Research Article

Modal Computation and Analysis Based on Phase Sequence of LLC Resonant DC-DC Converter

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LLC resonant DC-DC converter has wide working range, good voltage gains, and soft switching performance. So it is widely used in power transformation systems of new energy equipment such as UPS, electric vehicle, and photovoltaics (PV). The conduction loss in the main circuit constitutes the main loss occurred in the LLC resonant converter. It may greatly improve the power conversion efficiency of the converter by reducing the conduction loss. To solve the above problems, a method is proposed to calculate and design the parameters of the resonant element in this paper. This method abandons the traditional time sequence mathematical model, and, in order to reduce the effective value of the main loop current of resonant converter, the phase sequence modal analysis is applied. With ZVS as the constraint condition, the design parameters of resonance element that can minimize the conduction loss can be obtained. In the final part, an experimental prototype (300W) is designed and the effectiveness of the mentioned method is verified.

1. Introduction

With the global climate deterioration and the increase of energy demand, distributed generation (DG) which is utilized by the users was used to converse the clean energy to electricity, such as wind power and solar power systems. DG has gradually become the key equipment in solving problems of environment and energy. In the typical low voltage DC distributed system such as electric vehicle charging system and small photovoltaic power generation, the DC-DC converter is not only the physical interface of power interaction, but also the key equipment which can stabilize the DC generator voltage. Given that the output voltage and current of ESS have a wider range, it is necessary that DC-DC converter has a broad gain and load range, and it is supposed to be more efficient, stable, and reliable as introduced in literature [1–3].

At present, the requirements of high conversion efficiency and high power density have become critical factors when we design the DC/DC converter. Although it can effectively reduce the volume of the device by improving the switch operating efficiency of the converter, great switching losses that followed could result in the decrease in the conversion efficiency of the converter, and then it will be difficult to improve the overall power density of the converter. With the advancement of soft switch technology in recent decades, a lot of auxiliary networks for soft switch are developed to reduce the switching losses in the converter and to improve the conversion efficiency and the power density. However, some of these soft switch auxiliary networks have complex structure while others are difficult to control. Besides, some of them will cause losses. For this reason, all of them are not the best solution to this problem.

Three-elements LLC resonant converter is one of the hottest DC-DC converters. This converter is controlled by variable frequency conversion. Through the reasonable design, the switches of the primary side and the synchronous rectifier of the secondary side can work under the condition of soft switching without any auxiliary network. Therefore, it has great advantages in conversion efficiency.

The half-bridge LLC resonant converter is shown in Figure 1. There are two MOSFETs in the primary side, including $S_1$ and $S_2$, which is composed of crystal diode $D_{oss1}$, $D_{oss2}$ and shunt capacitor $C_{oss1}$, $C_{oss2}$. In the secondary
side, half-wave rectification is used, including two diodes \( D_1 \), \( D_3 \) and two capacitances. The resonant tank of the primary loop consists of resonant inductor \( L_r \), magnetic inductance \( L_m \), and resonant capacitance \( C_r \). The operating mode of the resonant tank not only depends on the working stage of converter, but also depends on the working frequency and the load of converter. Therefore, it is difficult to calculate and analyze the resonant characteristics of the LLC converter.

The elements involved in resonance include resonant capacitance \( C_r \), resonant inductor \( L_r \), and magnetic inductance \( L_m \). The values of these elements are quite important in calculating the loss of the main loop. Thus the method of calculating and designing of it has become a hot topic studied by many experts in this field. In literature [4–8], a detailed analysis of the basic working principle of the LLC resonant converter is conducted, and the basic calculation and design methods of resonance parameters are proposed. However, all of these methods mentioned above are based on timing sequence, while they lack enough accuracy in calculating the energy loss as introduced in literature [9–13]. Therefore, it is impossible to obtain the optimized parameters that can improve the power conversion efficiency by using timing models. It is also difficult to provide an intuitive interpretation, so inconvenience is caused for the application of the project as introduced in literature [14–22]. However, through the modal analysis based on the mode of the phase sequence, more accurate relation curves which can show the maximum load voltage gain varying over frequency can be obtained, and the accurate modeling and calculation can be realized for the operational model of converter, and the optimized parameters which minimize the power loss of the converter can be obtained.

2. Analysis on the Primary Loop of the Converter

In AC circuits, the phase angle can reflect the condition of the circuit at each moment, so it can reflect the change of signal. Compared to the analysis of the time sequence, phase sequence is often used as a method for the frequency-domain analysis, and it helps to make the analysis and calculation more accurate. The phase angle is adopted to perform modal dividing. The three fundamental operating modes of the LLC resonant converter can be broken down to three modes as follows:

P mode: the voltage of the magnetic inductance \( L_m \) is clamped in \( 2nV_o \); the primary side transfers energy to the secondary side.

O mode: the voltage of the magnetic inductance \( L_m \) is less than \( 2nV_o \); the primary side will not transfer energy to the secondary side.

N mode: the voltage of the magnetic inductance \( L_m \) is clamped in \(-2nV_o \); the primary side transfers energy to the secondary side.

When the primary side of the LLC resonant converter works under the ZVS (Zero Voltage Switch) condition, the soft switching can be realized with the switch tubes. In ideal condition, no energy will be consumed, and the resonant circuit is inductive and it works safely. Therefore, in most cases, the LLC resonant converter works under the ZVS condition and ZVC condition contains eight modes: P, O, PO, PON, PN, NP, NOP, and OPO. The operating frequency is \( 1/2\pi \sqrt{L_mC_r} \leq f_s \leq 1/2\pi \sqrt{(L_m + L_r)/C_r} \) during PO mode. As the parameters for each element in the main circuit have been designed, when the RMS value of the current passing through the main circuit becomes minimum, the conduction loss will be minimum. Thus, in this paper, the minimum RMS value of the resonant current in this mode represents the minimum conduction loss. Then, the optimization model can be built using ZVS condition as the constraint. At last, if we analyze them by using the formulas of \( \frac{Z_r}{C_r} = \sqrt{L_r/C_r}, m = (L_m + L_r)/(L_r), f_r = f_s, \) the optimum design can be obtained. In the PO mode, ideal waveforms of components at each stage are shown in Figure 2. The phase angle is used as the boundary; a period can be divided into six working modes. As the characteristics of the first half period are similar to that of the second half period, they are exactly the same except the direction of the current and voltage is reverse. Therefore, only the first half period is calculated and analyzed in the paper.

1. Stage 1 \([\theta_p \sim \theta_i]\). When the phase angle is \( \theta_p \), the terminal voltage of \( S_1 \) will decrease to zero, and \( D_{on} \) in the body diode \( S_1 \) will turn on. The phase angle of resonant current will vary until it has a certain value \( \theta_p \), and \( S_1 \) will turn on at the zero voltage. The equivalent circuit is shown in Figure 3.

The LLC resonant converter works in P mode at this moment, the voltage is normalized to \( 2nV_o \), and the current is normalized to \( 2nV_o/Z_r, L_r, \) and \( C_r \). Start resonating, voltage of magnetic inductance \( L_m \) is clamped on \( 2nV_o \), resonant current \( i_{2n} \) will change in sine wave shape, and its equation of
Figure 2: The ideal waveforms of each mode.

Figure 3: Working stage 1.

The state is $i_{L_r}(\theta) = I_{L_r} \sin(\theta)$; the state equation can be designed as

$$V_{in} - 2nV_o \cdot u_{C_r}(\theta) - 2nV_o = L_r \cdot \frac{di_{L_r}(\theta) \cdot (2nV_o/Z_r)}{dt}$$

$$= L_r \cdot \frac{2\pi f_r \cdot 2nV_o/Z_r \cdot dL_r \cdot \sin(\theta)}{d\theta}$$

$$= C_r \cdot \frac{du_{C_r}(t) \cdot 2nV_o}{dt} = C_r \cdot \frac{2\pi f_r \cdot 2nV_o du_{C_r}(\theta)}{d\theta} = \frac{2nV_o}{Z_r} \cdot i_{L_r}(\theta)$$

The normalized result is shown in

$$L_m \cdot \frac{di_{L_m}(t) \cdot 2nV_o/Z_r}{dt} = 2nV_o$$

(1)

The phase angle $\theta = \theta_{p0}$, $i_{L_r}(\theta_{p0}) = 0$, because $S_1$ has reached ZVS before. Then $S_1$ continues to be conductive,
and resonant current $i_c (\theta)$ passes through the circuit in a reserved direction, but the equation for the operating mode and the equivalent circuit remain unchanged. When the angle of phase reaches $\theta_p$, $i_c (\theta_p) = i_{c_m} (\theta_p)$, the diode will be turned off at the secondary side.

(2) Stage 2 [$\theta_{c0} \sim \theta_{c1}$]. The diodes are turned off at the secondary side, LLC converter enters the O mode, and the primary side will not transfer energy to the secondary side during this stage. $L_r$, $L_m$, and $C_r$ take part in resonance in this mode, and the working condition for each element in the circuit is shown in Figure 4.

During this stage $i_{c_0} = i_{c_m}$, all the waveforms belong to sine wave. Because in the P mode $i_c (\theta) = I_L \sin(\theta/\sqrt{m})$, then $2\pi f_r t = \theta$, $f_r = 1/2\pi \sqrt{L_r \cdot C_r}$. During the O mode, $2\pi f_{o0} t = \theta'$, $f_{o0} = 1/2\pi \sqrt{(L_r + L_m) \cdot C_r}$. Thus we have $\theta' = \theta/\sqrt{m}$. This paper assumes that in the mode O $i_c (\theta) = I_L \sin(\theta/\sqrt{m})$, we can perform differentiation for $\theta'/\sqrt{m}$ can be regarded as a whole), and $\sqrt{m}$ will not take part in differentiation. The equation for the mode is (3) as follows:

$$
\begin{align*}
(L_r + L_m) \cdot \frac{di_{c_0} (t)}{dt} + 2nV_o \cdot \frac{di_{c_m} (\theta')}{d\theta} = V_{in} - u_{c_0} (\theta') \cdot 2nV_o \\
C_r \cdot \frac{du_{c_m} (t)}{dt} = 2nV_o \cdot i_{c_m} (\theta') \\
The normalized result can be obtained as \\
\therefore u_{c_0} (\theta') = -\sqrt{m} \cdot I_L \cos \left( \frac{\theta}{\sqrt{m}} \right) + \frac{1}{M} \\
u_{c_m} (\theta') = \frac{m-1}{\sqrt{m}} \cdot I_L \cos \left( \frac{\theta}{\sqrt{m}} \right)
\end{align*}
$$

(3) Stage 3 [$\theta_{c1} \sim \theta_{c2}$]. When the phase angle is $\theta_{c1}$, $S_1$ will turn off. Parasitic capacitance $C_{o1}$ is charging while $C_{o2}$ is discharging. When phase angle is $\theta_{c2}$, the terminal voltage of $S_1$ will equal $V_{gs}$ and the terminal voltage of $S_2$ will equal zero, which can prepare for ZVS. The state of the circuit is shown in Figure 5. After that, the diode of secondary side $D_2$ will stay conductive. The next half cycle symmetrical to the first half of cycle will be initiated.

Here $L_r$, $L_m$, $C_r$, $C_{o1}$, and $C_{o2}$ work at the same time in resonance.

The state equation can be designed as

$$
\begin{align*}
(L_r + L_m) \cdot \frac{di_{c_0}}{d\theta} + 2nV_o \frac{di_{c_m}}{d\theta} = 2nV_o \cdot u_{c_0} (\theta) \\
= 2nV_o \cdot u_{c_2} (\theta) \\
C_{o1} \cdot 2nf_s \cdot \frac{d}{d\theta} \left[ V_{in} - 2nV_o \cdot u_{c_2} (\theta) \right] \\
= \frac{2nV_o}{Z_r} \cdot i_{c_0} (\theta) + C_{o2} \cdot 2nf_s \cdot \frac{du_{c_2}}{d\theta} \\
L_m \cdot 2nf_s \cdot \frac{di_{c_0}}{d\theta} = \frac{2nV_o}{Z_r} \cdot i_{c_0} (\theta) \\
\end{align*}
$$

Assume $C_{o1} = C_{o2} = C_{o}$, (4) and (5) can be combined and we have

$$
\begin{align*}
u_{c_0} (\theta) &= \frac{L_m + L_r}{Z_r} \cdot 2nf_s \cdot I_L \cos \left( \frac{\theta}{\sqrt{m}} \right) - \sqrt{m} \\
&= \frac{1}{M} \cdot I_L \cos \left( \frac{\theta_{c2}}{\sqrt{m}} \right)
\end{align*}
$$

According to the condition of ZVS, when the phase angle is $\theta_{c2}$, $S_2$ will turn on under ZVS, and $u_{c_0} (\theta_{c2}) = 0$, (6) can be transformed to

$$
\begin{align*}
\frac{1}{M} &= \sqrt{m} \cdot I_L \cos \left( \frac{\theta_{o2}}{\sqrt{m}} \right) - \frac{L_m + L_r}{Z_r} \cdot 2nf_s \\
\therefore I_L &= \frac{L_m + L_r}{Z_r} \cdot 2nf_s \cdot \frac{1}{M} \\
\end{align*}
$$

$I_L$ is the normalized value of the sinusoidal current in the mode O that equals normalized value of the current at the end of P mode. The first state and the last state of the resonant are symmetrical. Based on (2), (8) can be obtained:

$$
\begin{align*}
i_{c_m} (\theta) &= I_{c_m} \left( \theta - \theta_{p0} \right) \\
i_{c_m} (\theta_{p0}) &= -i_{c_m} (\theta_{p0} + \pi)
\end{align*}
$$

Uniting these equations, (9) can be obtained:

$$
\begin{align*}
i_{c_m} (\theta_{p0}) &= - \frac{\pi}{2} \left( \frac{2nV_o}{Z_r} \right) \\
i_{c_m} (\theta) &= \frac{1}{2} \sqrt{\left( \frac{P_o \cdot Z_r}{(2nV_o)^2} \right)} \\
i_{c_m} (\theta) &= I_{c_m} \sin \left( \theta_p \right)
\end{align*}
$$
Uniting (1), (7), (9), (10) can be obtained:

$$\frac{1}{M} = \sqrt{m} \cdot \frac{\pi}{2} \sqrt{\frac{P_o^2 Z_r^2}{(2nV_o)^4} + \frac{1}{(m-1)^2} \cdot \sin^2 \left( \frac{\theta_o}{\sqrt{m}} \right) \cdot \cos \left( \frac{\theta_o}{\sqrt{m}} \right) - \frac{L_m + L_r}{Z_r} \cdot 2\pi f_s \cdot \frac{V_o}{8nL_m f_r}}$$

Setting $M \geq 1$ in order to get a high voltage gain, (11) can be obtained:

$$\sqrt{m} \cdot \frac{\pi}{2} \sqrt{\frac{P_o^2 Z_r^2}{(2nV_o)^4} + \frac{1}{(m-1)^2} \cdot \sin^2 \left( \frac{\theta_o}{\sqrt{m}} \right) \cdot \cos \left( \frac{\theta_o}{\sqrt{m}} \right) - \frac{1}{m} \cdot \frac{\pi V_o}{4nZ_r}} \cdot f_n \leq 1$$

### 3. Optimization Model

According to Figure 6, the x-axis stands for the normalized working frequency $f_n = f_s / f_r$, and the y-axis stands for the voltage gain $M = V_o / V_{in}$. When $f_n$ varies max gain point (inflection point) to 1, the voltage gain should be greater than 1 ($M \geq 1$) in order to make sure we obtain the required output voltage and high power conversion efficiency. As shown in Figure 6, there are many working points which can meet requirements of gain under the value of same $m$ and different $Z_r$. Therefore, the sweep frequency is performed with different $m ((L_m + L_r)/L_r)$ value as shown in Figure 7 to analyze the optimization model. According to Figure 7, there are also many working points which can meet requirements of gain under the value of same $Z_r$ and different $m$. With the same value of $m$, the loss of resonant tank and the gain of voltage have same inflection point in terms of their curves. In order to improve conversion efficiency, a higher voltage gain and a lower loss of resonant tank are needed, considering that we should reduce $m$ value to increase voltage gain and increase $m$ value to reduce power loss. With the optimized model proposed in this paper, the balance point for the $m$ value can be obtained.

In the LLC resonant converter, the conductive loss constitutes most of losses in the total loss. The RMS value of the resonant current can reflect the conductive loss. If the RMS value of the resonant current can be minimized, then the conductive loss can be minimized. Thus the extreme values can be worked out with the RMS value of the resonant current as the objective function. Based on the analysis of the above section and (2) and (9), the actual effective value of the resonant current flowing through $L_r$ and $L_m$ can be obtained through

$$I_{L,RMS} = \pi \frac{P_o^2}{2\sqrt{2} (2nV_o)^2 + \frac{(2nV_o)^2}{(m-1)^2} \cdot Z_r^2}$$

$$I_{L_m,RMS} = \pi \frac{2nV_o}{2\sqrt{3} (m-1) \cdot Z_r}$$
Uniting (11) and (12) and considering the characteristics of the resonant converter, (13) can be obtained:

\[
\sqrt{m} \cdot \frac{\pi}{2} \cdot \left( \frac{P_o^2}{(2nV_o)^2} + \frac{1}{(m-1)^2} \right) \cdot \frac{\pi V_o}{4nZ_r} \cdot f_n \leq 1
\]

\[
I_{L_{RMS}} = \frac{\pi}{2\sqrt{2}} \left( \frac{P_o^2}{(2nV_o)^2} + \frac{(2nV_o)^2}{(m-1)^2 \cdot Z_r^2} \right) \quad (13)
\]

\[
I_{L_{RMS}} = \frac{\pi}{2\sqrt{3}} \frac{2nV_o}{Z_r} 
\]

In (11), the value of voltage gain can be set as the minimum value 1 \((M = 1)\). Then it can be treated as the problem of the maximum value of \(I_{L_{RMS}}, I_{L_{RMS}}\). \(V_o, P_o, n\) in (13) are all the known quantities; thus, the values of design required parameters such as \(m, Z_r, f_n\) can be obtained.

### Table 1: Design specifications of the prototype.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vin</td>
<td>300-350V</td>
</tr>
<tr>
<td>Vo</td>
<td>24V</td>
</tr>
<tr>
<td>Io</td>
<td>12.5A</td>
</tr>
<tr>
<td>Po</td>
<td>300W</td>
</tr>
<tr>
<td>(n)</td>
<td>7</td>
</tr>
<tr>
<td>(f_{\text{max}})</td>
<td>100kHz</td>
</tr>
<tr>
<td>(f_{\text{min}})</td>
<td>50kHz</td>
</tr>
</tbody>
</table>

Resonant elements can be calculated by

\[
Cr = \frac{1}{2\pi Z_r \cdot f_n \cdot f_{\text{max}}}
\]

\[
L_r = \frac{1}{(2\pi f_n \cdot f_{\text{max}})^2 \cdot Cr}
\]

\[
L_m = (m - 1) \cdot L_r
\]

### 4. Experimental Verification

To verify the effectiveness of the proposed method in this paper, a 300W LLC resonant converter experimental prototype is designed. Design specifications are listed in Table 1.
### Table 2: Resonant tank parameters of the prototype.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_m$</td>
<td>335$\mu$H</td>
</tr>
<tr>
<td>$L_r$</td>
<td>86$\mu$H</td>
</tr>
<tr>
<td>$C_r$</td>
<td>57nF</td>
</tr>
</tbody>
</table>

Based on the method proposed in this paper, the necessary variables of the resonant parameters can be obtained as:

\[ m = 4.9 \]
\[ Z_r = 39 \quad (Q = 0.42) \]
\[ f_n = 0.72 \quad (15) \]

Based on (14), the resonant tank parameters can be obtained which are listed in Table 2.

The experimental prototype developed in this paper is shown as in Figure 8.

As shown in Figure 9, during the heavy load condition, the resonant current $i_{L_r}$ of the primary loop, the drive voltage $V_{gs1}$ of the switch tube $S_1$, the drain-source voltage $V_{coss1}$ is consistent with the ideal waveform. As shown in Figure 10, the resonant current $i_{L_r}$ will change in ways approximate to a triangular wave shape. The waveform is same as the ideal waveform, and ZVS can be realized. Under such working condition, $i_{L_r}$ will be close to $i_{L_m}$, and there is just a little current flowing through the transformer. As shown in Figure 11, the main circuit voltage $V_S$ of the converter reaches zero before $i_{L_r}$ does; it proves that the converter realize ZVS and the converter has a good performance.

The power conversion efficiency of the converter is analyzed using the power analyzer, as shown in Figure 12. The output power of the converter varies within a small range close to the rated power, and the max efficiency is 97%. As the output power which varies within a small range close to the rated power accounts for 92% and above, it is proved that if the parameters are obtained using the proposed design method in this paper, higher power conversion efficiency of converter can be realized.

### 5. Summary

In aviation power supply, electric vehicles, photovoltaic power generation, and other fields, DC-DC converter is the physical port that achieves energy-interaction between DC bus and distributed power supply and energy storage system (ESS), so its working capability and the power conversion efficiency is of great significance. LLC resonant DC-DC converter has a broader range of work, higher voltage gain, and a good performance of soft switching; thus, the converter is widely concerned and applied. The main loss of LLC converter is the conduction loss of the main circuit. The parameter values of the resonant elements have a decisive effect on the conduction loss. Therefore, to optimize calculation and design of the parameters is the main way to improve the conversion efficiency of the LLC resonant converter. The traditional mathematical model of time sequence is abandoned in this paper, while a method based on the modal analysis of the phase sequence is proposed. The optimization goal of this method is to reduce the RMS value of current in the main circuit, and ZVS is used as the constraint condition. Using this method can not only ensure the LLC converter works smoothly but also improve the power conversion efficiency and realize the optimal design of converter. Finally,
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a 300W experimental prototype is designed to verify the effectiveness of the proposed method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Jianjun Hao and Yuejin Ma contributed equally to this work.

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