A New Calculating Method of Pressure Response Time of Pneumatic Brake Pipe Based on Experiments

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Air brake system is one of the common braking methods for buses and trucks; its excellent performance guarantees the safety of the vehicle and the stability of the braking. As an important part of the pneumatic brake system, the brake pipe is an important factor influencing the pressure response time of the pneumatic brake system. Based on the exploratory experiment of pneumatic brake pipe, the influence of pipe length, pipe diameter, inlet sonic conductance, initial pressure, and supply pressure on pipe pressure response time was analyzed by fuzzy gray correlation analysis method. The results show that tube length is the most important factor affecting the pressure response time. Combined with the analysis results of gray correlation degree, the experimental scheme of the response time of the pneumatic brake pipe was designed by the response surface experimental design method. Based on the multiparameter analysis method, the influence of the experimental parameters on the pipe pressure response time was analyzed. Based on the experimental data, the form of calculation formula is derived by dimension analysis method, which provides a theoretical basis for the selection of pneumatic brake pipes and the design of air brake system.

1. Introduction

As one of the most commonly used braking methods, pneumatic braking has been widely used in buses and trucks [1]. With the rise of electric vehicles and autonomous driving technology, brake-by-wire and electric control have gradually become the development trend, but due to its high cost, the popularity in developing countries is very low [2]. Therefore, air brake is still the main braking method for vehicles such as buses and trucks. GB 7258-2017 [3] and GB 12676-2014 [4] clearly define the range of pressure response time of the pneumatic brake circuit to ensure safe driving of the vehicle. Therefore, the pressure response time is one of the key parameters for the air brake system to meet the vehicle braking requirements. The brake circuit uses pipes to connect the key components in different arrangements. Kenji [5] studied the influence of the brake pipe on the braking performance of the vehicle. According to the inner diameter of different pipes and its influence on the braking force of each wheel, the braking force optimization strategy was proposed. Karthikeyan [6] studied the pressure response time of the pneumatic brake system by analyzing the valves and pipes of the pneumatic brake system and built a control model for the pressure response time of the electropneumatic brake based on the model prediction algorithm. The validity of the model provides a theoretical reference for the development of electropneumatic braking. Mithun [7,8] and others established a model of the pneumatic brake system and studied its pressure response time using AMESim, Simulink, and MWork. Qin [9] verified the pressure response time delay causing the longitudinal or lateral braking distance by establishing a model of the pressure response time of the pneumatic brake system. Different circuit layouts such as pipe diversion and confluence will directly affect the pressure response time of the brake system, affecting the braking performance and stability. Wang [10] et al. demonstrated that the pipeline in the pneumatic brake system accounted for 30% of the total time delay in the response delay of the entire brake system. Therefore, it is important to analyze the pressure response of pipelines and study the influence of different parameters on their pressure response.
### Table 1: Types of pneumatic pipe.

<table>
<thead>
<tr>
<th>Pipe types</th>
<th>Features</th>
<th>Positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy supply</td>
<td>Supply compressed air</td>
<td>Between the air compressor and the air tank</td>
</tr>
<tr>
<td>Control line</td>
<td>Control air circuit opening and closing</td>
<td>Between the air tank and the relay valve</td>
</tr>
<tr>
<td>Actuation line</td>
<td>Actuator</td>
<td>Between the pedal valve and the relay valve</td>
</tr>
</tbody>
</table>

![Schematic diagram of one-quarter brake circuit.](image)

According to the arrangement and function of the pneumatic brake pipe in the pneumatic brake system, the pneumatic brake pipes are mainly divided into three types, as shown in Table 1.

Table 1 shows that the pneumatic lines are divided into an energy supply line, a control line, and an actuating line in the pneumatic brake system. Figure 1 is a schematic diagram of a quarter circuit in a pneumatic brake system, which shows the pneumatic circuit from the air supply to the brake chamber, three of which are shown in the figure [10].

For pneumatic pipelines, many scholars study the calculation method of pipelines. Cai [11] applied the distributed parameter method to establish the pipe model to calculate the pressure loss and time response of the pipe and adopted the upwind differential method discrete pipe model. The method has first-order accuracy and high requirements on the calculation step length. Luo [12] used the equivalent idea of gas volume to simplify the theoretical aerodynamic model and analyzed the variation of the multivariate index with time. Jun [13] built a distributed model of one-dimensional pipe based on state equation, motion equation, and continuous equation and calculated the pressure loss and response delay of long pipe and tested the two ends of the cylinder with pipe. Zielke [14] uses the method of characteristics to obtain the relationship of the gas transient flow in the frequency domain and solve the transient response of the pipe. Cengel and Cimbala [15] calculated the relationship between pressure loss and mass flow in a pneumatic system and proposed that the resistance of the pneumatic system is proportional to the length of the pipe and the aerodynamic viscosity. Mohammad [16], based on the two-fluid conservation equation, established a homogeneous two-phase gas pipe model to accurately calculate the transient changes of the natural gas pipe system.

According to these descriptions above, the analysis of the pressure response time of pneumatic brake pipe mainly has some deficiencies as follows. (1) The experimental design method in the international standard only gives the design scheme of the general pneumatic circuit, which cannot provide theoretical support for the selection of experimental parameters, resulting in lack of basis for the selection of parameters in the experimental design. (2) The research on the pressure response time of pneumatic brake pipes mainly focuses on the flow characteristics of pneumatic brake pipes, which is insufficient for guiding the design of pneumatic brake systems. Therefore, there is a lack of a formula for calculating the pressure response time of a pneumatic brake pipe with appropriate accuracy and convenient calculation.

In this paper, the pneumatic brake pipe is taken as the research object, and the pressure response time of the pneumatic brake pipe is studied based on the experiment. The parameters affecting the pressure response time of the pneumatic brake pipe are analyzed. Combined with the requirements of the pneumatic brake system for the pressure response time of the pneumatic brake pipe, the calculation formula of the pressure response time is derived, which provides a theoretical reference for the design of the pneumatic brake pipe.

### 2. Experimental Design of Pressure Response Time of Brake Pipe

In order to design the experimental scientifically, it is necessary to clarify the parameters affecting the pressure response time of the pneumatic brake pipe and its laws. Grey correlation degree [17] is a method for analyzing the degree of association between variables in the system. The principle is to compare the shape of curves at different points for influence parameters and target parameters. The higher the degree of similarity, the greater the degree of correlation. Therefore, to determine the influence law of the influence parameters on the brake pipe pressure response time, the experimental data was obtained by exploratory brake pipe experiment as shown in Figure 2, and the influence law of each parameter on the brake pipe pressure response time was determined by analyzing the experimental data, and the parameters of the later pneumatic brake pipe experiment were determined to design a more scientific and rigorous experimental program.
2.1. Key Influence Parameters of Pressure Response Time. In this paper, the experimental data of brake pipe pressure response time is obtained based on the exploratory experiment of pneumatic brake pipe. The influence law of influence parameters on pressure response time is analyzed by fuzzy gray correlation degree. The key influence parameters of pressure response time of pneumatic brake pipe are determined, which provides a theoretical reference for the selection of the parameters of the pipe's pressure response experiment.

Influence Parameters Setting. The matrix composed of the factors affecting system behavior in the grey correlation degree is a comparison sequence and is as follows:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{c1} & \cdots & a_{cn} \end{pmatrix}$$ (1)

Here, $A$ is a comparison sequence matrix, $c$ is a combination of comparison sequences, and $n$ is an element type.

Considering the influencing factors involved in the air brake system, the tube length $L$, the pipe diameter $d$, the inlet sonic conductance $C$, the initial pressure $p_0$, and the supply pressure $p$ are taken as matrix elements, and $c=5$ in the matrix.

The matrix of gray correlation degree reflecting the behavior characteristics of the system is a reference sequence and is as follows:

$$B_x = [b_1 \ b_2 \ \cdots \ b_c]$$ (2)

Figure 3 is the pressure response curves of the pneumatic brake pipe at PS3 by experiment. It is clear that the higher the pressure is, the longer the pressure reaches the set point. When the pressure difference between the inside and outside of the brake pipe is larger, the pressure rises faster, and as the pressure rises, the rising trend becomes slower. When the pressure is 0.5MPa, the pressure response time reaches 500 ms, and the delay is large, which will seriously affect the execution time of the actuator at the end of the brake pipe.
Table 2: Exploratory experiment parameter range setting.

<table>
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<th>Parameter type</th>
<th>Parameter range</th>
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<tr>
<td>d/mm</td>
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<tr>
<td>L/m</td>
<td>5-20</td>
</tr>
<tr>
<td>C/dm³/s (s bar)</td>
<td>1-4</td>
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<tr>
<td>pᵢ/MPa</td>
<td>0.1-0.3</td>
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<tr>
<td>p/MPa</td>
<td>0.4-0.65</td>
</tr>
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</table>

Here, B is a reference sequence matrix, x is the number of the reference sequences, c is a combination of reference sequences, and elements of the reference sequence are consistent with that of the comparison sequence. The pressure response time t is selected as a reference sequence characterizing the characteristics of the pipe.

The reference sequence was obtained by experiments from the parameters of the comparison sequence. According to the engineering application, the range of experimental parameters is set as shown in Table 2. To reasonably set the experimental data and accurately reflect the influence of parameters on the pressure response time, the value of the experimental parameters is determined by the central composite design method [18]. According to the central composite design method, 10 experimental setting schemes were determined as shown in Table 3, and the corresponding composite design method [18]. According to the central composite design method, the range of experimental parameters on the pressure response time, the value of the experimental parameters is set as shown in Table 2, and the corresponding composite design method [18].

Dimensionless Influence Parameters. The gray correlation degree needs to be compared between the parameters; then the dimensions of the comparison parameters are the same degree needs to be compared between the parameters; then the dimensions of the comparison parameters are the same.

\[ X_i = (x(1), x(2), \ldots, x(n)) \]  

(3)

\[ x(k)' = \frac{x(k)}{\max_k x(k)} \]  

(4)

In (4), k=1,2,3…n.

Calculation of Correlation Coefficient Affecting Parameters. The above-mentioned dimensionless sequence matrix is applied to calculate the correlation coefficient. The correlation coefficient is calculated as follows:

\[ \delta(c) = \frac{\min_{c} \min_{1,0} |a_{cn} - b_c| + \alpha \max_{c} \max_{1,0} |a_{cn} - b_c|}{|a_{cn} - b_m| + \alpha \max_{c} \max_{1,0} |a_{cn} - b_c|} \]  

(5)

Here, \( \delta(c) \) indicates the correlation coefficient between the elements of the c-row and n-th columns of the comparison sequence and the c-th element of the reference sequence. \( \min \min_{1,0} |a_{cn} - b_c| \) represents the minimum value of the absolute value of the difference between each influence parameter and the reference sequence. \( \max \max_{1,0} |a_{cn} - b_c| \) represents the maximum value of the absolute value of the difference between each influence parameter and the reference sequence. \( \alpha \) is the resolution factor: \( \alpha \in (0,1) \). Since different combinations correspond to different degrees of association, this is not convenient for comparative analysis of parameters. Therefore, the average of the correlation coefficients of the main parameters is used as an evaluation index to measure the degree of association:

\[ r_i = \frac{1}{c} \sum_{k=1}^{c} \delta(k), \quad k = 1,2, \ldots, c \]  

(6)

Table 4 shows the average correlation degree of each parameter calculated.

Fuzzy Membership Calculation. Fuzzy membership degree [19] is a kind of fuzzy evaluation. The closer the membership degree is to 1, the higher the degree of the factor belongs to the thing; the closer to 0 it is, the lower the degree of the factor belongs to the thing. The influence of each influence parameter on the pressure response is ambiguous, and the membership degree can accurately reflect the influence law of the factors.

\[ r_2 = \frac{\sum_{c=1}^{2} a_{cn} b_c}{\sqrt{\sum_{c=1}^{2} a_{cn}^2 \sum_{c=1}^{2} b_c^2}} \]  

(7)

Here, \( r_2 \) indicates membership; \( a_{cn} \) is an element of comparison sequence matrix; \( b_c \) is reference sequence matrix elements. The calculation results of the membership degree are shown in Table 5.

Fuzzy Relevance. To ensure the reliability of the analysis results, the fuzzy correlation degree is used as the final evaluation index through combining the correlation coefficient and the membership degree. This provides a theoretical reference for experimental design.

\[ r = \frac{r_1 + r_2}{2} \]  

(8)

Here, \( r \) is fuzzy relevance. The fuzzy relevance of each parameter is shown in Table 6.

It can be seen from Table 6 that the pipe length has the greatest influence on the pressure response time, although the pipe length and pipe diameter are the structural parameters of the pneumatic brake pipe, and the pipe diameter has secondary effect on the pressure response time. As the inlet sonic conductance of the pipe, the influence of the sonic conductance on the pipe pressure response is inferior to that of the length of the pipe; two factors related to pressure have less effect on the pressure response time, especially the initial pressure.

Therefore, pipe length, pipe diameter, inlet sonic conductance, and gas supply pressure cause the fuzzy correlation degree to exceed 0.7, which are parameters with obvious influence degree. Compared with other parameters, the initial pressure has no obvious influence on the pressure response time.

2.2. Design of Experimental Scheme for Pressure Response Time of Pneumatic Brake Pipe. Based on the fuzzy gray correlation analysis, the experimental design of the pneumatic brake pipe pressure response time is conducted by using the CCD design of experimental method [20] in the response surface. In CCD,
pipe diameter is selected as \{8, 10, 12\} mm, pipe length is selected as \{5, 12.5, 20\} m, inlet sonic conductance is selected as \{1, 2.2, 3.4\} dm\(^3\)/s-bar, and gas supply pressure is selected as \{0.3, 0.4, 0.5\} MPa.

According to QC/T 35-2011 [21] “Car and trailer air pressure control device performance requirements and bench test method,” GB 12676-2014 [4] “commercial vehicle and trailer brake system technical requirements and test methods,” and other standards, the specified experimental methods and conditions are connected to the test circuit according to the design scheme of the test bench, and the circuit is connected according to its functional requirements. The test bench for the pipe pressure response time is shown in Figure 4.

3. Experimental Data Analysis of Pressure Response Time of Pneumatic Brake Pipe

3.1. Experimental Data Processing Method Based on Experimental Standard. GB 12676-2014 “Technical requirements and test methods for brake systems for commercial vehicles and trailers” stipulates that when the pressure measured from the start of the brake pedal to the control line joint reaches 10% of the stable value, pressure response time of pneumatic brake pipe shall not exceed 0.2 s; when it reaches 75% of the steady state value, pressure response time of pneumatic brake pipe shall not exceed 0.4 s.

Confirmation of the Start of the Pressure Response. When the solenoid valve is opened to inflate the pipe, the pressure change in the pipe is not obvious due to the error of the pressure sensor and the response delay. Since the standard clearly defines the pressure stability value of 10%, the time when the inlet pressure of the pipe reaches 10% of the supply pressure is taken as the initial response time, which is recorded as \(t_1\).

Confirmation of the End of the Pressure Response. Since the standard specifies the time requirement that the pressure stability value reaches 75%, the time at which the pressure at the outlet end of the pipe reaches 75% of the supply pressure is selected as the end point of the pressure response time, which is recorded as \(t_2\).

The reading formula for the pressure response time of the inflation process is

\[ t_c = t_2 - t_1 \]  

The average of the three measured pressure response times is taken as the final pressure response time, which helps to reduce system errors.

Experimental Data on Line Pressure Response Time. Figure 5 shows the manner in which the pressure response time is
Table 7: Multiparameters experimental data.

<table>
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<th>No.</th>
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</table>

Figure 5: Reading of pressure response time.

3.2. Influence of Multiparameter Variation on Pressure Response Time of Pneumatic Brake Pipe. In order to further confirm the influence of each parameter on the pipe pressure response time, a multiparameter analysis method was used to clarify the interaction between the parameters. Figure 6 shows the curve of the line pressure response time when the two parameters change. The two curves in the figure represent the pressure response time when the maximum and minimum values of each experimental parameter are taken. As shown in Figures 6(a), 6(c), and 6(e), in the two-parameter variation of pipe diameter and pipe length, pipe diameter and gas supply pressure, and pipe length and gas supply pressure, the larger the pipe diameter, the larger the pipe length, the larger the gas supply pressure, and the pipe pressure response time increases. Therefore, when the pipe length, pipe diameter, and gas supply pressure are constant, the pipe pressure response time increases. The larger the pipe length, the greater the gas supply pressure, and the pipe pressure response time increases. Therefore, when the pipe length, pipe diameter, and gas supply pressure change simultaneously, the pipe pressure response time shows a positive correlation trend, and the pipe length has the most significant influence. As shown in Figures 6(d) and 6(f), when the tube length changes, the sonic flow conductance is larger, and the pipe pressure response time is smaller. When the supply pressure changes, the sonic conductance is larger, and the pipe pressure response time is smaller. Therefore, when the pipe length and the supply pressure are constant, the pipe pressure response time
Figure 6: Effect of multiparameter changes on inflation time: (a) pipe diameter and length; (b) pipe diameter and inlet conditions; (c) pipe diameter and supply pressure; (d) pipe length and entry conditions; (e) tube length and gas supply pressure; (f) entrance conditions and supply pressure.

As shown in Figure 6(b), the influence trend of the two-parameter change of the pipe diameter and the inlet sonic conductance is completely different from that of the pipe diameter and the inlet sonic conductance. When the sonic conductance is 1 dm$^3$/(s·bar), the pressure response time when pipe diameter is 8 mm is greater than that when pipe diameter is 9 mm. The pipe pressure response time decreases first and then increases with the increase of pipe diameter. When the sonic conductance is 3.4 dm$^3$/(s·bar), as the pipe diameter increases, the pressure response time decreases, so the sonic conductance and the pipe diameter are coupled. As the pipe diameter increases, the volume acts more than

and the inlet sonic conductance are negatively correlated.
Table 8: Comparison of sonic flow conductance values.

<table>
<thead>
<tr>
<th>No.</th>
<th>d/mm</th>
<th>L/m</th>
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<td>3.4</td>
<td>5.61</td>
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</table>

the flow rate, and the pipe pressure response time increases. Moreover, when the sonic conductance is large enough, the pipe diameter limits the flow rate. As the pipe diameter increases, the flow velocity gradually increases, and the sonic conductance dominates. Therefore, as the sonic conductance increases, the pipe pressure response time decreases.

The influence of pipe length, pipe diameter, and gas supply pressure on pipe pressure response time is unidirectional. The pipe pressure response time increases with the increase of pipe length, pipe diameter, and supply pressure; the influence of sonic conductance on the pipe pressure response time is negatively correlated; that is, as the sonic flow conductance increases, the pipe pressure response time decreases; there is a coupling effect between the pipe diameter and the inlet sonic conductance; when the pipe diameter or sonic conductance is in the leading role, the pipe pressure response time is only affected by the dominant parameters.

4. Formula Derivation of Pressure Response Time of Pneumatic Brake Pipe

To facilitate the calculation of the pipe pressure response time, the dimension analysis method is used to derive the formula for calculating the pressure response time.

4.1. Formula Derivation of Pressure Response Time of Pneumatic Brake Pipe

Eckersten [22] proposed a formula for calculating the sonic conductance of pipes. It is verified that the average error of the sonic conductance of the pneumatic brake pipe is less than 5% by experiments and simulations. It has been widely used in various fields of industrial production. Therefore, the sonic conductance of the pipes is calculated through this formula and is compared with the sonic conductance of the pipe inlet to confirm the dominant factors.

The formula is as follows:

\[
C_g = \frac{0.029d^2}{\sqrt{L/d^{1.25} + 510}} \tag{10}
\]

Here, L indicates the length of the tube; d indicates the diameter of the pipe; Cg represents the sonic flow conductance of the pipe.

The calculation results of each experimental group are shown in Table 8.

In Table 8, Cr represents the sonic conductance at the entrance, and Cg represents the sonic conductance of the pipe. When \( C_g \geq C_r \), sonic conductance of inlet plays a leading role in pressure response time. The pressure response time decreases as the sonic conductance increases. When \( C_g < C_r \), the pipe diameter plays a major role in the pressure.
response time, and the pressure response time increases as the pipe diameter increases. The results show that when the pipe diameter is 8 mm and the pipe length is 20 m, \( C_g < C_r \); for the remaining groups, \( C_g > C_r \). Using the dimension analysis method to derive the formula for calculating the pressure response time, the unit of the calculation formula is guaranteed to be \( s \), and the other parameters involved are the pipe diameter \( d (m) \), the pipe length \( L (m) \), the sonic flow conductance \( C (m^3/(s \cdot Pa)) \), and gas supply pressure \( p (Pa) \).

**Table 9: Supplementary experimental data.**

<table>
<thead>
<tr>
<th>No.</th>
<th>( d (mm) )</th>
<th>( L (m) )</th>
<th>( Cr (d/m^3/(s \cdot bar)) )</th>
<th>( Cg (d/m^3/(s \cdot bar)) )</th>
<th>( p (MPa) )</th>
<th>( t_1 (ms) )</th>
<th>( t_2 (ms) )</th>
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</table>

response time and the pressure response time increases as the pipe diameter increases. The results show that when the pipe diameter is 8 mm and the pipe length is 20 m, \( C_g < C_r \); for the remaining groups, \( C_g > C_r \). Using the dimension analysis method to derive the formula for calculating the pressure response time, the unit of the calculation formula is guaranteed to be \( s \), and the other parameters involved are the pipe diameter \( d (m) \), the pipe length \( L (m) \), the sonic flow conductance \( C (m^3/(s \cdot Pa)) \), and gas supply pressure \( p (Pa) \).

\[ t \sim \frac{dLp}{C} \rightarrow s \sim \frac{m \cdot m \cdot Pa}{m^3}(s \cdot Pa) = \frac{s \cdot Pa^2}{m} \quad (11) \]

In order to get \( s \), it is necessary to eliminate \( Pa \) and \( m \). Since the unit of \( d \) and \( L \) is \( m \), the formula form of the molecule becomes \( \frac{L^4p}{C^2} \); and the atmospheric pressure \( Pa \) is introduced to eliminate \( Pa \) of the molecule. The formula for calculating the pressure response time is shown as follows:

\[ t = a_1 + a_2 \frac{L^4p}{dC} + a_3 \frac{dL^2p}{Cp_a} \quad (12) \]

**Table 10: Calculation formula \((C_g \geq C_r)\).**

<table>
<thead>
<tr>
<th>Application ranges</th>
<th>Formula forms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_g \geq C_r )</td>
<td>( t = 0.087 + 0.009 \frac{dLp}{Cp_a} + 2.58 \frac{dL^2p}{Cp_a^2} )</td>
</tr>
</tbody>
</table>

The unit of \( L \) on the molecule is \( m \). In order to eliminate the denominator unit \( m \), the formula form of the molecule becomes \( L^4p \); and the atmospheric pressure \( Pa \) is introduced to eliminate \( Pa \) of the molecule. The formula for calculating the pressure response time is

\[ t = a_1 + a_2 \frac{L^4p}{Cp_a} \quad (14) \]

When \( C_g < C_r \), only two sets of experimental data satisfy the size relationship of the sonic conductance in the experimental groups. To improve the accuracy of the formula fitting, the experimental data of the sonic conductance relationship is supplemented, as shown in Table 9.

4.2. Fitting Calculation Formula for Pressure Response Time of Pneumatic Brake Pipe. Based on the above pressure response time calculation formula, the coefficients of the formula are fitted by Istopf* software. The coefficients of the pressure response time formula are fitted and the optimal calculation coefficient is determined by selecting an appropriate algorithm.

\( C_g \geq C_r \). The Levenberg-Marquardt global optimization algorithm is used to optimize the pressure response time formula coefficients. When the number of iterations is 25, the calculation results are optimal, as shown in Table 10.

From Table 10, it can be seen that when the absolute value of the correlation coefficient is closer to 1, the calculation
error of the formula is smaller, and the linear correlation between variables is higher. The decision coefficient is called the goodness of fit and determines the degree of correlation of the formula. The closer the decision coefficient is to 1, the higher the reference value of the formula is. In the table, the correlation coefficient of the formula is 0.97 and the coefficient of determination is 0.94, which are closer to 1. This indicates that the formula has a good fitting effect.

Figure 7 shows the calculation error of the calculation formula. The errors of several calculated values in the figure are above 80 ms, which is larger compared with the experimental value. Moreover, the average error of all calculation results is 30–40 ms.

Only when the calculation formula error is less than 5%, the calculation formula has engineering application value. When the error judgment pressure response time is ranged from 600 ms to 800 ms, the accuracy of the formula meets the requirements, in which the response time of the control line in the pneumatic brake system must not exceed 400 ms, because this calculation formula does not apply to the response time calculation of a pneumatic brake system. It can be seen from the formula of the derivation of the parameter influence that d and L in the molecule are squared to obtain the units, and the influence parameters are integers, which reduces the computational difficulty but limits the computational accuracy of the formula. Thus, the indexes of d, L, C, and p are used as the undetermined coefficient on the basis of the unchanged calculation formula formation as follows:

\[ t = a_1 + a_2 \frac{d^{a_1} L^{a_2} p^{a_3}}{C^{a_4} p_a^{a_5}} \]  

(15)

Here, the undetermined coefficient of the calculation formula is increased and the number of calculation formulas is reduced.

The Levenberg-Marquardt global optimization algorithm is used to optimize the pressure response time formula coefficients. When the number of iterations is 27, the calculation results are optimal, as shown in Table 12.

<table>
<thead>
<tr>
<th>Application ranges</th>
<th>Formula forms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_g \geq C_r )</td>
<td>[ t = 0.075 + 167 \frac{d^{1+\alpha} L^{2+\beta} p^{1+\gamma}}{C^{a_4} p_a^{a_5}} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Units</th>
<th>Correlation coefficient</th>
<th>Decision coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d ) m</td>
<td>( L ) m</td>
<td>( C ) m³/(s·Pa)</td>
</tr>
</tbody>
</table>

When \( C_g < C_r \), the Levenberg-Marquardt global optimization algorithm is used to optimize the pressure response time formula coefficients. When the number of iterations is 17, the calculation results are optimal, as shown in Table 12.

When \( C_g < C_r \), it is clear from Table 12 that, for the calculation formula of the pressure response time of the pipe inflation, the pipe diameter d is in the denominator, indicating that the increase of the pipe diameter will reduce the pressure response time. The tube length L has an index of...
4, indicating that the change in tube length can significantly affect the pressure response time. The correlation coefficient and the decision coefficient of the calculation formula are 0.94 and 0.88, respectively, which indicates that the formula has certain calculation accuracy, but the coefficient of determination is less than 0.9, which lacks reference value.

Figure 9 shows that the calculation error of individual groups is more than 40 ms, and the calculated average error is 26 ms. When \( C_g < C_r \), the pipe pressure response time is generally below 400 ms; then the calculation error of the formula is more than 7%. Therefore, the calculation error of the formula does not meet the accuracy requirement and is not applicable to the calculation of the pressure response time of the pneumatic brake pipe. To improve the accuracy of the calculation formula, the original formula is optimized. The exponents of \( d \), \( L \), \( C \), and \( p \) are set to the undetermined coefficients, and the form of the formula remains unchanged. The improved formula is as follows:

\[
t = a_1 + a_2 \frac{L^a_1 p^a_2}{C_p^{a_3}}
\]

(16)

The above formula contains six undetermined coefficients, and the complexity of the formula form is slightly increased, but the probability of improving the accuracy is increased. The Levenberg-Marquardt global optimization algorithm is used to optimize the pressure response time formula coefficients. When the number of iterations is 34, the calculation results are optimal, as shown in Table 13.

The correlation coefficient and the decision coefficient of the formula in Table 13 are 0.99 and 0.99, respectively, and the accuracy and reference value of the improved formula are significantly improved. The improved formula improves the calculation accuracy by increasing the number of parameter indices, so the improved formula is reasonable.

Figure 10 shows an analysis of the calculation formula of the improved pipe inflation. In Figure 10(a), the solid line indicates the experimental value, the dash line indicates the formula calculation value, and Figure 10(b) shows the calculation error of the calculation formula. Figure 10(a)
show that the trend of the experimental value and the calculated value are consistent, and there is a small deviation at the corners of the two curves. The maximum error of the calculation formula in Figure 10(b) is 3.5ms, which is much smaller than the calculation error of the premodification formula. The error is 1%~2% and the accuracy is reliable. Therefore, when $C_g < C_r$, the inflation calculation formula is applicable to the pneumatic brake line, which provides a theoretical reference for the calculation of the pipe pressure response time.

### 4.3. Summary of Calculation Formula for Pressure Response Time of Pneumatic Brake Pipe

The form of the calculation formula is derived based on the dimension analysis method, and the optimization algorithm is used to obtain the undetermined coefficient of the calculation formula; then the calculation formula of the pressure response time is fitted. The calculation formula of the pneumatic line pressure response time is summarized in Table 14.

The calculation formula obtained by this method consists of constant term and fractional term. The error of calculation formula is about 8 ms. It can judge the increase and decrease of pressure response time according to the parameter change, and it is easy to determine the parameters of the pipe.
5. Conclusions

In this paper, under the engineering application of pneumatic brake system, the calculation formula of the pressure response time of pneumatic brake pipe is obtained based on the experiment. The results show that the calculation formula is accurate and has engineering reference value.

Based on the experiment of pneumatic brake pipe, the influence law of each parameter on the pressure response time of pneumatic brake pipe is quantitatively analyzed by fuzzy correlation analysis method. The pipe length is the parameter that has the greatest influence on the pressure response time, and the supply pressure is the parameter that has the least effect on the pressure response time.

The multiparameter variation analysis method is used to analyze the change of pressure response time when the parameters change. The results show that the tube length and supply pressure are positively correlated with the pressure response time of the pipe, and the tube diameter and the inlet sonic conductance are coupled. When the pipe diameter or sonic conductance are in the leading role, the pressure response time is only affected by the dominant parameters, which lays a theoretical foundation for the derivation of the calculation formula.

The calculation formula based on the experimental data directly shows the influence trend of each parameter on the pressure response time, and the formula is simple. By comparing the sonic conductance of the inlet and the pipe, the calculation formula is segmented to ensure the calculation accuracy, which provides a theoretical reference for the design of the pneumatic pipe.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References
