Research Article

Research on Multiple Griffith Cracks at the Interface of Fine-Grained Piezoelectric Coating/Substrate

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The behavior of a fine-grained piezoelectric coating/substrate with multiple Griffith interface cracks under electromechanical loads is investigated. In this work, double coupled singular integral equations are proposed to solve the fracture problems. Both the singular integral equation and single-valued conditions are simplified into an algebraic equation and solved by numerical calculation. Thereby, the intensity factors of electric displacement and stress obtained are used to obtain the expression of the energy release rate. Furthermore, numerical results of the energy release rate with material parameters are demonstrated. Based on the obtained results, it could be concluded that the energy release rate is closely related to the size of the interface cracks and the mechanical-electrical loading. For a bimaterial structure, the fine-grained piezoelectric structure exhibited better material performance compared to the large one.

1. Introduction

Piezoelectric bimaterials have been widely applied in many fields, such as sensors, actuators, filters, and storage devices. Owing to the reliability of composite materials, double-layer bonded bimaterial composite structures have played an important role in smart structures. However, piezoelectric composites tend to crack, fracture, and debond at the interface during the process of fabricating defects and load conditions [1]. Therefore, the issue of defects of piezoelectric materials under electromechanical load has attracted wide attention [2–7]. Viun et al. [8] handled the interface problems of bi-piezoelectric materials under tensile mechanical loads and electric displacements. The influence of material parameters on finite-sized piezoelectric plates is analyzed by a finite-element method. Li et al. [9] analyzed the mechanical properties of static and dynamic interface cracks of functionally graded coatings/substrates. In this work, the Fourier transform and Laplace transform were used as the fundamental solutions to solve the problem under static and dynamic loads. Soh et al. [10] studied the behavior of bi-piezoelectric ceramic strips with mode III central interface cracks, and the influences of thickness and material coefficients of piezoelectric strips on intensity factors and stress were obtained. Shavlakadze [11] demonstrated the electroelastic contact problem of piezoelectric plates with elastic coatings. By using the method of analytic function theory, the problem was reduced to a singular integrodifferential equation with fixed singularity. Shin and Kim [12] studied the transient response of mode III interface cracks between the piezoelectric layer and the functional orthotropic material layer. The integral equations were solved by the integral method, and it was concluded that the material parameters could prevent the transient fracture of the interface crack. Gherrous and Ferdjani [13] investigated a Griffith interface crack of bi-piezoelectric materials, and the singular integral equations were reduced into algebraic equations by using Chebyshev polynomials. Nan and Wang [14] studied the effect of residual surface stress on the stress and electric field strength of conductive cracks in piezoelectric nanomaterials. The numerical results showed that the residual stress had a significant effect on the piezoelectric nanomaterials, and the piezoelectric nanomaterials had stronger electromechanical coupling compared to the
2. Problem Formulation

As shown in Figure 1, the fine-grained ceramic powder was sprayed uniformly to the surface of the piezoelectric substrate by plasma spraying technology to form a coating. A fine-grain piezoelectric coating/substrate structure was obtained by polarization treatment of coating. The thickness of the coating is \( h_1 \) and of the substrate is \( h_2 \). The rectangular coordinate system was built with the \( X \)-axis along the structure interface and the \( Y \)-axis along the structure thickness direction. The multiple Griffith interface cracks are 2-dimensional and all within the same orientation and occupying the intervals \( x \in (a_j, b_j) \) \((j = 1, 2, \ldots, n)\), respectively. The fine-grained piezoelectric coating and piezoelectric substrate are polarized along the \( Z \)-axis, and they are transversely isotropic.

Assume that the problem of the fine-grained piezoelectric coating/substrate interface cracks is considered as only the out-of-plane displacement \( W \) and the in-plane electric fields \( \Phi \) such that

\[
\begin{align*}
U &= V = 0, \\
W &= W(x, y), \\
\Phi &= \Phi(x, y).
\end{align*}
\]

In this case, the constitutive equations become [4]

\[
\begin{align*}
\sigma_{xx} &= c_{44}^{(k)} \frac{\partial W}{\partial x} + \epsilon_{15}^{(k)} \frac{\partial \Phi}{\partial x}, \\
D_x &= \epsilon_{15}^{(k)} \frac{\partial W}{\partial x} - \epsilon_{11}^{(k)} \frac{\partial \Phi}{\partial x}, \\
\sigma_{yx} &= c_{44}^{(k)} \frac{\partial W}{\partial y} + \epsilon_{15}^{(k)} \frac{\partial \Phi}{\partial y}, \\
D_y &= \epsilon_{15}^{(k)} \frac{\partial W}{\partial y} - \epsilon_{11}^{(k)} \frac{\partial \Phi}{\partial y},
\end{align*}
\]

where \( \sigma_{xx} \) and \( \sigma_{yx} \) are the stress components, \( D_x \) and \( D_y \) are the electric displacement components, \( c_{44}^{(k)} \) is the elastic modulus, \( \epsilon_{15}^{(k)} \) and \( \epsilon_{11}^{(k)} \) are the piezoelectric and dielectric constants, and the superscript \( k \) \((k = 1, 2)\) stands for the fine-grained piezoelectric coating and piezoelectric substrate, respectively.

According to Park, the governing equations can be written as follows:

\[
\begin{align*}
c_{44}^{(k)} v^2 W + \epsilon_{15}^{(k)} v^2 \Phi &= 0, \\
\epsilon_{15}^{(k)} v^2 W - \epsilon_{11}^{(k)} v^2 \Phi &= 0, \quad k = 1, 2,
\end{align*}
\]
where $\nabla^2$ is the two-dimensional Laplacian operator.

Assume that there is no loading on the surface of the coating/substrate and the antiplane shear stress $\sigma_{z2}$ and in-plane electric displacement $D_y$ are loaded on the surface of the Griffith interface cracks. Therefore, the mixed boundary conditions of the problems can be written as follows:

\[
\sigma_{z2}(x, h_1) = \sigma_{z2}(x, -h_2) = 0, \quad (8)
\]
\[
D_y(x, h_1) = D_y(x, -h_2) = 0, \quad (9)
\]
\[
\sigma_{z2}(x, 0^+) = \sigma_{z2}(x, 0^-), \quad x \notin (a_j, b_j), \quad (10)
\]
\[
D_y(x, 0^+) = D_y(x, 0^-), \quad x \notin (a_j, b_j), \quad (11)
\]
\[
W(x, 0^+) = W(x, 0^-), \quad x \notin (a_j, b_j), \quad (12)
\]
\[
\Phi(x, 0^+) = \Phi(x, 0^-), \quad x \notin (a_j, b_j), \quad (13)
\]
\[
\sigma_{z2}(x, 0^+) = -\sigma_0, \quad x \in (a_j, b_j), \quad (14)
\]
\[
D_y(x, 0^+) = -D_0, \quad x \in (a_j, b_j). \quad (15)
\]

\section{3. Solution to the Problem}

By Fourier integral transform of equations (6) and (7), the loading displacement and electric field are obtained, respectively:

\[
W(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ A_{kk} e^{ia y} + B_{kk} e^{-ia y} \right] e^{-i\alpha x} \, da, \quad (16)
\]
\[
\Phi(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ A_{kk} e^{ia y} + B_{kk} e^{-ia y} \right] e^{-i\alpha x} \, da, \quad (17)
\]

where $A_{kk}(\alpha), B_{kk}(\alpha), A_{kk}(\alpha), \text{ and } B_{kk}(\alpha)$ are the unknown functions ($k = 1, 2$).

Substitution formulas (16) and (17) into equations (4) and (5), the stress and the electric displacement are obtained, respectively:

\[
\sigma_{z2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ c_{44}^{(k)} \left( A_{kk} e^{ia y} - B_{kk} e^{-ia y} \right) + e_{4}^{(k)} \left( A_{kk} e^{ia y} - B_{kk} e^{-ia y} \right) \right] e^{-i\alpha x} \, da, \quad (18)
\]
\[
D_y = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ c_{15}^{(k)} \left( A_{kk} e^{ia y} - B_{kk} e^{-ia y} \right) + e_{15}^{(k)} \left( A_{kk} e^{ia y} - B_{kk} e^{-ia y} \right) \right] e^{-i\alpha x} \, da. \quad (19)
\]

In order to solve the problem easily, we utilize continuously distributed dislocations to simulate collinear cracks. For this, we introduce the density function as follows:

\[
g_{W}^{(j)}(x) = \frac{d}{dx} \left[ W^{(j)}(x, 0^+) - W^{(j)}(x, 0^-) \right], \quad (20)
\]
\[
g_{D}^{(j)}(x) = \frac{d}{dx} \left[ \Phi^{(j)}(x, 0^+) - \Phi^{(j)}(x, 0^-) \right], \quad j = 1, 2, \ldots, n. \quad (21)
\]

According to equations (12) and (13), the dislocation density function should make the following single-valued conditions valid:

\[
\int_{a_i}^{b_i} g_{W}^{(j)}(x) \, dt = 0, \quad (22)
\]

Using the boundary conditions (8) and (9), we obtain

\[
A_{11}(\alpha) = B_{11}(\alpha) e^{-2i\alpha h_1},
\]
\[
A_{21}(\alpha) = B_{21}(\alpha) e^{-2i\alpha h_1},
\]
\[
B_{12}(\alpha) = A_{12}(\alpha) e^{2i\alpha h_1},
\]
\[
B_{22}(\alpha) = A_{22}(\alpha) e^{2i\alpha h_1}. \quad (23)
\]

Inserting equations (16) and (17) into equations (20) and (21) and inserting equations (18) and (19) into the boundary conditions (8)–(11), (14), and (15), we obtain

\[
c_{44}^{(1)} \left( A_{11}(\alpha) e^{ia h_1} - B_{11}(\alpha) e^{-ia h_1} \right)
+ e_{4}^{(1)} \left( A_{21}(\alpha) e^{ia h_1} - B_{21}(\alpha) e^{-ia h_1} \right) = 0, \quad (24)
\]
\[
c_{15}^{(2)} \left( A_{12}(\alpha) e^{ia h_1} - B_{12}(\alpha) e^{-ia h_1} \right)
+ e_{15}^{(2)} \left( A_{22}(\alpha) e^{ia h_1} - B_{22}(\alpha) e^{-ia h_1} \right) = 0, \quad (25)
\]
\[
e_{15}^{(1)} \left( A_{11}(\alpha) e^{ia h_1} - B_{11}(\alpha) e^{-ia h_1} \right)
+ e_{11}^{(1)} \left( A_{21}(\alpha) e^{ia h_1} - B_{21}(\alpha) e^{-ia h_1} \right) = 0, \quad (26)
\]
\[
e_{15}^{(2)} \left( A_{12}(\alpha) e^{ia h_1} - B_{12}(\alpha) e^{-ia h_1} \right)
+ e_{11}^{(2)} \left( A_{22}(\alpha) e^{ia h_1} - B_{22}(\alpha) e^{-ia h_1} \right) = 0, \quad (27)
\]
\[
c_{44}^{(1)} \left( A_{11}(\alpha) - B_{11}(\alpha) \right) + e_{44}^{(1)} \left( A_{21}(\alpha) - B_{21}(\alpha) \right)
= e_{44}^{(2)} \left( A_{12}(\alpha) - B_{12}(\alpha) \right) + e_{15}^{(2)} \left( A_{22}(\alpha) - B_{22}(\alpha) \right), \quad (28)
\]
\[
e_{44}^{(1)} \left( A_{11}(\alpha) - B_{11}(\alpha) \right) - e_{11}^{(1)} \left( A_{21}(\alpha) - B_{21}(\alpha) \right)
= e_{15}^{(2)} \left( A_{12}(\alpha) - B_{12}(\alpha) \right) - e_{11}^{(2)} \left( A_{22}(\alpha) - B_{22}(\alpha) \right), \quad (29)
\]
\[ A_{11}(\alpha) + B_{11}(\alpha) - A_{12}(\alpha) - B_{12}(\alpha) = \frac{i}{\alpha} \int_{a_j}^{b_j} g_\alpha^{(j)}(t)e^{\text{iat}} \, dt = F_1, \]

\[ A_{21}(\alpha) + B_{21}(\alpha) - A_{22}(\alpha) - B_{22}(\alpha) = \frac{i}{\alpha} \int_{a_j}^{b_j} g_\alpha^{(j)}(t)e^{\text{iat}} \, dt = F_2. \]

Equations (25)–(32) can be written in the form of matrix as follows:

\[ \mathbf{A} \cdot \mathbf{B} = \mathbf{C}, \]

where

\[
\begin{bmatrix}
 c_{44}^{(1)} e^{\text{iat}} & -c_{44}^{(1)} e^{\text{iat}} & 0 & 0 & e_{15}^{(1)} e^{\text{iat}} & -e_{15}^{(1)} e^{\text{iat}} & 0 & 0 \\
 0 & 0 & c_{44}^{(2)} e^{\text{iat}} & -c_{44}^{(2)} e^{\text{iat}} & 0 & 0 & e_{15}^{(2)} e^{\text{iat}} & -e_{15}^{(2)} e^{\text{iat}} \\
 -c_{15}^{(1)} e^{\text{iat}} & -e_{15}^{(1)} e^{\text{iat}} & 0 & 0 & -e_{11}^{(1)} e^{\text{iat}} & -e_{11}^{(1)} e^{\text{iat}} & 0 & 0 \\
 0 & 0 & e_{15}^{(2)} e^{\text{iat}} & -e_{15}^{(2)} e^{\text{iat}} & 0 & 0 & -e_{11}^{(2)} e^{\text{iat}} & -e_{11}^{(2)} e^{\text{iat}} \\
 e_{15}^{(1)} & -e_{15}^{(1)} & -e_{15}^{(2)} & e_{15}^{(2)} & -e_{11}^{(1)} & -e_{11}^{(1)} & 0 & 0 \\
 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1
\end{bmatrix}
\]

\[ \mathbf{B} = [A_{11}, B_{11}, A_{12}, B_{12}, A_{21}, B_{21}, A_{22}, B_{22}]^T, \]

\[ \mathbf{C} = [0, 0, 0, 0, 0, 0, F_1, F_2]^T. \]

Solving the linear equation (32), we obtain

\[ A_{11}(\alpha) = \frac{H_1 F_1 + H_2 F_2}{H}, \]

\[ A_{12}(\alpha) = \frac{H_3 F_1 + H_4 F_2}{H}, \]

\[ B_{11}(\alpha) = \frac{H_5 F_1 + H_6 F_2}{H}, \]

\[ B_{12}(\alpha) = \frac{H_7 F_1 + H_8 F_2}{H}, \]

\[ A_{21}(\alpha) = \frac{H_9 F_1 + H_{10} F_2}{H}, \]

\[ A_{22}(\alpha) = \frac{H_{11} F_1 + H_{12} F_2}{H}, \]

\[ B_{21}(\alpha) = \frac{H_{13} F_1 + H_{14} F_2}{H}, \]

\[ B_{22}(\alpha) = \frac{H_{15} F_1 + H_{16} F_2}{H}. \]

The expressions of \( H \) and \( H_q, q = 1, 2, \ldots, 16 \), are given in Appendix.

Obviously, the unknown functions \( A_{m1}(\alpha), A_{m2}(\alpha) \) and \( B_{m1}(\alpha), B_{m2}(\alpha) \) \( (m = 1, 2) \) depend on \( F_1 \) and \( F_2 \) and, consequently, on the density functions \( g_\alpha^{(j)}(x) \) and \( g_\alpha^{(j)}(x) \). Once \( F_1 \) and \( F_2 \) are determined, the stress expressions and electric displacement expressions could be obtained.

For this, by substituting equations (16) and (17) into equations (4) and (5), we obtain

\[ \sigma_{yz}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( c_{44}^{(2)}(\alpha)e^{\text{iat}} \right) \left( H_3 - H_2e^{-2\text{iat}} \right) F_1 \]

\[ + \left( H_4 - H_3e^{-2\text{iat}} \right) F_2 \]

\[ \cdot \left( H_{11} - H_{15}e^{-2\text{iat}} \right) F_1 \]

\[ + \left( H_{12} - H_{16}e^{-2\text{iat}} \right) F_2 \] \[
\cdot e^{-\text{iat} \alpha} d\alpha, \]

\[ D_y(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( e_{15}^{(2)}(\alpha)e^{\text{iat}} \right) \left( H_3 - H_2e^{-2\text{iat}} \right) F_1 \]

\[ + \left( H_4 - H_3e^{-2\text{iat}} \right) F_2 \]

\[ \cdot \left( H_{11} - H_{15}e^{-2\text{iat}} \right) F_1 \]

\[ + \left( H_{12} - H_{16}e^{-2\text{iat}} \right) F_2 \] \[
\cdot e^{-\text{iat} \alpha} d\alpha. \]

Next, we need to transform equations (35) and (36) into singular integral equations of the first kind with Cauchy
kernel. In another form, the integral equations (35) and (36) will be

\[
\sigma_{yz}(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \left[ (e_{x, y}^{(2)} H_3 - e_{x, y}^{(2)} H_7) + e_{11}^{(2)} H_{11} - e_{11}^{(2)} H_{15} \right] F_1 + (e_{44}^{(2)} H_4 - e_{44}^{(2)} H_8) + e_{11}^{(2)} H_{12} - e_{11}^{(2)} H_{16} \right] e^{-j\alpha x} d\alpha = -\sigma_0,
\]

(37)

\[
D_y(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \left[ (e_{x, y}^{(2)} H_3 - e_{x, y}^{(2)} H_7) + e_{11}^{(2)} H_{11} - e_{11}^{(2)} H_{15} \right] + (e_{44}^{(2)} H_4 - e_{44}^{(2)} H_8) + e_{11}^{(2)} H_{12} - e_{11}^{(2)} H_{16} \right] e^{-j\alpha x} d\alpha = -D_0,
\]

(38)

where

\[
Q_1(\alpha) = e_{44}^{(2)} H_3 - e_{44}^{(2)} H_7 + e_{11}^{(2)} H_{11} - e_{11}^{(2)} H_{15},
\]

\[
Q_2(\alpha) = e_{44}^{(2)} H_4 - e_{44}^{(2)} H_8 + e_{11}^{(2)} H_{12} - e_{11}^{(2)} H_{16},
\]

\[
Q_3(\alpha) = e_{11}^{(2)} H_{12} - e_{11}^{(2)} H_{16}.
\]

Equations (37) and (38) can be rewritten in the following forms:

\[
\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_1 F_1 e^{-j\alpha x} d\alpha
\]

(40)

\[
+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_2 F_2 e^{-j\alpha x} d\alpha = -\sigma_0,
\]

\[
\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_3 F_1 e^{-j\alpha x} d\alpha
\]

(41)

\[
+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_4 F_2 e^{-j\alpha x} d\alpha = -D_0.
\]

By substituting the expressions of \(F_1\) and \(F_2\) into equations (40) and (41), we obtain

\[
\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_1 F_1 e^{-j\alpha x} d\alpha \left( \int_{\alpha_j}^{b_f} g_W^{(j)}(t) e^{ja t} dt \right) e^{-j\alpha x} d\alpha
\]

\[
+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_2 F_2 e^{-j\alpha x} d\alpha \left( \int_{\alpha_j}^{b_f} g_W^{(j)}(t) e^{ja t} dt \right) e^{-j\alpha x} d\alpha = -\sigma_0,
\]

\[
\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_3 F_1 e^{-j\alpha x} d\alpha \left( \int_{\alpha_j}^{b_f} g_W^{(j)}(t) e^{ja t} dt \right) e^{-j\alpha x} d\alpha
\]

(42)

\[
+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) Q_4 F_2 e^{-j\alpha x} d\alpha \left( \int_{\alpha_j}^{b_f} g_W^{(j)}(t) e^{ja t} dt \right) e^{-j\alpha x} d\alpha = -D_0.
\]

Through changing the order of the integration, we obtain

\[
\frac{1}{2\pi} \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt
\]

\[
+ \frac{1}{2\pi} \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt = -\sigma_0,
\]

\[
\frac{1}{2\pi} \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt
\]

(43)

\[
+ \frac{1}{2\pi} \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt = -D_0.
\]

On the cracks’ surface,

\[
\int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt
\]

\[
+ \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt = -\pi\sigma_0,
\]

\[
\int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt
\]

(45)

\[
+ \int_{\alpha_j}^{b_f} \left( \int_{-\infty}^{+\infty} \left( \frac{|a|e^{j\alpha y}}{H} \right) e^{-j\alpha x} d\alpha \right) g_W^{(j)}(t) dt = -\pi D_0.
\]

(46)
where

\[ Q_{1x} = e^{(2)}_{44} e^{(2)}_{44} (e^{(1)}_{44} - e^{(1)}_{15}) - (e^{(1)}_{15} e^{(2)}_{44} - e^{(1)}_{15} e^{(2)}_{15}) \]

\[ Q_{2x} = e^{(1)}_{15} e^{(2)}_{44} (e^{(1)}_{11} - e^{(1)}_{12}) + e^{(1)}_{11} e^{(2)}_{44} (e^{(2)}_{44} - e^{(2)}_{11}) \]

\[ Q_{3x} = e^{(1)}_{12} e^{(2)}_{15} (e^{(1)}_{11} - e^{(1)}_{12}) - (e^{(1)}_{15} e^{(2)}_{15} - e^{(1)}_{15} e^{(2)}_{15}) \]

\[ Q_{4x} = e^{(1)}_{12} e^{(2)}_{15} (e^{(1)}_{11} - e^{(1)}_{12}) + e^{(1)}_{11} e^{(2)}_{15} (e^{(2)}_{15} - e^{(2)}_{11}) \]

\[ H_x = (e^{(2)}_{15} - e^{(1)}_{15})^2 - (e^{(2)}_{44} - e^{(2)}_{15})(e^{(1)}_{11} - e^{(1)}_{12}) \]

\[ k_1(x,t) = \int_0^\infty \left( \frac{Q_1(a)}{H} - \frac{Q_1}{H_x} \right) \sin(\alpha(t-x))da, \]

\[ k_2(x,t) = \int_0^\infty \left( \frac{Q_2(a)}{H} - \frac{Q_2}{H_x} \right) \sin(\alpha(t-x))da, \]

\[ k_3(x,t) = \int_0^\infty \left( \frac{Q_3(a)}{H} - \frac{Q_3}{H_x} \right) \sin(\alpha(t-x))da, \]

\[ k_4(x,t) = \int_0^\infty \left( \frac{Q_4(a)}{H} - \frac{Q_4}{H_x} \right) \sin(\alpha(t-x))da. \]

Ultimately, equations (4) and (5) are reduced to coupling the first-kind Cauchy singular integral equations.

In equations (45) and (46), \( g^{(j)}_W(t) \) and \( g^{(j)}_B(t) \) are unknown functions with the following conditions:

\[ \int_a^b g^{(j)}_W(t)dt = 0, j = 1, 2, \ldots, n. \]

\[ \int_a^b g^{(j)}_B(t)dt = 0, j = 1, 2, \ldots, n. \]

By the principle of superposition and mixed boundary conditions, equations (45) and (46) can be formulated as

\[ \sum_{j=1}^n \int_a^b \left( \frac{Q_{1x}}{H_x} \frac{1}{t_j - x_k} + k_1(x_k,t_j) \right) g^{(j)}_W(t_j)dt_j + \int_a^b \left( \frac{Q_{2x}}{H_x} \frac{1}{t_j - x_k} + k_2(x_k,t_j) \right) g^{(j)}_B(t_j)dt_j = -\pi \sigma_0, \]

\[ x_k \in (a_k, b_k), j = 1, 2, \ldots, n. \]

(49)

\[ \sum_{j=1}^n \int_a^b \left( \frac{Q_{3x}}{H_x} \frac{1}{t_j - x_k} + k_3(x_k,t_j) \right) g^{(j)}_W(t_j)dt_j + \int_a^b \left( \frac{Q_{4x}}{H_x} \frac{1}{t_j - x_k} + k_4(x_k,t_j) \right) g^{(j)}_B(t_j)dt_j = -\pi D_0, \]

\[ x_k \in (a_k, b_k), j = 1, 2, \ldots, n. \]

(50)

4. Resolution of Singular Integral Equations

4.1. Normalization of Singular Integral Equations. For the convenience of numerical calculations, we define the normalized quantities as follows:

\[ r_j = \frac{x_j - c_{0j}}{a_{0j}}, \]

\[ s_j = \frac{t_j - c_{0j}}{a_{0j}}. \]

(51)

\[ g^{(j)}_W(x) = f^{(j)}_1(x), \]

\[ g^{(j)}_B(x) = f^{(j)}_2(x), \]

(52)

\[ k_i(x,t) = L_i(r,s), \quad i = 1, 2, 3, 4, \]

(53)

Using equations (51) and (52), the integral equations (49) and (50) can be expressed as
\[
\sum_{j=1}^{n} \left( \int_{-1}^{1} \left( \frac{Q_{s}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{1}(r_{k}, s_{j}) \right) \cdot f_{1}^{(j)}(s_{j}) ds_{j} + \int_{-1}^{1} \left( \frac{Q_{s}^{*}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{2}(r_{k}, s_{j}) \right) f_{1}^{(j)}(s_{j}) ds_{j} \right) = -\pi \sigma_{0}, \tag{54}
\]

\[
\sum_{j=1}^{n} \left( \int_{-1}^{1} \left( \frac{Q_{s}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{3}(r_{k}, s_{j}) \right) \cdot f_{1}^{(j)}(s_{j}) ds_{j} + \int_{-1}^{1} \left( \frac{Q_{s}^{*}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{4}(r_{k}, s_{j}) \right) f_{1}^{(j)}(s_{j}) ds_{j} \right) = -\pi D_{0}. \tag{55}
\]

The dislocation density functions \( f_{1}^{(j)}(s_{j}) \) and \( f_{2}^{(j)}(s_{j}) \) \((j = 1, 2, \ldots, n)\) in equations (54) and (55) have the squareroot-type singularity. So we can express \( f_{1}^{(j)}(s_{j}) = (1/\sqrt{1 - s_{j}^{2}}) \cdot g_{1}^{(j)}(s_{j}) \sigma_{0}, \) where \( g_{1}^{(j)}(s_{j}) \) is a continuous function defined in the interval \([-1, 1]\). In the same way, \( f_{2}^{(j)}(s_{j}) \) may be expressed as

\[
f_{1}^{(j)}(s_{j}) = \frac{g_{1}^{(j)}(s_{j})}{\sqrt{1 - s_{j}^{2}}} \pi \sigma_{0}, \tag{56}
\]

\[
f_{2}^{(j)}(s_{j}) = \frac{g_{2}^{(j)}(s_{j})}{\sqrt{1 - s_{j}^{2}}} \pi D_{0}, \quad j = 1, 2, \ldots, n. \tag{57}
\]

Using the Chebyshev collocation method, one can transform equations (54), (55), and (48) into the system of algebraic equations as follows:

\[
\frac{1}{N} \sum_{k=0}^{N} \chi_{k} \sum_{j=1}^{n} \left[ \left( \frac{Q_{s}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{1}(r_{kq}, s_{j}) \right) g_{1}^{(j)}(s_{j}) \right] = -1, \tag{58}
\]

\[
\frac{1}{N} \sum_{k=0}^{N} \chi_{k} \sum_{j=1}^{n} \left[ \left( \frac{Q_{s}^{*}}{a_{0} \xi + a_{0} \xi + c_{0} - a_{0} a_{0} r_{k} - c_{0}} + a_{0} L_{2}(r_{kq}, s_{j}) \right) g_{2}^{(j)}(s_{j}) \right] = -1, \tag{59}
\]

\[
\sum_{k=0}^{N} \chi_{k} \tilde{g}_{1}^{(j)}(s_{j}) = 0, \tag{60}
\]

where

\[
\chi_{0} = \chi_{N} = \frac{1}{2},
\]

\[
\chi_{1} = \cdots = \chi_{N-1} = 1,
\]

\[
r_{kq} = \cos \left( \frac{2q - 1}{2N} \pi \right),
\]

\[
s_{jR} = \cos \left( \frac{N \pi}{N} \right).
\]

Equations (58)-(60) are solved numerically to get the solutions of \( g_{1}^{(j)}(s_{j}) \) and \( g_{2}^{(j)}(s_{j}) \), which can permit to get the stress and electric displacement.

4.2. Fracture Parameters. The intensity factors are defined by [4]

\[
K_{o}^{(j)} = \lim_{x \to a_{j}} \sqrt{2\pi \left( a_{j} - x \right)} \sigma_{xy}(x, 0), \tag{62}
\]

\[
K_{h}^{(j)} = \lim_{x \to b_{j}} \sqrt{2\pi \left( x - b_{j} \right)} \sigma_{xy}(x, 0), \tag{63}
\]

\[
K_{D}^{(j)} = \lim_{x \to a_{j}} \sqrt{2\pi \left( a_{j} - x \right)} D_{y}(x, 0), \tag{64}
\]

\( N \) is the node number of the quadrature formula and \( s_{jR} \) and \( r_{kq} \) are the zero points of the first and second kinds of Chebyshev polynomials. Equations (58)-(60) are solved numerically to get the solutions of \( g_{1}^{(j)}(s_{j}) \) and \( g_{2}^{(j)}(s_{j}) \), which can permit to get the stress and electric displacement.
\[
K_{Dj}^b = \lim_{x \to b_j} \frac{1}{\pi} \left( x - b_j \right) D_y(x, 0), \quad j = 1, 2, \ldots, n.
\]

According to equations (45) and (46), the singular parts of \( \sigma_{yz}(x, 0) \) and \( D_y(x, 0) \) are

\[
\lim_{x \to a_j} \sigma_{yz}(x, 0) = \frac{Q_{1s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{Q_{2s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j,
\]

\[
\lim_{x \to b_j} D_y(x, 0) = \frac{Q_{1s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{Q_{2s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j.
\]

Substituting equations (56) and (57) into equations (66)–(69), we obtain

\[
\lim_{x \to a_j} \sigma_{yz}(x, 0) = \frac{\sigma_0}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{\sigma_0}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j.
\]

\[
\lim_{x \to b_j} D_y(x, 0) = \frac{\sigma_0}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{\sigma_0}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j.
\]

Substituting equations (56) and (57) into equations (66)–(69), we obtain

\[
\lim_{x \to a_j} \sigma_{yz}(x, 0) = \frac{Q_{1s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{Q_{2s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j\]

\[
\lim_{x \to b_j} D_y(x, 0) = \frac{Q_{1s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{1}^{(j)}(s_j) ds_j + \frac{Q_{2s}}{H_s} \lim_{s_j \to -1} \int_{-1}^{1} f_{2}^{(j)}(s_j) ds_j.
\]

Substituting equations (70)–(73) into equations (62)–(65), we obtain

\[
K_{\sigma}^g = \frac{\sigma_0}{\pi a_0} \left[ Q_{1s} g_1^{(j)}(-1) + Q_{2s} g_2^{(j)}(-1) \right],
\]

\[
K_{\sigma}^b = \frac{\sigma_0}{\pi a_0} \left[ Q_{1s} g_1^{(j)}(1) + Q_{2s} g_2^{(j)}(1) \right],
\]

\[
K_Y^g = D_0 \frac{\sigma_0}{\pi a_0} \left[ Q_{1s} g_1^{(j)}(-1) + Q_{2s} g_2^{(j)}(-1) \right],
\]

\[
K_Y^b = D_0 \frac{\sigma_0}{\pi a_0} \left[ Q_{1s} g_1^{(j)}(1) + Q_{2s} g_2^{(j)}(1) \right], \quad j = 1, 2, \ldots, n.
\]

The energy release rate of piezoelectric materials has the following form [24]:

\[
G = \frac{1}{4} \left[ \omega_1 K_{\sigma}^g + \omega_2 K_{\sigma} K_Y + \omega_3 K_Y^2 \right],
\]

where

\[
K = [K_{\sigma}, K_Y],
\]

\[
\omega_1 = \frac{Q_{1s}}{H_s A},
\]

\[
\omega_2 = \frac{Q_{2s} - Q_{1s}}{H_s A},
\]

\[
\omega_3 = -\frac{Q_{2s}}{H_s A},
\]

\[
A = \left[ \begin{array}{cc}
\frac{Q_{1s}}{H_s} & \frac{Q_{2s}}{H_s} \\
\frac{Q_{2s}}{H_s} & \frac{Q_{1s}}{H_s}
\end{array} \right].
\]
5. Numerical Process and Results

This section is divided into two parts. Firstly, some numerical applications to verify the obtained solutions are given, in the case when we have a single interface crack. Then, the influence of material constants on the energy release rate is studied when we have multiple interface cracks.

Initially, we assume that the piezoelectric substrate is the PZT-5H ceramic. Its parameters are given as [2]

\[
\begin{align*}
\varepsilon_{44}^{(2)} &= 3.53 \times 10^{10} \text{ N/m}^2, \\
\varepsilon_{33}^{(2)} &= 17.0 \text{ C/m}^2, \\
\varepsilon_{11}^{(2)} &= 1.51 \times 10^{-8} \text{ C/Vm,} \\
G_{cr} &= 5 \text{ N/m,}
\end{align*}
\]

where superscript 2 represents the substrate material.

5.1. A Single Interface Crack. To validate the solution, we chose the case that material 1 is a fine-grained coating made of PZT-5H ceramics by plasma spraying technology and material 2 is normal PZT-5H ceramics (where 1 and 2 represent the fine-grained coating and substrate material, respectively), which means that material 1 and material 2 have approximately the same piezoelectric and dielectric constants under suitable conditions, and the elastic modulus of material 1 is larger than that of material 2. So \( c_{44}^{(1)}/c_{44}^{(2)} = 2.5, h_1/h_2 = 1/100, \) and the single interface crack of \( l_0 = 5 \text{ mm} \) loaded by an antiplane stress of \( \sigma_0 = 4.2 \text{ MPa} \) and a variable electric displacement \( D_0 \) are considered. The curves of the energy release rate \( G/G_{cr} \) as a function of \( D_0 \) under different values of \( l_0/h_1 \) are plotted (Figure 2). Then, the same interface crack is loaded by an electric displacement \( D_0 \) of \( 10^{-3} \text{ c/m}^2 \) and a variable stress \( \sigma_0 \). For different values of \( l_0/h_1 \), the curves \( G/G_{cr} \) as a function of \( \sigma_0 \) are plotted (Figure 3).

5.1.1. Studies of the Effect of Stress \( \sigma_0 \) and Electric Displacement \( D_0 \) on Energy Release Rate. Effects of electric displacement \( D_0 \) and stress \( \sigma_0 \) on the energy release rate for different \( l_0/h_1 \) are shown in Figures 2 and 3, respectively.

5.2. Multiple Interface Cracks. The interaction between multiple interface cracks is investigated. The distance between double interface crack centers is chosen as \( a/l_0 = a/l_1 = 6/5 \) (where \( l_0 = l_1 = 5 \text{ mm}, l_2 = 3 \text{ mm}, \) and \( h_1 = 5 \text{ mm} \)

![Figure 2: Effects of electric displacement \( D_0 \) on the energy release rate for different \( l_0/h_1 \).](image)

![Figure 3: Effects of stress \( \sigma_0 \) on the energy release rate for different \( l_0/h_1 \).](image)

![Figure 4: Effects of stress \( \sigma_0 \) on the energy release rate for double identical interface cracks.](image)
and other material parameters are the same as in the previous selection). The influences of electric displacement $D_0$ on the normalized energy release rate $G/G_{cr}$ are shown in Figures 4 and 5 and the change rules of the normalized energy release rate $G/G_{cr}$ and the value of $l/h_1$ are shown in Figures 6–8 for the interaction between three identical and different Griffith cracks. The rules of the normalized energy release rate $G/G_{cr}$ versus the value of $l/h_1$ are shown in Figures 9 and 10 (where $a/l_0 = a/l_1 = 6/5$, $l_0 = l_1 = 5$ mm, $l_2 = 3$ mm, $l_3 = 2.5$ mm, $h_1 = 5$ mm, and $G_0 = \lim_{l/h_1 \to 0} G$ and other material parameters are the same as in the previous selection).

5.2.1. Studies of the Effect of Stress $\sigma_0$ and Electric Displacement $D_0$ on Energy Release Rate for Double Griffith Interface Cracks. Effects of electric displacement $D_0$ and stress $\sigma_0$ on the energy release rate for double Griffith interface cracks are shown in Figures 4 and 5.

5.2.2. Studies of the Influence of $l/h_1$ on Energy Release Rate for Double Interface Cracks. Effects of $l/h_1$ on the energy release rate for double Griffith interface cracks are shown in Figures 6–8.

5.2.3. Studies of the Influence of $l/h_1$ on Energy Release Rate for Three Interface Cracks. Effects of $l/h_1$ on the energy release rate for three interface cracks are shown in Figures 9 and 10.
6. Results and Discussion

Figure 2 displays the variation of the normalized energy release rate \( G/G_{cr} \) against the applied electric displacement \( D_0 \). It can be clearly seen that by varying \( l_0/h_1 \), the curves \( G/G_{cr} \) are parabolas, and as \( D_0 \) decreases from zero, \( G/G_{cr} \) always decreases. However, when \( D_0 \) increases from zero, \( G/G_{cr} \) firstly increases and then begins to decrease. The results show that negative electric displacement loading always prevents crack propagation and positive electric displacement loading may prompt or prevent crack propagation, which are consistent with the conclusions of Pak et al. [4, 10, 13]. However, the peak energy release rate of the fine-grained piezoelectric coating/substrate structure is higher than that of the large-grained piezoelectric coating/substrate structure.

Figure 3 indicates the effects of applied stress \( \sigma_0 \) on the normalized energy release rate \( G/G_{cr} \). It can be easily seen that as stress \( \sigma_0 \) increases from zero, \( G/G_{cr} \) always increases. When the stress \( \sigma_0 \) decreases from zero, the \( G/G_{cr} \) firstly decreases and then increases, and it indicates the initial closure of a crack followed by a type of crack propagation due to compressive loading. The results indicate that stress loading may prompt or retard crack propagation.

The diversification of the normalized energy release rate \( G/G_{cr} \) versus the applied electric displacement \( D_0 \) for double identical and different interface cracks is shown in Figures 4 and 5. It is found that the variation of the energy release rate against the applied electric displacement is similar to that shown in Figure 2. However, the normalized energy release rate of the double interface crack has a larger peak compared to that shown in Figure 2. Moreover, the peak values of the energy release rate near the crack tips \( b_1, a_1 \) are larger than those near the crack tips \( a_1, b_2 \). Because of symmetry when the length of double interface cracks is equal, the variations of the energy release rate near the crack tips \( b_1, a_1 \) and \( a_1, b_2 \) are the same.

Figures 6–8 present the effects of the values of \( l/h_1 \) (where \( l \) stands for the crack length) on the normalized energy release rate \( G/G_0 \) for double identical and different interface cracks. We found that the energy release rate increased with the increase of \( l/h_1 \). The change rate of the inner crack tips is higher with the variation of \( l/h_1 \) than that of the outer crack tips. The lines in Figure 8 intersect with the variation of \( l/h_1 \) in this case; if the double cracks have the same length, they are consistent with previous conclusions.

Figures 9 and 10 illustrate the change rule of the normalized energy release rate \( G/G_0 \) and \( l/h_1 \) for three identical and different interface cracks. The results reflect that the values of the energy release rate susceptible to \( l/h_1 \) near the inner crack tips are larger than those that approach the outer crack tips, regardless of the number of cracks. At the same time, we noticed that when the value of thickness of coating \( h_1 \) was small, the value of the energy release rate was large, which indicated that the structure was more safe when the coating was thin. This reflects the superiority of the fine-grained piezoelectric material structure because it can make the structure thinner.

7. Conclusions

In this paper, multiple Griffith cracks at the interface of a fine-grained piezoelectric coating/substrate are studied. The variation of the energy release rate with applied stress, electric displacement, crack length, and coating thickness is demonstrated. Based on the change rule of the energy release rate and various parameters, the following conclusions are drawn:

1. The interaction between cracks, applied electromechanical loads, and coating thickness has a great influence on the energy release rate for Griffith interface cracks. Moreover, the negative electric
displacement loading always prevents crack propagation, and positive electric displacement loading may prompt or delay crack propagation. Furthermore, the appropriate stress loading can restrain the crack growth. Therefore, in the practical application of engineering, we can choose the appropriate load to ensure the safety of the structure.

(2) For a bimaterial structure, compared with previous works [10, 13], we found that the peak of the energy release rate at the crack tips of a fine-grained piezoelectric coating/substrate interface is larger than that at the crack tips of a large-grained piezoelectric material/substrate structure interface, and thus, the fine-grained piezoelectric coating/substrate structure has safer material properties. Therefore, in order to better resist fractures, fine-grained piezoelectric materials are a better choice in engineering.

**Nomenclature**

- \(a_j, b_j\): Left and right tips of arbitrary interface cracks
- \(A_{ik} (\alpha)\): Unknown functions satisfying the loading displacement \(W(x, y)\)
- \(A_{2k} (\alpha)\): Unknown functions satisfying the electric field \(\Phi(x, y)\)
- \(B_{ik} (\alpha)\): Unknown functions satisfying the loading displacement \(W(x, y)\)
- \(B_{2k} (\alpha)\): Unknown functions satisfying the electric field \(\Phi(x, y)\)
- \(c_{44}^{(k)}\): Elastic modulus
- \(C\): The constant terms of linear equations with unknown functions \(A_{ik} (\alpha), B_{ik} (\alpha), A_{2k} (\alpha), B_{2k} (\alpha)\)
- \(D_0\): Electric displacement
- \(D_x\) and \(D_y\): Electric displacement components
- \(F_1\): Integral expression with unknown functions \(g^{(j)}_w(x)\)
- \(F_2\): Integral expression with unknown functions \(g^{(j)}_w(x)\) and \(g^{(j)}_1 (s_j)\) and \(g^{(j)}_2 (s_j)\)
- \(g^{(j)}_w(x)\): Unknown functions with the condition \(\int_{a_j}^{b_j} g^{(j)}_w (t) dt = 0\)
- \(G_{cr}\): Critical crack extension force
- \(G\): Energy release rate
- \(h_1\): Thickness of the coating
- \(h_2\): Thickness of the substrate
- \(H_s\): Limit value of \(H\) when \(\alpha\) tends to infinity
- \(H_q\): Combination of \(e^{-2|\alpha|h_1}, e^{-|\alpha|h_2}, \) and \(e^{-4|\alpha|h_1}\)
- \(H_2\): Combination of \(e^{-2|\alpha|h_1}\) and \(e^{-2|\alpha|h_2}\)
- \(K_{oa}\): Stress intensity factor at the left tip of the arbitrary interface crack
- \(K_{ob}\): Stress intensity factor at the right tip of the arbitrary interface crack
- \(K_{D1}\): Electric displacement intensity factor at the left tip of the arbitrary interface crack
- \(K_{D2}\): Electric displacement intensity factor at the right tip of the arbitrary interface crack
- \(K\): Intensity factors
- \(l\): Crack length
- \(N\): The node number of the quadrature formula
- \(Q_t\): Limit value of \(Q\) when \(\alpha\) tends to infinity
- \(Q_1\): Linear combination of \(H_q\)
- \(r_{kq}\): Zero points of the second kind of Chebyshev polynomials
- \(s_{jr}\): Zero points of the first kind of Chebyshev polynomials
- \(U\) and \(V\): In-plane displacement
- \(W\): Out-of-plane displacement
- \(X\): \(X\)-axis of the rectangular space coordinate system
- \(Y\): \(Y\)-axis of the rectangular space coordinate system
- \(Z\): \(Z\)-axis of the rectangular space coordinate system
- \(\sigma_0\): Stress
- \(\sigma_{xx}\) and \(\sigma_{yy}\): Stress components
- \(\epsilon_{15}^{(k)}\): Piezoelectric constants
- \(\epsilon_{11}^{(k)}\): Dielectric constants
- \(\Phi\): In-plane electric fields
- \(\nabla^2\): Two-dimensional Laplacian operator
- \(\Lambda\): Matrix composed of \(Q_t\) and \(H_q\).

**Appendix**

The expressions of \(H\) and \(H_q, q = 1, 2, ..., 16\), are as follows:
\( a_1 = (1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_{44}^{(2)}(1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_{44}^{(1)}, \)
\( a_2 = (1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_1^{(2)}(1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_1^{(1)}, \)
\( a_3 = -(1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_1^{(2)}(1 - e^{-2|\alpha|h_1})(1 + e^{-2|\alpha|h_1})c_1^{(1)}, \)
\( H = a_2^2 - a_1a_3, \)
\( H_1 = \frac{e^{-2|\alpha|h_1}(a_2^2 - a_1a_3) + e^{-2|\alpha|h_1}(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{44}^{(1)}a_2)}{(1 + e^{-2|\alpha|h_1})} \)
\( H_2 = \frac{e^{-2|\alpha|h_1}(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2)}{(1 + e^{-2|\alpha|h_1})} \)
\( H_3 = (1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{15}^{(1)}a_2), \)
\( H_4 = (1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2), \)
\( H_5 = a_2^2 - a_1a_3 + (1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{44}^{(1)}a_2) \)
\( H_6 = \frac{(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2)}{(1 + e^{-2|\alpha|h_1})} \)
\( H_7 = e^{-2|\alpha|h_1}(1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{15}^{(1)}a_2), \)
\( H_8 = e^{-2|\alpha|h_1}(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2), \)
\( H_9 = e^{-2|\alpha|h_1}(a_2^2 - a_1a_3) + e^{-2|\alpha|h_1}(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{44}^{(1)}a_2) \)
\( H_{10} = \frac{e^{-2|\alpha|h_1}(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2)}{(1 + e^{-2|\alpha|h_1})} \)
\( H_{11} = (1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{44}^{(1)}a_2), \)
\( H_{12} = (1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{44}^{(1)}a_2), \)
\( H_{13} = a_2^2 - a_1a_3 + (1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{44}^{(1)}a_3 - c_{44}^{(1)}a_2) \)
\( H_{14} = \frac{(1 + e^{-2|\alpha|h_1})(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2)}{(1 + e^{-2|\alpha|h_1})} \)
\( H_{15} = e^{-2|\alpha|h_1}(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{44}^{(1)}a_2), \)
\( H_{16} = e^{-2|\alpha|h_1}(1 - e^{-4|\alpha|h_1})(c_{15}^{(1)}a_3 - c_{15}^{(1)}a_2). \)

Data Availability

All the numerical calculation data used to support the findings of this study can be obtained by calculating the equations in this paper, and piezoelectric material parameters are taken from reference [1]. The codes used in this paper are available from the corresponding author upon request.
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References


