Research Article

New Method of State-Space Formulation for Degenerate Circuit and Coupling Circuit

Chung-Cheng Chen ¹, Jian Ke,¹ and Yen-Ting Chen^{2,3}

¹City College of Dongguan University of Technology, No. 1, Wenchang Road, Liaobu Town, Dongguan, Guangdong, China

²Department of Automatic Control Engineering, Feng Chia University, No. 100, Wenhwa Rd., Seatwen, Taichung 40724, Taiwan

³Department of Electrical Engineering, Graduate School, National Chung Hsing University, 145 Xingda Rd., South Dist., Taichung City 402, Taiwan

Correspondence should be addressed to Chung-Cheng Chen; ccc49827@ms25.hinet.net

Received 5 January 2019; Revised 29 March 2019; Accepted 16 April 2019; Published 17 June 2019

Academic Editor: Gaetano Zizzo

Copyright © 2019 Chung-Cheng Chen et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The state-space formulation overcomes many limitations of traditional differential equation approach and is utilized as alternative to many traditional approaches in the modern electrical field. This paper proposes a new method of finding the state equation for degenerate circuit and coupling circuit that have not been systematically solved now. This paper also introduces some sound improvements to solve complicated dependent-source circuits. Four comparative examples are demonstrated to show the significant merits that our method owns over the traditional approaches.

1. Introduction

The state-space formulation plays a significant part to probe the significant properties including robustness and stability in the electrical related theories [1, 2]. The state-space formulation overcomes many nonsystematic limitations of traditional differential equation approach in the modern electrical field [3, 4].

The work proposed in [4] has focused on reducing the computational complexity of the state equation by substituting capacitors with voltage generators and inductors with current generators for the electrical circuits. At this point, the circuit contains only generators and resistors. The claim cannot work for the electrical circuit with coupling circuit due to the appearance of both inductor and mutual differential item Mdi_L/dt . Apply the substitution theorem such that the capacitors (inductors) are changed with the current sources (the voltage sources), respectively, in our previous result [5]. The substitution theorem prevents the derivative elements Ldi_L/dt and Cdv_C/dt from being in the electrical system. However, [5] cannot derive the state equation for electrical circuit with coupling elements due to the coupling term Mdi_L/dt . In order to overcome this shortcoming, we

propose a new method of substituting the coupling term with $Mdi_{L2}/dt = (M/L_2)v_{L2}$, $Mdi_{L1}/dt = (M/L_1)v_{L1}$, and hence the problem of the coupling differential item Mdi_L/dt can be solved.

References [4] and [5] have claimed that the proposed method of finding the state equation cannot to be applied to degenerate circuits in which there are fewer state variables than energy-storage elements (capacitors and inductors), where the excess components are called the redundancy elements. The study of degenerate networks with the redundancy element will need more mathematical work for advanced electrical networks theories. On the other hand, a new method of finding the state equations for degenerate circuit is successfully presented in this paper. In order to solve the difficulty for degenerate circuits, we propose a new method of putting the redundancy element $R_e(t)$ into evidence in proper current or voltage terms $i_{C2} = (\pm C_2/C_1)i_{C1} \pm C_2\dot{R}_e(t)$ and $v_{L2} = (\pm L_2/L_1)v_{L1} \pm L_2\dot{R}_e$ and hence the problem of the redundancy element can be solved.

Reference [3, page131] had claimed that the problem will yield increased difficulties in using network analysis to obtain the state equation for the electrical network with a dependent

source. The existing studies [6, page683] [7, page521] [8] show that the designs of the state equation become very complex for dependent-source circuit. The main characteristic of this paper is exploited in its systematic and unified structure that solves the limitations of those traditional approaches.

Up to now, the existing conventional approaches for the derivation of state-space formulation are summarized as follows.

Method 1. Apply the Thevenin equivalent theorem to get the state equation [9]. The conventional standard methods of deriving the Thevenin equivalent circuit need two steps as follows: (Step 1) Let the load device be open-circuited and the Thevenin-voltage parameter is calculated as the open-circuit voltage across the load device. From the existent literatures [10, 11], it is obvious to see that the calculation of the open-circuit voltage is tedious when the network has the complicated devices consisting the dependent sources and coupling devices based on using the Kirchhoff's voltage law (Kirchhoff's current law). (Step 2) The left work of finding Thevenin equivalent circuit is the derivation of the Thevenin-resistance parameter. There are mainly three techniques for the derivation of Thevenin-resistance in existent researches as follows: (Technique 1) Set all independent sources to be zero with the load device being disconnected and then the equivalent resistance of the zero-energy circuit at the load device is the desired Thevenin-resistance parameter via the conventional node-voltage matrix method [12]. Technique 1 cannot be utilized when some voltage (current) source is not in series (parallel) with a resistor based on the intrinsic properties of the conventional node-voltage (mesh-current) matrix method. (Technique 2) Find the open-circuit voltage V_{OC} and the short-circuit current I_{SC} across the load device, then the Thevenin-resistance parameter is equal to $Z_{TH} = V_{OC}/I_{SC}$ [10, 13]. Technique 2 is not only impractical but also at the risk of encountering the interminate case 0/0. For some circuits consisting only dependent sources and resistors, one needs to apply the alternative method because the ratio of the open-circuit voltage V_{OC} to the short-circuit current I_{SC} is interminate form 0/0. (Technique 3) Firstly set all independent sources to be zero with the load device being disconnected. Then, use additionally the independent current (voltage) source I_t (V_t) to the load device and thus find the load voltage (current) voltage (current) V_k (I_k) by $Z_{TH} = V_k/I_t$ ($Z_{TH} = V_t/I_k$) [11]. Technique 3 is very restricted to certain conditions due to the requirement of setting all independent sources to be zero. Moreover, it also is obvious to see that using conventional Technique 3 is very messy based on using the Kirchhoff's voltage law (Kirchhoff's current law).

Method 2. Obtain the state-space formulation for electrical circuits based on the superposition theorem [14]. In particular, it formulates an n-order circuit and splits this into n first-order circuit, each of which is solved separately. However, the splitting method is not easily be applied to get the state-space formulation when the network has the complicated devices consisting the bridging feedback element or dependent sources devices due to the intrinsic properties.

Method 3. Formulate the state equation for electrical via the normal tree theory [15]. The method defines a complex normal tree in order to select the state variables. Moreover, it builds a set of equations via the nonsystematic Kirchhoff's law.

Method 4. Get the state-space formulation for electrical circuits based on using filling matrices [16, 17]. The method is only limited to nondegenerate circuits. Moreover, the operations of inverting the matrices make the method be impractical.

To solve all above shortcomings of the conventional approaches, we have proposed an efficient method for the calculation of the state-space formulation in a straightforward way. The significant novelties of this proposed approach include the following: (1) it can easily be applied to the electrical circuits that voltage (current) source is not in series (parallel) with a resistor, mutual-inductance element, and the dependent-source element without using tedious Kirchhoff's law; (2) it does not formulate an n-order circuit and split this into n first-order circuit; (3) it is conceptually simpler since it does not need the use of normal tree theory; (4) it is applicable to both nondegenerate and degenerate circuits; (5) it does not need impractical operations of inverting the matrices.

2. New Method of Finding the State Equations

View the capacitor voltages and the inductor currents as state variables. The main structure of our proposed new simple unifying approach is given below.

Step 1. Apply the substitution theorem to replace the capacitors with the current sources and the inductors with the voltage sources. Using the source-transformation theorem changes all voltage elements to be current elements and all current elements to be voltage elements, respectively.

If the voltage element is not in series with a resistor and the current element is not in parallel with a resistor, we add a zero resistor to be in series with the voltage element and an infinite resistor to be in parallel with the current element.

If the desired circuit contains the coupling elements, the coupling element can be replaced with $Mdi_{L2}/dt = (M/L_2)v_{L2}$, $Mdi_{L1}/dt = (M/L_1)v_{L1}$. If the considered circuit is degenerate with redundancy voltage $v(t)$ or current $i(t)$, then we elaborate in such a way to show the redundancy element into equation $i_{C2} = (\pm C_2/C_1)i_{C1} \pm C_2\dot{v}(t)$ and/or $v_{L2} = (\pm L_2/L_1)v_{L1} \pm L_2\dot{i}(t)$ where current generator i_{C2} substitutes capacitor $C2$ and voltage source v_{L2} substitutes inductor $L2$ on a case by case basis.

Step 2. Draw up the matrix node-voltage equations or matrix mesh-current equations by the following matrix equations:

$$\begin{bmatrix} Y_{11} & -Y_{12} & \cdots & -Y_{1N} \\ -Y_{21} & Y_{22} & \cdots & -Y_{2N} \\ \vdots & \vdots & & \vdots \\ -Y_{N1} & -Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$= \begin{bmatrix} \pm I_{S11} \pm I_{S12} \pm \cdots \\ \pm I_{S21} \pm I_{S22} \pm \cdots \\ \vdots \\ \pm I_{SN1} \pm I_{SN2} \pm \cdots \end{bmatrix}$$

(1)

$$\begin{bmatrix} Z_{11} & -Z_{12} & \cdots & -Z_{1N} \\ -Z_{21} & Z_{22} & \cdots & -Z_{2N} \\ \vdots & \vdots & & \vdots \\ -Z_{N1} & -Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

(2)

$$= \begin{bmatrix} \pm V_{S11} \pm V_{S12} \pm \cdots \\ \pm V_{S21} \pm V_{S22} \pm \cdots \\ \vdots \\ \pm V_{SN1} \pm V_{SN2} \pm \cdots \end{bmatrix}$$

where V_{sij} and I_{sij} are denoted as the additional voltage sources and the current sources respectively, Z_{ij} and Y_{ij} are denoted as the impedance and the admittances, respectively, and V_i and I_i are denoted as the node-voltage variables and the mesh-current variables, respectively. Moreover, I_{ii} , $i = 1, 2, \dots, N$ are in clockwise direction.

If the dependent current elements and voltage elements are concerned in the listed matrix, then the dependent elements must be changed to be node-voltage elements or mesh-current elements and then the easy moving-term processes are performed.

Step 3. Applying our invented easy matrix operations changes the node-voltage elements or mesh-current elements to the state elements v_{Ck} , i_{Lk} . Some easy and significant matrix operations are outlined as follows:

(rule i)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(3)

$$\begin{bmatrix} A & B \cdot k & C \\ D & E \cdot k & F \\ G & H \cdot k & I \end{bmatrix} \begin{bmatrix} x \\ \frac{y}{k} \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(rule ii)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x - k \\ y \\ z \end{bmatrix} = \begin{bmatrix} J - Ak \\ K - Dk \\ L - Gk \end{bmatrix}$$

(4)

(rule iii)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(5)

$$\begin{bmatrix} A & B + A & C \\ D & E + D & F \\ G & H + G & I \end{bmatrix} \begin{bmatrix} x - y \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(rule iv)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(6)

$$\begin{bmatrix} B & A & C \\ E & D & F \\ H & G & I \end{bmatrix} \begin{bmatrix} y \\ x \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(rule v)

$$\begin{bmatrix} A & B & C \\ D & 0 & F \\ G & 0 & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(7)

$$\begin{bmatrix} D & F \\ G & I \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} K \\ L \end{bmatrix}$$

(rule vi)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(8)

$$\begin{bmatrix} -J & B & C \\ -K & E & F \\ -L & H & I \end{bmatrix} \begin{bmatrix} \frac{x}{x^2} \\ \frac{y}{x} \\ \frac{z}{x} \end{bmatrix} = \begin{bmatrix} -A \\ -D \\ -G \end{bmatrix}$$

(rule vii)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} \Rightarrow$$

(9)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G + D & H + E & I + F \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L + K \end{bmatrix}$$

Step 4. Reorganize the elements i_{Ck} , v_{Lk} to build a new matrix equation from the right-hand side of original listed matrix equation. Use the Cramer's rule to obtain the elements i_{Ck} and v_{Lk} . Then substitute the elements i_{Ck} with $C_k(dv_{Ck}/dt)$ and the elements v_{Lk} with $L_k(di_{Lk}/dt)$, respectively, to get the state equation.

FIGURE 1

FIGURE 2

3. Demonstrated Examples

In order to show the significant merits that our method owns over the traditional approaches [3, 4, 18], four comparative examples are proposed in this section.

Example 1. Consider the electrical network [4] shown in Figure 1. The presented approach will be compared to the state-space formulation proposed in [4]. The state equation is easily determined by using only four-step approach in unified process.

Step 1. Applying the substitution theorem and source-transformation theorem yields Figures 2 and 3.

Step 2. Draw up the matrix node-voltage equations and solve the dependent element by easy moving-term processes.

FIGURE 3

$$i_L = \frac{v_{22} - v_L}{R_{CCC}} \quad (13)$$

Applying (rule iii) to (10) yields

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{2R} & 0 & -\frac{1}{2R} \\ 0 & \frac{1}{R_{CCC}} & 0 \\ -\frac{1}{2R} & 0 & \frac{1}{3R} + \frac{1}{2R} \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{22} \\ v_{33} \end{bmatrix} = \begin{bmatrix} +\frac{v_S}{R} - i_{C1} \\ i_{C1} - i_{C2} + \frac{v_L}{R_{CCC}} \\ i_{C2} \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{2R} & 0 + \frac{1}{R} + \frac{1}{2R} & -\frac{1}{2R} \\ 0 & \frac{1}{R_{CCC}} & 0 \\ -\frac{1}{2R} & 0 - \frac{1}{2R} & \frac{1}{3R} + \frac{1}{2R} \end{bmatrix} \begin{bmatrix} v_{11} - v_{22} \\ v_{22} \\ v_{33} \end{bmatrix} = \begin{bmatrix} +\frac{v_S}{R} - i_{C1} \\ i_{C1} - i_{C2} + \frac{v_L}{R_{CCC}} \\ i_{C2} \end{bmatrix} \quad (14)$$

Step 3. Using our invented easy matrix operations changes the node-voltage elements to the state elements v_{Ck}, i_{Lk} based on the following equations:

$$v_{C1} = v_{11} - v_{22} \quad (11)$$

$$v_{C2} = v_{22} - v_{33} = -(v_{33} - v_{22}) \quad (12)$$

From (rule iii), (14) is changed to be

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{2R} & 0 + \frac{1}{R} + \frac{1}{2R} - \frac{1}{2R} & -\frac{1}{2R} \\ 0 & \frac{1}{R_{CCC}} & 0 \\ -\frac{1}{2R} & 0 - \frac{1}{2R} + \frac{1}{3R} + \frac{1}{2R} & \frac{1}{3R} + \frac{1}{2R} \end{bmatrix} \begin{bmatrix} v_{11} - v_{22} \\ v_{22} \\ v_{33} - v_{22} \end{bmatrix} = \begin{bmatrix} +\frac{v_S}{R} - i_{C1} \\ i_{C1} - i_{C2} + \frac{v_L}{R_{CCC}} \\ i_{C2} \end{bmatrix}$$

$$= \begin{bmatrix} +\frac{v_s}{R} - i_{C1} \\ i_{C1} - i_{C2} + \frac{v_L}{R_{CCC}} \\ i_{C2} \end{bmatrix} \quad (15)$$

Using (rule ii) and (rule iii) to (15) gets

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{2R} & R_{CCC} \left(0 + \frac{1}{R} + \frac{1}{2R} - \frac{1}{2R}\right) & -\frac{1}{2R} \\ 0 & \frac{1}{R_{CCC}} R_{CCC} & 0 \\ -\frac{1}{2R} & R_{CCC} \left(0 - \frac{1}{2R} + \frac{1}{3R} + \frac{1}{2R}\right) & \frac{1}{3R} + \frac{1}{2R} \end{bmatrix} \begin{bmatrix} v_{11} - v_{22} \\ \frac{v_{22} - v_L}{R_{CCC}} \\ v_{33} - v_{22} \end{bmatrix} = \begin{bmatrix} +\frac{v_s}{R} - i_{C1} - \frac{1}{R} v_L \\ i_{C1} - i_{C2} + \frac{v_L}{R_{CCC}} - \frac{v_L}{R_{CCC}} \\ i_{C2} - \frac{1}{3R} v_L \end{bmatrix} \quad (16)$$

i.e.,

$$\begin{bmatrix} \frac{3}{2R} & 0 & \frac{1}{2R} \\ 0 & 1 & 0 \\ -\frac{1}{2R} & 0 & -\frac{5}{6R} \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_L \\ v_{C2} \end{bmatrix} = \begin{bmatrix} \frac{v_s}{R} - i_{C1} - \frac{1}{R} v_L \\ i_{C1} - i_{C2} \\ i_{C2} - \frac{1}{3R} v_L \end{bmatrix} \quad (17)$$

Step 4. Reorganize the elements i_{Ck} , v_{Lk} to build a new matrix equation from the right-hand side of original listed matrix equation.

$$\begin{bmatrix} -1 & 0 & -\frac{1}{R} \\ 1 & -1 & 0 \\ 0 & 1 & -\frac{1}{3R} \end{bmatrix} \begin{bmatrix} i_{C1} \\ i_{C2} \\ v_L \end{bmatrix} = \begin{bmatrix} \frac{3}{2R} & \frac{1}{2R} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{2R} & -\frac{5}{6R} & 0 \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{R} \\ 0 \\ 0 \end{bmatrix} v_s \quad (18)$$

Use the Cramer's rule to obtain the elements i_{Ck} (v_{Lk}) and then get the desired state equation.

$$\begin{bmatrix} C_1 \frac{dv_{C1}}{dt} (i.e. i_{C1}) \\ C_2 \frac{dv_{C2}}{dt} (i.e. i_{C2}) \\ L \frac{di_L}{dt} (i.e. v_L) \end{bmatrix} = \begin{bmatrix} -\frac{3}{4R} & -\frac{3}{4R} & \frac{3}{4} \\ -\frac{4R}{3} & -\frac{4R}{3} & -\frac{1}{4} \\ -\frac{3}{4} & \frac{1}{4} & -\frac{3R}{4} \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{4R} \\ \frac{1}{4R} \\ \frac{3}{4} \end{bmatrix} v_s \quad (19)$$

Example 2. Reference [4] claimed that the state equation was mainly obtained by substituting capacitors with voltage

generators and inductors with current generators for the electrical circuit. At this point, the circuit contains only generators and resistors. The claim cannot work for the electrical circuit with coupling circuit due to the appearance of both inductor and mutual differential item. Consider the electrical network with coupling elements shown in Figure 4, where the mutual inductance is denoted as M .

From the general circuit theory, the related equivalent circuit of coupling circuit is given by Figure 5.

Since

$$v_{L2} = L_2 \frac{di_{L2}}{dt} \quad (20)$$

and

$$v_{L1} = L_1 \frac{di_{L1}}{dt} \quad (21)$$

the dependent coupling element can be replaced with

$$\frac{di_{L2}}{dt} = \frac{1}{L_2} v_{L2} \quad (22)$$

and

$$\frac{di_{L1}}{dt} = \frac{1}{L_1} v_{L1} \quad (23)$$

Apply the new approach to obtain the state equations as follows.

Step 1. Applying the substitution theorem and source-transformation theorem gets Figure 6.

FIGURE 4

FIGURE 5

Step 2. Draw up the matrix node-voltage equations and solve the dependent source by simple moving-term processes.

$$i_{L1} = \frac{v_1 - v_s}{R_1} \quad (25)$$

$$i_{L2} = \frac{v_2}{R_2} \quad (26)$$

$$v_C = v_3 \quad (27)$$

$$\begin{bmatrix} \frac{1}{R_{CCC}} + \frac{1}{R_1} & 0 & -\frac{1}{R_{CCC}} \\ 0 & \frac{1}{R_{CCC}} + \frac{1}{R_2} & -\frac{1}{R_{CCC}} \\ -\frac{1}{R_{CCC}} & -\frac{1}{R_{CCC}} & \frac{2}{R_{CCC}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_s}{R_1} - \frac{v_{L1} + (M/L_2)v_{L2}}{R_{CCC}} \\ -\frac{(M/L_1)v_{L1} + v_{L2}}{R_{CCC}} \\ \frac{v_{L1} + (M/L_2)v_{L2}}{R_{CCC}} + \frac{(M/L_1)v_{L1} + v_{L2}}{R_{CCC}} - i_C \end{bmatrix} \quad (24)$$

From (rule ii), we get

$$\begin{bmatrix} \frac{1}{R_{CCC}} + \frac{1}{R_1} & 0 & -\frac{1}{R_{CCC}} \\ 0 & \frac{1}{R_{CCC}} + \frac{1}{R_2} & -\frac{1}{R_{CCC}} \\ -\frac{1}{R_{CCC}} & -\frac{1}{R_{CCC}} & \frac{2}{R_{CCC}} \end{bmatrix} \begin{bmatrix} v_1 - v_s \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_s}{R_1} - \frac{v_{L1} + (M/L_2)v_{L2}}{R_{CCC}} - \frac{v_s}{R_{CCC}} - \frac{v_s}{R_1} \\ -\frac{(M/L_1)v_{L1} + v_{L2}}{R_{CCC}} \\ \frac{v_{L1} + (M/L_2)v_{L2}}{R_{CCC}} + \frac{(M/L_1)v_{L1} + v_{L2}}{R_{CCC}} - i_C + \frac{v_s}{R_{CCC}} \end{bmatrix} \quad (28)$$

Step 3. Use our invented easy matrix operations changes the node-voltage elements to the state elements v_{Ck} , i_{Lk} based on the following equations:

FIGURE 6

Adding first row and second row to third row yields

$$\begin{bmatrix} \frac{1}{R_{CCC}} + \frac{1}{R_1} & 0 & -\frac{1}{R_{CCC}} \\ 0 & \frac{1}{R_{CCC}} + \frac{1}{R_2} & -\frac{1}{R_{CCC}} \\ \frac{1}{R_1} & \frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} v_1 - v_s \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\frac{v_{L1} + (M/L_2)v_{L2}}{R_{CCC}} - \frac{v_s}{R_{CCC}} \\ -\frac{(M/L_1)v_{L1} + v_{L2}}{R_{CCC}} \\ -i_C \end{bmatrix} \quad (29)$$

Multiplying first row and second row with R_{CCC} yields

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \frac{1}{R_1} & \frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_C \end{bmatrix} = \begin{bmatrix} -v_{L1} - \frac{M}{L_2}v_{L2} - v_s \\ -\frac{M}{L_1}v_{L1} - v_{L2} \\ -i_C \end{bmatrix} \quad (30)$$

Step 4. Reorganize the elements i_{Ck} , v_{Lk} to build a new matrix equation from the right-hand side of original listed matrix equation.

$$\begin{bmatrix} -1 & -\frac{M}{L_2} & 0 \\ -\frac{M}{L_1} & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{L1} \\ v_{L2} \\ i_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \frac{1}{R_1} & \frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_s \quad (31)$$

Use the Cramer's rule to obtain the elements i_{Ck} (v_{Lk}) and then get the desired state equation.

$$\begin{bmatrix} L_1 \frac{di_{L1}}{dt} \text{ (i.e. } v_{L1}) \\ L_2 \frac{di_{L2}}{dt} \text{ (i.e. } v_{L2}) \\ C \frac{dv_C}{dt} \text{ (i.e. } i_C) \end{bmatrix} = \begin{bmatrix} \frac{1}{\Delta} & -\frac{M}{\Delta L_2} & \left(\frac{M}{L_2} - 1\right) \frac{1}{\Delta} \\ -\frac{M}{\Delta L_1} & \frac{1}{\Delta} & \left(\frac{M}{L_1} - 1\right) \frac{1}{\Delta} \\ -\frac{1}{R_1} & -\frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{\Delta} \\ -\frac{M}{\Delta L_1} \\ 0 \end{bmatrix} v_s \quad (32)$$

where $\Delta \equiv M^2/L_1 L_2 - 1$.

FIGURE 7

Example 3. Reference [4] has claimed that the proposed method of finding the state equation is only applicable to nondegenerate circuits. An example for degenerate circuit is presented by using only four-step approach in unified process. Consider the degenerate circuit shown in Figure 7 consisting of an inductive cutset and a capacitive loop [18, page784]. For the sake of simplicity, let all numerical values be equal to one.

Since

$$\begin{aligned} i_{C2} &= C_2 \frac{dv_{C2}}{dt} = C_2 \frac{d}{dt} (v_{C1} - v_{S2}) \\ &= \frac{C_2}{C_1} C_1 \frac{dv_{C1}}{dt} - C_2 \frac{dv_{S2}}{dt} = \frac{C_2}{C_1} i_{C1} - C_2 \dot{v}_{S2} \end{aligned} \quad (33)$$

and

$$v_{L2} = L_2 \frac{di_{L2}}{dt} = L_2 \frac{d}{dt} (-i_{L1} - i_s) = \frac{-L_2}{L_1} v_{L1} - L_2 \dot{i}_s \quad (34)$$

We will substitute the inductor L_2 and capacitor C_2 with voltage source $(-L_2/L_1)v_{L1} - L_2 \dot{i}_s$ and current source $(C_2/C_1)i_{C1} - C_2 \dot{v}_{S2}$, respectively, as follows.

Step 1. Applying the substitution theorem and source-transformation theorem obtains Figure 8.

Step 2. Draw up the matrix node-voltage equations and solve the dependent source by simple moving-term processes.

$$\begin{bmatrix} \frac{2}{R_{ccc}} + \frac{1}{R_1} & -\frac{1}{R_{ccc}} & -\frac{1}{R_{ccc}} \\ -\frac{1}{R_{ccc}} & \frac{1}{R_{ccc}} + \frac{1}{R_2} & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{L1}}{R_{ccc}} + \frac{v_{S2}}{R_{ccc}} \\ \frac{-v_{L1}}{R_{ccc}} + i_s + \frac{L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_s \\ \frac{-v_{S2}}{R_{ccc}} - i_s - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (35)$$

Step 3. Use our invented easy matrix operations changes the node-voltage elements to the state elements v_{Ck}, i_{Lk} based on the following equations:

$$v_{C1} = v_1 \quad (36)$$

$$i_{L1} = \frac{v_1 - v_2 - v_{L1}}{R_{ccc}} \quad (37)$$

Applying (rule i) to (35) yields

$$\begin{bmatrix} \frac{2}{R_{ccc}} + \frac{1}{R_1} & \frac{1}{R_{ccc}} & -\frac{1}{R_{ccc}} \\ -\frac{1}{R_{ccc}} & -\frac{1}{R_{ccc}} - \frac{1}{R_2} & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{L1}}{R_{ccc}} + \frac{v_{S2}}{R_{ccc}} \\ \frac{-v_{L1}}{R_{ccc}} + i_s + \frac{L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_s \\ \frac{-v_{S2}}{R_{ccc}} - i_s - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (38)$$

FIGURE 8

From (rule iii), we get

$$\begin{bmatrix} \frac{1}{R_{ccc}} + \frac{1}{R_1} & \frac{1}{R_{ccc}} & -\frac{1}{R_{ccc}} \\ \frac{1}{R_2} & -\frac{1}{R_{ccc}} - \frac{1}{R_2} & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 + v_1 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{L1}}{R_{ccc}} + \frac{v_{S2}}{R_{ccc}} \\ \frac{-v_{L1}}{R_{ccc}} + i_S + \frac{L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} i_S \\ \frac{-v_{S2}}{R_{ccc}} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (39)$$

Applying (rule i) to (40) yields

$$\begin{bmatrix} \frac{1}{R_{ccc}} + \frac{1}{R_1} & 1 & -\frac{1}{R_{ccc}} \\ \frac{1}{R_2} & -1 & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ \frac{-v_2 + v_1 - v_{L1}}{R_{ccc}} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{S2}}{R_{ccc}} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} i_S \\ \frac{-v_{S2}}{R_{ccc}} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (41)$$

From (rule ii), (39) is changed to be

$$\begin{bmatrix} \frac{1}{R_{ccc}} + \frac{1}{R_1} & \frac{1}{R_{ccc}} & -\frac{1}{R_{ccc}} \\ \frac{1}{R_2} & -\frac{1}{R_{ccc}} - \frac{1}{R_2} & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 + v_1 - v_{L1} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{S2}}{R_{ccc}} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} i_S \\ \frac{-v_{S2}}{R_{ccc}} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (40)$$

i.e.,

$$\begin{bmatrix} \frac{1}{R_{ccc}} + \frac{1}{R_1} & 1 & -\frac{1}{R_{ccc}} \\ \frac{1}{R_2} & -1 & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} + \frac{v_{S2}}{R_{ccc}} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} i_S \\ \frac{-v_{S2}}{R_{ccc}} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (42)$$

Adding third row to first row yields

$$\begin{bmatrix} \frac{1}{R_1} & 1 & \frac{1}{R_3} \\ \frac{1}{R_2} & -1 & 0 \\ -\frac{1}{R_{ccc}} & 0 & \frac{1}{R_{ccc}} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_S \\ -\frac{v_{S2}}{R_{ccc}} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \end{bmatrix} \quad (43)$$

Multiplying third row with R_{ccc} yields

$$\begin{bmatrix} \frac{1}{R_1} & 1 & \frac{1}{R_3} \\ \frac{1}{R_2} & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - i_{C1} - i_S - \frac{C_2}{C_1} i_{C1} + C_2 \dot{v}_{S2} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_S \\ -v_{S2} \end{bmatrix} \quad (44)$$

Multiplying third row with $-1/R_3$ to first row obtains

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} & 1 & 0 \\ \frac{1}{R_2} & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - \left(1 + \frac{C_2}{C_1}\right) i_{C1} - i_S + C_2 \dot{v}_{S2} + \frac{v_{S2}}{R_3} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_S \\ -v_{S2} \end{bmatrix} \quad (45)$$

From (rule v), we get

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} & 1 \\ \frac{1}{R_2} & -1 \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \end{bmatrix} = \begin{bmatrix} \frac{v_{S1}}{R_1} - \left(1 + \frac{C_2}{C_1}\right) i_{C1} - i_S + C_2 \dot{v}_{S2} + \frac{v_{S2}}{R_3} \\ +i_S + \frac{L_1 + L_2}{L_1 R_2} v_{L1} + \frac{L_2}{R_2} \dot{i}_S \end{bmatrix} \quad (46)$$

Step 4. Reorganize the elements i_{Ck} , v_{Lk} to build a new matrix equation from the right-hand side of original listed matrix equation.

$$\begin{bmatrix} -\left(1 + \frac{C_2}{C_1}\right) & 0 \\ 0 & \frac{L_1 + L_2}{L_1 R_2} \end{bmatrix} \begin{bmatrix} i_{C1} \\ v_{L1} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} & 1 \\ \frac{1}{R_2} & -1 \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \end{bmatrix} + \begin{bmatrix} -\frac{1}{R_1} & -\frac{1}{R_3} & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{S1} \\ v_{S2} \\ i_S \end{bmatrix} + \begin{bmatrix} 0 & -C_2 & 0 \\ 0 & 0 & -\frac{L_2}{R_2} \end{bmatrix} \begin{bmatrix} \dot{v}_{S1} \\ \dot{v}_{S2} \\ \dot{i}_S \end{bmatrix} \quad (47)$$

Use the Cramer's rule to obtain the elements i_{Ck} (v_{Lk}) and then get the desired state equation.

$$\begin{bmatrix} C_1 \frac{dv_{C1}}{dt} \text{ (i.e. } i_{C1}) \\ L_1 \frac{di_{L1}}{dt} \text{ (i.e. } v_{L1}) \end{bmatrix} = \begin{bmatrix} -\frac{C_1}{C_1 + C_2} \left(\frac{1}{R_1} + \frac{1}{R_3}\right) & -\frac{C_1}{C_1 + C_2} \\ \frac{L_1}{L_1 + L_2} & -\frac{L_1 R_2}{L_1 + L_2} \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \end{bmatrix} + \begin{bmatrix} \frac{C_1}{C_1 + C_2} \frac{1}{R_1} & \frac{C_1}{C_1 + C_2} \frac{1}{R_3} & -\frac{C_1}{C_1 + C_2} \\ 0 & 0 & -\frac{L_1 R_2}{L_1 + L_2} \end{bmatrix} \begin{bmatrix} v_{S1} \\ v_{S2} \\ i_S \end{bmatrix} + \begin{bmatrix} 0 & \frac{C_1 C_2}{C_1 + C_2} & 0 \\ 0 & 0 & -\frac{L_1 L_2}{L_1 + L_2} \end{bmatrix} \begin{bmatrix} \dot{v}_{S1} \\ \dot{v}_{S2} \\ \dot{i}_S \end{bmatrix} \quad (48)$$

Example 4. It has been stated in [3, page131] that the problem will yield increased difficulties in using network analysis to obtain the state equation for the electrical circuit with a dependent element. Consider the electrical circuit with dependent element demonstrated in Figure 9 [3, page130].

Step 1. Applying the substitution theorem and source-transformation theorem yields Figures 10 and 11.

FIGURE 9

FIGURE 10

FIGURE 11

From (rule i), we get

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_{CCC}} & 0 \\ 0 & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} = \begin{bmatrix} i_s - i_C + \frac{v_L}{R_{CCC}} \\ i_C + g_m v_L \end{bmatrix} \quad (52)$$

Applying (rule iii) to (52) yields

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_{CCC}} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 + v_1 \end{bmatrix} = \begin{bmatrix} i_s - i_C + \frac{v_L}{R_{CCC}} \\ i_C + g_m v_L \end{bmatrix} \quad (53)$$

Step 2. Draw up the matrix node-voltage equations and solve the dependent source by simple moving-term processes.

i.e.,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_{CCC}} & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_s - i_C + \frac{v_L}{R_{CCC}} \\ i_C + g_m v_L \end{bmatrix} \quad (49)$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_{CCC}} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_C \end{bmatrix} = \begin{bmatrix} i_s - i_C + \frac{v_L}{R_{CCC}} \\ i_C + g_m v_L \end{bmatrix} \quad (54)$$

From (rule ii), (54) is changed to be

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_{CCC}} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 - v_L \\ v_C \end{bmatrix} = \begin{bmatrix} i_s - i_C - \frac{v_L}{R_1} \\ i_C + v_L \left(g_m - \frac{1}{R_2} \right) \end{bmatrix} \quad (55)$$

Step 3. Using our invented easy matrix operations changes the node-voltage elements to the state elements v_{Ck}, i_{Lk} based on the following equations:

$$v_c = -v_2 + v_1 \quad (50)$$

$$i_L = \frac{v_1 - v_L}{R_{CCC}} \quad (51)$$

From (rule i), we get

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_{CCC}}\right) \times R_{CCC} & 0 \\ \left(\frac{1}{R_2}\right) \times R_{CCC} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} \frac{v_1 - v_L}{R_{CCC}} (i.e. i_L) \\ v_C \end{bmatrix} = \begin{bmatrix} i_S - i_C - \frac{v_L}{R_1} \\ i_C + v_L \left(g_m - \frac{1}{R_2}\right) \end{bmatrix} \quad (56)$$

i.e.,

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} i_S - i_C - \frac{v_L}{R_1} \\ i_C + v_L \left(g_m - \frac{1}{R_2}\right) \end{bmatrix} \quad (57)$$

Step 4. Reorganize the elements i_{Ck} , v_{Lk} to build a new matrix equation from the right-hand side of original listed matrix equation.

$$\begin{bmatrix} -1 & -\frac{1}{R_1} \\ 1 & g_m - \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} i_C \\ v_L \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i_S \quad (58)$$

Use the Cramer's rule to obtain the elements i_{Ck} (v_{Lk}) and then get the desired state equation.

$$\begin{aligned} & \begin{bmatrix} C \frac{dv_{C1}}{dt} (i.e. i_C) \\ L \frac{di_{L1}}{dt} (i.e. v_L) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{R_1 R_2 \Delta} & \frac{1}{\Delta} \left(g_m - \frac{1}{R_2}\right) \\ \frac{1}{R_2 \Delta} & -\frac{1}{\Delta} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{\Delta} \left(-g_m + \frac{1}{R_2}\right) \\ \frac{1}{\Delta} \end{bmatrix} i_S \end{aligned} \quad (59)$$

where $\Delta \equiv -g_m + 1/R_1 + 1/R_2$.

4. Conclusion

Modern circuit analysis approaches model the electrical circuit by integral-differential equation with several non-systematic limitations that are overcome by the state-space formulation in the modern electrical field. The main contribution of this paper is to present a new method of finding the state equation for degenerate circuit and coupling circuit that have not been systematically solved now. Moreover, this paper also proposes some strong improvements to solve the dependent-source circuit via four demonstrated examples. We will extend the proposed result to construct the low-pass, band-pass and high-pass filter in state-space environment in near future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the Institute-Level Major Scientific Research and Training Projects (Lead Doctor special projects) for City College of Dongguan University of Technology, Grant no. 2017YZDYB01Z, Grant Title: Create a new theory and technology of electric circuit and electronics science based on Chen's Electrical Unifying Approach. This work was also supported by City College of Dongguan University of Technology, Grant nos. 201713844031, 2019QJY006Z, and 201813844009.

References

- [1] B. D. Gilbert, F. Julian, G. Xavier, D. Salvatore, and A. S. Jon, "Generalized voltage-based state-space modeling of M," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 6, no. 2, pp. 707–725, 2018.
- [2] M. Song, C. Gao, M. Shahidehpour, Z. Li, J. Yang, and H. Yan, "State space modeling and control of aggregated TCLs for regulation services in power grids," *IEEE Transactions on Smart Grid*, 2018.
- [3] N. S. Nise, *Control Systems Engineering*, Addison-Wesley, New York, NY, USA, 1995.
- [4] T. Martinez-Marin, "State-space formulation for circuit analysis," *IEEE Transactions on Education*, vol. 53, no. 3, pp. 497–503, 2010.
- [5] C. C. Chen, T. L. Chien, Y. C. Chen, W. J. Lin, and S. H. Yang, "A new simple unifying approach of finding the state equation model and its practical application," *Chaos, Solitons & Fractals*, vol. 42, no. 4, pp. 2464–2472, 2009.
- [6] E. S. Kuh and R. A. Rohrer, "The state-variable approach to network analysis," *Proceedings of the IEEE*, vol. 53, no. 7, pp. 672–686, 1965.
- [7] C. A. Desoer and E. S. Kuh, *Basic Circuit Theory*, McGraw-Hill, New York, NY, USA, 1969.
- [8] R. A. Rohrer, *Circuit Theory: An Introduction to the State Variable Approach*, McGraw-Hill, New York, NY, USA, 1970.
- [9] L. J. Tung and B. W. Kwan, *Circuit Analysis*, World Scientific, Singapore, 2001.
- [10] J. W. Nilsson and S. A. Riedel, *Electric Circuit*, Prentice Hall, NJ, USA, 2015.
- [11] C. K. Alexander and M. N. O. Sadiku, *Fundamentals of Electric Circuits*, McGraw-Hill, New York, NY, USA, 2018.
- [12] D. R. Cunningham and J. A. Stuller, *Basic Circuit Analysis*, Houghton Mifflin, Boston, MA, USA, 1991.
- [13] R. C. Dorf and J. A. Svoboda, *Introduction to Electric Circuits*, John Wiley and Sons, NJ, USA, 2006.

- [14] D. L. Skaar, "Using the superposition method to formulate the state variable matrix for linear networks," *IEEE Transactions on Education*, vol. 44, no. 4, pp. 311–314, 2001.
- [15] W. K. Chen, *State-Variable Techniques*, CRC Press, Boca Raton, FL, USA, 2000.
- [16] S. Natarajan, "A systematic method for obtaining state equations using MNA," *IEE Proceedings Part G Circuits, Devices and Systems*, vol. 138, no. 3, pp. 341–346, 1991.
- [17] D. C. Karamousantas, G. E. Chatzarakis, G. N. Korres, and P. J. Katsikas, "Obtaining state equations for planar nondegenerate linear electric circuits using mesh analysis with virtual voltage sources," *International Journal of Electrical Engineering Education*, vol. 45, no. 3, pp. 239–250, 2008.
- [18] W. K. Chen, *The Circuits and Filter Handbook*, CRC Press, Boca Raton, FL, USA, 1995.

Submit your manuscripts at
www.hindawi.com