Research Article

Modeling of Cutting Forces with a Serrated End Mill

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Received 26 November 2018; Accepted 5 February 2019; Published 11 March 2019

Academic Editor: Xiaoliang Jin

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The paper presents a mechanistic cutting force model of serrated end mill to predict cutting forces. Geometric model of serrated end mill is established, which covers variable helix end mill geometries. In this model, the serration of helical cutting flutes is expressed spatially and the wave of serration is defined to be a sine wave. The spatial vector is applied to define chip thickness so as to enhance the spatial expressiveness of the model, which is perpendicular to the curvature of each flute. Each helical flute is scatted into a series of infinitesimal cutting edges. The infinitesimal cutting forces depend on three cutting force coefficients and three edge force coefficients in the tangential, radial, and axial directions at every cutting element. By integrating the infinitesimal cutting forces along each cutting edge, the milling forces with serrated end mill can be predicted. The model feasibility of the serrated end mill is verified by comparing the predicted and measured cutting forces. Moreover, the model is also verified such that it can also predict cutting forces with other types of end mills, such as variable helix serrated end mill, variable helix end mill, and regular end mill.

1. Introduction

Nowadays, the manufacturing industry demands high productivity because of the competitiveness environment. In manufacturing industry, milling is one of the most important metal cutting, which is based on removing material by cutting mechanism. The efficiency and stability of milling are conflicted in almost all cases. Therefore, it is necessary to study the mechanism for predicting the cutting forces during milling process, which contributes to optimizing the parameters, designing the clamps and tools, analyzing the deformation, and suppressing the vibration during the milling process [1].

Researches on end milling have been done in recent years, which is most focused on the research of mechanics and dynamics models with regular end mills. Merchant [2] proposed a mathematical model, in which the well-known orthogonal cutting model equations were developed. His research provided a good foundation for the later researchers. By analyzing the milling process, Martelotli [3] presented a force model which firstly expressed the cutting thickness with the geometric method. Sutherland and DeVor [4] established a cutting force model to predict the cutting forces and surface error. System deflections which appear on the chip load are studied by the model, and a method is proposed to balance cutting forces and system deflections. Although the proposed model was predicted accurately, it did not consider the characteristic of flexible thin plates during end milling. Budak et al. [5] proposed a method to obtain cutting force coefficients from orthogonal cutting data, which eliminated the calibration of cutter geometry in the experiment. Budak's method can be used in the multiaxis milling with complex cutter geometries. Li et al. [6] considered the effects of end cutting edge and tool nose radius in the first slice of each tooth, whereas they analyzed other slices in the force model without considering the effects. Liu et al. [7, 8] developed a theoretical dynamic cutting force model with peripheral helical end mills. The model analyzed oblique cutting method, size effect of undeformed chip thickness, and the effective rake angle effect. S. Ratchev et al. [9, 10] proposed an analytical method to predict machining dimensional form error of low rigidity components. FE model was integrated in the force model to analyze the part deflection.

In order to obtain high efficiency in milling, some special cutters were invented, such as serrated end mills. Such tools were widely used because of the significant efficiency in material removal rate during milling.
Serrated end mills are mostly used in rough milling, which have variation of cutter radius along the flute. Because of the serrated cutting edge profiles, chip load is irregularly distributed both along and between the helical flutes, which contributes in increasing the stability of serrated end mills. Campomanes [11] proposed a mechanics and dynamics model of serrated end mills. In the model, average cutting force coefficients and approximate chip thickness were presented, which was first modeled by Altintas [12]. Campomanes found that when helical flutes were replaced by sinusoidally varying serrated cutting edges, chatter stability can be improved. Merdol and Altintas [13] established a theoretical model about serrated tools. Serrated flute design knots had been fitted to a cubic spline, which were projected on helical flutes. Dombovari et al. [14] proposed a model to predict milling forces with serrated tools. Moreover, the Semidiscretization Method was applied to gain the stability of serrated end mills. In the paper, the practical advantages of serrated tools were confirmed considering the effect of serration. Grabowski et al. [15] presented a method to predict milling forces and studied the stability of complex tools. In the recent study, Ozkirimli et al. [16] proposed a generalized model to analyze the chatter stability for complex milling operations. Sultan and Okafor [17] established a cutting force prediction model of wavy-edge bull-nose helical end mill. It studied the effects of geometric parameters of WEBNHE on the predicted cutting force components and the resultant cutting force. Techranizadeh and Budak [18] investigated the effect of serration and proposed a method for optimizing serration shapes.

The previous researches concentrated on serrated end mill separately. Besides, the expression of chip thickness with serrated end mill was not explained clearly. Moreover, the industry process needs higher accuracy and higher efficiency lately. In order to satisfy the demands, end mills with special geometric structure have been widely used. Therefore, it is significant to study the force model with special end mills, which can be utilized to optimize the cutting parameters, design the cutting tools, and analyze the stability during the milling process.

The paper establishes a mechanistic cutting force model to predict cutting forces with serrated end mill during end milling process. First, it establishes a geometric model of serrated end mill considering pitch angles and helix angles in each helical cutting flute. In the model, the serration of helical cutting flute is expressed distinctly, which is determined as a sine wave. The chip thickness is also explained in geometry, which can be calculated by a sine wave with amplitude and wavelength. Radius of serrated tool changes along axial direction, which can be calculated by considering the serration wave geometry.

2. Modeling of Serrated End Mill

Serrated end mill has wavy flank surface along the flutes. As shown in Figure 1, the wave can be determined as a sine wave with amplitude A and wave length λ. Radius of serrated tool changes along axial direction, which can be calculated by considering the serration wave geometry.

2.1. Serration. By varying the local radius $R_j$ of jth flute at level z, the serration of serrated end mill can be obtained.

\[ R_j(z) = R - \Delta R_j(z) \]  

(1)

$R$ is the radius of cylindrical serrated end mill shank envelope, and $z$ is the axial depth. $\Delta R_j(z)$ is the variation, which can be calculated by a sine wave with amplitude and wave length.

\[ \Delta R_j(z) = A - A \sin (\gamma(z,j)) \]  

(2)

$A$ is the peak-to-peak amplitude. $\gamma(z,j)$ is the angle of the serration wave in the jth tooth at level of z, which is shown in Figure 1 and obtained by

\[ \gamma(z,j) = 2\pi \left( \frac{z \cos (\beta_j)}{\lambda} \right) - \sum_{k=1}^{j-1} \phi_{pj} \]  

(3)

As shown in Figure 1, $\sum_{k=1}^{j-1} \phi_{pj}$ is the phase shift of serration wave and $\phi_{pj}$ is the pitch angle between the subsequence flutes at the tip of end mill.

2.2. Geometric Model of the Helical Cutting Edges. As shown in Figure 1, the radial immersion angle of point $P$ can be obtained as

\[
\phi_j(t, z) = \begin{cases} 
\phi_{10} - \psi_j(z) & j = 1 \\
\phi_{10} + \sum_{p=2}^{N} \phi_{pj} - \psi_j(z) & 1 < j \leq N
\end{cases}
\]  

(4)

$\phi_{10}$ is the rotation angle at level z=0, which can be seen as a reference edge. It can be determined by the spindle speed $n$, time $t$, and the initial angular position $\phi_0$.

\[ \phi_{10} = 2\pi nt + \phi_0 \]  

(5)

As shown in Figure 1, $\psi_j(z)$ is the radial lag angle, which is related to axial depth $z$, radius $R$ of cylindrical serrated end
mill shank envelope, and local helix angle $\beta_j$ in each flute. It can be calculated by

$$\psi_j (z) = z \tan \frac{\beta_j}{R} \tag{6}$$

When the peak-to-peak amplitude $A$ is zero, the proposed model can predict cutting forces with variable helix end mill. What is more, the proposed model can turn to be regular cutting force model if $A=0$ and local helix angle $\beta_j$ is invariable.

As shown in cylindrical serrated end mill in Figure 2, the point $P$ can be determined by vector $\vec{P}_j(t, z)$ from origin $O$.

$$\vec{P}_j (t, z) = P_x (t, z) \vec{i} + P_y (t, z) \vec{j} + P_z (t, z) \vec{k} \tag{7}$$

$$P_x (t, z) = R_j (z) \sin \phi_j (t, z)$$

$$P_y (t, z) = R_j (z) \cos \phi_j (t, z)$$

$$P_z (t, z) = \frac{R \psi_j (z)}{\tan \beta_j} \tag{8}$$

The axial immersion angle $\kappa_j(t, z)$ is associated with the scalar product of the tangent and normal vectors, which can be obtained by

$$\kappa_j (t, z) = a \cos \frac{\vec{T}_j (t, z) \cdot \vec{V}_j (t, z)}{|| \vec{T}_j (t, z) ||} \tag{9}$$

The unit vector points outward in $XY$ plane, which can be calculated as

$$\vec{V}_j (t, z) = \sin \phi_j (t, z) \vec{i} + \cos \phi_j (t, z) \vec{j} \tag{11}$$
Substituting (10) and (11) into (9), the full definition of axial immersion angle \( \kappa_j(t, z) \) is obtained as

\[
\kappa_j(t, z) = a 
\cdot \cos \left[ \frac{(d/dz) R_j(z)}{\sqrt{[(d/dz) R_j(z)]^2 + [(\tan \beta_j/R) R_j(z)]^2 + 1}} \right] (12)
\]

2.3. Chip Thickness. During milling process, the chip thickness is identified to be perpendicular to the curvature of the serrated flute. As shown in Figure 3, chip thickness can be represented as the local distance between the previous and the cutting surfaces approximately in the direction of the local normal vector \( \overrightarrow{n}_j(t, z) \) of the flute.

The chip thickness between the \( j \)th and the \((j+1)\)th flutes can be obtained by

\[
h_{y, j}(t, z) \approx \Delta r_j(t, z) \overrightarrow{n}_j(t, z) (13)
\]

The equation cannot strictly represent the physical chip thickness. In Figure 3, the local movement of the \( j \)th flute relative to the \((j+1)\)th flute at the same angular position can be expressed as

\[
\Delta r_j(t, z) = r_j(t, z) - r_{j-1}(t, z) = \begin{cases} 
(R_j(z) - R_{j-1}(z)) \sin \phi_j(t, z) + f_{ij}(t, z) & j \geq 2 \\
0 & j = 1 
\end{cases}
\]

where the feed can be calculated as follows:

\[
f_{ij}(t, z) = \frac{V_f \varphi_j(z)}{2\pi n} (15)
\]

Between the \( j \)th and \((j+1)\)th flutes of serrated end mill in the axial direction, the pitch angle can be shown as

\[
\varphi_j(z) = \begin{cases} 
2\pi + \phi_j(t, z) - \phi_N(t, z) = \phi_{p1} - z \frac{\tan \beta_{j+N-1} - \tan \beta_j}{R} & j = 1 \\
\phi_j(t, z) - \phi_{j-1}(t, z) = \phi_{p1} - z \frac{\tan \beta_j - \tan \beta_{j-1}}{R} & 1 < j \leq N
\end{cases} (16)
\]
The unit normal vector of the jth flute at level z for uneven radius can be obtained by

\[
\mathbf{n}_j(t, z) = \begin{bmatrix}
\sin \kappa_j(t, z) \sin \phi_j(t, z) \\
\sin \kappa_j(t, z) \cos \phi_j(t, z) \\
\cos \kappa_j(t, z)
\end{bmatrix}
\]  \hspace{1cm} (17)

The effective chip thickness along axial direction can be evaluated as

\[
h_j(t, z) = G_j(t, z) h_{g,j}(t, z)
\]  \hspace{1cm} (18)

Since the milling is effective only when the edge of end mill is cutting, \( G_j(t, z) \) determines whether the jth tooth is cutting.

\[
G_j(t, z) = \begin{cases} 
1 & \phi_{st,j} \leq \phi_j(t, z) \leq \phi_{ex,j} \\
0 & \phi_j(t, z) < \phi_{st,j} \text{ or } \phi_j(t, z) > \phi_{ex,j}
\end{cases}
\]  \hspace{1cm} (19)

\( \phi_{st,j} \) and \( \phi_{ex,j} \) are the positive entry angle and the positive exit angle, which can be obtained by the integral interval for the effective cutting region. As shown in Figure 4, the intersection \( C = (\phi_{st,j}(t), \phi_{ex,j}(t)) \) can be obtained by the intervals \( A = (\phi_j(t, 0), \phi_j(t, a_p)) \) and \( B = (\phi_a, \phi_{ex}) \).

\[
C = (\phi_j(t), \phi_j(t)) \rightarrow \begin{cases} 
\phi_{st,j}(t) = \max(\phi_j(t, 0), \phi_a) \\
\phi_{ex,j}(t) = \min(\phi_j(t, a_p), \phi_{ex})
\end{cases}
\]  \hspace{1cm} (20)

where \( \phi_a \) and \( \phi_{ex} \) can be calculated by radial depth of cut \( a_e \) and radius of cylindrical serrated end mill shank envelope in different milling modes.

\[
\phi_{st} = \pi - \arccos \left( 1 - \frac{a_e}{R} \right),
\]

\[
\phi_{ex} = \pi \quad \text{down}
\]

\[
\phi_{st} = 0,
\]

\[
\phi_{ex} = \arccos \left( 1 - \frac{a_e}{R} \right) \quad \text{up}
\]  \hspace{1cm} (21)

2.4. Cutting Force Model. The cutting forces with serrated end mill are modeled similar to that with regular end mill when the influence of serration geometry in cutting mechanism is not considered. The helical flutes are scattered into a series of infinitesimal cutting edges. The differential tangential, radial, and axial cutting forces (\( dF_t, dF_r, dF_a \)) can be calculated by the differential chip thickness \( h_j(z, t) \) and the differential flute segment length \( db \), which can be obtained by (25).

\[
dF_t = G_j(z, t) \left(K_{tc} h_{g,j}(z, t) + K_{re} \right) db
\]

\[
dF_r = G_j(z, t) \left(K_{rc} h_{g,j}(z, t) + K_{re} \right) db
\]  \hspace{1cm} (22)

\[
dF_a = G_j(z, t) \left(K_{ac} h_{g,j}(z, t) + K_{ac} \right) db
\]

\[
db = \frac{dz}{\sin(\kappa_j)}
\]  \hspace{1cm} (23)

Cutting force coefficients \( (K_{tc}, K_{rc}, K_{ac}) \) and edge force coefficients \( (K_{tc}, K_{rc}, K_{ac}) \) in the tangential, radial, axial directions can be obtained by the experiment. As shown in Figure 5, differential forces \( (dF_t, dF_r, dF_a) \) can be converted into the components in X-Y-Z coordinates.
3. Experimental Verification

The cutting force model proposed by the paper was verified with a series of cutting experiments as follows. To achieve stable milling process, milling conditions were selected. The cutting forces were recorded by the dynamometer (Kistler9257A), and the sampling frequency was 10000 Hz. The tests were conducted with workpiece material Al7075 alloy. The tests were conducted with four types of end mills which were shown in Figure 6 and detailed in Table 1. According to the approach, cutting tests were conducted with constant axial depth of cut and radial depth of cut at different feed rates. Groove milling was adopted. The spindle speed was set as 2000 rpm, and the axial depth of cut was 1 mm. The feed rates were {0.01, 0.015, 0.02, 0.025, 0.03} mm/tooth. The formulas of the average cutting forces can be deduced as

\[
\mathbf{F} = \mathbf{F}_c f_c + \mathbf{F}_e
\]
Table 1: Geometry parameters of four types of end mills.

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>Diameter (mm)</th>
<th>Pitch angle (°)</th>
<th>Helix angle (°)</th>
<th>Amplitude (mm)</th>
<th>Wavelength (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regular serrated end mill</td>
<td>10</td>
<td>90° -90° -90°</td>
<td>35° -35° -35°</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Variable helix serrated end mill</td>
<td>10</td>
<td>96° -84° -96° -84°</td>
<td>44° -41° -44° -41°</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Variable helix end mill</td>
<td>10</td>
<td>96° -84° -96° -84°</td>
<td>44° -41° -44° -41°</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>4</td>
<td>Regular end mill</td>
<td>10</td>
<td>90° -90° -90°</td>
<td>35° -35° -35°</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Some comparisons of simulated and measured cutting forces with regular serrated end mill (No. 1) were shown by Figures 9–13.

In the condition of down-milling, the experiments were conducted at various depths of cut, spindle speeds, and feed velocities. As shown in the results, it can be found that the simulated and measured cutting forces were essentially consistent in the peak and the approximate trend. Because of the self-excited vibration during milling process, some errors existed between the simulated and measured cutting forces within each cycle.

3.2. Variable Helix Serrated End Mill. In the proposed model, radial immersion angle took pitch angle and helix angle into account. As a result, it can be also utilized in predicting the cutting forces with variable helix serrated end mill (No. 2). Some comparisons of simulated and measured cutting forces with variable helix serrated end mill (No. 2) were shown in Figures 14–16.

Some tests were conducted with variable helix serrated end mill. The experimental program can be obtained in...
Table 2: Cutting force coefficients and edge force coefficients.

<table>
<thead>
<tr>
<th>No.</th>
<th>$K_{tc}$</th>
<th>$K_{re}$</th>
<th>$K_{ae}$</th>
<th>$K_{te}$</th>
<th>$K_{re}$</th>
<th>$K_{ae}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>757.4</td>
<td>-260.4</td>
<td>-37.1</td>
<td>17.7</td>
<td>-12.9</td>
<td>-4.3</td>
</tr>
<tr>
<td>2</td>
<td>754.3</td>
<td>-252.5</td>
<td>-39.7</td>
<td>15.8</td>
<td>-11.4</td>
<td>-3.9</td>
</tr>
<tr>
<td>3</td>
<td>760.7</td>
<td>-255.8</td>
<td>-41.3</td>
<td>14.4</td>
<td>-12.0</td>
<td>-4.5</td>
</tr>
<tr>
<td>4</td>
<td>762.5</td>
<td>-253.4</td>
<td>-43.2</td>
<td>15.0</td>
<td>-11.8</td>
<td>-4.1</td>
</tr>
</tbody>
</table>

Table 3: Milling force experimental program.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial depth of cut (mm)</th>
<th>Radial depth of cut (mm)</th>
<th>Spindle speed (r/min)</th>
<th>Feed velocity (mm/min)</th>
<th>Mode of milling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2000</td>
<td>120</td>
<td>Down</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2000</td>
<td>80</td>
<td>Down</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2000</td>
<td>120</td>
<td>Down</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2000</td>
<td>160</td>
<td>Down</td>
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<td>4</td>
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</tr>
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<td>7</td>
<td>4</td>
<td>4</td>
<td>2000</td>
<td>160</td>
<td>Down</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>4</td>
<td>2500</td>
<td>120</td>
<td>Down</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>4</td>
<td>2500</td>
<td>80</td>
<td>Down</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>4</td>
<td>2500</td>
<td>120</td>
<td>Down</td>
</tr>
<tr>
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<td>2500</td>
<td>160</td>
<td>Down</td>
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<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>2500</td>
<td>80</td>
<td>Down</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>4</td>
<td>2500</td>
<td>120</td>
<td>Down</td>
</tr>
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<tr>
<td>15</td>
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<td>4</td>
<td>3000</td>
<td>120</td>
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<tr>
<td>16</td>
<td>3</td>
<td>4</td>
<td>3000</td>
<td>80</td>
<td>Down</td>
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</tr>
<tr>
<td>21</td>
<td>4</td>
<td>4</td>
<td>3000</td>
<td>160</td>
<td>Down</td>
</tr>
</tbody>
</table>

Figure 7: Average cutting forces at different feed rates in X, Y, and Z directions.

Table 1. Figures 15–18 showed that the simulated and measured cutting forces with variable helix serrated end mill also had essential consistence in the peak and the approximate trend. The results verified the reliability of the model in predicting cutting forces with variable helix serrated end mill.

3.3. Comparison of Cutting Forces. Serrated end mills (No. 1 and No. 2) have smaller cutting forces than variable helix end mill (No. 3) and regular end mill (No. 4) in the same cutting parameters because of the serration wave, which can be observed in Figures 17 and 18. In above, serrated end mill is
Figure 8: Photograph of experimental setup.

Figure 9: Comparison of measured and simulated cutting forces for test 4.

Figure 10: Comparison of measured and simulated cutting forces for test 6.

Figure 11: Comparison of measured and simulated cutting forces for test 12.

Figure 12: Comparison of measured and simulated cutting forces for test 19.

more suitable to be used in rough cutting process than regular end mill and variable helix end mill.

4. Conclusions

The paper presents an analytical model, which can be utilized to predict cutting forces with serrated end mills during end milling process. The serrated end model redefines the serration and covers variable helix end mill geometries. Moreover, the model has a good spatial expressiveness and research expansibility. The following should be noted:
(1) The geometric model of serrated end mill takes pitch angle and helix angle into account in each helical cutting flute. The force model can be utilized to predict cutting forces with variable helix serrated end mill.

(2) Chip thickness of serrated end mill is developed in the force model, which applied spatial vector to define chip thickness so as to enhance the spatial expressiveness of the model. The characteristic of chip thickness is expressed with serrated end mill, which is discontinuous during milling process.

(3) The proposed model can also be turned to be regular cutting model and is verified by experiments. Comparing with cutting forces with regular end mill and variable helix end mill, cutting forces with serrated end mills are smaller. Because of the serration wave, serrated end mill is more suitable to be used in rough cutting process.

So as to verify the viability of the proposed model, various cutting tests with regular serrated end mill and variable helix serrated end mill were conducted. By comparing the measured and predicted cutting forces, the variability of the proposed model was verified. The proposed model can also be utilized to analyze the stability during end milling process and extended to be a generalized model with complex tool geometries furtherly. Besides, the proposed model also provides a reference for cutting mechanism with serrated tools, which contributes to tool optimization and design.
Nomenclature

$X\cdot Y\cdot Z$: Coordinate system of end mill as shown in the figure

$\Delta F_t, \Delta F_r, \Delta F_a$: Differential cutting forces of end mill in tangential, radial, and axial directions

$F_x, F_y, F_z$: Cutting forces in $X\cdot Y\cdot Z$

$R$: Radius of cylindrical serrated end mill shank envelope

$R_j(z)$: Radius of serrated end mill

$\Delta R_j(z)$: Variation of the radius

$A$: Amplitude of wave line in the serration

$\lambda$: Wavelength of the serration

$\gamma(z, j)$: Angle of the serration wave in the $j$th tooth at level of $z$

$P$: A cutting point on cutting edge

$n$: Spindle speed

$N$: Flute number

$a_r, a_p$: Radial depth of cut and axial depth of cut

$\beta_j$: Helix angle of the $j$th flute

$\psi_j(z)$: Radial lag angle of the $j$th flute at elevation $z$
\( \phi_{10} \) \quad \text{Initial angle of the first flute at elevation} \quad z=0

\( \phi_0^i \) \quad \text{Initial angular position} \quad \phi_{pj}^i \quad \text{Pitch angle between the } j\text{-th flute and the} \quad j\text{th flute at the tip of end mill} \quad \phi_{j}(t, z) \quad \text{Radial immersion angle of the } j\text{th flute at elevation} \quad z

\( \varphi_j(z) \) \quad \text{Pitch angle between the } j\text{th flute and the} \quad j+1\text{th flute of end mill at elevation} \quad z

\( \phi_{pj}^n \) \quad \text{Pitch angle between the } j\text{th flute and the} \quad N\text{th flute number} \quad \phi_{ex}, \phi_{et} \quad \text{Entry angle and exit angle} \quad \kappa_j(t, z) \quad \text{Axial immersion angle of } j\text{th flute at elevation} \quad z

\( P_j(t, z) \) \quad \text{The vector from the origin } O \text{ to point } P \quad \tau_j(t, z) \quad \text{The tangent to the serrated flute} \quad \nu_j(t, z) \quad \text{The radial to the serrated flute} \quad \pi_j(t, z) \quad \text{The axial to the serrated flute} \quad \tau_j(z, t) \quad \text{The vector from the origin } O \text{ to tool tip} \quad \Delta \tau_j(z, t) \quad \text{The local movement of the } j\text{th flute relative to the } (j+1)\text{th flute}

\( h_{pj}(t, z) \) \quad \text{Instantaneous cutting thickness between the} \quad j\text{th and the } (j+1)\text{th flutes} \quad h_j(t, z) \quad \text{Effective chip thickness of the } j\text{th flute at } z \quad \text{along axial direction} \quad G_j(t, z) \quad \text{Switching function} \quad d_z \quad \text{Differential height in the segment} \quad db \quad \text{Differential flute segment length} \quad f_j(t, z) \quad \text{Feed of } j\text{th tooth} \quad V_f \quad \text{Velocity of end mill}

\( K_{tc}, K_{tr}, K_{ac} \) \quad \text{Cutting force coefficients in tangential, radial, and axial directions} \quad K_{tc}, K_{tr}, K_{ac} \quad \text{Edge force coefficients in tangential, radial, and axial directions.}

**Data Availability**

The data used to support the findings of this study are restricted by the "973" National Basic Research Program of China. The data used to support the findings of this study are currently under embargo.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

The authors are grateful to the “973” National Basic Research Program of China for supporting the work grant no. 2014CB046603. The National Science Foundation for Young Scientists of China for supporting this work grant no. 51505331. And Lei Wang is thanked for providing experimental help and language help.

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