Modeling of the Tail Slap for an Underwater Projectile within Supercavitation

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Abstract

Detailed process of the tail slap for an underwater projectile in supercavitation is studied in this paper. Firstly, the horizontal equation of motion for the projectile and a dynamic model of the projectile’s tail slap are introduced through a mathematical derivation, including a simple harmonic motion model of the projectile within the supercavitation layer and the tail slap reflection movement model inside the cavity. Subsequently, the MATLAB software package is used to model the projectile’s motion. Characteristics for force from the projectile tail on the water layer during a single tail slap and the behavior of the projectile’s angular motion as well as the characteristics of projectile tail motion through multiple tail slaps are investigated. It is found that the force magnitude of the water on the projectile tail decreases gradually while the relative motion angle of the projectile tail into the water layer and the contact time increase obviously as the projectile repeatedly slaps the cavity interface. When the projectile velocity decreases into a certain range, i.e., less than 50 m/s, the contact time of the tail in the water layer increases dramatically, and the supercavitation can tend to collapse and lead to unstable projectile motion. With increasing lateral deviation and angular velocity of the projectile tail, the number of tail slaps and the angle of reflection both increase significantly. In the case of supercavitation, the parameters of different tail slap points tend to be stable with time.

1. Introduction

When an underwater vehicle moves within a certain speed high enough, its surface pressure is lower than the saturated vapor pressure of local water, thus forming a cavity that can surround nearly the entire projectile. This phenomenon is called natural supercavitation [1, 2]. When supercavitation occurs, only a small part of the tip of the vehicle is in contact with liquid water, and the rest only contacts the vapor in the cavity. In this case, surface friction is greatly reduced, which can ensure that the underwater vehicle can maintain high speed efficiently, greatly improving its performance [3, 4]. However, since the cavity itself is inside the interface of liquid phase and gas phase, the instability of this two-phase flow is obvious [5]. Meanwhile, the environmental pressure of the vehicle is affected by multiple external factors, and there are problems such as perturbation of hydrodynamic coefficients and unknown interaction from wake flow [6]. These further affect the persistence of the cavity and the force on the vehicle and greatly increase the complexity of the motion of the supercavitation vehicle [7, 8].

An underwater high-speed projectile is a typical supercavitation vehicle. Tail swing is very likely to occur when the underwater high-speed projectile moves in the supercavitation cavity, due to external disturbance or its own unstable characteristics [9]. That is, slaps occur between the tail and the upper and lower walls of the cavity, and the projectile body attitude is corrected to maintain stability by these slap forces. Rand [10] made a very valuable pioneering exploration on the tail slap dynamics analysis of supercavitating projectile. Subsequently, Zhao et al. [11] researched tail slap phenomenon during projectile horizontal movement. Their work included the influences of initial disturbance on tail slap motion, the cavity shape, and ballistic characteristics as well as the tail slap motion characteristics of supercavitating projectiles with different center of mass positions. Li et al. [12] studied the tail slap of supercavitation vehicles for different motion conditions and compared the effects of gravitational
acceleration on tail slaps at different speeds. Zhang [13] et al. developed an improved method to obtain the length of a projectile that was immersed in a cavity boundary. He et al. [14] established the equations of motion for tail slap coupling in supercavitation vehicles. They studied the fluid-solid coupling response of the supercavitating projectile during the tail slap process and analyzed the influence of fluid-solid coupling on the attitude and dynamic characteristics of the projectile. Zou et al. [15] established an analysis model of supercavitating projectile motion through the gas mass conservation equation and studied factors such as gravity and angle of attack on the tail slap dynamics of a supercavitating vehicle. Mirzaei et al. [16, 17] established a nonlinear dynamic model to simulate the process of two-dimensional planar force acting on the tail of a supercavitation vehicle. They provided an equation to model the case in which the tail of the high-speed supercavitating projectile slaps on the cavity wall surface and established an empirical model for predicting the tail slap of the projectile. Kulkarni et al. [18] directly simulated the slaps of the projectile tail and cavity walls and predicted the behavior of slaps between the projectile’s tail and the cavity walls. Zhao et al. [19] established a computational fluid dynamics model under different initial disturbances and studied the ballistic characteristics when tail slaps occur. Zhang et al. [20] investigated the tail-slap loads of supercavitating projectiles based on Logvinovich’s Principle. They concluded that the impact duration increases while frequency of impacts decreases as the slenderness ratio of projectiles increases.

Tail slap has been extensively studied as a main stable or quasistable motion model for supercavitating bodies in water. The motion characteristics of tail slap remain unclear due to the instability of the two-phase flow of the supercavitation and the complexity of projectile motion in the cavity. Although plenty of works have been carried out to study the motion characteristic of tail slap as stated above, only minor of them are focused on the detailed process of the motion of the tail slaps, which is always significant for the application of a supercavitating projectile. In view of this, the characteristics of projectile forward motion and tail slap motion are studied through a simplified supercavitating projectile with both single tail slap in the water layer and multiple tail slaps in the cavitation being taken into account. A mathematical model is introduced to demonstrate the tail slap process, and influence of parameters including reflection angle, impact time, and cavity radius on tail slaps is discussed. Diagrams of tail slaps on various conditions are proposed to provide a different perspective to sketch the motion of projectiles in supercavitation, which may provide a reference for a better understanding of the interesting tail slap phenomenon.

2. Dynamic Model of Supercavitating Tail Slap

2.1. The Model of Forward Motion. A supercavitating projectile is affected by hydrodynamic force, vapor force in the cavitation, and external disturbances during movement. The movement is also closely related to the shape of the projectile and characteristics of the fluid medium. The actual dynamic characteristic of the projectile is very complicated. Considering that the supercavitating projectile is in horizontal motion, the following assumptions are made according to Rand [10]:

(1) the effect of gravity on the trajectory of the projectile’s center of mass is negligible;
(2) the projectile is treated as a rigid body and its elastic deformation is not taken into account;
(3) in the process of projectile motion, the projectile body can move forward steadily during the contact between the tail and the cavitation liquid-vapor interface, regardless of the influence of vapor in the supercavitation;
(4) the projectile nose is always located at the central line of the cavity. The projectile body swings around the nose and does not rotate around its own axis of symmetry during its motion;
(5) the cavity’s shape is symmetrical with respect to the horizontal line of motion of the projectile, and the diameter of the cavity decreases with decreasing projectile velocity. The cavity radius at the tail end of the projectile is \( r \). It is generally acknowledged that the cavity radius for moving body increases as the increase of velocity; therefore a linear relationship with the forward velocity \( \dot{x} \) of the projectile is introduced to simplify the prediction model of cavity radius:

\[
r (\dot{x}) = C_1 \dot{x} + C_2
\]  

where \( C_1 \) and \( C_2 \) are undetermined coefficients. These coefficients are determined based on the given projectile velocity and cavitation parameters (generally obtained from simulations or experiments).

In the above assumptions, the motion of the supercavitating projectile is simplified into a rigid body motion model containing only translation and oscillation. When the projectile tail is not in contact with the liquid, its motion is modeled as translation in the \( x \)-direction and rotation about the projectile’s nose at a constant angular velocity, as illustrated in Figure 1. The horizontal distance from the fixed launching datum to the center of mass is set as \( x \), and the projectile body is subject to drag force \( F_D \) at its nose. A simplified model of supercavitating projectile motion is obtained according to these assumptions, as shown in Figure 1.

The displacement of the projectile’s center of mass due to small inclination angle \( \alpha \) is ignored. The axis of the projectile and cavity centerline coincides in the \( x \)-direction and the following equation can be obtained according to the experimental results of May et al. [21] and Newton’s second law:

\[
m\ddot{x} = -F_D = \frac{1}{2} \rho A \dot{x}^2 k \cos^2 \alpha
\]  

where \( \rho \) denotes the density of water, \( A \) is the cross-sectional area of the nose, \( \dot{x} \) is the forward speed as above, \( \dot{x} \) is the acceleration, \( m \) is the mass of the projectile, and \( k \) is the dimensionless parameter (generally set as 0.9).
Setting \( \epsilon = \rho A k / 2m \), when \( \alpha \) tends to be 0, the above equation can be turned into

\[
\lim_{\alpha \to 0} -\frac{F_D}{m} = \lim_{\alpha \to 0} -\frac{\rho A k}{2m} x^2 \cos^2 \alpha = \lim_{\alpha \to 0} -\epsilon \dot{x}^2 \cos^2 \alpha = -\epsilon \dot{x}^2
\]  
(3)

From (3), we find

\[
\ddot{x} = -\epsilon \dot{x}^2
\]  
(4)

where \( \dot{x} = \frac{dx}{dt} \). Letting \( \dot{x}_0 \) denote the initial velocity of the projectile, the following equations can be obtained by solving the differential equation to give

\[
x(t) = \frac{1}{\epsilon} \ln \left( 1 + \epsilon \dot{x}_0 t \right)
\]  
(5)

\[
\dot{x}(t) = \frac{\dot{x}_0}{1 + \epsilon \dot{x}_0 t}
\]  
(6)

By substituting (6) into (1), we get

\[
r(t) = C_1 \frac{\dot{x}_0}{1 + \epsilon \dot{x}_0 t} + C_2
\]  
(7)

For the above mathematical model, the initial conditions for homogeneous projectile are given as follows [10]: the projectile length is \( L = 0.12 \text{ m} \); the projectile mass is \( 0.15 \text{ kg} \); the projectile tail diameter is \( D = 0.015 \text{ m} \); the nose diameter is \( D_0 = 0.002 \text{ m} \); the taper angle of nose is \( 25^\circ \) and the corresponding cross-sectional area is \( A = 3.14159 \times 10^{-6} \text{ m}^2 \); the distance from the tail of the projectile to its center of mass is \( a = 0.05577 \text{ m} \); and the moment of inertia of the projectile about its center of mass is \( I = 0.000777 \text{ kg m}^2 \). Suppose that the initial velocity of the projectile is \( \dot{x}_0 = 1500 \text{ m/s} \) with the diameter of the cavity at the projectile tail being \( r = 0.05 \text{ m} \), and \( \dot{x} = 100 \text{ m/s} \) with \( r = 0.03 \text{ m} \). According to (1), \( C_1 = 0.0001429 \) and \( C_2 = 0.02857 \), i.e., \( r(\dot{x}) = 0.00001429 \dot{x} + 0.02857 \). The MATLAB software package is used to simulate forward motion of the projectile, and results for velocity and displacement versus time are demonstrated in Figure 2. It is found that the present results are in good agreement with that got from Rand [10], as shown in Figure 2. The projectile’s velocity rapidly drops from 1500 m/s to 391.9 m/s within the first 0.2 s, during which the displacement of the projectile is 142.5 m. Then, the velocity slowly decreases, and the displacement slowly increases, reaching 51.2 m/s and 358.3 m at 2 s, respectively. In general, the minimum speed range required for the projectile to maintain a supercavitation (i.e., cavitation number \( \sigma \leq 0.1 \)) is about 40~50 m/s, so that the projectile may lose stability after 2 s of motion due to the collapse of supercavitation; its subsequent motion characteristics at lower velocity are beyond the scope of this article.

### 2.2. Modeling Tail Slap Motion

Based on the above-mentioned forward motion model for the projectile, motion characteristics of its tail slap will be discussed next. When the forward motion is disturbed by a random and instantaneous perturbation from surroundings or the cavitation’s instability,
the projectile is modeled as a rigid body whose nose is fixed and the tail swings around the nose based on previous. The tail will slap repeatedly on the supercavity interface around the projectile and a single tail slap is taken into account to study the slap details firstly.

If $\delta$ indicates the maximum distance that the projectile tail immersed into the water layer through the cavity interface during slap (Figure 3), then

$$\delta = L \sin \alpha + \frac{D}{2} \cos \alpha - r \quad (8)$$

where $L$ is the length of the moving body and $D$ is the diameter of the projectile tail.

According to the above model, thickness of the water layer disturbed by the projectile is defined as $\lambda \delta$ (where $\lambda$ is a coefficient less than 1) and the cross-sectional area of fluid layer is written as $A_1 = \lambda \delta \cdot D$. The longitudinal and lateral forces of the cavity boundary acting on the projectile tail are $R_D$ and $R_L$, respectively, as shown in Figure 3. The following equations can be obtained due to the conservation of momentum:

$$R_D = \rho A_1 \dot{x}^2 (1 - \cos \alpha) \quad (9)$$

$$R_L = \rho A_1 \dot{x}^2 \sin \alpha \quad (10)$$

Then, the swing of the projectile body around the nose satisfies the following equation:

$$I \ddot{\alpha} = -R_D \alpha \sin \alpha - R_L \alpha \cos \alpha \quad (11)$$

When $\alpha$ is small, $\sin \alpha$ can be approximated as $\alpha$ and $\cos \alpha$ as 1. By substituting (9) and (10) into (11), we get

$$I \ddot{\alpha} + \rho \lambda [L\alpha + \frac{D}{2} - r(\dot{x})] \ D \dot{x}^2 \alpha = 0 \quad (12)$$

Let $\Phi$ be the relative movement angle of the projectile tail slap; then we have $\Phi = \alpha - [2r(\dot{x}) - D]/2L > 0$. Eq. (12) can be written as

$$I \ddot{\Phi} + \rho \lambda D \dot{x}^2 \dot{\Phi}^2 + \rho \lambda D \dot{x}^2 \left[ r(\dot{x}) - \frac{D}{2} \right] \Phi = 0 \quad (13)$$

If the slap time is short, $\Phi$ is small and $r(\dot{x})$ can be seen as constant. Neglecting the term with $\Phi^2$ of (13), we can simplify the equation and end up with

$$I \ddot{\Phi} + \omega_0^2 \Phi = 0 \quad (14)$$

where $\omega_0^2 = (\rho \lambda D \dot{x}^2 / I) [r(\dot{x}) - D/2]$.

The equation represents a process of the moment when $\Phi$ is 0 (the initial slap point of the tail passes through the cavitation wall and into the water layer) to the moment when $\Phi$ again crosses 0 (the initial slap point of the tail passes back into the cavitation bubble and exits the water layer). Therefore, the duration of the slap is half of the simple harmonic vibration period, that is, $\Delta T = T/2 = \pi/\omega_0$. Through solving the above simple harmonic equations, there is

$$\Phi = A_0 \cos (\omega_0 t + \varphi) \quad (15)$$

where $A_0 = \omega L / \omega_0$ is the amplitude of the simple harmonic vibration of the projectile in the water layer and $\varphi = -\pi/2$ is the initial phase of the simple harmonic vibration.

The forces on the projectile in the water layer can then be expressed as

$$F = -Lm A_0 \omega_0^2 \cos (\omega_0 t + \varphi) \quad (16)$$

Based on the kinetic model and the above analysis, it is assumed that each tail slap of the supercavitating projectile is an instantaneous dynamic process and that the angular velocity is constant and the slap reflection angle is equal to the incident angle before and after a single tail slap. Let us define that the initial position of the projectile tail is located at the center line of the cavitation offset $\Delta x$ (initial lateral deviation) and swings upward with the nose at the center. According to the change of the cavitation cavity size with time, the location of the next slap point can be predicted. Therefore, the movement of the projectile tail in the supercavitation is essentially a slap problem with shrinking boundary (Figure 4). The parameters of the first slap point are given: $\omega$ is the angular velocity of the projectile body around the nose (the angular velocity of the tail slap),
\[ \theta_1 \] is the incident angle, \( t_1 \) is the time from the initial point to the first slap point, and \( r_1 \) is the cavitation radius of the slap point. Then, there is \( r_1' = r_1 - D/2 \), so that

\[ \begin{align*}
\theta_1 &= \arctan \frac{dx}{\omega L t_1} \\
 t_1 &= \sqrt{(r_1 - D/2)^2 - dx^2} / \omega L \\
r_1 &= \sqrt{dx^2 + (\omega L t_1)^2 + D/2}
\end{align*} \tag{17} \]

As shown in Figure 4, \( \theta_1 \) is the angle between the center of the circle E at the symmetric position of the first slap point and the line OE at the center O of the cavity and the direction line EF at the first slap point. From the first slap point to the second slap point, the projectile's motion in the cavity is due to slaps between the projectile tail and the cavity boundary. When the projectile moves towards the second slap point, the cavity at the first slap point shrinks from E point along the EO direction. The cavity and projectile intersect at point A, and the central position of the projectile tail is point B. In the direction of the EF line, the following can be obtained:

\[ \begin{align*}
\left( r_1 - \frac{D}{2} \right) \cos \theta_1 &- \sqrt{\left( r_2 - \frac{D}{2} \right)^2 - \left( r_1 - \frac{D}{2} \sin \theta_1 \right)^2} \\
+ \omega L (t_2 - t_1) &= 2 \left( r_1 - \frac{D}{2} \right) \cos \theta_1
\end{align*} \tag{18} \]

By analogy, the relationship between the \( n \)th slap and the \((n+1)\)th slap is

\[ \left( r_n - \frac{D}{2} \right) \cos \theta_n - \sqrt{\left( r_{n+1} - \frac{D}{2} \right)^2 - \left( r_n - \frac{D}{2} \sin \theta_n \right)^2} \tag{19} \]

\[ + \omega L (t_{n+1} - t_n) = 2 \left( r_n - \frac{D}{2} \right) \cos \theta_n \]

According to the iteration of the above equation, the cavitation radius and the tail slap reflection angle at the occurrence of the \((n+1)\)th tail slap are as follows:

\[ r_{n+1} = C_1 \frac{\dot{x}_0}{1 + \varepsilon x_{\text{ef}} t_{n+1}} + C_2 \]

\[ \theta_{n+1} = \arcsin \left( \frac{r_n - D/2 \sin \theta_n}{r_{n+1} - D/2} \right) \tag{20} \]

By obtaining the above parameters, the state change of the projectile when the tail swings in the supercavitation can be obtained.

### 3. Results and Discussion

Figure 5 shows the curves of force and relative motion angle for projectile tail in water layer with time during repeated tail slaps composed of multiple single tail slaps. The force amplitude of the projectile tail in harmonic motion in the water layer decreases with the continuous occurrence of the tail slap and the attenuation trend gradually decreases with time. Among them, the force amplitude of the first tail slap is 10 N, while that of the second tail slap decreases to about 4 N, decreasing by 60%. The relative motion angle which demonstrates the amplitude of the projectile tail in the water
MATLAB is further used to simulate the whole process of projectile tail slap movement through a 2 s time interval. The specific working conditions and corresponding tail slap frequency are shown in Table 1. Figures 7 and 8 show the schematic diagram of projectile tail motion relative to the central axis of the cavity at different angular velocities ($\omega = 1, 2 \text{ rad/s}$) and different initial lateral deviation conditions ($\Delta x = 0.01 \text{ m}, 0.02 \text{ m}$), respectively. Under the same initial side deviation $\Delta x$, the number of tail slaps increases with the increase of angular velocity $\omega$, and the effect of cavitation superposition gradually becomes obvious, as shown in Figure 7. When the angular velocity $\omega$ of tail slap is constant, the number of tail slaps increases as $\Delta x$ increases (Table 1), and the tail slap trajectory is closer to the cavitation bubble wall.

Table 2 shows the variation of cavitation radius $r_n$, tail slap reflection angle $\theta_n$, and tail slap occurrence time $t_n$ under typical conditions ($\Delta x = 0.01 \text{ m}, \omega = 2 \text{ rad/s}$). During the tail slap sequence from 0 to 2s, the cavity radius decreases and the rate of speed loss decreases, as the projectile moves forward and the tail slap occurs. The radius decreases from 0.0368 m for the first slap to 0.0324 m for the second slap. The cavitation radius of the 5th tail slap is 0.0302 m, while the cavitation radius from the 6th to the 11th is stable at around 0.029 m. The reflection angle gradually increases with the continued tail slaps, and the increasing trend decreases constantly, which is basically consistent with the trend of the cavitation radius. The time interval between two adjacent tail slaps also decreases slowly from 0.2096 s, which is about 0.1777 s between the 10th and 11th tail slaps; that is, the time interval between tail slaps is generally stable in the range of 0.17 s to 0.21 s.

Figure 9 shows the cavitation shrinkage curve and diagram of the tail slap process with different tail slap angular velocities ($\omega = 1, 2, 3 \text{ rad/s}$) and different initial lateral
Figure 7: The motion trajectory of the projectile tail over 2 s with different tail slap speeds.

Figure 8: Motion trajectory of the projectile tail over 2 s with different initial lateral deviations.

Table 1: Tail slap frequency of projectiles with different conditions.

<table>
<thead>
<tr>
<th>Statistical time</th>
<th>Lateral deviation</th>
<th>$\omega = 1$ rad/s</th>
<th>Slap frequency $\omega = 2$ rad/s</th>
<th>Slap frequency $\omega = 3$ rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>In 1 s</td>
<td>$\Delta x = 0$ m</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = 0.01$ m</td>
<td>3</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = 0.02$ m</td>
<td>4</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>In 2 s</td>
<td>$\Delta x = 0$ m</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = 0.01$ m</td>
<td>3</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = 0.02$ m</td>
<td>5</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>
deviations ($\Delta x = 0.01, 0.02, 0.03$ m), which further explains the tail slap frequency and tail slap points depicted in Figures 7 and 8. Figure 10 depicts the variation of the reflection angle of slap points caused by the projectile’s lateral deviations. When the lateral deviation is zero, the projectile is reflected along the vertical direction of the cavitation center line, and its reflection angle is zero. The reflection angle gradually increases as the lateral deviation increases. In comparison to the curve with lateral deviations of 0.01 m and 0.02 m, when the angular velocity remains unchanged, slap points increase with the increase of lateral deviation.

4. Conclusion

In this paper, a simplified model of the tail slap motion of a supercavitating projectile in water is established. In the case that the angular velocity energy loss is not considered, motion of the projectile is modeled as translation in the $x$-direction and rotation about the projectile’s nose at a constant angular velocity. The forward speed and the forward path of the projectile are inversely proportional functions and logarithmic functions with time as the independent variable, respectively. The projectile spends less time and has a relatively smaller relative motion angle during the tail slap process, and the angular velocity remains constant when it is not in contact with the cavitation bubble’s liquid surroundings. This makes the rebound an instantaneous process when the projectile tail
slaps the cavitation bubble’s boundary. The problem of the projectile tail slap motion under this condition is a dynamic boundary slap problem in which the boundary is constantly changing, the variation law is known, and the incident angle is equal to the reflection angle. The position of the tail slap point is determined by the cavitation radius of the previous slap point, the time at which the tail slap occurs, and the incident angle. From the tail slap simulation, the following conclusions can be drawn:

1. The force of the projectile in the water layer decreases with the increase of tail slap time and number of slaps, resulting in an increase of the contact time between the tail and the liquid water, and relative motion angle of the projectile in the water layer. However, the contact time and relative swing angle of projectile in the water layer are relatively small.

2. Within the 2 s simulated time, the contact time of the projectile with the water layer is much smaller than the projectile’s tail slap harmonic period, and the slap of the projectile and the cavitation wall can be regarded as an instantaneous effect. When the stay time of projectile tail in the water layer continues to increase dramatically (t > 2 s), the projectile velocity drops below 50 m/s; that is, supercavitation possibly collapses and projectile motion instability may then occur.

3. With the initial lateral deviation of the projectile and the increase of the angular velocity of the projectile tail, the time of the first contact between the projectile tail and the cavitation boundary advances, and the number of slaps between the projectile tail and the cavitation wall surface also increases. Furthermore, the trajectory map of the projectile’s tail slap becomes more complex, the reflection angle of the slap points gradually increases, and the trajectory of the projectile gets closer to the contour of the cavity. In the case of supercavitation, parameters such as the cavitation radius of the tail slap point, the reflection angle, and time intervals between two tail slaps tend to be stable as the number of tail slaps increases.

Data Availability

(1) The reference data in Figure 2 (Rand’s results) were from the paper “Impact Dynamics of a Supercavitating Underwater Projectile” at http://audiophile.tam.cornell.edu/randpdf/ahsum.pdf. (2) The geometry data and some initial motional parameters for the projectile were from the paper “Impact Dynamics of a Supercavitating Underwater Projectile” at http://audiophile.tam.cornell.edu/randpdf/ahsum.pdf. (3) All other data in this paper were generated at School of Energy and Power Engineering, Nanjing University of Science and Technology. Derived data supporting the findings of this study are available from the corresponding author [Xujian LYU at XJLYU@njust.edu.cn] on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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