Research Article

A Three-Dimensional Cooperative Guidance Law Based on Consensus Theory for Maneuvering Targets

Wenjie Zhang ¹, Xiao Du ², and Qunli Xia ¹

¹School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China

²China Academy of Launch Vehicle Technology, Beijing 100076, China

Correspondence should be addressed to Qunli Xia; 1010@bit.edu.cn

Received 27 April 2019; Accepted 25 June 2019; Published 2 July 2019

Academic Editor: Francesco Conte

Copyright © 2019 Wenjie Zhang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, a cooperative guidance law based on consensus theory is proposed for multiple flight vehicles against a maneuvering target in a three-dimensional plane. The proposed guidance law has three orientation components: the one along the line-of-sight (LOS) ensures that all flight vehicles reach the target simultaneously, and the other two normal to the LOS in longitudinal and lateral plane guarantee the accurate interception. The target maneuverings are estimated by a finite-time convergence disturbance observer (FTDOB) and are compensated into the kinematic equation to achieve more accurate interception. Using the Lyapunov stability theorem, time-to-go of all flight vehicles' convergence is proved. Mathematical simulations verify the effectiveness of the proposed cooperative guidance law.

1. Introduction

Consensus theory has an early application in computer science [1]. Recent years, with the development of cooperative guidance, it has been gradually applied to aircrafts. Multi-agent systems have huge potential in various applications such as path planning, penetrating, and intercepting [2–6].

Nowadays defense systems have been equipped in most of the important strategic and tactical military targets [7], so it is hard for a traditional single flight vehicle to meet the needs of warfare missions [6]. Modern warfare emphasizes systematization; the communication among different flight vehicles are required. The cooperation between different flight vehicles will enhance the overall combat capability, increase penetration capability, improve the hit probability, etc. Therefore, the research of multiagent cooperative attack has a very important engineering significance.

The core of multiple flight vehicles cooperative attack is to control the impact time consistently. The authors in literature [8] propose an impact time control guidance law (ITCG) in a two-dimensional plane; a time error feedback term is added to the traditional proportional guidance law to accomplish simultaneous arrival. The authors in literature [9] adopt a polynomial form to realize impact time control

for a stationary target. A method to control impact time by adjusting proportional navigation coefficient is put forward in literature [10] for a stationary target in a two-dimensional plane. However, the method above requires giving impact time in advance and it is usually hard to get. Therefore, these kinds of methods cannot be regarded as an intelligent multiple flight vehicle cooperative guidance to some degree and they are in fact open-loop cooperative guidance law. On the basis of these studies, the authors in literature [11] propose a two-level cooperative guidance architecture for multi-flight vehicles attack; the architecture is composed of local guidance for each flight vehicle on the lower level and a coordination strategy on the upper level. The coordinate strategy can be realized by decentralized coordination and centralized coordination respectively. Then the impact time is given through the coordinate variable. Therefore there is no need to specify a time in advance. But the lower level for each missile is still using the existing ITCG and the method has certain complexities and limitations.

Actually, most of the above guidance laws are not real closed-loop cooperative guidance laws or only focus on stationary targets or are established in a two-dimensional plane; motivated by this, a closed-loop cooperative guidance

FIGURE 1: Kinematic geometry.

law for intercepting maneuvering targets in a three-dimensional plane is proposed in this paper based on multiagent consensus theory. A kinematic equation between the flight vehicle and target in a three-dimensional plane is derived and a FTDOB is used to estimate the target maneuvering and compensate in the kinematic equation for accurate interception to maneuvering targets. Through the guidance law, all flight vehicles reach agreement quickly. It is not necessary to preset an impact time, so all of them exchange time-to-go information with each other to accomplish closed-loop cooperative guidance. It is worth pointing out that if one of flight vehicles fails, the system can also work, so the proposed guidance law has strong robustness.

This paper is organized as follows: In Section 2, relative kinematic and cooperative guidance model are described in a three-dimensional coordinate system for a maneuvering target; some important definitions and lemmas are also given. In Section 3, the cooperative guidance problem of multiple flight vehicles is formulated based on consensus theory, and convergence of the system is verified. The results of mathematical simulation are shown in Section 4, and the main simulation results are discussed. Finally, the conclusions are given in Section 5.

2. Preliminary Concepts

2.1. Problem Formulation. Taking the location where multiple flight vehicles attack against a maneuvering target into consideration, the relative kinematic geometry model is shown in Figure 1. OXY is the inertial reference coordinate system; suppose there are n flight vehicles; the subscript i ($i = 1, 2, \dots, n$) denotes the state variable of the i -th flight vehicle. M_i represents the i -th flight vehicle, and T represents the target. r_i, q_i are the relative distance and LOS angle between the i -th flight vehicle and the target. V_i, θ_i, a_i denote the speed, flight path angle, and normal acceleration command of the i -th flight vehicle while V_t, θ_t represent the target speed and target flight path angle.

The i -th flight vehicle's and target's relative kinematic equation can be described as

$$\dot{r}_i = V_t \cos(\theta_t - q_i) - V_i \cos(\theta_i - q_i) \quad (1)$$

FIGURE 2: Relative motion in three-dimensional plane.

$$r_i \dot{q}_i = V_t \sin(\theta_t - q_i) - V_i \sin(\theta_i - q_i) \quad (2)$$

$$\dot{\theta}_i = \frac{a_i}{V_i} \quad (3)$$

where a_i represents the acceleration component normal to velocity vector.

In the three-dimensional guidance space, the relative motion between i -th flight vehicle and target are exhibited in Figure 2. $M_i xyz$ is in the inertial reference coordinate system $oxyz$, and $M_i x_L y_L z_L$ is in the LOS coordinate system $ox_L y_L z_L$. q_p and q_y are the LOS heading angle and LOS azimuth angle, respectively. The acceleration components of the i -th flight vehicle along three orientations of LOS frame are $\mathbf{u}_i = [u_{r,i} \ u_{q,i} \ u_{y,i}]$, while for the target they are $\mathbf{w} = [w_r \ w_q \ w_y]$.

Define the relative velocity vectors between the i -th flight vehicle and target as follows.

$$\mathbf{V}_i = \begin{bmatrix} V_{r,i} \\ V_{q,i} \\ V_{y,i} \end{bmatrix} = \begin{bmatrix} \dot{r}_i \\ r_i \dot{q}_{p,i} \\ -r_i \dot{q}_{y,i} \cos(q_{p,i}) \end{bmatrix} \quad (4)$$

In (4), $V_{r,i}, V_{q,i}, V_{y,i}$ represent the velocity components along and perpendicular to the LOS of the flight vehicle in longitudinal and lateral plane, respectively. We can obtain (5) through differentiating (4) to time.

$$\frac{d\mathbf{V}_i}{dt} = \boldsymbol{\omega} \times \mathbf{V}_i + \frac{\partial \mathbf{V}_i}{\partial t} = \mathbf{w} - \mathbf{u}_i \quad (5)$$

In (5), $d\mathbf{V}_i/dt$ is the derivative of $\mathbf{V}_i$ to time in inertial system, $\partial \mathbf{V}_i/\partial t$ is the derivative of $\mathbf{V}_i$ to time in LOS system, and $\boldsymbol{\omega}$ is the rotation angular velocity of LOS system to inertial system. The following can be obtained.

$$\boldsymbol{\omega} = \begin{bmatrix} 0 & -\dot{q}_{p,i} & \dot{q}_{y,i} \cos q_{p,i} \\ \dot{q}_{p,i} & 0 & -\dot{q}_{y,i} \sin q_{p,i} \\ -\dot{q}_{y,i} \cos q_{p,i} & \dot{q}_{y,i} \sin q_{p,i} & 0 \end{bmatrix} \quad (6)$$

Substituting (6) into (5), we can obtain the following.

$$\begin{aligned} \ddot{r}_i - r_i \dot{q}_{p,i}^2 - r_i \dot{q}_{y,i}^2 \cos^2 q_{p,i} &= w_r - u_{r,i} \\ r_i \ddot{q}_{p,i} + 2\dot{r}_i \dot{q}_{p,i} + r_i \dot{q}_{y,i}^2 \sin q_{p,i} \cos q_{p,i} &= w_q - u_{q,i} \\ -r_i \ddot{q}_{y,i} \cos q_{p,i} - 2\dot{r}_i \dot{q}_{y,i} \cos q_{p,i} + 2\dot{r}_i \dot{q}_{p,i} \dot{q}_{y,i} \sin q_{p,i} \\ &= w_y - u_{y,i} \end{aligned} \quad (7)$$

By combining (7) with (4), the i -th flight vehicle kinematic equation in the LOS coordinate system can be described as follows.

$$\begin{bmatrix} \dot{r}_i \\ \dot{V}_{r,i} \\ \dot{V}_{q,i} \\ \dot{V}_{y,i} \end{bmatrix} = \begin{bmatrix} V_{r,i} \\ \frac{V_{q,i}^2}{r_i} + \frac{V_{y,i}^2}{r_i} + w_r - u_{r,i} \\ -\frac{V_{r,i} V_{q,i}}{r_i} - \frac{V_{y,i}^2 \tan q_{p,i}}{r_i} + w_q - u_{q,i} \\ -\frac{V_{r,i} V_{y,i}}{r_i} - \frac{V_{y,i} V_{q,i} \tan q_{p,i}}{r_i} + w_y - u_{y,i} \end{bmatrix} \quad (8)$$

In a salvo attack scenario of multiple flight vehicles to a target, the time-to-go of them should reach agreement. The i -th flight vehicle's time-to-go is defined as follows.

$$t_{go,i} = \frac{r_i}{V_{r,i}} \quad (9)$$

Then there is

$$t_{imp,i} = t + t_{go,i} \quad (10)$$

where t is the current flight time of i -th flight vehicle and $t_{imp,i}$ is the attacking time. We can obtain (11) and (12) by differentiating (9) and (10), respectively.

$$\dot{t}_{go,i} = -1 + \frac{V_{q,i}^2}{V_{r,i}^2} - \frac{r_i}{V_{r,i}^2} u_{r,i} + w_r \quad (11)$$

$$\dot{t}_{imp,i} = 1 + \dot{t}_{go,i} = \frac{V_{q,i}^2}{V_{r,i}^2} - \frac{r_i}{V_{r,i}^2} u_{r,i} + w_r \quad (12)$$

To realize simultaneous arrival, each flight vehicle's time-to-go should be controlled, the control objective is designing $u_{r,i}$ to make $t_{go,i}$ converge to zero uniformly so that all flight vehicles attack target simultaneously.

2.2. Theoretical Basis. For multiagent system consensus study, graph theory has been widely used. In the paper, a weighted graph $\Theta = (v, \varepsilon, A)$ is used to represent the communication topology, where $v = \{1, 2, 3, \dots, n\}$ is the nonempty node set, the number of node is rank, and $\varepsilon \subset v \times v = \{(i, j) : i, j \in v\}$ is the edge set. The weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{n \times n}$, $i, j = 1, 2, 3, \dots, n$, where a_{ij} is the weight of edge from i to j . The graph is called an undirected graph if and only if $(i, j) \in \varepsilon \iff (j, i) \in \varepsilon$, and $a_{ij} = a_{ji} > 0$, $a_{ij} = 0$ if $i = j$. Otherwise, it is called a directed graph. Node j is the neighbor of node i ; $N_i = \{j \in v : (i, j) \in \varepsilon, j \neq i\}$ could denote the neighbor set of node i .

Lemma 1 (see [12]). If $y_i > 0$ ($i = 1, 2, \dots, n$) and $0 < p < 1$, then there is $(\sum_{i=1}^n y_i)^p \leq \sum_{i=1}^n y_i^p$.

Lemma 2 (see [13]). $L(A)$ is the graph Laplacian of $\Theta(A)$, which is defined by the following.

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq i}^n a_{ik}, & j = i \\ -a_{ij} & j \neq i \end{cases} \quad (13)$$

$L(A)$ has the following properties:

(1) 0 is an eigenvalue of $L(A)$, and $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbf{R}^n$ is the corresponding eigenvector.

(2) $x^T L(A)x = (1/2) \sum_{i,j=1}^n a_{ij}(x_j - x_i)^2$, because $L(A)$ is semipositive definiteness, so all eigenvalues of $L(A)$ are real and not less than zero; that is, $0 \leq \lambda_1(L) < \lambda_2(L) \leq \dots \leq \lambda_n(L)$.

(3) If $\Theta(A)$ is connected, the second smallest eigenvalue of $L(A)$, which is denoted by $\lambda_2(L)$, is larger than zero.

(4) The algebraic connectivity of $\Theta(A)$ is equal to $\min_{x \neq 0, \mathbf{1}^T x = 0} (x^T L(A)x / x^T x) > 0$; therefore, if $\mathbf{1}^T x = 0$, $x^T L(A)x \geq \lambda_2(L(A))x^T x$.

Lemma 3 (see [14]). Consider the following system.

$$\dot{x} = f(x, t), \quad x \in U_0 \subset \mathbf{R}^n \quad (14)$$

Suppose that there is a positive definite function $V(x)$ which is continuous differentiable; it is defined in a neighborhood of the origin. For the appropriate real numbers $c > 0$ and $0 < \beta < 1$, there is $\dot{V}(x) + cV^\beta(x) \leq 0, x \in \bar{U} \setminus \{0\}$; then the system could converge to origin in finite time, and the upper bound of the convergence time is satisfied.

$$T(x_0) \leq \frac{V^{1-\beta}(x_0)}{c(1-\beta)} \quad (15)$$

Consider the single-input-single-output (SISO) system as follows:

$$\dot{x} = u + d \quad (16)$$

where x is the state variable, u is the control variable, and d represents uncertainty. Assume that d is n -th differentiable and satisfies $d^n < D, D > 0$. The uncertainty d in system (16) is estimated by a FTDOB in [15] shown as

$$\dot{z}_0 = v_0 + u$$

$$v_0 = -\lambda_n L^{1/(n+1)} |z_0 - x|^{n/(n+1)} \operatorname{sgn}(z_0 - x) + z_1$$

$$\dot{z}_1 = v_1$$

$$v_1 = -\lambda_{n-1} L^{1/n} |z_1 - v_0|^{(n-1)/n} \operatorname{sgn}(z_1 - v_0) + z_2$$

$$\vdots$$

$$\dot{z}_{n-1} = v_{n-1}$$

$$v_{n-1} = -\lambda_1 L^{1/2} |z_{n-1} - v_{n-2}|^{1/2} \operatorname{sgn}(z_{n-1} - v_{n-2}) + z_n$$

$$\dot{z}_n = -\lambda_0 L \operatorname{sgn}(z_n - v_{n-1})$$

$$\begin{aligned}\hat{x} &= z_0, \\ \hat{d} &= z_1, \dots, \hat{d}^{(n-1)} = z_n\end{aligned}\quad (17)$$

where $\lambda_i > 0 (i = 1, 2, \dots, n)$ is the parameter to be designed and $\hat{x}, \hat{d}, \dots, \hat{d}^{(n-1)}$ are the estimations of $x, d, \dots, d^{(n-1)}$, respectively. According to the theory in [15], Lemma 4 is obtained as follows.

Lemma 4. The estimation error dynamics of FTDOB 3 are governed by the following.

$$\begin{aligned}\dot{\sigma}_0 &= -\lambda_n L^{1/(n+1)} |\sigma_0|^{n/(n+1)} \operatorname{sgn}(\sigma_0) + \sigma_1 \\ \dot{\sigma}_1 &= -\lambda_{n-1} L^{1/n} |\sigma_1 - \dot{\sigma}_0|^{(n-1)/n} \operatorname{sgn}(\sigma_1 - \dot{\sigma}_0) + \sigma_2 \\ &\vdots \\ \dot{\sigma}_{n-1} &= -\lambda_1 L^{1/2} |\sigma_{n-1} - \dot{\sigma}_{n-2}|^{1/2} \operatorname{sgn}(\sigma_{n-1} - \dot{\sigma}_{n-2}) + \sigma_n \\ \dot{\sigma}_n &\in -\lambda_0 L \operatorname{sgn}(\sigma_n - \dot{\sigma}_{n-1}) + [-L, L]\end{aligned}\quad (18)$$

In (18), $\sigma_0 = z_0 - x$, $\sigma_1 = z_1 - d$, $\dots$, $\sigma_n = z_n - d^{(n-1)}$ are estimation errors; they are finite-time stable, which means there exists a time constant $T > 0$ making $\sigma_i = 0$ ($i = 1, 2, \dots, n$) for all $t > T$.

3. Three-Dimensional Cooperative Guidance Law Design

A salvo attack cooperative guidance law for a maneuvering target is designed in this subsection. The i -th flight vehicle's acceleration command along three orientations of LOS coordinate system are $u_{r,i}$, $u_{q,i}$, and $u_{y,i}$, respectively. $u_{r,i}$ is given based on the finite-time consensus theory to achieve simultaneous arrival and $u_{q,i}$, $u_{y,i}$ are given based on traditional proportional guidance law to achieve precise interception.

Firstly, we design $u_{r,i}$ to ensure all flight vehicles' time-to-go reaches agreement.

On the basis of the finite-time convergence theory and multiagent consensus theory, a guidance command along LOS for i -th flight vehicle is illustrated as follows.

$$\begin{aligned}u_{r,i} &= -\frac{V_{r,i}^2}{r_i} \left[\sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(t_{go,j} - t_{go,i}) \right. \\ &\quad \left. - b_i \operatorname{sig}^\alpha(t_{imp,i} - t_{imp}^*) \right]\end{aligned}\quad (19)$$

In (19), $\operatorname{sig}^\alpha(t_{go,j} - t_{go,i}) = \operatorname{sgn}(t_{go,j} - t_{go,i}) \cdot |t_{go,j} - t_{go,i}|^\alpha$, $0 < \alpha < 1$, is the parameter to be designed, t_{imp}^* is the desired or expected impact time, b_i is a pinning gain; if i -th flight vehicle can acquire the message of the expected impact time, $b_i > 0$; otherwise $b_i = 0$.

Theorem 5. If multiple flight vehicles' communication topology $\Theta = (v, \varepsilon, A)$ is connected, the flight vehicles' time-to-go will reach agreement through (19).

Proof. $e_i = t_{imp,i} - t_{imp}^*$ is the impact time error; the error dynamics can be written as follows.

$$\begin{aligned}\dot{e}_i &= \dot{t}_{imp,i} \\ &= \sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(t_{go,j} - t_{go,i}) - b_i \operatorname{sig}^\alpha(t_{imp,i} - t_{imp}^*)^\alpha\end{aligned}\quad (20)$$

Construct the Lyapunov function as follows.

$$V(t) = \frac{1}{2} \sum_{i=1}^n e_i^2 \quad (21)$$

It can be known for proper β and c , $V(t)$ will reach zero in finite time according to Lemma 3, which illustrates that time-to-go of all flight vehicles can reach agreement in finite time.

Differentiating (21) to time, there is

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^n e_i \dot{e}_i = \sum_{i=1}^n e_i \left[\sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(t_{go,j} - t_{go,i}) \right. \\ &\quad \left. - b_i \operatorname{sig}^\alpha(t_{imp,i} - t_{imp}^*)^\alpha \right] \\ &= \sum_{i=1}^n e_i \left[\sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(e_j - e_i) - b_i \operatorname{sig}^\alpha(e_i) \right] = \frac{1}{2} \\ &\quad \cdot \sum_{i=1}^n e_i \sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(e_j - e_i) + \frac{1}{2} \sum_{i=1}^n e_i \sum_{j=1}^n a_{ij} \operatorname{sig}^\alpha(e_j \\ &\quad - e_i) - \sum_{i=1}^n b_i |e_i|^{\alpha+1} = \frac{1}{2} \sum_{i,j=1}^n a_{ij} (e_i - e_j) \operatorname{sig}^\alpha(e_i \\ &\quad - e_j) - \sum_{i=1}^n b_i |e_i|^{\alpha+1} = -\frac{1}{2} \sum_{i,j=1}^n a_{ij} |e_j - e_i|^{\alpha+1} \\ &\quad - \sum_{i=1}^n b_i |e_i|^{\alpha+1} = -\frac{1}{2} \\ &\quad \cdot \sum_{i,j=1}^n \left((a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2 \right)^{(\alpha+1)/2} - \sum_{i=1}^n b_i |e_i|^{\alpha+1} \\ &\leq 0\end{aligned}\quad (22)$$

From (22) there is

$$\begin{aligned}\dot{V}(t) &\leq -\frac{1}{2} \sum_{i,j=1, i \neq j}^n \left((a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2 \right)^{(\alpha+1)/2} \\ &\leq -\frac{1}{2} \left(\sum_{i,j=1, i \neq j}^n (a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2 \right)^{(\alpha+1)/2}\end{aligned}\quad (23)$$

(23) can be rewritten as

$$\dot{V}(t) \leq -\frac{1}{2} \left(\frac{\sum_{i,j=1,i \neq j}^n (a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2}{V(t)} V(t) \right)^{(\alpha+1)/2} \quad (24)$$

Let $B = [(a_{ij})^{2/(\alpha+1)}] \in \mathbf{R}^{n \times n}$; from Lemma 1, there is

$$\sum_{i,j=1,i \neq j}^n (a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2 = 2e^T L(B) e \quad (25)$$

As $1^T e = 0$, there is

$$\begin{aligned} \frac{\sum_{i,j=1,i \neq j}^n (a_{ij})^{2/(\alpha+1)} |e_j - e_i|^2}{V(t)} &= \frac{2e^T L(B) e}{(1/2) e^T e} \\ &\geq \frac{2\lambda_2(L(B)) e^T e}{(1/2) e^T e} = 4\lambda_2(L(B)) \end{aligned} \quad (26)$$

In (26), $\lambda_2(L(B))$ is the algebraic connectivity of graph $\Theta(B)$, and $\lambda_2(L(B))$ is larger than zero when $\Theta(B)$ is connected; (19) can then be expressed as

$$\begin{aligned} \dot{V}(t) &\leq -\frac{1}{2} (4\lambda_2(L(B)) V(t))^{(\alpha+1)/2} \\ &= -2^\alpha K^{(\alpha+1)/2} V^{(\alpha+1)/2}(t) \end{aligned} \quad (27)$$

where $K = \lambda_2(L(B))$. Thus, according to Lemma 3, $V(t)$ will reach zero in finite time.

$$T_{reach} \leq \frac{2^{1-\alpha} V^{(1-\alpha)/2}(0)}{(1-\alpha) K^{(\alpha+1)/2}} \quad (28)$$

The convergence of the guidance law along LOS is proved. Another important assignment is to design acceleration command normal to LOS; this makes precise interception.

The traditional proportional navigation guidance law is adopted in longitudinal and lateral plane as follows:

$$u_{q,i} = NV_{r,i} \dot{q}_{p,i}, \quad N = 3 \quad (29)$$

$$u_{y,i} = NV_{r,i} \dot{q}_{y,i}, \quad N = 3 \quad (30)$$

where $\dot{q}_{p,i}$ is the LOS angular rate of i -th flight vehicle in longitudinal plane and $\dot{q}_{y,i}$ is the LOS angular rate of i -th flight vehicle in lateral plane.

For the estimation of target maneuverings, from (8) there is

$$\begin{aligned} \dot{V}_{r,i} &= \frac{V_{q,i}^2}{r_i} + \frac{V_{y,i}^2}{r_i} + w_r - u_{r,i} \\ \dot{V}_{q,i} &= -\frac{V_{r,i} V_{q,i}}{r_i} - \frac{V_{y,i}^2 \tan q_{p,i}}{r_i} + w_q - u_{q,i} \\ \dot{V}_{y,i} &= -\frac{V_{r,i} V_{y,i}}{r_i} - \frac{V_{y,i} V_{q,i} \tan q_{p,i}}{r_i} + w_y - u_{y,i} \end{aligned} \quad (31)$$

A second-order FTDOB is applied to estimate the target maneuverings $\mathbf{w} = [w_r \ w_q \ w_y]$ in system (31). It can be constructed as

$$\begin{aligned} \dot{z}_{01} &= v_{01} + \frac{V_{q,i}^2}{r_i} + \frac{V_{y,i}^2}{r_i} - u_{r,i} \\ v_{01} &= -\lambda_{21} L_1^{1/3} |z_{01} - V_{r,i}|^{2/3} \operatorname{sgn}(z_{01} - V_{r,i}) + z_{11} \\ \dot{z}_{11} &= v_{11} \\ v_{11} &= -\lambda_{11} L_1^{1/2} |z_{11} - v_{01}|^{1/2} \operatorname{sgn}(z_{11} - v_{01}) + z_{21} \\ \dot{z}_{21} &= -\lambda_{01} L_1 \operatorname{sgn}(z_{21} - v_{11}) \end{aligned} \quad (32)$$

$$\begin{aligned} \dot{z}_{02} &= v_{02} - \frac{V_{r,i} V_{q,i}}{r_i} - \frac{V_{y,i}^2 \tan q_{p,i}}{r_i} - u_{q,i} \\ v_{02} &= -\lambda_{22} L_2^{1/3} |z_{02} - V_{q,i}|^{2/3} \operatorname{sgn}(z_{02} - V_{q,i}) + z_{12} \\ \dot{z}_{12} &= v_{12} \end{aligned} \quad (33)$$

$$\begin{aligned} v_{12} &= -\lambda_{12} L_2^{1/2} |z_{12} - v_{02}|^{1/2} \operatorname{sgn}(z_{12} - v_{02}) + z_{22} \\ \dot{z}_{22} &= -\lambda_{02} L_2 \operatorname{sgn}(z_{22} - v_{12}) \\ \dot{z}_{03} &= v_{03} - \frac{V_{r,i} V_{y,i}}{r_i} - \frac{V_{y,i} V_{q,i} \tan q_{p,i}}{r_i} - u_{y,i} \\ v_{03} &= -\lambda_{23} L_3^{1/3} |z_{03} - V_{y,i}|^{2/3} \operatorname{sgn}(z_{03} - V_{y,i}) + z_{13} \\ \dot{z}_{13} &= v_{13} \\ v_{13} &= -\lambda_{13} L_3^{1/2} |z_{13} - v_{03}|^{1/2} \operatorname{sgn}(z_{13} - v_{03}) + z_{23} \\ \dot{z}_{23} &= -\lambda_{03} L_3 \operatorname{sgn}(z_{23} - v_{13}) \end{aligned} \quad (34)$$

and λ_i can be chosen as in literature [15]. The parameter L is used to adjust gain to change response speed. From Lemma 4, $z_{11} \rightarrow w_r$, $z_{12} \rightarrow w_q$, $z_{13} \rightarrow w_y$ in finite time; the target maneuverings could be estimated. Therefore, the i -th flight vehicle's kinematic equation is changed as

$$\begin{aligned} \dot{V}_{r,i} &= \frac{V_{q,i}^2}{r_i} + \frac{V_{y,i}^2}{r_i} + z_{11} - u_{r,i} \\ \dot{V}_{q,i} &= -\frac{V_{r,i} V_{q,i}}{r_i} - \frac{V_{y,i}^2 \tan q_{p,i}}{r_i} + z_{12} - u_{q,i} \\ \dot{V}_{y,i} &= -\frac{V_{r,i} V_{y,i}}{r_i} - \frac{V_{y,i} V_{q,i} \tan q_{p,i}}{r_i} + z_{13} - u_{y,i} \end{aligned} \quad (35)$$

Remark. Equation (35) can be rewritten as

$$\dot{V}_{r,i} = \frac{V_{q,i}^2}{r_i} + \frac{V_{y,i}^2}{r_i} + u_{r,i}^*$$

□

FIGURE 3: Communication topology.

$$\begin{aligned}\dot{V}_{q,i} &= -\frac{V_{r,i}V_{q,i}}{r_i} - \frac{V_{y,i}^2 \tan q_{p,i}}{r_i} + u_{q,i}^* \\ \dot{V}_{y,i} &= -\frac{V_{r,i}V_{y,i}}{r_i} - \frac{V_{y,i}V_{q,i} \tan q_{p,i}}{r_i} + z_{13} + u_{y,i}^*\end{aligned}\quad (36)$$

where $u_{r,i}^* = -u_{r,i} + z_{11}$, $u_{q,i}^* = u_{q,i} - z_{12}$, $u_{y,i}^* = u_{y,i} - z_{13}$; for $u_{q,i}^*$ and $u_{y,i}^*$, this is a kind of target maneuvering compensation based on traditional proportional guidance, like augmented proportional navigation in literature [16]. Through (35) or (36), the guidance law could accomplish accurate interception for maneuvering targets.

4. Simulation Results

In order to reveal the performance of the proposed guidance law, a salvo attack from three flight vehicles to a maneuvering target is considered in this subsection. The three attacking flight vehicles exchange time-to-go messages with each other and add them in the proposed guidance law. The communication topology between the three flight vehicles is exhibited in Figure 3, in which the communication is connected; that is, $a_{ij} = 1$, if $i \neq j$.

The weighted adjacency matrix between the three flight vehicles can be obtained from Figure 3.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (37)$$

The initial conditions for the three flight vehicles and target are illustrated in Table 1.

The parameters in the simulation are designed as $\alpha = 0.5$, $\lambda_{0i} = 1.1$, $\lambda_{1i} = 1.5$, $\lambda_{2i} = 2$, $i = 1, 2, 3$, $L = 4$. The target acceleration components along three orientations of LOS are $w_r = 0.5$, $w_q = 1$, $w_y = 0.1$. Parameter b is divided into two cases. For $b = 0$, the impact time of all flight vehicles is uncertain, and for $b = 2$, all flight vehicles fly in a designated time.

Then simulation results are obtained for $b = 0$.

Figures 4 and 5 display longitudinal and lateral trajectory curves of flight vehicles and target, respectively. The three flight vehicles start off from different locations, and they all hit the maneuvering target. Figure 6 shows flight vehicles-to-target distance curve versus time. At the beginning, they are diverse, flight vehicle 1 has the longest distance, and flight

FIGURE 4: Longitudinal trajectory curves between flight vehicles and the target.

FIGURE 5: Lateral trajectory curves between flight vehicles and the target.

FIGURE 6: Flight vehicle-to-target distance curves.

TABLE 1: Initial parameters of flight vehicles and target.

Object	Location (m)	Flight path angle (°)	Velocity (m/s)
Vehicle1	(0,6000,100)	40	300
Vehicle2	(0,3000,0)	30	300
Vehicle3	(0,1000,-100)	20	300
Target	(8000,0,0)	0	10

FIGURE 7: Impact time curves of flight vehicles.

FIGURE 9: Acceleration command curves along LOS of flight vehicles.

FIGURE 8: Time-to-go curves of flight vehicles.

FIGURE 10: Acceleration command curves perpendicular to LOS in longitudinal plane of flight vehicles.

vehicle 3 has the shortest. Through the proposed cooperative guidance law, distances of three flight vehicles to target reach agreement and converge to zero at the same time, which means they all hit the target simultaneously; the impact time is 33.06s. Figures 7 and 8 display the flight vehicles' impact time and time-to-go curves versus time. At the beginning, they are greatly different which is consistent with distance difference. Flight vehicle 1 has maximal impact time and time-to-go while flight vehicle 3 has the minimum ones.

Through the multiagent consensus theory, they finally reach agreement in around 1.5s; the convergence rate is relatively fast. Figures 9–11 show the control command along three orientations of LOS. The command will not converge to zero for maneuvering target. Figures 12–14 display the estimation of target maneuvering; they all have a relatively accurate estimation for target maneuvering.

FIGURE 11: Acceleration command curves perpendicular to LOS in lateral plane of flight vehicles.

FIGURE 14: Estimation of target maneuvering curves perpendicular to LOS in lateral plane.

FIGURE 12: Estimation of target maneuvering curves along LOS.

FIGURE 13: Estimation of target maneuvering curves perpendicular to LOS in longitudinal plane.

FIGURE 15: Longitudinal trajectory curves between flight vehicles and the target.

For $b = 2$, all flight vehicles will fly in a designated time. We choose the expected impact time as $t_{imp}^* = 40s$; simulation results are shown as follows.

From Figures 15–24, it can be observed that the flight vehicles also reach agreement under the designed cooperative guidance law and hit the target simultaneously in the designated time, 40s. The time-to-go converges uniformly in around 4s. The target maneuvering can be estimated accurately.

5. Conclusions

Based on multiagent consensus theory, the paper designs a guidance law for cooperative attack which accomplishes a simultaneous arrival in a three-dimensional plane for maneuvering targets. The target acceleration components along three orientations of LOS coordinate system are estimated by FTDOB, and they are compensated in the kinematic equation. The flight vehicles' acceleration components along

FIGURE 16: Lateral trajectory curves between flight vehicles and the target.

FIGURE 19: Acceleration command curves along LOS of flight vehicles.

FIGURE 17: Flight vehicle-to-target distance curves.

FIGURE 20: Acceleration command curves perpendicular to LOS in longitudinal plane of flight vehicles.

FIGURE 18: Time-to-go curves of flight vehicles.

FIGURE 21: Acceleration command curves perpendicular to LOS in lateral plane of flight vehicles.

FIGURE 22: Estimation of target maneuvering curves along LOS.

FIGURE 23: Estimation of target maneuvering curves perpendicular to LOS in longitudinal plane.

the LOS are designed through consensus theory. The convergence is verified by Lyapunov stability theorem. However, for the acceleration perpendicular to the LOS in longitudinal and lateral plane, the traditional proportional navigation guidance law is applied. Simulation results show that the target maneuvering estimators have a fast convergence speed, so they could estimate target maneuverings rapidly and accurately. And all flight vehicles accomplish precise interception for the maneuvering target simultaneously, and the time-to-go of them converges uniformly quickly, which demonstrates the effectiveness of the cooperative guidance law.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

FIGURE 24: Estimation of target maneuvering curves perpendicular to LOS in lateral plane.

Acknowledgments

This work was supported in part by the Joint Equipment Fund of the Ministry of Education (No. 6141A02022340).

References

- [1] N. A. Lynch, *Distributed Algorithms*, Morgan Kaufman Publisher, San Francisco, Calif, USA, 1997.
- [2] M. H. Degroot, "Reaching a consensus," *Journal of the American Statistical Association*, vol. 69, no. 345, pp. 118–121, 1974.
- [3] P. R. Chandler, S. Rasmussen, and M. Pachter, "UAV cooperative path planning," in *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit 2000*, pp. 14–17, 2000.
- [4] J. Wohletz, "Cooperative, dynamic mission control for uncertain, multi-vehicle autonomous systems," in *Proceedings of the IEEE Conference on Decision and Control*, pp. 1–10, 2002.
- [5] C. Schumacher and P. R. Chandler, "Task allocation for wide area search munitions via network flow optimization," in *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit 2001*, 2001.
- [6] A. Richards, J. Bellingham, M. Tillerson, and J. How, "Coordination and control of multiple UAVs," in *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit 2002*, 2002.
- [7] P. Zarchan, "Tactical and strategic missile guidance," in *Progress in Astronautics and Aeronautics*, vol. 157, AIAA, Washington, DC, USA, 1994.
- [8] I.-S. Jeon, J.-I. Lee, and M.-J. Tahk, "Impact-time-control guidance law for anti-ship missiles," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 2, pp. 260–266, 2006.
- [9] I. Jeon and J. Lee, "Impact-time-control guidance law with constraints on seeker look angle," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 53, no. 10, pp. 2621–2627, 2017.
- [10] I.-S. Jeon, J.-I. Lee, and M.-J. Tahk, "Homing guidance law for cooperative attack of multiple missiles," *Journal of Guidance, Control, and Dynamics*, vol. 33, no. 1, pp. 275–280, 2010.
- [11] S. Zhao and R. Zhou, "Cooperative guidance for multimissile salvo attack," *Chinese Journal of Aeronautics*, vol. 21, no. 6, pp. 533–539, 2008.

- [12] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.
- [13] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
- [14] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Transactions on Automatic Control*, vol. 43, no. 5, pp. 678–682, 1998.
- [15] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *International Journal of Control*, vol. 76, no. 9-10, pp. 924–941, 2003.
- [16] V. Garber, "Optimum intercept laws for accelerating targets," *AIAA Journal*, vol. 6, no. 11, pp. 2196–2198, 1968.