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Research Article

Optimization Problem of Insurance Investment Based on Spectral Risk Measure and RAROC Criterion

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This paper introduces spectral risk measure (SRM) into optimization problem of insurance investment. Spectral risk measure could describe the degree of risk aversion, so the underlying strategy might take the investor's risk attitude into account. We establish an optimization model aiming at maximizing risk-adjusted return of capital (RAROC) involved with spectral risk measure. The theoretical result is derived and empirical study is displayed under different risk measures and different confidence levels comparatively. The result shows that risk attitude has a significant impact on investment strategy. With the increase of risk aversion factor, the investment ratio of risk asset correspondingly reduces. When the aversive level increases to a certain extent, the impact on investment strategies disappears because of the marginal effect of risk aversion. In the case of VaR and CVaR without regard for risk aversion, the investment ratio of risk asset is increasing significantly.

1. Introduction

Underwriting business and investment business are two main fund sources of an insurance company. In recent years, more and more insurers have paid attention to the efficiency of investment business because of increasing competition among insurance companies, continuing decline in underwriting profits and gradual relaxation of insurance investment policies.

The relationship between return and risk needs to be fully balanced in insurance investment, in which mean-risk optimization is the most commonly used criterion. For the measurement of risk, variance is a common choice. Early studies, for example, Lambert and Hofflander [1], Kahane and Nye [2], and Briys [3], established optimal portfolio model for property insurance under mean-variance criterion. Later, due to the limitation of variance, new risk measures were proposed constantly and mean-risk models were also extended in various backgrounds; see [4–9]. In particular, ruin probability and some down-side risk measures such as VaR and CaR were introduced into insurance business to find the optimal investment strategy. Guo and Li [10] used mean-VaR model to analyze the choice of optimal portfolios for insurers. Chen et al. [11] investigated an investment-reinsurance problem under dynamic Value-at-Risk (VaR) constraint. Zeng et al. [12] established two mean-CaR models to study reinsurance-investment problem of insurers and obtained the explicit expressions of the optimal deterministic rebalance reinsurance-investment strategies and mean-CaR efficient frontiers.

Risk measures used in the above literatures indeed describe different risk characteristics of the assets, but they do not take investors' risk attitude into account. Spectral risk measures (SRM) proposed by Acerbi et al. [13] characterize investors' risk aversion and have been applied to fields of banks and securities, for example, Adam et al. [14] and Diao et al. [15] and the references therein. However, to the best of our knowledge, there is no literature which studied optimal investment problem in insurance business based on SRM. On the other hand, mean is generally used to describe the return, but the insurer needs to determine the amount of capital based on entire risk situation of company. The risk-adjusted return on capital (RAROC) takes into account the capital return adjusted by risk, which makes up for the shortcomings

from average-return principle; see [16–18] and the references therein.

This paper will introduce spectral risk measure into optimal investment model with RAROC as optimization target, construct optimization model, and give its theoretical and empirical study. The rest of this paper is organized as follows. Section 2 illustrates spectral risk measure and insurance return model used here. Section 3 finds the solution of the optimization problem. The empirical application is displayed in Section 4. Section 5 concludes the paper.

2. Spectral Risk Measure and Insurance Return Model

2.1. Spectral Risk Measure

Definition 1 (see [19]). Suppose that random variable X represents the loss of assets, and its distribution function can be denoted as $F(x) = Pr(X \le x)$. Spectral risk measure with confidence level $p = 1 - \alpha$ ($\alpha \epsilon (0,1)$) is defined as follows:

$$\rho = \int_0^1 \phi(p)\, q_p dp, \tag{1}$$

where $\phi(p) : (0,1) \longmapsto \mathscr{R}$ is a weight function or risk spectral function and $q_p = \inf\{x \mid F(x) \ge p\}$ is p-quantile of distribution function. SRM is a coherent risk measure when $\phi(p)$ satisfies nonnegativity, normalization, and increasingness.

Specially, ρ is Value at Risk (VaR) if $\phi(p) = \{0, p \ne 1-\alpha;\ \infty, p = 1-\alpha\}$. ρ corresponds to Conditional Value at Risk (CVaR) if $\phi(p) = (1/\alpha) I_{\{p \ge 1-\alpha\}}$. If $\phi(p) = (\gamma e^{-(1-p)\gamma/\alpha}/\alpha(1-e^{-\gamma})) I_{\{1-\alpha \le p \le 1\}}$, ρ is exponential spectral risk measure; if $\phi(p) = \{(\beta(\alpha - 1 + p)^{\beta-1}/\alpha^{\beta}) I_{\{1-\alpha \le p \le 1\}}, \beta > 1;\ (\beta(1-p)^{\beta-1}/\alpha^{\beta}) I_{\{1-\alpha \le p \le 1\}}, 0 < \beta < 1\}$, ρ is power spectral risk measure, where $\gamma > 0$ is the coefficient of absolute risk aversion and $\beta > 0$ is the coefficient of relative risk aversion.

Proposition 2 (see [20]). *Suppose that R denotes income variable, and then $X = -R$ denotes the loss variable. If R follows normal distribution assumption, we can get*

$$SRM(R) = -E(R) + T(\alpha)\,\sigma(R). \tag{2}$$

$$\text{Specially: } VaR(R) = -E(R) + \Phi^{-1}(p)\,\sigma(R) \tag{3}$$

$$\text{and } CVaR(R) = -E(R) + \frac{f(\Phi^{-1}(p))}{\alpha}\sigma(R), \tag{4}$$

where $T(\alpha) = \int_0^1 \Phi^{-1}(p)\phi(p)dp$, $\Phi^{-1}(p)$ is p-quantile of standard normal distribution and $f(.)$ is probability density function of standard normal distribution.

2.2. Insurance Return Model. Suppose that insurers invest in N assets, one of which is risk-free asset, and others are risk assets. Therefore, the total profit is given as

$$R_p = r_b R_0 + gR_0\left(1 - \sum_{i=1}^{N-1} k_i\right) r_0 + gR_0 \sum_{i=1}^{N-1} k_i r_i, \tag{5}$$

where R_0, r_b, g denote premium charged by insurers, the rate of underwriting profit, and investment ratio, respectively. Constant r_0 denotes the rate of risk-free asset return. And random variable r_i $(i = 1, 2, \cdots, N-1)$ denotes the rate of risk asset return with $N(\mu_i, \sigma_i^2)$ assumption. k_i is the investment weight of the i-th risk asset and we assume that $0 < \sum_{i=1}^{N-1} k_i < 1$.

Let $K = (R_0,\ gR_0k_1,\ gR_0k_2, \cdots,\ gR_0k_{N-1})^T$ and $r = (r_b + gr_0, r_1 - r_0, \cdots, r_{N-1} - r_0)^T$ with mean μ and covariance matrix Σ. Then we have $R = r^T K$, and $E(R) = \mu^T K$, $\sigma(R) = \sqrt{K^T \Sigma K}$. ρ_c is the upper limit of risk the insurer can bear, that is, $SRM(R_p) \le \rho_c$.

3. Optimal Investment Strategy for Insurers Based on SRM-RAROC Criterion

In this section, we establish SRM-RAROC optimization model and derive the optimal solution under normal distribution assumption.

3.1. Optimization Model. Here the investment performance evaluation is measured by risk-adjusted return on capital (RAROC) instead of the absolute amount of income as follows:

$$RAROC = \frac{E(R_p)}{SRM(R_p)}. \tag{6}$$

Thus, the optimization model can be formulated as

$$\begin{aligned} \max \quad & RAROC = \frac{E(R_p)}{SRM(R_p)} \\ \text{s.t.} \quad & 0 < \sum_{i=1}^{N-1} k_i < 1 \\ & SRM(R_p) = -E(R_p) + T(\alpha)\,\sigma(R_p) \le \rho_c \\ & E(R_p) = \mu^T K \\ & \sigma(R_p) = \sqrt{K^T \Sigma K}. \end{aligned} \tag{7}$$

3.2. Solution of Optimization Model

Step 1 (simplifying optimization model). Define θ as n-dimension vector, $\theta = (\theta_1, \theta_2, \cdots, \theta_n)^T$, where $\theta_1 = 1/(1 + g\sum_{J=1}^{N-1} k_j)\theta_i = gk_{i-1}/(1 + g\sum_{J=1}^{N-1} k_j)$, $i = 2, 3, \cdots, n$. And then K can be rewritten as $K = R_0(1 + g\sum_{J=1}^{N-1} k_j)\theta$, $I^T\theta = 1$, where I is n-dimension vector, $I = (1, 1, \cdots, 1)^T$.

Let $r_p = r^T\theta$, then $\mu_p = E(r_p) = \mu^T\theta$, $\sigma_p = var(r_p) = \sqrt{\theta^T\Sigma\theta}$. With $1 - \alpha$ confidence level, $SRM(r_p) = -\mu^T\theta + T(\alpha)\sqrt{\theta^T\Sigma\theta}$. Then we have

$$E\left(R_p\right) = \mu^T K = \mu^T R_0 \left(1 + g\sum_{J=1}^{N-1} k_j\right)\theta = R_0\left(1 + g\sum_{J=1}^{N-1} k_j\right) E\left(r_p\right) \tag{8}$$

$$\text{and } SRM\left(R_p\right) = -E\left(R_p\right) + T(\alpha)\,\sigma\left(R_p\right) = R_0\left(1 + g\sum_{J=1}^{N-1} k_j\right) SRM\left(r_p\right). \tag{9}$$

So RAROC can be rewritten as

$$RAROC = \frac{E\left(R_p\right)}{SRM\left(R_p\right)} = \frac{R_0\left(1 + g\sum_{J=1}^{N-1} k_j\right) E\left(r_p\right)}{R_0\left(1 + g\sum_{J=1}^{N-1} k_j\right) SRM\left(r_p\right)} = \frac{E\left(r_p\right)}{SRM\left(r_p\right)} \tag{10}$$

And model (7) can be transformed as

$$\begin{aligned} \max \quad & RAROC = \frac{E\left(r_p\right)}{SRM\left(r_p\right)} \\ \text{s.t.} \quad & I^T\theta = 1 \\ & SRM\left(r_p\right) = -\mu_p + T(\alpha)\,\sigma_p \le \rho_c \\ & \mu_p = E\left(r_p\right) = \mu^T\theta \\ & \sigma_p = \sigma\left(r_p\right) = \sqrt{\theta^T\Sigma\theta} \end{aligned} \tag{11}$$

Step 2 (effective frontier curve equation of mean-SRM space). Effective frontier in mean-risk space refers to the portfolio that maximizes the return at a certain level of risk or minimizes the risk at a certain level of return. Therefore, mathematical expression of curve equation of effective frontier can be given as

$$\begin{aligned} \min \quad & \left(-\mu^T\theta + T(\alpha)\sqrt{\theta^T\Sigma\theta}\right) \\ \text{s.t.} \quad & \mu_p = \mu^T\theta \\ & I^T\theta = 1. \end{aligned} \tag{12}$$

Solving model (12) by Lagrange multiplier method yields

$$\theta = \frac{1}{d}\Sigma^{-1}\left(\left(c\mu_p - b\right)\mu + \left(a - b\mu_p\right)I\right), \tag{13}$$

where $a = \mu^T\Sigma^{-1}\mu$, $b = \mu^T\Sigma^{-1}I = I^T\Sigma^{-1}\mu$, $c = I^T\Sigma^{-1}I$, $d = ac - b^2$.

So then, $\sigma_p{}^2 = \theta^T\Sigma\theta = (c\mu_p{}^2 - 2b\mu_p + a)/d$.
Therefore the effective frontier curve equation is

$$SRM\left(r_p\right) = -\mu_p + T(\alpha)\sqrt{\frac{1}{d}\left(c\mu_p{}^2 - 2b\mu_p + a\right)}. \tag{14}$$

Assume that μ_{opt} be the optimal return for a given risk ρ based on formula (14); the corresponding optimal portfolio weights on effective frontier curve can be solved as

$$\theta_{opt} = \frac{1}{d}\Sigma^{-1}\left(\left(c\mu_{opt} - b\right)\mu + \left(a - b\mu_{opt}\right)I\right). \tag{15}$$

Step 3 (RAROC maximized portfolio under SRM constraints). Let $RAROC = \mu_p/SRM(r_p) = u$; that is, the slope u of line $\mu_p = uSRM(r_p)$ will be maximized in the processing of optimization. From portfolio theory in finance, we know that maximum value is obtained when the line is tangent to the effective leading edge. The tangent point is the optimal portfolio when the tangent point is on the left of the constraint line while the intersection of the constraint line and the effective frontier is the optimal portfolio when the tangent point is on the right of the constraint line.

Let $(\mathrm{SRM_T}, \mu_\mathrm{T})$ denote the intersection portfolio. It is obvious that $\mathrm{SRM_T} = \rho_\mathrm{c}$ at the intersection point. From effective frontier curve equation, we can find that

$$\mu_\mathrm{T} = \frac{\left(\mathrm{bT}^2 + \mathrm{d}\rho_\mathrm{c}\right) + \mathrm{T}\sqrt{\mathrm{d}\left(2\mathrm{b}\rho_\mathrm{c} + \mathrm{c}\rho_\mathrm{c}{}^2 + \mathrm{a} - \mathrm{T}^2\right)}}{\mathrm{cT}^2 - \mathrm{d}}. \tag{16}$$

Let $(\mathrm{SRM_{tg}}, \mu_\mathrm{tg})$ denote tangent portfolio. The formula $\partial\mathrm{SRM}/\partial\mu_\mathrm{p} = 1/\mathrm{u}$ is true for tangent point when the line is tangent to the effective frontier. So it follows that

$$-1 + \frac{\mathrm{T}(\alpha)\left(\mathrm{c}\mu_\mathrm{p} - \mathrm{b}\right)}{\mathrm{d}\sqrt{(1/\mathrm{d})\left(\mathrm{c}\mu_\mathrm{p}{}^2 - 2\mathrm{b}\mu_\mathrm{p} + \mathrm{a}\right)}} = \frac{-\mu_\mathrm{p} + \mathrm{T}(\alpha)\sqrt{(1/\mathrm{d})\left(\mathrm{c}\mu_\mathrm{p}{}^2 - 2\mathrm{b}\mu_\mathrm{p} + \mathrm{a}\right)}}{\mu_\mathrm{p}}, \tag{17}$$

which results in the following tangent point portfolio:

$$\left(\mathrm{SRM_{tg}}, \mu_\mathrm{tg}\right) = \left(\frac{\sqrt{\mathrm{a}}}{\mathrm{b}}\left(\mathrm{T} - \sqrt{\mathrm{a}}\right), \frac{\mathrm{a}}{\mathrm{b}}\right). \tag{18}$$

Summarily, the optimal solution of optimization model can be expressed as

$$\left(\mathrm{SRM_{opt}}, \mu_\mathrm{opt}\right) = \begin{cases} \left(\mathrm{SRM_{tg}}, \mu_\mathrm{tg}\right) & \text{if } \rho_\mathrm{tg} \le \rho_\mathrm{c} \\ \left(\mathrm{SRM_T}, \mu_\mathrm{T}\right) & \text{if } \rho_\mathrm{tg} > \rho_\mathrm{c}, \end{cases} \tag{19}$$

and the optimal portfolio weight is

$$\theta_\mathrm{opt} = \frac{1}{\mathrm{d}}\Sigma^{-1}\left(\left(\mathrm{c}\mu_\mathrm{opt} - \mathrm{b}\right)\mu + \left(\mathrm{a} - \mathrm{b}\mu_\mathrm{opt}\right)\mathrm{I}\right). \tag{20}$$

Therefore, the optimal investment ratio of each risk asset is

$$\mathrm{k_{i-1}} = \frac{\theta_\mathrm{i}}{\mathrm{g}\theta_1} \quad \mathrm{i} = 2, 3, \cdots, \mathrm{N}, \tag{21}$$

Table 1: Descriptive statistical analysis of risk assets.

	Chang'an Vehicle (000625)	JoinTo Energy (000600)	Yili (600887)
Median	0.196729	0.005992	-0.070959
Maximum	1.341006	1.124433	1.299169
Minimum	-1.631535	-1.184555	-1.196948
Standard deviation	0.866075	0.632811	0.698381
Skewness	-0.545350	-0.132884	0.049686
Kurtosis	2.653994	2.993299	2.709674
J-B statistic	0.545562	0.034779	0.039235
p value	0.761260	0.982761	0.980574

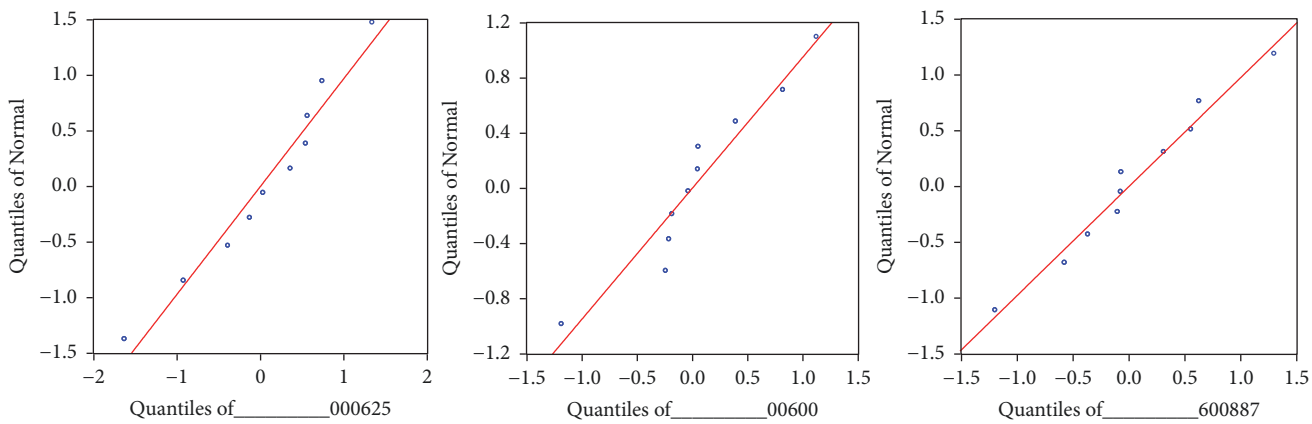

Figure 1: QQ chart of each risky asset's return.

and the corresponding proportion of investment in risk-free assets is

$$1 - \sum_{i=1}^{N-1} k_i = 1 - \frac{1-\theta_1}{g\theta_1} = \frac{g\theta_1 + \theta_1 - 1}{g\theta_1}. \quad (22)$$

4. Data Analysis

4.1. Data Selection. In this section, we assume that insurers invest in one risk-free asset and three security risk assets. Bank deposit is regarded as risk-free asset and the selected risk assets are JoinTo Energy (000600), Chang'an Vehicle (000625), and Yili (600887). Yearly data is chosen from January 1, 2006, to December 31, 2016. The data for China Life Insurance Company Ltd is calculated from the company's annual report and semi-annual report. Data of risk assets is obtained from Guotai An CSMAR series of research databases.

4.2. Descriptive Statistics of Risk Asset Data. We conduct descriptive statistical analysis of risk assets; see Table 1 and Figure 1.

It can be found from Table 1 that the distribution of return for three risk assets presents a certain degree of skewness and flatter peak than normal distribution. From QQ chart and the fact that JB statistic of each risky asset return rate is less than 5.99, which is 95% quantile of $\chi^2(2)$, we can conclude that the return of each risky asset is subject to a normal distribution approximately.

4.3. Calculation of Related Variables

(1) Investment Ratio and Rate of Underwriting Profit. The investment ratio is an important indicator to measure the level of capital utilization of an insurance company, which can be calculated by investment assets divided by total assets. The rate of underwriting profit can be calculated by the difference between total profit and investment profit divided by underwriting income. Based on investment data and underwriting data of China Life Insurance Company Ltd, we obtain that $g = 94.08\%$ and $r_b = -0.1776$ here.

(2) Rate of Return for Risk-Free Asset and Risky Asset. From China Life Insurance Company Ltd's data of the amount of bank deposit and bank deposit return in 2006–2016, we can calculate the mean of the return rate $E(r_0) = 0.0428$ as risk-free return rate. Each risky asset return is calculated by $r_i = \ln(P_{it}/P_{it-1})$, where P_{it} and P_{it-1} denote i-th asset's price at time t and t-1, respectively. So it can be calculated from Guotai An CSMAR Series Research Database that the average return of JoinTo Energy (000600), Chang'an Vehicle (000625), and Yili (600887) are 0.0580, 0.0516 and 0.0409, respectively.

4.4. Optimal Insurance Investment Strategy Based on SRM-RAROC Criterion. In this section, we conduct empirical analysis for optimal insurance investment strategy. The confidence level is set to 90% and 95% and the upper limit of risk is assumed to be 0.02. Based on formulas in Section 2, we calculate the optimal weights of the assets under different confidence levels. The results are displayed in Table 2.

Table 2: Optimal investment strategy under confidence level.

$\alpha = 0.05,\ \rho_c = 0.02$					
Risk measure	Factor of risk aversion	Risk-free asset	JoinTo Energy (000600)	Chang'an Vehicle (000625)	Yili (600887)
VaR	—	0.7136	0.2538	-0.0903	0.1229
CVaR	—	0.8062	0.1812	-0.0674	0.0799
Exponential SRM	$\gamma = 0.2$	0.8150	0.1743	-0.0652	0.0758
	$\gamma = 0.4$	0.8261	0.1657	-0.0625	0.0707
	$\gamma > 0.6$	0.8400	0.1548	-0.0590	0.0642
Power SRM	$\beta = 1.1$	0.8177	0.1723	-0.0646	0.0746
	$\beta = 1.2$	0.8342	0.1594	-0.0605	0.0669
	$\beta > 1.3$	0.8400	0.1548	-0.0590	0.0642
$\alpha = 0.1,\ \rho_c = 0.02$					
Risk measure	Factor of risk aversion	Risk-free asset	JoinTo Energy (000600)	Chang'an Vehicle (000625)	Yili (600887)
VaR	—	0.6357	0.3148	-0.1096	0.1591
CVaR	—	0.7350	0.2370	-0.0850	0.1130
Exponential SRM	$\gamma = 0.2$	0.7393	0.2336	-0.0840	0.1110
	$\gamma = 0.4$	0.7437	0.2302	-0.0829	0.1089
	$\gamma \geq 3.5$	0.8335	0.1599	-0.0606	0.0673
Power SRM	$\beta = 1.1$	0.7405	0.2327	-0.0837	0.1104
	$\beta = 1.2$	0.7457	0.2286	-0.0824	0.1080
	$\beta \geq 3$	0.8335	0.1599	-0.0606	0.0673

Remark: the negative value means short-selling.

From Table 2, we can obtain the following conclusion.

(1) Generally, the optimal investment proportion in risk asset is decreasing as the factor of risk aversion increases, which reflects the effect of risk attitude on investment strategy. When the factor of risk aversion reaches some fix value, the optimal investment proportion stays in a roughly identical level, which shows the existence of marginal effect from risk aversion extent.

(2) Compared with the results under different risk measures, risk aversion attitude shows a significant effect on the choice and assignment among risk assets and risk-free asset; and hence the risk attitude should not be ignored.

(3) As confidence level becomes bigger, the optimal investment proportion in risky asset increases and the one in risk-free asset decreases. This is a natural conclusion since bigger confidence level will make the value of risk be smaller, which results in the increasing trend of investing in risk assets.

5. Conclusions

This paper constructed an insurance optimization model including spectral risk measure and risk-adjusted return of capital and conducted theoretical and empirical analysis. The main innovation of this paper is introduction of spectral risk measure in insurance business, which makes the risk attitude of the investor be considered in decision-making. The result tells us that more risk aversion will decrease the investment ratio in risk asset and increase the interest of investing in risk-free asset. However, the impact on investment strategies will disappear when the level of risk aversion increases to a certain extent. Furthermore, both confidence level and the threshold of risk play a significant role on optimal strategy. For convenience, here we suppose that the underwriting insurance return follows a deterministic process and the policy constraints for insurance investment are not involved. It may be also of interest to extend the research to the case with stochastic insurance surplus process and/or the government policy constraint. We will explore these problems in the following studies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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