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Research Article

Adaptive Fuzzy Command Filtered Control for Chua's Chaotic System

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In this paper, we propose the command filtered adaptive fuzzy backstepping control (AFBC) approach for Chua's chaotic system with external disturbance. Based on two proposed first-order command filters, the convergence of tracking errors as well as the problem of "explosion of complexity" in traditional backstepping design procedure is solved. In the command filtered AFBC design, we do not need to calculate the complicated partial derivatives of the virtual control inputs. Fuzzy logic systems (FLSs) are used to identify the system uncertainties in real time. Based on Lyapunov stability criterion, the proposed controller can guarantee that all signals in the closed-loop system keep bounded, and the tracking errors converge to a small region eventually. Finally, simulation studies have been provided to verify the effectiveness of the proposed method.

1. Introduction

It is well known that adaptive backstepping control (ABC) is an effective technique for controlling nonlinear systems in parameter strict-feedback form [1]. For ABC of strict-feedback nonlinear systems without system uncertainties and external disturbances, this issue has been studied by using many control approaches [2–4]. Based on the sliding mode filters, [5, 6] estimated the command derivatives in the design of ABC. Linear filters for derivative generation were considered in [7]. Then, Farrell et al. in [8] introduced a command filtered backstepping control (CFBC) method, in which some new approaches were given to indicate that the virtual tracking errors between the signals of the command filtered and standard ABC methods were of $\circ(1/W)$, where W represented the frequency of the command filter. Up to now, many command filter control methods have been reported [9–11]. The above literature only addressed the nonadaptive case for nonlinear feedback systems. Design of ABC with complicated situations was given in, for instance, [12–20]. It should be mentioned that the dimension of the input variables of the estimated system must be extended to include the reference trajectory and its first derivatives. However, the aforementioned works studied the approximation problem of the command derivatives, but the resulting implementation does not achieve the theoretical guarantees of the ABC design. That is to say, new approaches are expected to solve this problem.

It has been shown that modeling of plant systems is badly affected by system uncertainties, i.e., parameter uncertainties, modeling errors, external disturbances, etc. This strongly motivates the study to design a robust, flexible, and effective controller, which can suppress complexities that demean the exhibition of the plants [21–37]. For backstepping control of nonlinear systems subject to system uncertainties, some control methods have been proposed, for example, in [38–43]. On the other hand, to tackle system uncertainties, scientists and researchers have proposed a lot of intelligent methods such as fuzzy logic systems (FLSs), neural networks, and neurofuzzy systems. In these methods, FLSs have been shown to be most successful and popular [44–48]. Following later advancement in intelligent control techniques, adaptive fuzzy controllers were developed such as fuzzy gain scheduled PID controller, fuzzy model reference adaptive controller, and self-organizing fuzzy controller. Adaptive fuzzy backstepping control (AFBC) methods also have been reported recently, for example, in [2, 4, 38, 45, 49, 50]. In [4], AFBC has been established for fractional-order strict-feedback systems. In [45], AFBC has been given for uncertain nonlinear systems with input saturation. Wang et al. introduced a command filtered AFBC approach for uncertain nonlinear systems, where the "explosion of complexity" problem in backstepping design and chartering phenomenon were solved [49]. However, in their work, external disturbance was not considered, and a complicated second-order filter was used in the controller design.

Motivated by above discussion, this paper will investigate the control uncertain Chua's chaotic system with external disturbance by means of command filtered AFBC. Combining the ABC method and command filter, a robust command filter AFBC is established. The proposed method can guarantee that all signals in the closed-loop system remain bounded, and the tracking errors converge to a small neighborhood of the origin eventually. The main contributions of this paper can be summarized as follows.

(1) The proposed command filter AFBC method works well even in the presence of full unknown system structure and external disturbance. A simple first-order filter has been introduced. Compared with the filter introduced in [49], our method is simpler and easier to be established. The proposed command filter guarantees that the commanded tracking error as well as its first derivative satisfies our control objective.

(2) By using the proposed command filter, the conventional "explosion of complexity" problem can be avoided. That is to say, the complicated calculation of partial derivative of the virtual control input is unnecessary. Our controller and adaptation laws are more concise compared with dynamic surface control approach, for example, in [51, 52].

The structure of this paper is arranged as follows. Section 2 gives the description of FLSs. The description of the problem and the controller design as well as the stability analysis are included in Section 3. The simulation results are indicated in Section 4. Finally, Section 5 gives a brief conclusion of this paper.

2. Description of FLS

A FLS contains four parts, i.e., the knowledge base, fuzzifier, fuzzy inference engine basing on the fuzzy rules, and defuzzifier. The j-th fuzzy rule is written as

$\mathscr{R}^{(j)}$: if x_1 is E_1^j, x_2 is $E_2^j, \ldots, x_n$ is E_n^j, then $\hat{f}(\mathbf{x}(t))$ is C^j $(j = 1, 2, \ldots, N)$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ and $\hat{f}(\mathbf{x}(t)) \in \mathbb{R}$ are, respectively, the input and the output of fuzzy logic systems. E_i^j and C^j $(i = 1, 2, \ldots, n)$ are fuzzy sets belonging to $\mathbb{R}$. The output of fuzzy logic systems can be expressed by

$$\hat{f}(\mathbf{x}(t)) = \frac{\sum_{j=1}^{N} \theta_j(t) \left[\prod_{i=1}^{n} \mu_{E_i^j}(x_i(t))\right]}{\sum_{j=1}^{N} \left[\prod_{i=1}^{n} \mu_{E_i^j}(x_i(t))\right]}, \tag{1}$$

where $\theta_j(t)$ is a value where fuzzy membership function μ_{C^j} is maximum. Generally, we can consider that $\mu_{C^j}(\theta_j(t)) = 1$, and fuzzy basic function is $\varphi_j(\mathbf{x}(t)) = \prod_{i=1}^{n} \mu_{E_i^j}(x_i(t)) / \sum_{j=1}^{N} [\prod_{i=1}^{n} \mu_{E_i^j}(x_i(t))]$. Let $\boldsymbol{\varphi}(\mathbf{x}(t)) = [\varphi_1(\mathbf{x}(t)), \varphi_2(\mathbf{x}(t)), \ldots, \varphi_N(\mathbf{x}(t))]^T$, $\boldsymbol{\theta}(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_N(t)]^T$, and then output of fuzzy logic systems can be written as

$$\hat{f}(\mathbf{x}(t)) = \boldsymbol{\theta}^T(t)\boldsymbol{\varphi}(\mathbf{x}(t)). \tag{2}$$

Lemma 1. *Suppose that $h(\mathbf{x})$ is a continuous function defined on compact set Ω; for any constants $\varepsilon > 0$, there exists a fuzzy logic system approximating function $\hat{f}(\mathbf{x})$ forming (2) such that*

$$\sup_{\Omega} \left| h(\mathbf{x}(t)) - \hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\varphi}(\mathbf{x}(t)) \right| \leq \varepsilon, \tag{3}$$

where $\hat{\boldsymbol{\theta}}(t)$ is an estimator of optimal vector $\boldsymbol{\theta}^*$.

3. Main Results

3.1. Problem Description. The controlled Chua's system is described as

$$\begin{aligned} \dot{x}_1(t) &= \alpha x_2(t) - \alpha x_1(t) - f(x_1(t)) + g_1(x_1(t)), \\ \dot{x}_2(t) &= x_3(t) + x_1(t) - x_2(t) + g_2(\overline{\mathbf{x}}_2(t)), \\ \dot{x}_3(t) &= \beta x_2(t) - \gamma x_3(t) + g_3(\mathbf{x}(t)) + d(t) + u(t), \\ y(t) &= x_1(t), \end{aligned} \tag{4}$$

with

$$f(x_1(t)) = a x_1(t) + b\left(\left|x_1(t) + 1\right| - \left|x_1(t) - 1\right|\right) \tag{5}$$

where α, β, γ, a, and b are system parameters, $y \in \mathscr{R}$ represents the system output, $g_1(x_1(t))$, $g_2(\overline{\mathbf{x}}_2(t))$, and $g(\mathbf{x}(t))$ are the system uncertainties with $\overline{\mathbf{x}}_2(t) = [x_1(t), x_2(t)]^T \in \mathscr{R}^2$, $\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t)]^T \in \mathscr{R}^3$, $d(t) \in \mathscr{R}$ is an unknown external disturbance, and $u(t) \in \mathscr{R}$ denotes the control input.

Define the output tracking error $e_1(t) = x_1(t) - x_1^c(t)$ where $x_1^c(t) \in \mathscr{R}$ is a known smooth enough referenced signal. The paper aims to design a proper controller $u(t)$ such that the tracking error $e_1(t)$ tends to an arbitrary small region.

3.2. Controller Design. To meet the control objective, a robust command filtered backstepping controller that contains three steps will be constructed.

Step 1. Consider the first dynamical equation in system (4):

$$\dot{x}_1(t) = \alpha x_2(t) + \Delta g_1(x_1(t)), \tag{6}$$

where $\Delta g_1(x_1(t)) = -\alpha x_1(t) - f(x_1(t)) + g_1(x_1(t))$ is an unknown nonlinear function. Thus, $\Delta g_1(x_1(t))$ can be approximated through FLS (2) as

$$\begin{aligned} \Delta g_1(x_1(t)) &= \boldsymbol{\theta}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) \\ &= \boldsymbol{\theta}_1^{*T}\boldsymbol{\varphi}_1(x_1(t)) + \varepsilon_1(t) \end{aligned} \tag{7}$$

where $\boldsymbol{\theta}_1^{*T}$ represents the optimal fuzzy parameter and $\varepsilon_1(t)$ is the optimal approximation error. Then, the virtual input can be designed as

$$z_2(t) = -\frac{1}{\alpha}\left[k_1 e_1(t) + \boldsymbol{\theta}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) + \hat{\varepsilon}_1(t)\arctan\left(\frac{\tilde{e}_1(t)}{\lambda_1}\right) - \dot{x}_1^c(t)\right] \quad (8)$$

where $\hat{\varepsilon}_1(t)$ is the estimation of the upper bound of the fuzzy approximation error $\varepsilon_1(t)$, k_1 and λ_1 are two positive design parameters, and $\tilde{e}_1(t)$ is the compensated tracking error that will be defined later. Then, it follows from (6), (7), and (8) that

$$\begin{aligned}\dot{e}_1(t) &= \alpha x_2(t) + \Delta g_1(x_1(t)) - \dot{x}_1^c(t)\\ &= \alpha(x_2(t) - z_2(t)) + \Delta g_1(x_1(t)) - \dot{x}_1^c(t)\\ &\quad + \alpha z_2(t)\\ &= -k_1 e_1(t) + \alpha(x_2(t) - z_2(t)) + \boldsymbol{\theta}_1^{*T}\boldsymbol{\varphi}_1(x_1(t))\\ &\quad + \varepsilon_1(t) - \boldsymbol{\theta}_1^T(t)\boldsymbol{\varphi}_1(x_1(t))\\ &\quad - \hat{\varepsilon}_1(t)\arctan\left(\frac{\tilde{e}_1(t)}{\lambda_1}\right)\\ &= -k_1 e_1(t) + \alpha(x_2^c(t) - z_2(t))\\ &\quad - \tilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) + \varepsilon_1(t)\\ &\quad - \hat{\varepsilon}_1(t)\arctan\left(\frac{\tilde{e}_1(t)}{\lambda_1}\right) + \alpha e_2(t)\end{aligned} \quad (9)$$

where $\tilde{\boldsymbol{\theta}}_1^T(t) = \boldsymbol{\theta}_1^T(t) - \boldsymbol{\theta}_1^{*T}$ is the fuzzy parameter estimation error, $e_2(t) = x_2(t) - x_2^c(t)$ is the command filtered tracking error, and x_2^c will be given later. The compensated tracking error signal can be defined as

$$\tilde{e}_1(t) = e_1(t) - \zeta_1(t) \quad (10)$$

where $\zeta_1(t)$ is an added term which is the solution of the following filter:

$$\dot{\zeta}_1(t) = -k_1\zeta_1(t) + \alpha(x_2^c(t) - z_2(t)) + \alpha\zeta_2(t) \quad (11)$$

where $\zeta_2(t)$ will be given in Step 2 and $x_2^c(t)$ is obtained by the filter

$$\dot{x}_2^c(t) = -\varpi_2(x_2^c(t) - z_2(t)) \quad (12)$$

with $\varpi_2 > 0$ being a design parameter. The initial condition for $x_2^c(t)$ is $x_2^c(0) = 0$. Thus, adaptation laws for $\boldsymbol{\theta}_1(t)$ and $\hat{\varepsilon}_1(t)$ are designed as

$$\dot{\boldsymbol{\theta}}_1(t) = c_{11}\tilde{e}_1(t)\boldsymbol{\varphi}_1(x_1(t)) - c_{11}c_{12}\boldsymbol{\theta}_1(t) \quad (13)$$

and

$$\dot{\hat{\varepsilon}}_1(t) = c_{41}\tilde{e}_1(t)\tanh\left(\frac{\tilde{e}_1(t)}{\lambda_1}\right) - c_{41}c_{42}\hat{\varepsilon}_1(t), \quad (14)$$

respectively, where c_{11}, c_{12}, c_{41}, and c_{42} are all positive design parameters.

Step 2. It follows from (4) that

$$\dot{x}_2(t) = x_3(t) + \Delta g_2(\bar{\mathbf{x}}_2(t)) \quad (15)$$

where $\Delta g_2(\bar{\mathbf{x}}_2(t)) = x_1(t) - x_2(t) + g_2(\bar{\mathbf{x}}_2(t))$ is an unknown nonlinear function, which can be approximated through FLS (2) by

$$\begin{aligned}\Delta g_2(\bar{\mathbf{x}}_2(t)) &= \boldsymbol{\theta}_2^T(t)\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t))\\ &= \boldsymbol{\theta}_2^{*T}\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t)) + \varepsilon_2(t).\end{aligned} \quad (16)$$

The virtual tracking errors, similar to that in Step 1, are defined as

$$\begin{aligned}e_2(t) &= x_2(t) - x_2^c(t),\\ \tilde{e}_2(t) &= e_2(t) - \zeta_2(t),\end{aligned} \quad (17)$$

with

$$\dot{\zeta}_2(t) = -k_2\zeta_2(t) + x_3^c(t) - z_3(t) + \zeta_3(t) \quad (18)$$

where $\zeta_2(0) = 0$, $\zeta_3(t)$ will be given in Step 3,

$$\dot{x}_3^c(t) = -\varpi_3(x_3^c(t) - z_3(t)), \quad (19)$$

$z_3(t)$ is the virtual control input designed as

$$\begin{aligned}z_3(t) &= -k_2 e_2(t) - \boldsymbol{\theta}_2^T(t)\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t))\\ &\quad - \hat{\varepsilon}_2(t)\arctan\left(\frac{\tilde{e}_2(t)}{\lambda_2}\right) + \dot{x}_2^c(t) - \alpha\tilde{e}_1(t)\end{aligned} \quad (20)$$

with k_2 and λ_2 being positive design parameters. Adaptation laws for $\boldsymbol{\theta}_2(t)$ and $\hat{\varepsilon}_2(t)$ can be given as

$$\dot{\boldsymbol{\theta}}_2(t) = c_{21}\tilde{e}_2(t)\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t)) - c_{21}c_{22}\boldsymbol{\theta}_2(t) \quad (21)$$

and

$$\dot{\hat{\varepsilon}}_2(t) = c_{51}\tilde{e}_2(t)\tanh\left(\frac{\tilde{e}_2(t)}{\lambda_1}\right) - c_{51}c_{52}\hat{\varepsilon}_2(t) \quad (22)$$

where c_{21}, c_{22}, c_{51}, and c_{52} are all positive design parameters. Based on above discussion, we have

$$\begin{aligned}\dot{e}_2(t) &= x_3(t) + \Delta g_2(\bar{\mathbf{x}}_2(t)) - \dot{x}_2^c(t)\\ &= x_3(t) - z_3(t) + \Delta g_3(\bar{\mathbf{x}}_2(t)) - \dot{x}_2^c(t) + z_3(t)\\ &= -k_2 e_2(t) + x_3(t) - z_3(t) + \boldsymbol{\theta}_2^{*T}\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t))\\ &\quad + \varepsilon_2(t) - \boldsymbol{\theta}_2^T(t)\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t))\\ &\quad - \hat{\varepsilon}_2(t)\arctan\left(\frac{\tilde{e}_2(t)}{\lambda_2}\right) - \alpha\tilde{e}_1(t)\\ &= -k_2 e_2(t) + x_3^c(t) - z_3(t) - \tilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\bar{\mathbf{x}}_2(t))\\ &\quad + \varepsilon_2(t) - \hat{\varepsilon}_2(t)\arctan\left(\frac{\tilde{e}_2(t)}{\lambda_2}\right) - \alpha\tilde{e}_1(t)\\ &\quad + e_3(t).\end{aligned} \quad (23)$$

Step 3. According to (4), we have

$$\dot{x}_3(t) = \Delta g_3(\mathbf{x}(t)) + d(t) + u(t) \tag{24}$$

with $\Delta g_3(\mathbf{x}(t)) = \beta x_2(t) - \gamma x_3(t) + g_3(\mathbf{x}(t))$ being an unknown nonlinear function that can be approximated by

$$\Delta g_3(\overline{\mathbf{x}}(t)) = \boldsymbol{\theta}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) = \boldsymbol{\theta}_3^{*T}\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) + \varepsilon_3(t). \tag{25}$$

The control input is designed as

$$\begin{aligned} u(t) = {} & -k_3 e_3(t) - \boldsymbol{\theta}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) \\ & - \left(\widehat{\varepsilon}_2(t) + \widehat{d}(t)\right)\arctan\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) + \dot{x}_3^c(t) \\ & - e_2(t) \end{aligned} \tag{26}$$

where k_3, λ_3 are two positive design parameters and $\widehat{d}(t)$ is the estimation of external disturbance $d(t)$. The compensated tracking errors are defined by

$$\begin{aligned} e_3(t) &= x_3(t) - x_3^c(t), \\ \widetilde{e}_3(t) &= e_3(t) - \zeta_3(t), \end{aligned} \tag{27}$$

where

$$\dot{\zeta}_3(t) = -k_3\zeta_3(t) - \zeta_2(t). \tag{28}$$

We design the following adaptation laws:

$$\dot{\boldsymbol{\theta}}_3(t) = c_{31}\widetilde{e}_3(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}(t)) - c_{31}c_{32}\boldsymbol{\theta}_3(t), \tag{29}$$

$$\dot{\widehat{\varepsilon}}_3(t) = c_{61}\widetilde{e}_3(t)\tanh\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) - c_{61}c_{62}\widehat{\varepsilon}_3(t) \tag{30}$$

and

$$\dot{\widehat{d}}(t) = c_{71}\widetilde{e}_3(t)\tanh\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) - c_{71}c_{72}\widehat{d}(t) \tag{31}$$

where $c_{31}, c_{32}, c_{61}, c_{62}, c_{71}$, and c_{72} are all positive design parameters. Thus,

$$\begin{aligned} \dot{e}_3(t) &= \Delta g_3(\overline{\mathbf{x}}(t)) - \dot{x}_3^c(t) + d(t) + u(t) \\ &= -k_3 e_3(t) + \boldsymbol{\theta}_3^{*T}\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) + \varepsilon_3(t) + d(t) \\ &\quad - \widehat{d}(t)\arctan\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) - \boldsymbol{\theta}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) \\ &\quad - \widehat{\varepsilon}_3(t)\arctan\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) - e_2(t) \\ &= -k_3 e_3(t) - \widetilde{\boldsymbol{\theta}}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) + \varepsilon_3(t) + d(t) \\ &\quad - \widehat{d}(t)\arctan\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) \\ &\quad - \widehat{\varepsilon}_3(t)\arctan\left(\frac{\widetilde{e}_3(t)}{\lambda_3}\right) - e_2(t). \end{aligned} \tag{32}$$

Here $\widetilde{d}(t) = \widehat{d}(t) - d(t)$ is the estimation error of $d(t)$.

To proceed, we need the following assumption and lemma.

Assumption 2. The external disturbance and the fuzzy approximate errors are bounded; i.e., there exist some positive constants d^* and ε_i^* such that $|d(t)| \le d^*$ and $\varepsilon_i(t) \le \varepsilon_i^*$.

Lemma 3 (see [49]). *Suppose that $\lambda > 0$; then it holds that*

$$|y| - y\tanh\left(\frac{y}{\lambda}\right) \le \kappa\lambda \tag{33}$$

where $\kappa = 0.2785$ is a constant.

3.3. Stability Analysis

3.3. Stability Analysis. According to the above analysis, the dynamical equations for the tracking errors are obtained as

$$\begin{aligned} \dot{\widetilde{e}}_1(t) &= \dot{e}_1(t) - \dot{\zeta}_1(t) \\ &= -k_1 e_1(t) + \alpha\left(x_2^c(t) - z_2(t)\right) \\ &\quad - \widetilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) + \varepsilon_1(t) \\ &\quad - \widehat{\varepsilon}_1(t)\arctan\left(\frac{\widetilde{e}_1(t)}{\lambda_1}\right) + \alpha e_2(t) - \dot{\zeta}_1(t) \\ &= -k_1 e_1(t) - \widetilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) + \varepsilon_1(t) \\ &\quad - k_1\zeta_1(t) - \widehat{\varepsilon}_1(t)\arctan\left(\frac{\widetilde{e}_1(t)}{\lambda_1}\right) + \alpha e_2(t) \\ &\quad - \alpha\zeta_2(t) \\ &= -k_1\widetilde{e}_1(t) + \alpha\widetilde{e}_2(t) - \widetilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) + \varepsilon_1(t) \\ &\quad - \widehat{\varepsilon}_1(t)\arctan\left(\frac{\widetilde{e}_1(t)}{\lambda_1}\right), \end{aligned} \tag{34}$$

$$\begin{aligned} \dot{\widetilde{e}}_2(t) &= \dot{e}_2(t) - \dot{\zeta}_2(t) \\ &= -k_2 e_2(t) + x_3^c(t) - z_3(t) - \widetilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) \\ &\quad + \varepsilon_2(t) - \widehat{\varepsilon}_2(t)\arctan\left(\frac{\widetilde{e}_2(t)}{\lambda_2}\right) - \alpha\widetilde{e}_1(t) \\ &\quad + e_3(t) - \dot{\zeta}_2(t) \\ &= -k_2 e_2(t) - \widetilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) + \varepsilon_2(t) - \zeta_3(t) \\ &\quad - \widehat{\varepsilon}_2(t)\arctan\left(\frac{\widetilde{e}_2(t)}{\lambda_2}\right) - \alpha\widetilde{e}_1(t) + e_3(t) \\ &\quad - k_2\zeta_2(t) \\ &= -k_2\widetilde{e}_2(t) - \widetilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) + \varepsilon_2(t) + \widetilde{e}_3(t) \\ &\quad - \widehat{\varepsilon}_2(t)\arctan\left(\frac{\widetilde{e}_2(t)}{\lambda_2}\right) - \alpha\widetilde{e}_1(t), \end{aligned} \tag{35}$$

$$\begin{aligned}
\dot{\tilde{e}}_3(t) &= \dot{e}_3(t) - \dot{\zeta}_3(t) \\
&= -k_3 e_3(t) - \tilde{\boldsymbol{\theta}}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) + \varepsilon_3(t) \\
&\quad - \hat{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right) - \dot{\zeta}_3(t) - e_2(t) \\
&\quad - \hat{\varepsilon}_3(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right) + d(t) \\
&= -k_3\tilde{e}_3(t) - \tilde{\boldsymbol{\theta}}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) + \varepsilon_3(t) + d(t) \\
&\quad - \hat{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right) - \tilde{e}_2(t) \\
&\quad - \hat{\varepsilon}_3(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right).
\end{aligned} \tag{36}$$

Remark 4. In this paper, to cancel the effect of the signals $x_i^c(t) - z_i(t)$, $i = 1,2,3$, three filters, i.e., (11), (18), and (28) have been introduced. It should be pointed out that the proposed filters can guarantee the boundedness of the added signals $\zeta_1(t), \zeta_2(t), \zeta_3(t)$. The stability analysis for these signals is presented in Theorem 5.

Theorem 5. *If the input signals $x_i^c(t) - z_i(t)$, $i = 1,2,3$, satisfy $|x_i^c(t) - z_i(t)| \le B$ where B is a positive constant, then the filters defined as (11), (18), and (28) have state variables bounded by*

$$\|\boldsymbol{\zeta}(t)\| \le \frac{A}{2\underline{k}}\left(1 - e^{-2\underline{k}t}\right) \tag{37}$$

where $\boldsymbol{\zeta}(t) = [\zeta_1(t), \zeta_2(t), \zeta_3(t)]^T \in \mathscr{R}^3$, $\underline{k} = (1/2)\min\{k_1, k_2, k_3\}$, and $A = B + \alpha$.

Proof. The Lyapunov function candidate is chosen as $V_1(t) = (1/2)\|\boldsymbol{\zeta}(t)\|^2$. According to (11), (18), and (28), the derivative of $V_1(t)$ with respect to time can be given as

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{3}\zeta_i(t)\dot{\zeta}_i(t) \\
&= -\sum_{i=1}^{3}k_i\zeta_i^2(t) + \alpha\zeta_1(t)\left(x_2^c(t) - z_2(t)\right) \\
&\quad + \zeta_2(t)\left(x_3^c(t) - z_3(t)\right) \\
&\le -2\underline{k}\|\boldsymbol{\zeta}(t)\|^2 + A\|\boldsymbol{\zeta}(t)\| \\
&\le -4\underline{k}V_1(t) + \sqrt{2}A\sqrt{V_1(t)}.
\end{aligned} \tag{38}$$

Thus, (38) implies that (37) holds. □

The main results of this section are included in the following theorem.

Theorem 6. *Consider system (4) under Assumption 2. The virtual control inputs are chosen as (8) and (20). The filters are given as (11), (12), (18), (19), and (28). The fuzzy parameters are updated by (13), (21), and (29). The estimation of the fuzzy approximation errors is updated by (14), (22), and (30). The estimation of $d(t)$ is given as (31). Then, the control input (26) guarantees that the tracking errors $\tilde{e}_1(t)$, $\tilde{e}_2(t)$, and $\tilde{e}_3(t)$ converge to a small region of zero if proper design parameters are chosen.*

Proof. Let the Lyapunov function candidate be

$$\begin{aligned}
V(t) &= \frac{1}{2}\sum_{i=1}^{3}\tilde{e}_i^2(t) + \sum_{i=1}^{3}\frac{1}{2c_{i1}}\tilde{\boldsymbol{\theta}}_i^T(t)\tilde{\boldsymbol{\theta}}_i(t) \\
&\quad + \sum_{i=1}^{3}\frac{1}{2c_{i+3,1}}\tilde{\varepsilon}_i^2(t) + \frac{1}{2c_{71}}\tilde{d}^2(t)
\end{aligned} \tag{39}$$

where $\tilde{\varepsilon}_i(t) = \hat{\varepsilon}_i(t) - \varepsilon_i^*$, $\tilde{d}(t) = \hat{d}(t) - d^*$ are estimation errors. It follows from (34), (35), (36), Assumption 2, and Lemma 3 that

$$\begin{aligned}
\sum_{i=1}^{3}\tilde{e}_i(t)\dot{\tilde{e}}_i(t) &= \sum_{i=1}^{3}\tilde{e}_i(t)\left[\varepsilon_i(t) - \hat{\varepsilon}_i(t)\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\right] \\
&\quad - \tilde{e}_1(t)\tilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) - \tilde{e}_2(t)\tilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) \\
&\quad - \tilde{e}_3(t)\tilde{\boldsymbol{\theta}}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) - \sum_{i=1}^{3}k_i\tilde{e}_i^2(t) + \tilde{e}_3(t)\left[d(t)\right. \\
&\quad \left. - \hat{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)\right] \le \sum_{i=1}^{3}\left[\left|\tilde{e}_i(t)\right|\varepsilon_i^*\right. \\
&\quad \left. - \tilde{e}_i(t)\hat{\varepsilon}_i(t)\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\right] - \tilde{e}_1(t)\tilde{\boldsymbol{\theta}}_1^T(t) \\
&\quad \cdot\boldsymbol{\varphi}_1(x_1(t)) - \tilde{e}_2(t)\tilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) - \tilde{e}_3(t)\tilde{\boldsymbol{\theta}}_3^T(t) \\
&\quad \cdot\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) - \sum_{i=1}^{3}k_i\tilde{e}_i^2(t) + \left|\tilde{e}_3(t)\right|d^* - \tilde{e}_3(t)\hat{d}(t) \\
&\quad \cdot\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right) = \sum_{i=1}^{3}\left[\left|\tilde{e}_i(t)\right|\varepsilon_i^*\right. \\
&\quad - \tilde{e}_i(t)\hat{\varepsilon}_i(t)\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right) \\
&\quad \left. - \tilde{e}_i(t)\varepsilon_i^*\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\right] - \tilde{e}_2(t)\tilde{\boldsymbol{\theta}}_2^T(t)\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t)) \\
&\quad - \tilde{e}_3(t)\tilde{\boldsymbol{\theta}}_3^T(t)\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t)) - \tilde{e}_1(t)\tilde{\boldsymbol{\theta}}_1^T(t)\boldsymbol{\varphi}_1(x_1(t)) \\
&\quad + \left|\tilde{e}_3(t)\right|d^* - \tilde{e}_3(t)\hat{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right) \\
&\quad + \sum_{i=1}^{3}\tilde{e}_i(t)\varepsilon_i^*\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right) - \sum_{i=1}^{3}k_i\tilde{e}_i^2(t) \\
&\quad - \tilde{e}_3(t)d^*\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)
\end{aligned}$$

$$
\begin{aligned}
&+\tilde{e}_3(t)\,d^*\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)\\
&\le\sum_{i=1}^{3}\left[-\tilde{e}_i(t)\,\hat{\varepsilon}_i(t)\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\right.\\
&\left.+\tilde{e}_i(t)\,\varepsilon_i^*\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\right]-\sum_{i=1}^{3}k_i\tilde{e}_i^2(t)-\tilde{e}_2(t)\\
&\cdot\tilde{\boldsymbol{\theta}}_2^T(t)\,\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t))-\tilde{e}_3(t)\,\tilde{\boldsymbol{\theta}}_3^T(t)\,\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t))-\tilde{e}_1(t)\\
&\cdot\tilde{\boldsymbol{\theta}}_1^T(t)\,\boldsymbol{\varphi}_1(x_1(t))-\tilde{e}_3(t)\,\hat{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)\\
&+\kappa\sum_{i=1}^{3}\lambda_i\varepsilon_i^*+\kappa\lambda_3 d^*+\tilde{e}_3(t)\,d^*\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)\\
&=-\sum_{i=1}^{3}\tilde{e}_i(t)\,\tilde{\varepsilon}_i(t)\arctan\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)-\sum_{i=1}^{3}k_i\tilde{e}_i^2(t)\\
&-\tilde{e}_3(t)\,\tilde{d}(t)\arctan\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)-\tilde{e}_2(t)\,\tilde{\boldsymbol{\theta}}_2^T(t)\\
&\cdot\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t))-\tilde{e}_3(t)\,\tilde{\boldsymbol{\theta}}_3^T(t)\,\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t))-\tilde{e}_1(t)\,\tilde{\boldsymbol{\theta}}_1^T(t)\\
&\cdot\boldsymbol{\varphi}_1(x_1(t))+\kappa\lambda_3 d^*+\kappa\sum_{i=1}^{3}\lambda_i\varepsilon_i^*.
\end{aligned}
\tag{40}
$$

By using the adaptations laws (13), (14), (21), (22), (29), (30), and (31), we have

$$
\begin{aligned}
&\sum_{i=1}^{3}\frac{1}{c_{i1}}\tilde{\boldsymbol{\theta}}_i^T(t)\,\dot{\tilde{\boldsymbol{\theta}}}_i(t)=\sum_{i=1}^{3}\frac{1}{c_{i1}}\tilde{\boldsymbol{\theta}}_i^T(t)\,\dot{\boldsymbol{\theta}}_i(t)\\
&=\tilde{e}_2(t)\,\tilde{\boldsymbol{\theta}}_2^T(t)\,\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t))+\tilde{e}_3(t)\,\tilde{\boldsymbol{\theta}}_3^T(t)\,\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t))\\
&\quad+\tilde{e}_1(t)\,\tilde{\boldsymbol{\theta}}_1^T(t)\,\boldsymbol{\varphi}_1(x_1(t))-\sum_{i=1}^{3}c_{i2}\tilde{\boldsymbol{\theta}}_i^T(t)\,\boldsymbol{\theta}_i(t)\\
&=\tilde{e}_2(t)\,\tilde{\boldsymbol{\theta}}_2^T(t)\,\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t))+\tilde{e}_3(t)\,\tilde{\boldsymbol{\theta}}_3^T(t)\,\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t))\\
&\quad+\tilde{e}_1(t)\,\tilde{\boldsymbol{\theta}}_1^T(t)\,\boldsymbol{\varphi}_1(x_1(t))\\
&\quad-\sum_{i=1}^{3}c_{i2}\tilde{\boldsymbol{\theta}}_i^T(t)\left(\tilde{\boldsymbol{\theta}}_i(t)+\boldsymbol{\theta}_i^*\right)\\
&\le\tilde{e}_2(t)\,\tilde{\boldsymbol{\theta}}_2^T(t)\,\boldsymbol{\varphi}_2(\overline{\mathbf{x}}_2(t))+\tilde{e}_3(t)\,\tilde{\boldsymbol{\theta}}_3^T(t)\,\boldsymbol{\varphi}_3(\overline{\mathbf{x}}(t))\\
&\quad+\tilde{e}_1(t)\,\tilde{\boldsymbol{\theta}}_1^T(t)\,\boldsymbol{\varphi}_1(x_1(t))-\sum_{i=1}^{3}\frac{c_{i2}}{2}\tilde{\boldsymbol{\theta}}_i^T(t)\,\tilde{\boldsymbol{\theta}}_i(t)\\
&\quad+\sum_{i=1}^{3}\frac{c_{i2}}{2}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^*,
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&\sum_{i=1}^{3}\frac{1}{2c_{i+3,1}}\tilde{\varepsilon}_i(t)\,\dot{\tilde{\varepsilon}}_i(t)=\sum_{i=1}^{3}\frac{1}{2c_{i+3,1}}\tilde{\varepsilon}_i(t)\,\dot{\hat{\varepsilon}}_i(t)\\
&\quad=\sum_{i=1}^{3}\tilde{\varepsilon}_i(t)\left[\tilde{e}_i(t)\tanh\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)-c_{i+3,2}\hat{\varepsilon}_i(t)\right]\\
&\quad=\sum_{i=1}^{3}\tilde{\varepsilon}_i(t)\,\tilde{e}_i(t)\tanh\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)\\
&\qquad-\sum_{i=1}^{3}c_{i+3,2}\tilde{\varepsilon}_i(t)\left(\tilde{\varepsilon}_i(t)+\varepsilon_i^*\right)\\
&\quad\le\sum_{i=1}^{3}\tilde{\varepsilon}_i(t)\,\tilde{e}_i(t)\tanh\left(\frac{\tilde{e}_i(t)}{\lambda_i}\right)-\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\tilde{\varepsilon}_i^2(t)\\
&\qquad+\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\varepsilon_i^{*2},
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&\frac{1}{c_{71}}\tilde{d}(t)\,\dot{\hat{d}}(t)\\
&\quad=\tilde{d}(t)\,\tilde{e}_3(t)\tanh\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)-c_{72}\tilde{d}(t)\left(\tilde{d}(t)+d^*\right)\\
&\quad\le\tilde{d}(t)\,\tilde{e}_3(t)\tanh\left(\frac{\tilde{e}_3(t)}{\lambda_3}\right)-\frac{c_{72}}{2}\tilde{d}^2(t)+\frac{c_{72}}{2}d^{*2}.
\end{aligned}
\tag{43}
$$

According to (40), (41), (42), and (43), the derivative of Lyapunov function (39) can be obtained as

$$
\begin{aligned}
\dot{V}(t)&\le-\sum_{i=1}^{3}k_i\tilde{e}_i^2(t)+\kappa\lambda_3 d^*+\kappa\sum_{i=1}^{3}\lambda_i\varepsilon_i^*\\
&\quad-\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\tilde{\varepsilon}_i^2(t)+\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\varepsilon_i^{*2}\\
&\quad-\sum_{i=1}^{3}\frac{c_{i2}}{2}\tilde{\boldsymbol{\theta}}_i^T(t)\,\tilde{\boldsymbol{\theta}}_i(t)+\sum_{i=1}^{3}\frac{c_{i2}}{2}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^*-\frac{c_{72}}{2}\tilde{d}^2(t)\\
&\quad+\frac{c_{72}}{2}d^{*2}\\
&=-\sum_{i=1}^{3}k_i\tilde{e}_i^2(t)-\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\tilde{\varepsilon}_i^2(t)\\
&\quad-\sum_{i=1}^{3}\frac{c_{i2}}{2}\tilde{\boldsymbol{\theta}}_i^T(t)\,\tilde{\boldsymbol{\theta}}_i(t)-\frac{c_{72}}{2}\tilde{d}^2(t)+\kappa\lambda_3 d^*\\
&\quad+\kappa\sum_{i=1}^{3}\lambda_i\varepsilon_i^*+\sum_{i=1}^{3}\frac{c_{i+3,2}}{2}\varepsilon_i^{*2}+\sum_{i=1}^{3}\frac{c_{i2}}{2}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^*\\
&\quad+\frac{c_{72}}{2}d^{*2}\\
&\le-\frac{C_1}{2}\sum_{i=1}^{3}\tilde{e}_i^2(t)-C_2\sum_{i=1}^{3}\frac{1}{2c_{i1}}\tilde{\boldsymbol{\theta}}_i^T(t)\,\tilde{\boldsymbol{\theta}}_i(t)\\
&\quad-C_3\sum_{i=1}^{3}\frac{1}{2c_{i+3,1}}\tilde{\varepsilon}_i^2(t)-\frac{C_4}{2c_{71}}\tilde{d}^2(t)+C_5
\end{aligned}
\tag{44}
$$

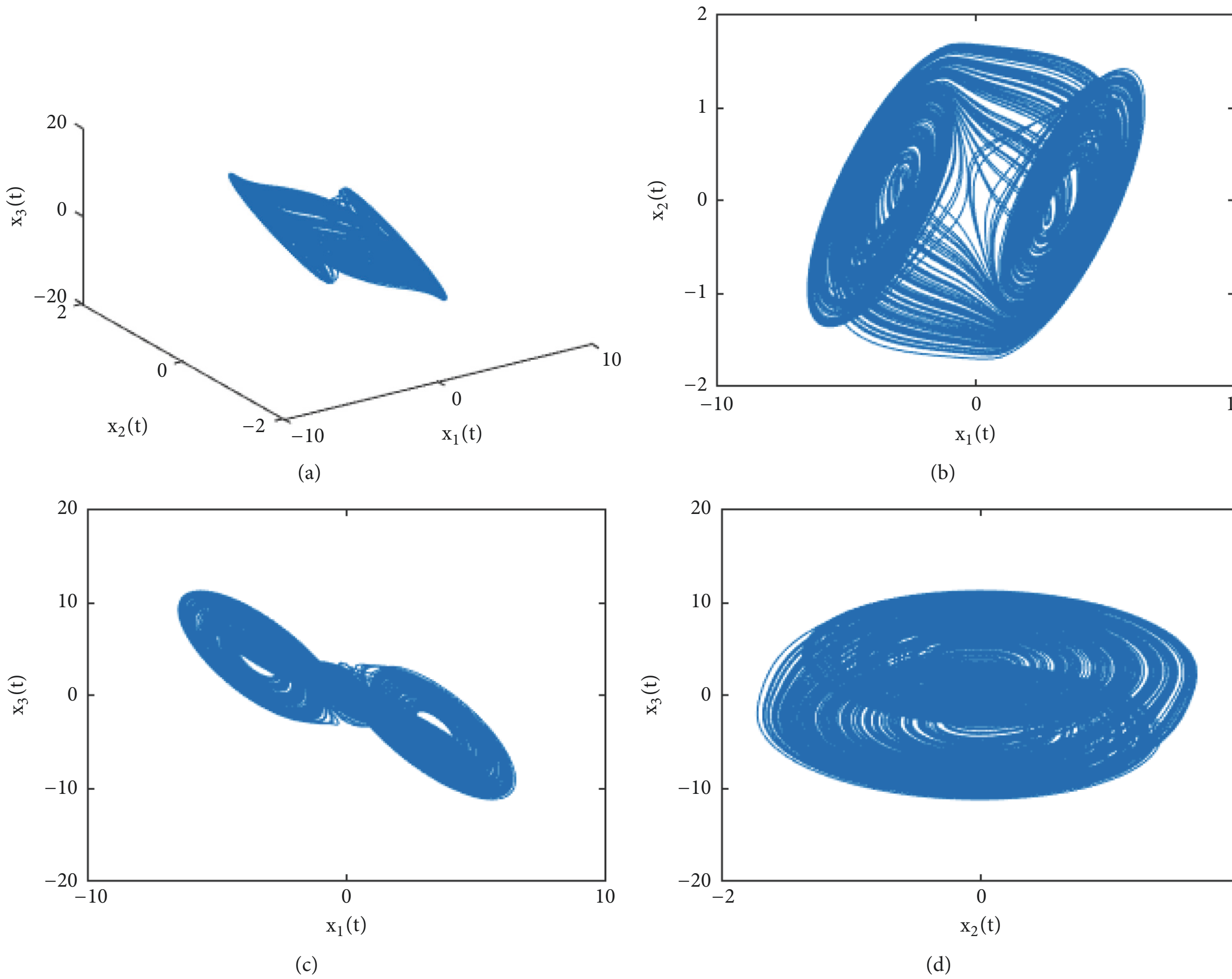

FIGURE 1: Dynamical behavior of system (4) in (a) $x_1 - x_2 - x_3$, (b) $x_1 - x_2$ plane, (c) $x_1 - x_3$ plane, and (d) $x_2 - x_3$ plane.

where $C_1 = 2\min\{k_1, k_2, k_3\}$, $C_2 = \min\{c_{11}c_{12}, c_{21}c_{22}, c_{31}c_{32}\}$, $C_3 = \min\{c_{41}c_{42}, c_{51}c_{52}, c_{61}c_{62}\}$, $C_4 = c_{71}c_{72}$, and $C_5 = \kappa\lambda_3 d^* + \kappa\sum_{i=1}^{3}\lambda_i\varepsilon_i^* + \sum_{i=1}^{3}(c_{i+3,2}/2)\varepsilon_i^{*2} + \sum_{i=1}^{3}(c_{i2}/2)\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* + c_{72}/2$ are positive constants.

According to (44) one knows that when $t \longrightarrow \infty$, $(1/2)\sum_{i=1}^{3}\tilde{e}_i^2(t) \le C_5/C_1$, $\sum_{i=1}^{3}(1/2c_{i1})\tilde{\boldsymbol{\theta}}_i^T(t)\tilde{\boldsymbol{\theta}}_i(t) \le C_5/C_2$, $\sum_{i=1}^{3}(1/2c_{i+3,1})\tilde{\varepsilon}_i^2(t) \le C_5/C_3$, and $(1/2c_{71})\tilde{d}^2(t) \le C_5/C_4$. That is to say, all signals in the closed-loop system will keep bounded. The tracking errors $\tilde{e}_1(t)$, $\tilde{e}_2(t)$, and $\tilde{e}_3(t)$ will eventually converge to a small region of zero if proper design parameters are chosen (small C_5 and large C_1). □

4. Simulation Example

Let the system parameters be $\alpha = 11.25, \beta = 18.6, \gamma = 0, a = -0.68, b = -0.545$, and the initial condition be $\mathbf{x}(0) = [1.5, -1, 0.5]^T$. Then, the uncontrolled system (4) (i.e., $g_i(\cdot) \equiv d(t) \equiv u(t) \equiv 0, i = 1, 2, 3$) shows complicated behavior, which is depicted in Figure 1.

In simulation, the system uncertainties are chosen as $g_1(x_1(t)) = \sin(x_1(t))$, $g_2(\bar{\mathbf{x}}_2(t)) = \sin\sqrt{x_1^2(t) + x_2^2(t)}$, and $g_3(\mathbf{x}(t)) = \cos\sqrt{x_1^2(t) + x_2^2(t) + x_3^2(t)}$. The disturbance is selected as $d(t) = 0.2\sin t + 0.1\cos t$. The referenced signal $x_1^c(t)$ is defined by

$$x_1^c(t) = \begin{cases} 0 & t \in [0, 6], \\ 1.5 & t > 6. \end{cases} \tag{45}$$

There are three FLSs used. For the first FLS, the input variable is $x_1(t)$, and we define 5 Gaussian membership function distributed on interval [-5 5]. For the second one, the input variables are $x_1(t)$ and $x_2(t)$. For each input, we define 5 Gaussian membership functions distributed on interval [-5 5]. For the last one, the input variables are $x_1(t)$, $x_2(t)$, and $x_3(t)$. For inputs $x_1(t)$ and $x_2(t)$, we define 5 Gaussian membership functions distributed on interval [-5 5], and for input $x_3(t)$, we define 5 Gaussian membership functions distributed on interval [-10 10]. The initial conditions for FLSs are $\boldsymbol{\theta}_1(0) = \mathbf{0}_{5\times1}$, $\boldsymbol{\theta}_2(0) = \mathbf{0}_{25\times1}$, and $\boldsymbol{\theta}_3(0) = \mathbf{0}_{125\times1}$.

The simulation results are presented in Figures 2–5. It has been shown that in Figure 2, the state $x_1(t)$ tracks the referenced signal $x_1^c(t) = 0$ for $t \le 5$ in about 2.5 seconds and tends to $x_1^c(t) = 1.5$ for $t > 5$ in a short time. The

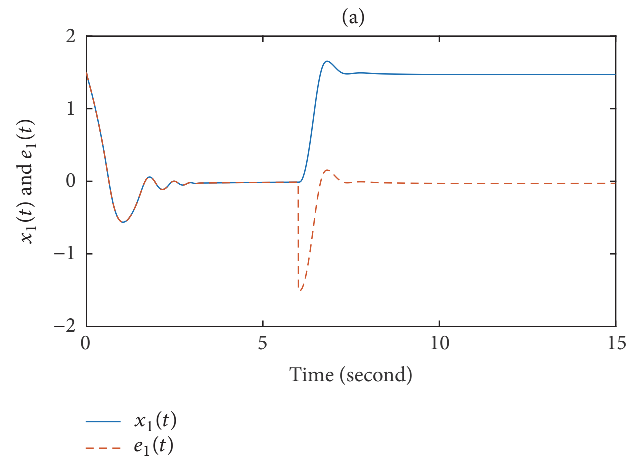

FIGURE 2: $x_1(t)$ and $e_1(t)$.

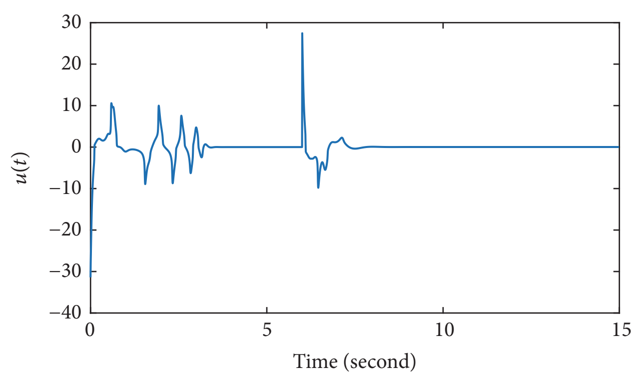

FIGURE 3: Control input $u(t)$.

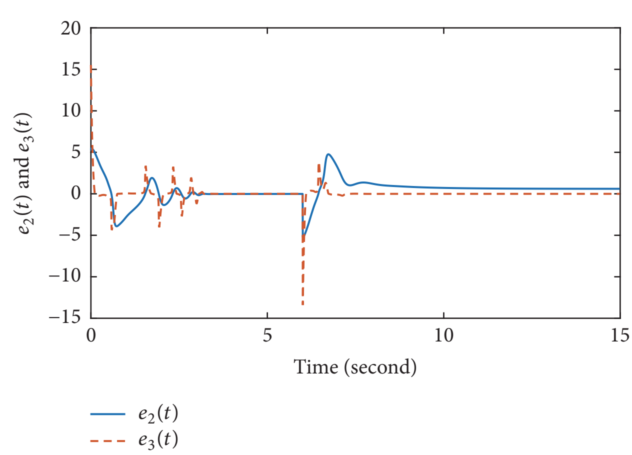

FIGURE 4: $e_2(t)$ and $e_3(t)$.

tracking error tends to zero rapidly. Figure 3 shows the time response of the control input $u(t)$. It should be pointed out that the proposed controller, the commonly used term sign$(\cdot)$, is not used in this paper. That is to say, our controller is smooth and bounded, just as indicated in Figure 3. Tracking errors $e_2(t)$ and $e_3(t)$ are presented in Figure 4. The fuzzy systems parameters are given in Figure 5. It is to know that

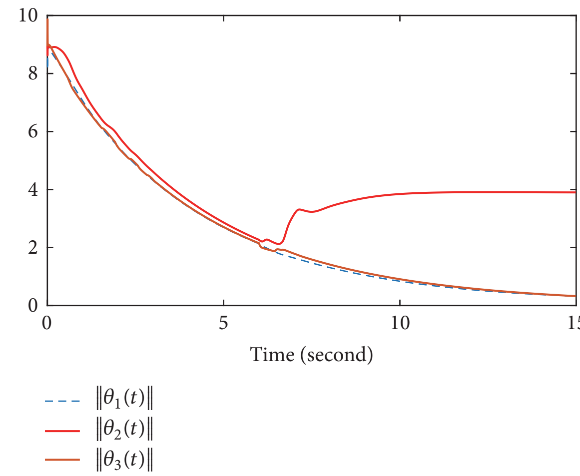

FIGURE 5: $\boldsymbol{\theta}_1(t)\|$, $\boldsymbol{\theta}_2(t)\|$, and $\boldsymbol{\theta}_3(t)\|$.

these simulation results are matched with Theorem 6, and the proposed method has good robustness.

5. Conclusions

In this paper, a command filtered AFBC method has been proposed for Chua's chaotic system with system uncertainties and external disturbances. It has been shown that the proposed method works well without the knowledge of any explicit uncertainty detection. One of the distinctive features of the proposed control approach consists in the fact that the problem of "explosion of complexity" in traditional backstepping design procedure is solved by the proposed first-order filter. In the stability analysis, Lyapunov stability criteria are used. The proposed command filtered AFBC can guarantee the convergence of tracking errors. Simulation results have verified our methods.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors do not have a direct financial relation with any commercial identity mentioned in their paper that might lead to conflicts of interest for any of the authors.

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