iG_iC_ie(t-d) + 2e^T(t)P_ie(t) \\
&+ \alpha\eta^TC_i^TG_i^TP_iG_iC_i\eta + \bar{\alpha}\eta_d^TC_i^TG_i^TP_iG_iC_i\eta_d \\
&+ \alpha\hat{\eta}^TC_i^TG_i^TP_iG_iC_i\hat{\eta} + \bar{\alpha}\hat{\eta}_d^TC_i^TG_i^TP_iG_iC_i\hat{\eta}_d + e^T(t) \\
&\cdot \sum_{j=1}^{N}\pi_{ij}P_je(t) + x^T(t)Qx(t) - x^T(t-d) \\
&\cdot Qx(t-d) + e^T(t)Qe(t) - e^T(t-d)Qe(t-d) \\
&+ d^2x^T(t)A_i^TZA_ix(t) - x^T(t)Zx(t) + 2x^T(t) \\
&\cdot Zx(t-d) - x^T(t-d)Zx(t-d) \\
&+ 4\alpha d^2\eta^TC_i^TG_i^TZG_iC_i\eta + 4\bar{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d \\
&+ 4\alpha d^2\hat{\eta}^TC_i^TG_i^TZG_iC_i\hat{\eta} + 4\bar{\alpha}d^2\hat{\eta}_d^TC_i^TG_i^TZG_iC_i\hat{\eta}_d \\
&+ 2d^2\left[(A_i - \alpha G_iC_i)e(t) - \bar{\alpha}G_iC_ie(t-d)\right]^T \\
&\times Z\left[(A_i - \alpha G_iC_i)e(t) - \bar{\alpha}G_iC_ie(t-d)\right] \\
&+ 2\alpha\bar{\alpha}d^2\left[G_iC_i(e(t-d) - e(t))\right]^T \\
&\cdot Z\left[G_iC_i(e(t-d) - e(t))\right] \\
&+ 8\alpha\bar{\alpha}d^2\eta^TC_i^TG_i^TZG_iC_i\eta \\
&+ 8\alpha\bar{\alpha}d^2\eta_d^TC_i^TG_i^TZG_iC_i\eta_d \\
&+ 8\alpha\bar{\alpha}d^2\hat{\eta}^TC_i^TG_i^TZG_iC_i\hat{\eta} \\
&+ 8\alpha\bar{\alpha}d^2\hat{\eta}_d^TC_i^TG_i^TZG_iC_i\hat{\eta}_d - e^T(t)Ze(t) + 2e^T(t) \\
&\cdot Ze(t-d) - e^T(t-d)Ze(t-d).
\end{aligned} \tag{64}$$

Based on conditions (20) and (21) and above inequalities, it is obtained that (64) is implied by

$$\begin{aligned}
\mathscr{L}V(x_t) \le{}& \xi^T(t)\left(\begin{bmatrix} \Omega_{i1} & \Omega_{i2} & Z & 0 \\ * & \Omega_{i3} & 0 & \Psi_{i3} \\ * & * & \Omega_{i4} & \Omega_{i5} \\ * & * & * & \Omega_{i6} \end{bmatrix}\right. \\
&+ 2\alpha\bar{\alpha}d^2\begin{bmatrix} 0 \\ -C_i^TG_i^T \\ 0 \\ C_i^TG_i^T \end{bmatrix} Z \begin{bmatrix} 0 \\ -C_i^TG_i^T \\ 0 \\ C_i^TG_i^T \end{bmatrix}^T \\
&\left. + 2d^2\begin{bmatrix} 0 \\ \left(A_i - \alpha C_i^TG_i\right)^T \\ 0 \\ -\bar{\alpha}C_i^TG_i^T \end{bmatrix} Z \begin{bmatrix} 0 \\ \left(A_i - \alpha C_i^TG_i\right)^T \\ 0 \\ -\bar{\alpha}C_i^TG_i^T \end{bmatrix}^T\right) \\
&\cdot \xi(t).
\end{aligned} \tag{65}$$

By using similar methods given in Theorems 3 and 6, condition (65) is obtained easily from inequality (59) with representation (23). This completes the proof. □

4. Numerical Examples

Example 1. Consider a delayed Markovian jump system of form (1), whose parameters are described to be

$$\begin{aligned}
A_1 &= \begin{bmatrix} -1.5 & 0.1 \\ 1 & -0.8 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.1 & 0.6 \\ 0.4 & 0.1 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -0.9 & -0.2 \\ 0.8 & -0.9 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} -0.2 & 0.5 \\ -0.3 & -0.2 \end{bmatrix}.
\end{aligned} \tag{66}$$

The transition rate matrix is given as

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.6 & -0.6 \end{bmatrix}. \tag{67}$$

Firstly, we design an observer with form (9). Letting $d = 0.3$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.2$, $\varepsilon_{d1} = 0.4$, and $\varepsilon_{d2} = 0.1$, by Theorem 3, the gains of observer (9) are computed as

$$\begin{aligned}
G_1 &= \begin{bmatrix} 0.0902 & 0.2853 \\ 0.2443 & 0.2980 \end{bmatrix}, \\
G_2 &= \begin{bmatrix} -0.1308 & -0.0389 \\ -0.0230 & -0.4683 \end{bmatrix}.
\end{aligned} \tag{68}$$

Applying the above designed observer, under the initial condition $x_0 = [1 \quad -1 \quad -1 \quad 1]^T$, we have the state responses of systems (1) and (10) given in Figure 1, while the simulation of the corresponding operation mode is given in Figure 2.

When another observer (47) is designed, its gains could be computed by Theorem 6 and given as

$$\begin{aligned}
G_1 &= \begin{bmatrix} 0.0855 & 0.3848 \\ 0.3415 & 0.2380 \end{bmatrix}, \\
G_2 &= \begin{bmatrix} -0.1079 & -0.0338 \\ -0.0148 & -0.5234 \end{bmatrix},
\end{aligned} \tag{69}$$

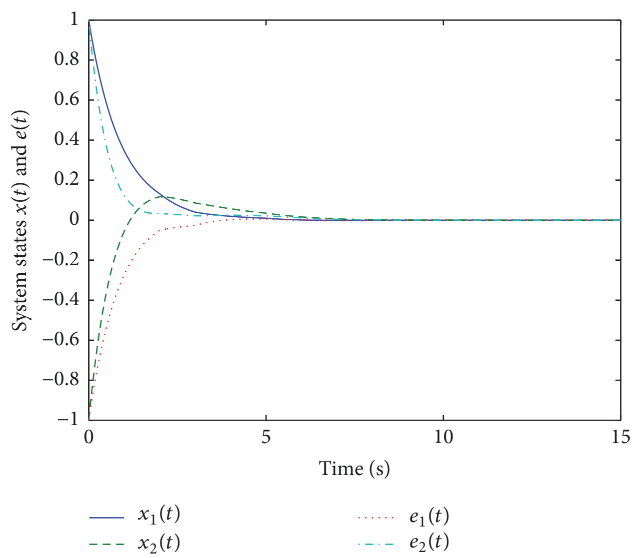

FIGURE 1: The state responses of systems (1) and (10).

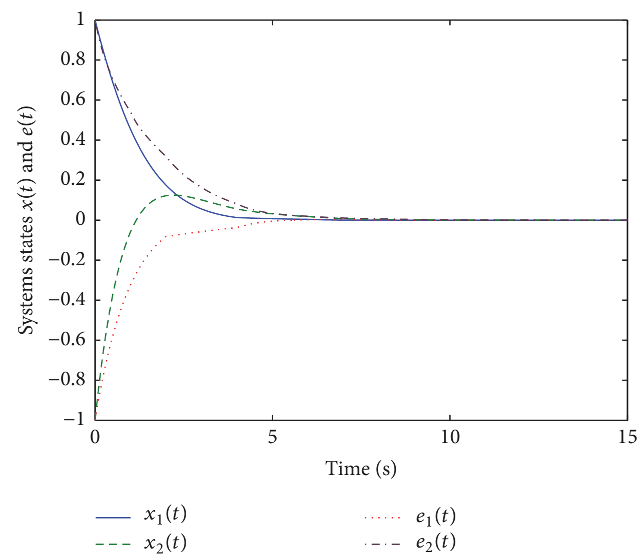

FIGURE 3: The state responses of systems (1) and (47).

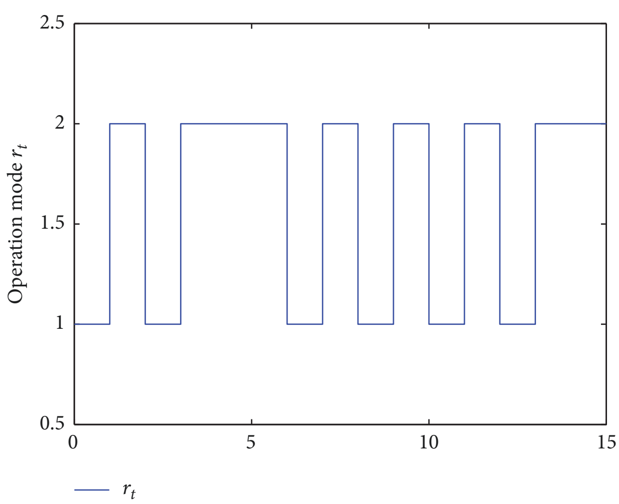

FIGURE 2: The simulation of operation mode.

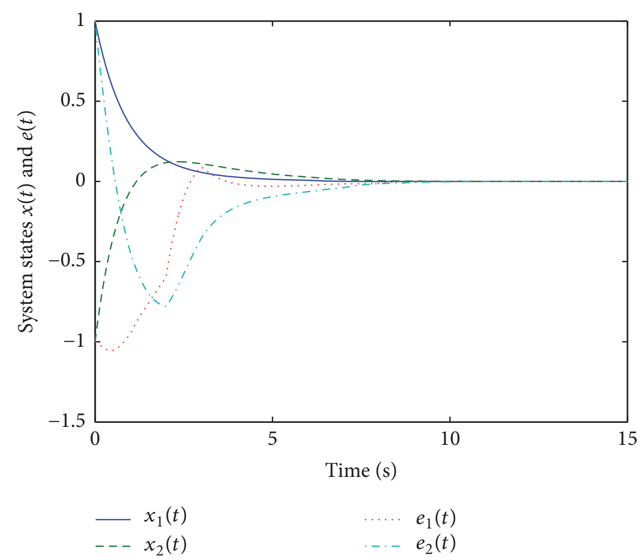

FIGURE 4: The state responses of systems (1) and (57).

where the corresponding parameters are the same as the above ones. At the same time, by Theorem 7, the gains of observer (57) are computed to be

$$G_1 = \begin{bmatrix} 0.2832 & 0.9648 \\ 0.2265 & 0.1486 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} 0.1489 & 0.8130 \\ -0.3814 & -2.0525 \end{bmatrix}. \quad (70)$$

Under the same initial condition, the state curves of the corresponding systems are simulated in Figures 3 and 4, respectively. Based on these simulations, it is seen that the designed observers are all useful and also demonstrate the utility of the proposed methods.

In order to further demonstrate the effects of output saturation existing in different forms, more comparisons among them will be done in the following. Firstly, we consider the relationship between probability α and allowable maximum d. Based on Theorems 3–7, the maximum allowable d along with α is given in Table 1, whose simulation is carried out in Figure 5. From this simulation, it is seen that observer (9) is the least conservative, while observer (56) has the largest conservatism.

Next, we consider the effects of parameters ε_i and ε_{di}. In order to demonstrate their effects clearly, without loss of generality, matrix A_1 is assumed to be

$$A_1 = \begin{bmatrix} -1 & 0.1 + \delta \\ 1 & -0.8 \end{bmatrix}, \quad (71)$$

where δ is a positive scalar to be determined, and the other matrices are constant. Based on Theorem 3, the allowable

Table 1: The allowable maximum d for different α.

α	0	0.3	0.5	0.8	1
Theorem 3	0.927	0.943	0.955	0.976	0.999
Theorem 6	0.918	0.928	0.936	0.958	0.999
Theorem 7	0.207	0.273	0.313	0.390	0.470

Table 2: The allowable $\delta_{\max}$ for different pair $(\varepsilon_1, \varepsilon_2)$.

	$\varepsilon_2 = 0.1$	$\varepsilon_2 = 0.4$	$\varepsilon_2 = 0.6$	$\varepsilon_2 = 0.9$
$\varepsilon_1 = 0.1$	0.9450	0.9370	0.9340	0.9300
$\varepsilon_1 = 0.4$	0.9000	0.8930	0.8900	0.8860
$\varepsilon_1 = 0.6$	0.8760	0.8690	0.8650	0.8610
$\varepsilon_1 = 0.9$	0.8440	0.8370	0.8340	0.8300

Table 3: The allowable $\delta_{\max}$ for different pair $(\varepsilon_1, \varepsilon_2)$.

	$\varepsilon_2 = 0.1$	$\varepsilon_2 = 0.4$	$\varepsilon_2 = 0.6$	$\varepsilon_2 = 0.9$
$\varepsilon_1 = 0.1$	0.9390	0.9330	0.9300	0.9260
$\varepsilon_1 = 0.4$	0.8950	0.8900	0.8870	0.8830
$\varepsilon_1 = 0.6$	0.8710	0.8660	0.8630	0.8590
$\varepsilon_1 = 0.9$	0.8400	0.8350	0.8320	0.8280

Table 4: The allowable $\delta_{\max}$ for different pair $(\varepsilon_1, \varepsilon_2)$.

	$\varepsilon_2 = 0.1$	$\varepsilon_2 = 0.4$	$\varepsilon_2 = 0.6$	$\varepsilon_2 = 0.9$
$\varepsilon_1 = 0.01$	0.1960	0.1220	0.0870	0.0430
$\varepsilon_1 = 0.05$	0.1690	0.1020	0.0690	0.0260
$\varepsilon_1 = 0.10$	0.1410	0.0800	0.0480	0.0050
$\varepsilon_1 = 0.40$	0.0390	—	—	—

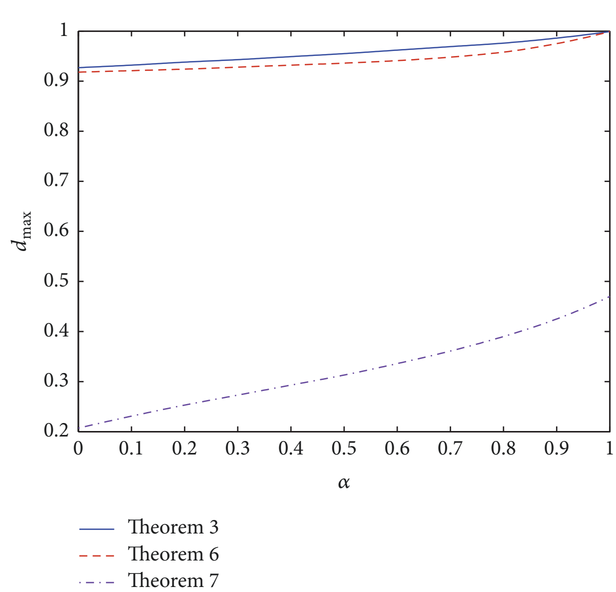

Figure 5: The curves of $d_{\max}$ along with α.

maximum $\delta \triangleq \delta_{\max}$ along with different pair $(\varepsilon_1, \varepsilon_2)$ is listed in Table 2. Similarly, Tables 3 and 4 are obtained by applying Theorems 6 and 7, respectively. Based on these tables, the simulation of correlation between pair $(\varepsilon_1, \varepsilon_2)$ and $\delta_{\max}$ is shown Figures 6 and 7. From Figure 6, it is seen that Theorem 3 related to observer (10) is less conservative than Theorem 6 corresponding to observer (47). From their forms, it is seen in this example that the delay term in output has a negative effect. However, the differences between them are very small. To the contrary, there is an advantage that the nondelay state of observer (47) is not necessarily available online, which could be replaced by a delay one with some probability. In this sense, it is said that observer (47) is better in terms of having less application constraints. As for Figure 7, it is concluded that observer (57) is the most conservative. The main reason is that a more output saturation is included and has a large negative effect in reducing performance. Similarly, we could also make a correlation between pair $(\varepsilon_{d1}, \varepsilon_{d2})$ and $\delta_{\max}$. Based on Theorems 3–7, the allowable $\delta_{\max}$ for different pair $(\varepsilon_{d1}, \varepsilon_{d2})$ are presented in Tables 5–7. From these tables, a similar conclusion could be obtained where Theorem 3 designed for observer (10) is superior over Theorems 6 and 7. Among them, it is found that the most conservative one is observer (57) obtained from Theorem 7. In addition, the corresponding simulations of such correlations between pair $(\varepsilon_{d1}, \varepsilon_{d2})$ and $\delta_{\max}$ could be obtained easily and will give the same conclusion, which are omitted here.

Table 5: The allowable $\delta_{\max}$ for different pair $(\varepsilon_{d1}, \varepsilon_{d2})$.

	$\varepsilon_{d2} = 0.1$	$\varepsilon_{d2} = 0.4$	$\varepsilon_{d2} = 0.6$	$\varepsilon_{d2} = 0.9$
$\varepsilon_{d1} = 0.1$	0.9470	0.9420	0.9390	0.9350
$\varepsilon_{d1} = 0.4$	0.9110	0.9080	0.9060	0.9030
$\varepsilon_{d1} = 0.6$	0.8890	0.8860	0.8840	0.8820
$\varepsilon_{d1} = 0.9$	0.8580	0.8550	0.8540	0.8510

Table 6: The allowable $\delta_{\max}$ for different pair $(\varepsilon_{d1}, \varepsilon_{d2})$.

	$\varepsilon_{d2} = 0.1$	$\varepsilon_{d2} = 0.4$	$\varepsilon_{d2} = 0.6$	$\varepsilon_{d2} = 0.9$
$\varepsilon_{d1} = 0.1$	0.9440	0.9390	0.9370	0.9330
$\varepsilon_{d1} = 0.4$	0.9060	0.9020	0.9000	0.8960
$\varepsilon_{d1} = 0.6$	0.8830	0.8790	0.8770	0.8730
$\varepsilon_{d1} = 0.9$	0.8510	0.8470	0.8450	0.8420

Table 7: The allowable $\delta_{\max}$ for different pair $(\varepsilon_{d1}, \varepsilon_{d2})$.

	$\varepsilon_{d2} = 0.1$	$\varepsilon_{d2} = 0.4$	$\varepsilon_{d2} = 0.6$	$\varepsilon_{d2} = 0.9$
$\varepsilon_{d1} = 0.01$	0.1950	0.1490	0.1210	0.0850
$\varepsilon_{d1} = 0.05$	0.1830	0.1380	0.1100	0.0730
$\varepsilon_{d1} = 0.10$	0.1660	0.1220	0.0940	0.0570
$\varepsilon_{d1} = 0.40$	0.0470	0.0110	—	—

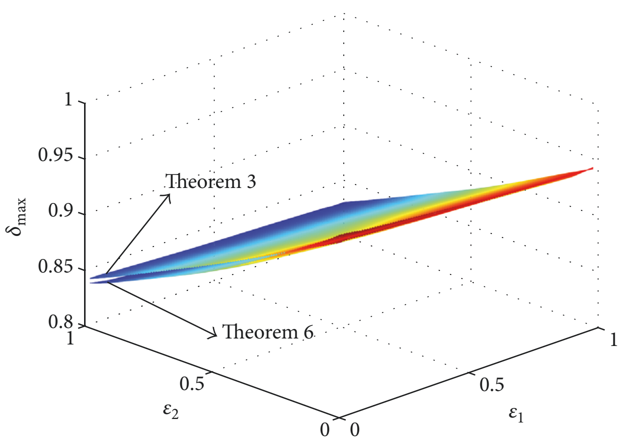

Figure 6: The simulation of correlation between pair $(\varepsilon_1, \varepsilon_2)$ and $\delta_{\max}$.

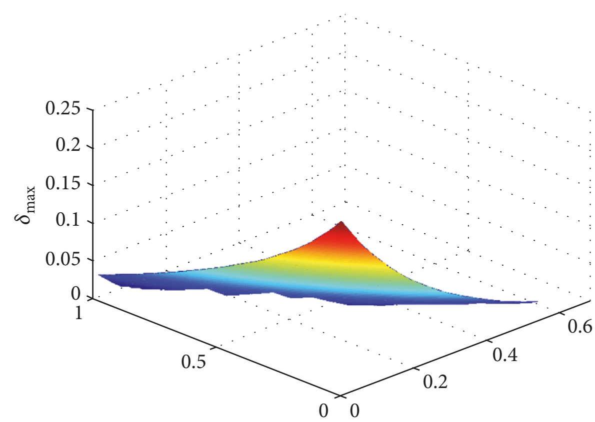

Figure 7: The simulation of correlation between pair (ϵ_1, ϵ_2) and $\delta_{\max}$.

5. Conclusion

In this paper, we have studied observer design problem of continuous-time Markovian jump systems with saturated output. First of all, a kind of state observer with output state saturation is proposed to be partially dependent. More precisely, both nondelay and delay sates are contained but occur asynchronously, whose probability distributions are embodied by the Bernoulli variable and taken into account in the observer design. By applying an improved inequality to deal with saturation terms, the existence conditions for observers with three kinds of output state saturations have been proposed with LMIs. Finally, a numerical example is used to testify the effectiveness and advantages of the presented methods.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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