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Research Article

A Computational Method of Active Earth Pressure from Finite Soil Body

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Nowadays, Coulomb and Rankine earth pressure theories have been widely applied to solve the earth pressure on a retaining structure. However, both of the theories established on the basis of the semi-infinite space assumption are not suitable for calculating the earth pressure from finite soil body. Therefore, this paper focuses on a theoretical study about the active earth pressure from finite soil body. Firstly, a common calculation model of finite soil body is established according to the results of previous studies. And then, based on Coulomb's theory and the wedge element method, an analytical solution of the unit active earth pressure from finite soil body is deduced without an assumption of its linear distribution in advance. Meanwhile, formulas of the active earth pressure strength coefficient and the application point of the resultant force are also deduced. Finally, the influence of parameters such as the frictional angle between the retaining wall back and backfill, slope angle of backfill, dip angle of the retaining wall back, the frictional angle between backfill and rock slope, and uniformly applied load on the backfill surface on the distribution of the unit active earth pressure and the application point of the resultant force is analyzed in detail.

1. Introduction

With wide applications of retaining walls in engineering, there have been more and more researches on the retaining walls. The methods to determine the active pressure on a retaining wall constitute a classical topic of soil mechanics [1]. Many scholars have studied this topic, and Coulomb and Rankine earth pressure theory are the most classical theories [2–4]. However, both of the theories are established on the basis of semi-infinite space assumption, which does not conform to actual conditions of many projects. When the retaining walls built in mountainous areas are often close to rock slope, which leads backfill behind the wall to form finite soil body, it is not suitable to calculate the earth pressure from finite soil body by Coulomb's and Rankine's theory. There are more and more problems about finite soil body in the design and construction of actual projects, but only a small number of scholars have conducted a preliminary study on them. According to the previous soil arch theory, Take and Valsangkar do an extensive series of centrifuge model tests to evaluate the use of flexible subminiature pressure cells and measure lateral earth pressures behind retaining walls of narrow backfill width [5]. Kniss et al. investigate the earth pressure against walls in narrow spaces using the finite element method [6]. Yang and Liu present finite element analyses of earth pressures in narrow retaining walls for both at-rest and active conditions, and the predicted data show a favorable agreement with measured data from centrifuge tests [7]. Fan and Fang present a numerical study on the behaviour of active earth pressures behind a rigid retaining wall with limited backfill space of various geometries [8]. On the basis of the limit equilibrium method with planar slip surfaces, Greco presents an analytical method to obtain a solution for the active thrust exerted by backfill of narrow width on gravity retaining walls [9]. Greco presents a limit equilibrium method, for calculating the active thrust on fascia retaining walls, where common methods cannot be used owing to the narrowness of the backfill [10].

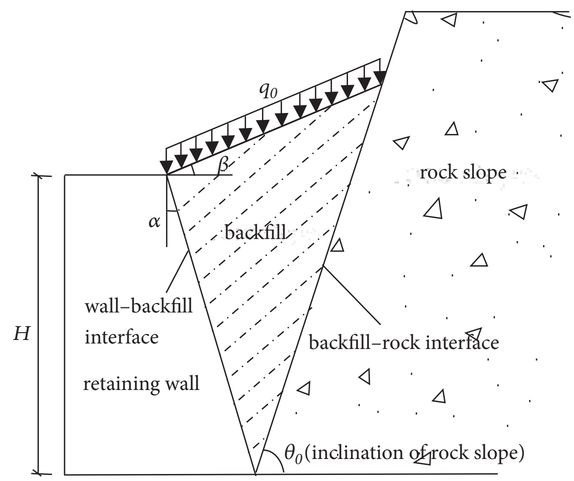


Figure 1: Schematic drawing of a rigid retaining wall near rock slope.

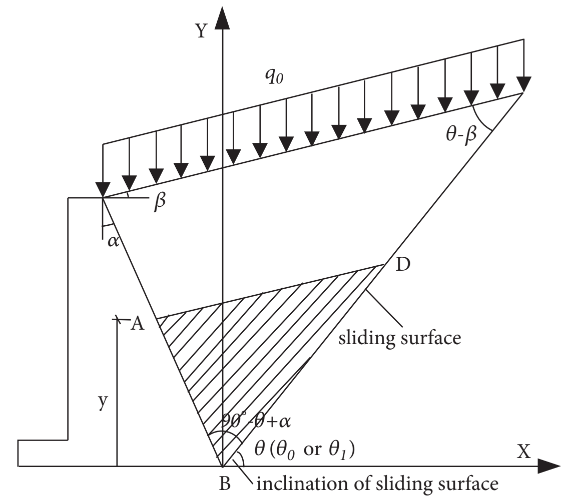


Figure 2: Sliding soil body.

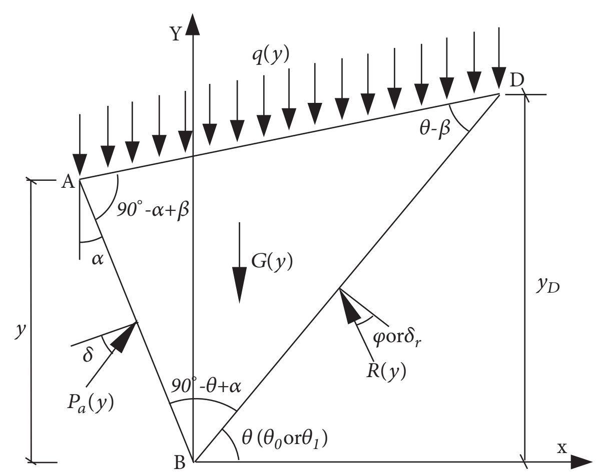


Figure 3: Force analysis of the partial sliding wedge ABD.

However, the above achievements are limited and insufficient. Therefore, based on Coulomb's theory and wedge element method, analytical solutions of the unit active earth pressure from finite soil body and the application point of the resultant force are deduced. Finally, the influence of parameters on the distribution of the unit active earth pressure from finite soil body and the application point of the resultant force is analyzed in detail.

2. Method of Analysis

A retaining wall model of finite soil body is shown in Figure 1. It is a common retaining wall near rock slope, which leads backfill behind the wall to form finite soil body. The retaining wall is a rigid retaining wall of height H and inclination α with the vertical. The cohesionless backfill surface is inclined at an angle β with the horizontal. q_0 is the uniform surcharge acting on the ground surface. Rock slope is inclined at an angle θ_0 with the horizontal.

2.1. Inclination θ of the Sliding Surface. According to Coulomb's theory based on semi-infinite space assumption, the inclination θ_1 of the Coulomb sliding surface is what makes the earth pressure reach the maximum, and it can be obtained when the Coulomb active earth pressure coefficient reaches the maximum as well. For the situation of finite soil body in Figure 1, the inclination θ of the sliding surface is determined as follows: when θ_0 is bigger than θ_1, backfill slides along the rock slope surface and the inclination θ of the sliding surface are taken equal to θ_0. When θ_0 is smaller than θ_1, the sliding surface is within backfill and the inclination θ of the sliding surface is taken equal to θ_1. θ_1 can be acquired from the following equation.

The expression of the Coulomb active earth pressure coefficient is shown as

$$K_a = \frac{\sin(\theta_1 - \varphi)\cos(\alpha - \theta_1)\cos(\alpha - \beta)}{\cos(\theta_1 - \alpha - \varphi - \delta)\cos^2\alpha\sin(\theta_1 - \beta)}, \qquad (1)$$

where φ is the internal friction angle of backfill and δ is the frictional angle between the back of the wall and backfill. To obtain the critical value of θ_1 which yields the maximum value of K_a, $dK_a/d\theta$ is set equal to 0.

2.2. Calculation Model. According to Coulomb's theory, the earth pressure against a retaining wall is due to the thrust exerted by a sliding soil body between the back of the wall and the sliding surface. The sliding soil body is shown as Figure 2. θ is the inclination of the sliding surface, and its value which is equal to θ_0 or θ_1 is determined as the above-mentioned method. ABD, a part of the sliding soil body at a depth y above the heel, is taken as an isolated unit for analysis and discussion. Its plane AD is parallel with the ground surface.

Forces analysis on the part wedge ABD of the thrust wedge is shown in Figure 3. $G(y)$ is the weight of wedge ABD, and $q(y)$ is the uniform vertical pressure on the top of wedge ABD. $P_a(y)$ is the force on the wall, namely, the active earth pressure, and the angle between $P_a(y)$ and the normal of the wall back is the frictional angle δ between the retaining wall back and backfill. $R(y)$ is the force on sliding surface. When the inclination θ of the sliding surface is equal to θ_1, the angle between $R(y)$ and the normal of the sliding surface is the internal frictional angle φ of backfill. When the inclination θ of the sliding surface is equal to θ_0, the angle between $R(y)$ and the normal of the sliding surface is the frictional angle δ_r between backfill and rock slope. If there is no test data about δ_r, δ_r is set to be 0.33 times as φ [11].

2.3. Establishment of Fundamental Equations. Figure 4 shows the force polygon considered in wedge ABD. $Q(y)$ is the resultant force acting on plane AD. According to sine theorem,

$$\frac{R(y)}{\cos(\alpha+\delta)} = \frac{P_a(y)}{\sin(\theta-\varphi)} = \frac{G(y)+Q(y)}{\sin(\alpha+\delta+\varphi-\theta)}. \tag{2}$$

Using the formula of Coulomb's active earth pressure,

$$P_a(y) = \frac{1}{2}K_a\gamma y^2 + K_a q(y)\frac{\cos\alpha\cdot\cos\beta}{\cos(\alpha-\beta)}y, \tag{3}$$

where γ is the unit weight of backfill. K_a is the active earth pressure coefficient, which is written as

$$K_a = \frac{\sin(\theta-\varphi)\cos(\alpha-\theta)\cos(\alpha-\beta)}{\cos(\theta-\alpha-\varphi-\delta)\cos^2\alpha\sin(\theta-\beta)}. \tag{4}$$

Introducing two variables $p_a(y)$ and $r(y)$, $p_a(y)$ is the unit active earth pressure, and $r(y)$ is the unit force on failure surface. There are two equations shown as

$$\begin{aligned} P_a(y) &= \int_0^y p_a(y)\,dy, \\ R(y) &= \int_0^{y_D} r(y)\,dy. \end{aligned} \tag{5}$$

Putting (3) into the first equation of (5) and, meanwhile, taking a derivative from both sides of the equation with respect to y, it can be obtained that

$$\begin{aligned} p_a(y) &= K_a\gamma y \\ &\quad + K_a\left[q(y)+q'(y)\,y\right]\frac{\cos\alpha\cdot\cos\beta}{\cos(\alpha-\beta)} \end{aligned} \tag{6}$$

$$\text{Let } p_a(y) = Kq(y), \tag{7}$$

where K is the unit active earth pressure coefficient. Putting (7) into (6), it can be obtained that

$$\begin{aligned} q'(y) &= \left(\frac{K}{K_a}\cdot\frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}-1\right)\cdot\frac{q(y)}{y} \\ &\quad - \frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}\cdot\gamma. \end{aligned} \tag{8}$$

Taking the moment equilibrium equation around the B point on the wedge ABD, it can be obtained that

$$\begin{aligned} &\int_0^y p_a(y)\cdot\cos\delta\cdot\frac{y}{\cos\alpha}dy + \frac{y\cdot\cos(\theta-\alpha)\cdot\cos\beta}{\cos\alpha\cdot\sin(\theta-\beta)} \\ &\quad\cdot q(y)\cdot\left[\frac{1}{2}\frac{\cos(\theta-\alpha)}{\cos\alpha\cdot\sin(\theta-\beta)}-\frac{\sin\alpha}{\cos\beta\cdot\cos\alpha}\right] \\ &\quad\cdot\cos\beta\cdot y + \frac{1}{2}\gamma\cdot\frac{y^2}{\cos^2\alpha} \\ &\quad\cdot\frac{\cos(\theta-\alpha)\cdot\cos(\alpha-\beta)}{\sin(\theta-\beta)}\cdot\frac{2}{3} \\ &\quad\cdot\left[\frac{1}{2}\frac{\cos(\theta-\alpha)}{\cos\alpha\cdot\sin(\theta-\beta)}-\frac{\sin\alpha}{\cos\beta\cdot\cos\alpha}\right]\cdot\cos\beta \\ &\quad\cdot y = \int_0^{y_D} r(y)\cdot\cos\varphi\cdot\frac{y}{\sin\theta}dy. \end{aligned} \tag{9}$$

Putting (2), the second equation of (5), and (7) into (9) and taking a derivative from both sides of the equation with respect to y, it can be obtained that

$$q'(y) = \left[A_1K - A_2K - 2\right]\cdot\frac{q(y)}{y} - \frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}\cdot\gamma, \tag{10}$$

where

$$\begin{aligned} A_1 &= \frac{\cos(\alpha+\delta)\cdot\cos\varphi\cdot\cos(\alpha-\beta)}{\sin(\theta-\varphi)\cdot\cos(\theta-\alpha)\cdot\cos^2\beta\cdot\left[\frac{1}{2}\frac{\cos(\theta-\alpha)}{\cos\alpha\cdot\sin(\theta-\beta)}-\frac{\sin\alpha}{\cos\beta\cdot\cos\alpha}\right]}, \\ A_2 &= \frac{\cos\delta\cdot\sin(\theta-\beta)}{\cos(\theta-\alpha)\cdot\cos^2\beta\cdot\left[\frac{1}{2}\frac{\cos(\theta-\alpha)}{\cos\alpha\cdot\sin(\theta-\beta)}-\frac{\sin\alpha}{\cos\beta\cdot\cos\alpha}\right]}. \end{aligned} \tag{11}$$

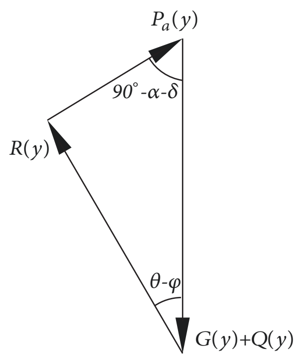


FIGURE 4: Static equilibrium of the partial sliding wedge ABD.

2.4. Solution of the Basic Equation. According to (8) with (10), the expression of the unit active earth pressure coefficient K can be obtained:

$$K = \frac{1}{A_1 - A_2 - A_3}, \tag{12}$$

where $A_3 = \cos(\alpha - \beta)/(\cos\alpha \cdot \cos\beta) \cdot 1/K_a$.

Letting $\lambda = K/K_a \cdot \cos(\alpha-\beta)/(\cos\alpha \cdot \cos\beta) - 1$ and solving (8), it can be obtained that

$$q(y) = Cy^{\lambda} + \frac{\cos(\alpha - \beta)}{(\lambda - 1)\cos\alpha \cdot \cos\beta} \cdot \gamma y, \tag{13}$$

where C is an integration constant; it can be determined by the boundary condition that $q(y) = q_0$ when $y = H$. It can be written as

$$C = q_0 H^{-\lambda} - \frac{\cos(\alpha - \beta)}{(\lambda - 1)\cos\alpha \cdot \cos\beta} \cdot \gamma \cdot H^{1-\lambda}. \tag{14}$$

According to (7), the unit active earth pressure can be obtained as

$$\begin{aligned} p_a(y) &= K\left[q_0 H^{-\lambda} - \frac{\cos(\alpha - \beta)}{(\lambda - 1)\cos\alpha \cdot \cos\beta} \cdot \gamma \cdot H^{1-\lambda}\right] \\ &\quad \cdot y^{\lambda} + K\frac{\cos(\alpha - \beta)}{(\lambda - 1)\cos\alpha \cdot \cos\beta} \cdot \gamma y. \end{aligned} \tag{15}$$

2.5. Application Point of the Resultant Earth Pressure. The height H_p of application point of the resultant earth pressure from the wall bottom is

$$\begin{aligned} H_p &= \frac{\int_0^H y p_a(y)\,dy}{\int_0^H p_a(y)\,dy} = \frac{2}{3}\left(\frac{\lambda + 1}{\lambda + 2}\right) \\ &\quad \cdot \frac{3q_0 + (\cos(\alpha - \beta)/(\cos\alpha \cdot \cos\beta))\gamma H}{2q_0 + (\cos(\alpha - \beta)/(\cos\alpha \cdot \cos\beta))\gamma H} H. \end{aligned} \tag{16}$$

3. Parametric Study

In this section, when the inclination of the sliding surface is θ_0, the influence of dip angle α of the retaining wall back, slope angle β of backfill, the frictional angle δ between the retaining wall back and backfill, the frictional angle δ_r between backfill and rock slope, and uniformly applied load q_0 on the backfill surface on the distribution of the unit active earth pressure and the application point of the resultant force is discussed as follows. θ_0, the inclination of the rock slope, is assumed to be 75°. Another condition can refer to [12].

When $H = 4.0$ m, $r = 18$ Kn/m^3, and $\theta_0 = 75°$, the active earth pressure distribution and the height H_p of application point of the resultant earth pressure with change in δr, in δ, in α, in β, and in q_0 are shown in Figures 5–14, respectively. As shown in Figure 5, the active earth pressure increases as δr increases near the heel of the wall; the active earth pressure increases as δr decreases in other places. As shown in Figure 7, the active earth pressure decreases as δ increases near the heel of the wall; the active earth pressure

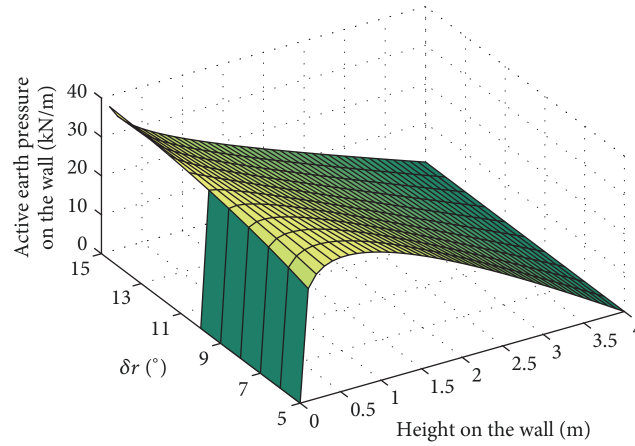


FIGURE 5: Distribution of active earth pressure with change in δr.

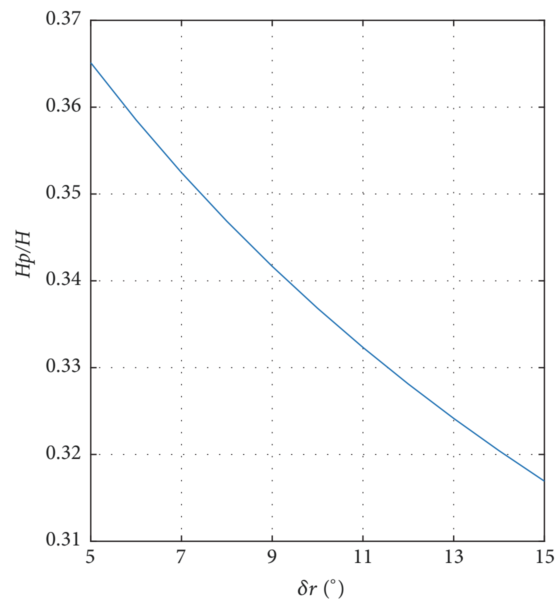


FIGURE 6: Application point of resultant earth pressure with change in δr.

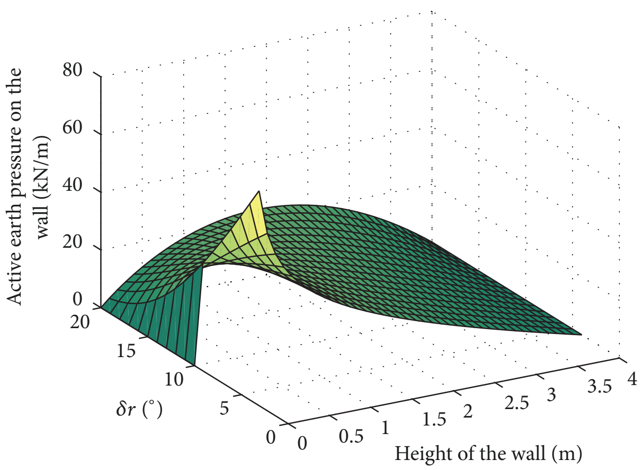


Figure 7: Distribution of active earth pressure with change in δ.

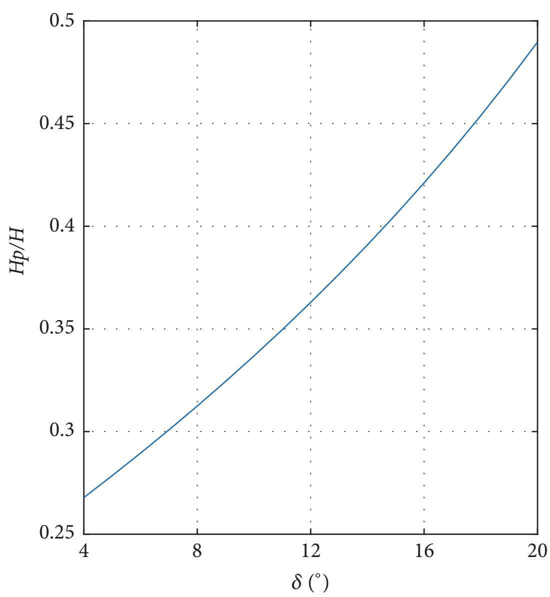


Figure 8: Application point of resultant earth pressure with change in δ.

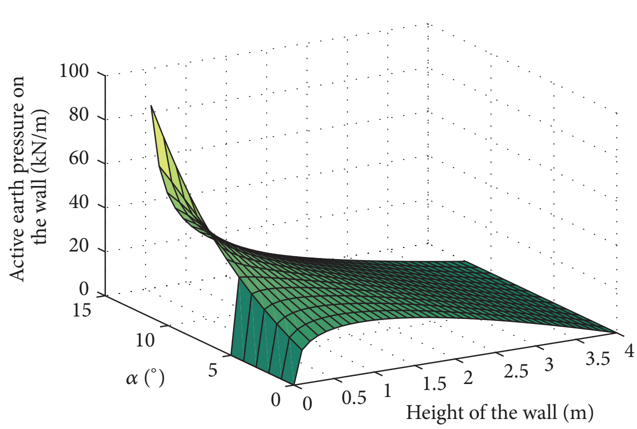


Figure 9: Distribution of active earth pressure with change in α.

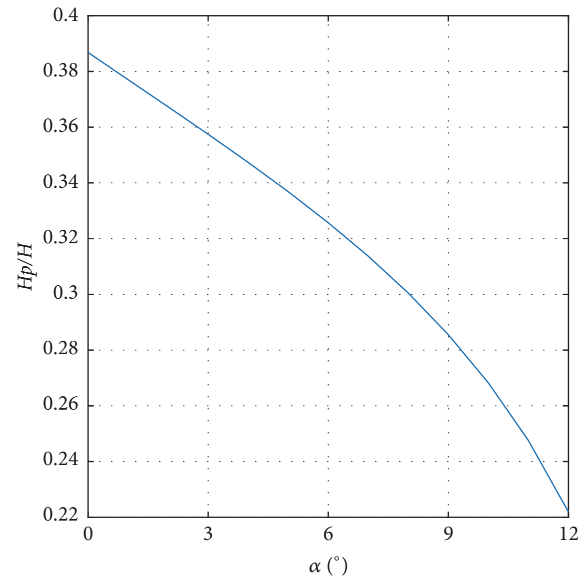


Figure 10: Application point of resultant earth change pressure with change in α.

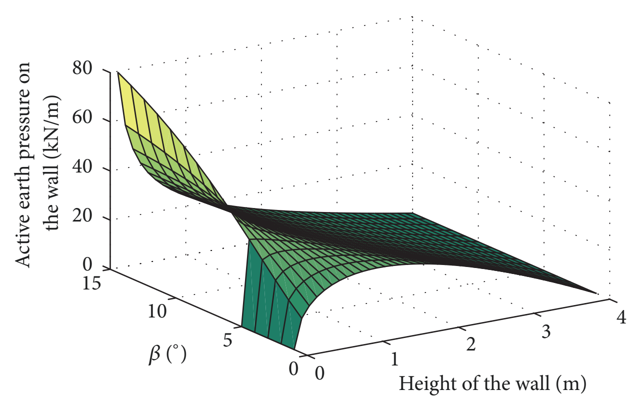


Figure 11: Distribution of active earth pressure with change in β.

increases as δ increases in other places. From Figures 9 and 11, it can be seen that the active earth pressure increases as α and β increase, respectively. As shown in Figure 13, the active earth pressure increases in proportion as q_0 increases. From Figures 6, 10, and 12, it can be seen that the height H_p of application point of resultant earth pressure decreases as δr, α, and β increase, respectively. From Figures 8 and 14, it can be seen that the height H_p of application point of the resultant earth pressure decreases as δ and q_0 increase, respectively.

From Figures 5, 7, 9, and 11, it is obvious that there is a critical value existing in dip angle α of the retaining wall back, slope angle β of backfill, the frictional angle δ between the retaining wall back and backfill, and the frictional angle δ_r between backfill and rock slope. The critical values divide the earth pressure distribution curves into two types. One type is a crooked nonlinear curve and another type is close to a line [12].

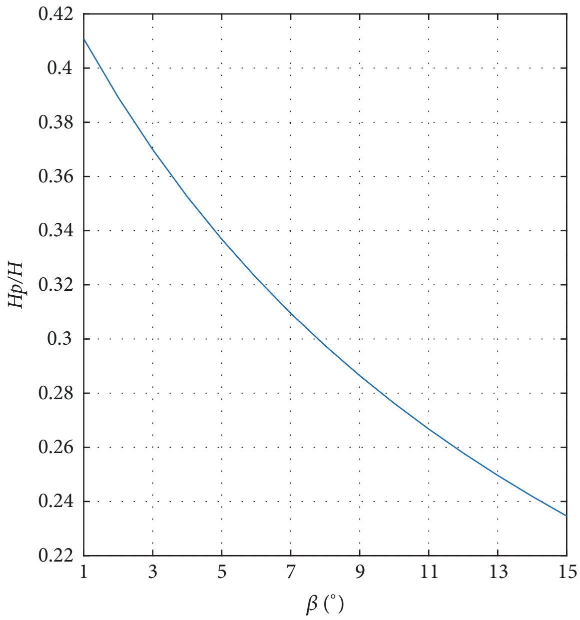


Figure 12: Application point of resultant earth pressure with change in β.

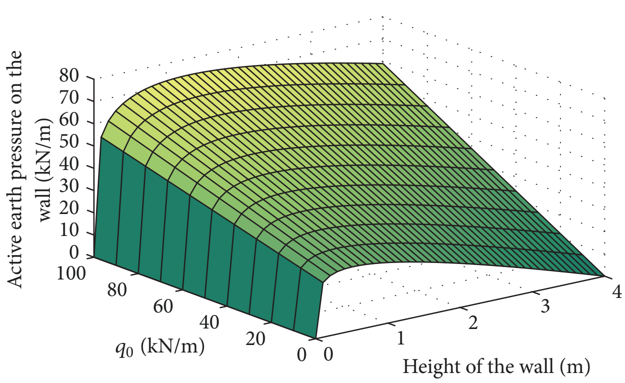


Figure 13: Distribution of active earth pressure with change in q_0.

4. Conclusions

(1) Based on Coulomb's theory and wedge element method, an analytical solution of the unit active earth pressure from finite soil body is deduced. Meanwhile, the formula of the application point of the resultant force is also deduced. When these formulas are applied, we firstly estimate whether backfill slides along rock slope or the sliding surface happens inside of backfill. And then whether the angle between $R(y)$ and the normal of the sliding surface is the internal frictional angle φ of backfill, or the frictional angle δ_r between backfill and rock slope is determined.

(2) By parametric study, it is found that the frictional angle between the retaining wall back and backfill, slope angle of backfill, dip angle of the retaining wall back, the frictional angle between backfill and rock slope, and uniformly applied load on the backfill surface have great influence on the active earth pressure distribution and the application point of the resultant earth pressure.

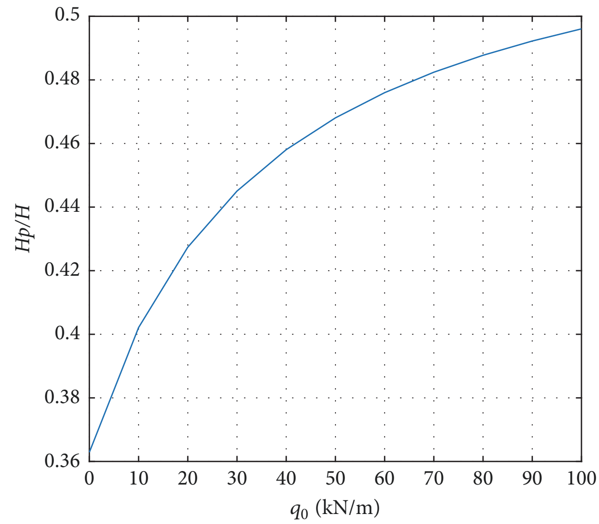


Figure 14: Application point of resultant earth pressure with change in q_0.

(3) According to the method of this paper, we can study the earth pressure from finite clayey soil body in the future.

Symbols

H: Height of retaining wall (Figure 1)
α: Angle that backfill wall interface makes with the vertical (Figure 1)
β: Angle that backfill surface makes with the horizontal (Figure 1)
q_0: Uniform surcharge on the ground surface (Figure 1)
θ_0: Angle that the backfill rock interface makes with the horizontal (Figure 1)
θ: Angle that the failure surface makes with the horizontal
K_a: Coulomb active earth pressure coefficient as defined in (1)
θ_1: Angle obtained according to the Coulomb active earth pressure coefficient in (1)
φ: Internal friction angle of backfill
δ: Frictional angle between the wall and backfill
δ_r: Frictional angle between backfill and rock slope
γ: Unit weight of backfill
K: Unit active earth pressure coefficient as defined in (12)
$G(y)$: Weight of wedge ABD (Figure 3)
$P_a(y)$: Active earth pressure on plane AD of wedge ABD (Figure 3)
$R(y)$: Force on failure surface BD of wedge ABD (Figure 3)
$q(y)$: Uniform vertical pressure on plane AD of wedge ABD (Figure 3)
$Q(y)$: Resultant force on plane AD of wedge ABD

$r(y)$: Unit force on failure surface BD of wedge ABD
A_1, A_2, A_3, λ: Terms as defined in Sections 2.2 and 2.3
H_p: Height of application point of the resultant earth pressure from the wall bottom as shown in (16).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] Y. Z. Wang, "Distribution of earth pressure on a retaining wall," *Géotechnique*, vol. 50, no. 1, pp. 83–88, 2000.

[2] A. Kezdi, "Earth pressure on retaining wall tilting about the toe," in *Proceedings of the Brussels Conference on Earth Pressure Problems*, vol. 1, pp. 166-132, 1958.

[3] T. Nakai, "Finite element computations for active and passive earth pressure problems of retaining wall," *Soils and Foundations*, vol. 25, no. 3, pp. 98–112, 1985.

[4] L. W. Wang, "Improvement of the method of determining the height of action point of Coulomb active earth pressure," *Mechanics in Engineering*, vol. 35, no. 6, pp. 55–58, 2013.

[5] W. A. Take and A. J. Valsangkar, "Earth pressures on unyielding retaining walls of narrow backfill width," *Canadian Geotechnical Journal*, vol. 38, no. 6, pp. 1220–1230, 2001.

[6] K. T. Kniss, K. H. Yang, S. G. Wright, and J. G. Zornberg, "Earth pressures and design consideration of narrow," in *Proceeding of the Conference of Texas Section-ASCE meeting*, Taylor, TX, USA, April 2007.

[7] K.-H. Yang and C.-N. Liu, "Finite element analysis of earth pressures for narrow retaining walls," *Journal of Geo Engineering*, vol. 2, no. 2, pp. 43–53, 2007.

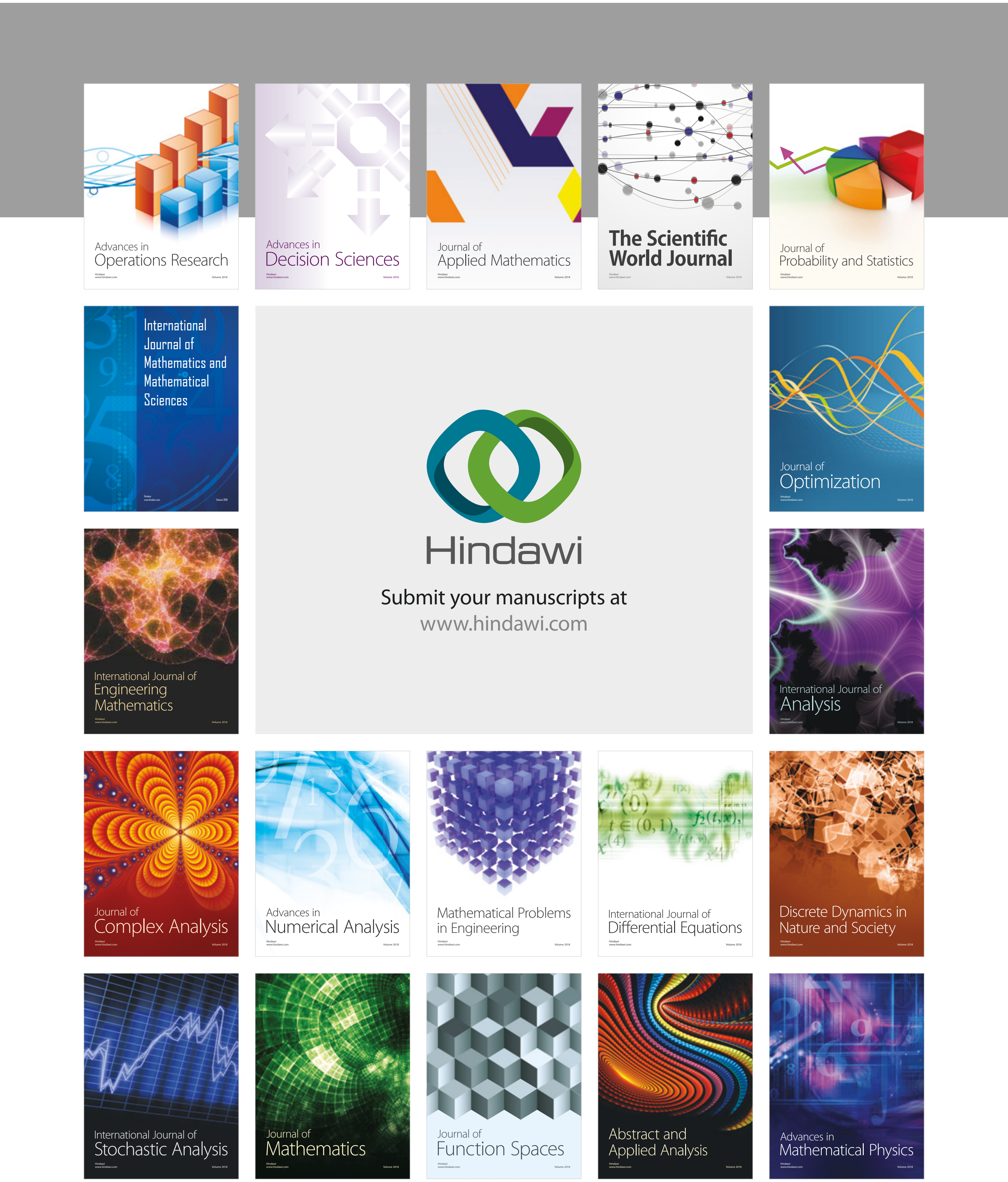
[8] C.-C. Fan and Y.-S. Fang, "Numerical solution of active earth pressures on rigid retaining walls built near rock faces," *Computers & Geosciences*, vol. 37, no. 7-8, pp. 1023–1029, 2010.

[9] V. Greco, "Active thrust on retaining walls of narrow backfill width," *Computers & Geosciences*, vol. 50, pp. 66–78, 2013.

[10] V. R. Greco, "Analytical solution of seismic pseudo-static active thrust acting on fascia retaining walls," *Soil Dynamics and Earthquake Engineering*, vol. 57, pp. 25–36, 2014.

[11] GB 50007-2011, "Code for design of building foundation," Housing and Urban-Rural Development, Beijing, 2011.

[12] J. Chen, M. Deng, and Y. Zhang, "Nonlinear active earth pressure distribution based on Coulomb's theory," *Applied Mechanics and Materials*, vol. 90-93, pp. 433–437, 2011.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