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Research Article

H_∞ Control for Mixed-Mode Based Switched Nonlinear Systems with Time-Varying Delay

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This paper investigated H_∞ performance of the switched nonlinear systems with time-varying delay, which contains mixed modes, important characteristics of switched delay systems. Most of the references ignored the existence of mixed modes in such systems, which is a fatal mistake. This paper will correct the mistake of the existing references. The mixed modes give the H_∞ control for switched nonlinear delay systems a challenge. With this difficulty, we take measures to handle the effects of mixed modes on such systems and achieve the H_∞ performance of such systems. Firstly, the H_∞ control analysis is divided into two cases, and the sufficient conditions for the exponential decay of the Lyapunov-Krasovskii functionals are given with the integral interval approach. Secondly, the combination of average dwell-time approach and subsystem activation rate is utilized to achieve the H_∞ performance for the mixed-mode based switched systems with time-varying delay. Finally, simulation examples show the efficiency of the switching control strategy.

1. Introduction

In recent decades, switched delay systems have emerged as a hot subject of research because of their applications in many fields, such as network control system, power system, and dynamic optimization [1, 2]. Lyapunov-Krasovskii functions are always utilized to analyze the stability for time delay systems [1, 3]. Average dwell time method is a common method for solving the asymptotic stability of switched delay system [4, 5]. However, switched time-varying delay system is hard to control because of the interaction between switching characteristics and time delay characteristics. It should be pointed out that the mixed modes are widespread in various switched delay systems, where the state depends on the present subsystem and the previous subsystem at the same time. It imposes great difficulty on the H_∞ control problem. Without loss of generality, most of the results are imposed constraints on the switching rules and time delays, which can avoid the mixed modes happening. Reference [6] is concerned with the robust tracking control for switched delay systems, but the strict switching constrained condition avoids the mixed modes happening. Reference [7] investigated the passivity for the switched delay systems under stochastic disturbance. Reference [8] investigated finite-time control for the switched delay systems via mode-based average dwell-time approach. It effectively avoided the mixed modes happening by the method that each mode had its respective dwell time. The above references have not solved the effects of mixed modes on such systems. Reference [9] proposed the concept for the mixed modes of the switched delay system. It effectively solved the stabilization problem of switched networked delay systems, which has aroused great concern. Reference [10] analyzed the effects of mixed modes on a class of switched delay systems and derived the stability condition for such system, but the proposed method is not applicable to nonlinear delay systems. Lyapunov functional approach is an effective strategy to the delay systems. Reference [11] proposed conditions to achieve the decay of solutions for the delay differential equation via the Lyapunov functional method. Reference [12] defined a novel Lyapunov function to guarantee the boundedness of solutions for a vector Linard equation. To the author's knowledge, H_∞ control issues on mixed-mode based switched delay systems have not been explored, which motivates us to further investigate this work.

On the other hand, the study on the switched systems between stable and unstable modes has been a hot subject in recent years. Reference [13] was concerned with the H_∞ performance of the uncertain switched neutral systems with stable and unstable modes, which first solved the H_∞ control problems on such systems. Reference [14] derived exponential stability conditions for the switched singular systems with stable and unstable modes, which first solved the exponential stability issue on such systems. Reference [15] derived the stability conditions of switched delay systems, which effectively eliminated effects of the unstable subsystems on such systems. However, [13–15] have not taken the mixed modes into account, which is a fatal mistake. If the switching between the stable subsystem and the unstable subsystem occurs over two times during the period of the time delay of the system, the mixed modes will be inspired. So mixed modes are widespread in the switched delay systems and they should not be ignored. If the mixed modes are ignored, the stability of such system will not be achieved and the transient performance of the system will be adversely affected. The combination of the mixed modes and the switched systems with stable modes and unstable modes makes the H_∞ control for such systems a challenge, where the state depends not only on the stable modes but also on the unstable modes. With the difficulty, we take the mixed modes and the H_∞ control strategy into the switched nonlinear delay systems with stabilizable subsystems and unstabilizable subsystems and derived the conditions for the H_∞ performance of the mixed-mode based switched nonlinear delay systems.

In this paper, the H_∞ control for the mixed-mode based switched nonlinear delay systems is investigated. This focused on solving the effect of the mixed modes on the switched system with the parameter constraints. The main contributions can be listed as follows. First, correcting the mistake of [13–15], the effects of mixed modes are first considered for the switched nonlinear delay systems with stabilizable subsystems and unstabilizable subsystems. It is more reasonable to the H_∞ analysis of the switched nonlinear systems with time-varying delays because the mixed modes are widespread in such systems. The mixed modes make the switching between the stabilizable subsystems and the unstabilizable subsystems more complex and give the H_∞ analysis of such systems a challenge. Second, due to the effects of mixed modes, the combination of the integral interval for the Lyapunov function and the estimation of quadrature is utilized to demonstrate the exponential decay of the Lyapunov functions. Third, sufficient criteria are given to achieve the H_∞ performance for the mixed-mode based switched delay systems via dwell-time strategy and the subsystem activation rate approach.

This paper is organized as follows. The problem formulation is introduced in Section 2. The main results are stated in Section 3. Simulation examples are in Section 4. Conclusion is provided in Section 5.

Notation. The notations utilized in this paper are quite standard. R^n and $R^{n\times n}$ refer to, respectively, the n-dimensional Euclidean space and $n\times n$ real matrices. $Q>0(\geq 0)$ denotes that Q is positive (semipositive) definite. For a square matrix Q, $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ are its maximum eigenvalue and minimum eigenvalue, respectively.

2. Problem Formulation

Consider the switched nonlinear delay systems:

$$\begin{aligned}\dot{x}(t) &= f_{\sigma(t)}(x(t)) + A_{\sigma(t)}x(t) + D_\sigma x(t-\tau(t)) \\ &\quad + B_{\sigma(t)}u(t) + \omega(t) \\ y(t) &= C_{\sigma(t)}x(t) \\ x(t) &= \phi(t), \quad t\in[-\tau,0]\end{aligned} \tag{1}$$

where $x(t)\in\mathfrak{R}^n$ denotes the state vector, $u(t)\in\mathfrak{R}^r$ denotes the control input, $\omega(t)\in\mathfrak{R}^n$ denotes the exogenous disturbance which satisfies $L_2[0,\infty)$, $y(t)\in\mathfrak{R}^q$ denotes the output, $f_i(x(t)) : R^n \longrightarrow R^n$ denotes a nonlinear function, $\phi(t)$ denotes initial condition defined on $[-\tau,0]$, $\sigma(t) : [0,\infty) \longrightarrow \Omega = \{1,2\ldots N\}$, and $\tau(t)$ is the time-varying delay. $0<\tau(t)<\tau$ for a constant τ and $0<\dot{\tau}(t)<\delta$ for a constant δ; A_i, D_i, and B_i are the matrices of the system and B_i is invertible matrix. Define that ith subsystem is stabilizable subsystem and jth subsystem is unstabilizable subsystem, where $i\in\underline{N}$ and $j\in\Omega\setminus\underline{N}$.

Assumption 1. For any $x(t)\in\mathfrak{R}^n$ and $\hat{x}(t)\in\mathfrak{R}^n$, nonlinear function $f_i(x(t))$ meets the global Lipschitz condition:

$$\left\|f_i(x(t)) - f_i(\hat{x}(t))\right\| \leq M_i\left\|(x(t)-\hat{x}(t))\right\| \tag{2}$$

where $M_i\in R^{n\times n}$ is a given Lipschitz constant matrix.

The performance index is as follows:

$$\int_0^\infty e^{-\alpha t}x^T(t)Qx(t)\,dt \leq \gamma^2\int_0^\infty \omega^T(t)\omega(t)\,dt \tag{3}$$

where Q represents a positive weighting matrix, $\gamma>0$ represents the disturbance attenuation level, and α is positive scalar.

The controller of system (1) is chosen as

$$u(t) = K_{\sigma(t)}x(t) - B_{\sigma(t)}^T\left(B_{\sigma(t)}B_{\sigma(t)}^T\right)^{-1}f_{\sigma(t)}(0) \tag{4}$$

where $K_{\sigma(t)}$ is the controller gain.

Remark 2. In general, assuming that $f_{\sigma(t)}(0)=0$, the controller of the system will be $u(t)=K_{\sigma(t)}x(t)$. However, in order to handle system (1) which dissatisfied such characteristic, we modify the controller. In order to deal with $f_{\sigma(t)}(0)\neq 0$, the controller should contain $B_{\sigma(t)}^T(B_{\sigma(t)}B_{\sigma(t)}^T)^{-1}f_{\sigma(t)}(0)$. Then the condition of the H_∞ performance will be given in Section 3.

Taking controller (4) into system switched (1), the closed-loop system can be shown as

$$\begin{aligned}\dot{x}(t) &= \left(A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)}\right)x(t) + D_{\sigma(t)}x(t-\tau(t)) \\ &\quad + \left(f_{\sigma(t)}(x) - f_{\sigma(t)}(0)\right) + \omega(t) \\ x(t) &= \phi(t), \quad t\in[-\tau,0]\end{aligned} \tag{5}$$

If $\sigma(t) = i$, the system will be represented as

$$\begin{aligned}\dot{x} &= \left(A_i + B_i K_i\right) x(t) + D_i x(t - \tau(t)) \\ &\quad + \left(f_i(x(t)) - f_i(0)\right) + \omega(t)\end{aligned} \tag{6}$$

3. Main Result

Considering switched system (5), if there exists unstabilizable system in the subsystems, the H_∞ control of switched system will be complicated. Taking $z(t) = \dot{x}(t)$, one has

$$\begin{aligned}z(t) &= \left(A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)} + D_{\sigma(t)}\right) x(t) \\ &\quad - D_{\sigma(t)} \int_{t-\tau(t)}^{t} z(s)\, ds + \omega(t)\end{aligned} \tag{7}$$

Consider Lyapunov-Krasovskii functional as follows:

$$\begin{aligned}V_{\sigma(t)}(t) &= x^T(t) Q_{\sigma(t)} x(t) \\ &\quad + \int_{-\tau}^{0} \int_{t+\theta}^{t} z^T(s) e^{\chi_{\sigma(t)}(t-s)} R z(s)\, ds\, d\theta\end{aligned} \tag{8}$$

where Q_l and R are the symmetric positive matrices, $V_i(t)$ denotes the stabilizable subsystems, and $V_j(t)$ corresponds to the unstabilizable subsystems of (1). The parameters are chosen as $\chi_i = -\alpha$ and $\chi_j = \beta$. According to average dwell-time strategy, the unstabilizable modes activation rate satisfies

$$\Delta_u = \frac{T_u(T_1, T_2)}{T_1 - T_2} < \frac{\alpha - \alpha^*}{\alpha + \beta} \tag{9}$$

where $\alpha^* \in (0, \alpha)$, T_u represents the dwell time of unstabilizable subsystems during $[T_1, T_2)$, and Δ_u represents the unstabilizable subsystems activation rate.

Then (8) is rewritten as

$$\begin{aligned}V_{\sigma(t)}(t) &= x^T(t) Q_{\sigma(t)} x(t) \\ &\quad + \int_{t-\tau}^{t} (s - t + \tau) z^T(s) e^{\chi_{\sigma(t)}(t-s)} R z(s)\, ds\end{aligned} \tag{10}$$

The system is assumed that it is switched from the lth mode to the kth mode at the instant t_k and $(t - \tau) \in [t_{k-1}, t_k)$. When $t \in [t_k, t_k + \tau)$, one has

$$\begin{aligned}V_k(t) &= x^T(t) Q_k x(t) \\ &\quad + \int_{t-\tau}^{t_k} (s - t + \tau) z^T(s) e^{\chi_l(t-s)} R z(s)\, ds \\ &\quad + \int_{t_k}^{t} (s - t + \tau) z^T(s) e^{\chi_k(t-s)} R z(s)\, ds\end{aligned} \tag{11}$$

When $t \in [t_k + \tau, t_{k+1})$,

$$\begin{aligned}V_k(t) &= x^T(t) Q_k x(t) \\ &\quad + \int_{t-\tau}^{t} (s - t + \tau) z^T(s) e^{\chi_k(t-s)} R z(s)\, ds\end{aligned} \tag{12}$$

Remark 3. When $t \in [t_k, t_k + \tau)$, $V_k(t)$ depends not only on the kth mode but also on the lth mode, which is called mixed modes. If $t_{k+1} - t_k \le \tau$, the mixed modes must be considered. It makes the H_∞ analysis of switched delay systems a challenge. References [6–8] avoid the occurrence of mixed modes by constraining the switching point $t_{k+1} - t_k > \tau$. This has strict constraints on switching rules and time delays. The essential difference of this paper is to analyze the case $t_{k+1} - t_k \le \tau$ and to overcome the effects of mixed modes on system (6). The result of [15] can be regarded as a special case of this paper, where Case 1 has not happened.

In order to eliminate the effects of mixed modes on H_∞ analysis of systems (5), the parameter constraints of switched system (5) are given as follows.

Lemma 4. *For given constants $\alpha > 0, \beta > 0$, if there exist symmetric positive definite matrices Q_l and R, any matrices U_l, T_l with appropriate dimensions, and positive parameters q, q_0, and τ, such that the conditions*

$$\Theta_i = \begin{bmatrix} \eta_{11i} & \eta_{12i} & -U_i D_i & U_i & U_i \\ * & \eta_{22i} & -T_i D_i & T_i & T_i \\ * & * & \eta_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{13}$$

$$\overline{\Theta}_j = \begin{bmatrix} \overline{\eta}_{11j} & \overline{\eta}_{12j} & -U_j D_j & U_j & U_j \\ * & \overline{\eta}_{22j} & -T_j D_j & T_j & T_j \\ * & * & \overline{\eta}_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{14}$$

$$H_{j,k} = \begin{bmatrix} \varrho_{11j} & (1 - \delta_k) D_j R \\ * & 0 \end{bmatrix} \ge -q_0 I \tag{15}$$

hold, then when $\sigma(t) = i \in \underline{N}$, we have

$$V_i(t) \le e^{-\alpha(t-t_0)} V_i(t_0) - \int_{t_0}^{t} e^{-\alpha(t-s)} \Gamma(s)\, ds \tag{16}$$

When $\sigma(t) = j \in \Omega \setminus \underline{N}$, we have

$$V_j(t) \le e^{\beta(t-t_0)} V_j(t_0) - \int_{t_0}^{t} e^{\beta(t-s)} \Gamma(s)\, ds \tag{17}$$

where $k = 1, 2, \delta_1 = 0, \delta_2 = \delta$,
$\eta_{11i} = (A_i + B_i K_i + D_i)^T U_i^T + U_i(A_i + B_i K_i + D_i) + (\alpha + 1) Q_i + M_i^T M_i$
$\eta_{12i} = Q_i - U_i + (A_i + D_i)^T T_i^T$
$\eta_{22i} = -T_i^T - T_i + \tau R$
$\eta_{33} = -[\tau^{-1} e^{-\alpha\tau} - q(\alpha - \alpha^*) e^{\beta d}] R$
$\overline{\eta}_{11j} = (A_j + B_j K_j + D_j)^T U_j^T + U_j(A_j + B_j K_j + D_j) + (1 - \beta) Q_j + M_j^T M_j$
$\overline{\eta}_{12j} = Q_j - U_j + (A_j + D_j)^T T_j^T$

$\overline{\eta}_{22j} = -T_j^T - T_j + \tau R$
$\overline{\eta}_{33} = -\tau^{-1}e^{-\alpha\tau}R$
$\varrho_{11j} = (A_j + B_jK_j)^T R + R(A_j + B_jK_j) - \beta R + M_j^T R + RM_j$
$\Gamma(s) = x^T(s)Qx(s) - \gamma^2\omega^T(s)\omega(s)$

Proof. The Lyapunov functionals (11) will be analyzed in two cases, $t_{k+1} - t_k \le \tau$ and $t_{k+1} - t_k > \tau$.

Case 1. $t_{k+1} - t_k \le \tau$.

In this case, the switched system covers $[t_k, t_{k+1})$. Differentiating functional (11), one can obtain

$$\begin{aligned}\dot{V}_i(t) &= 2x^T(t)Q_iz(t) + \tau z^T(t)Rz(t) \\ &\quad - \int_{t-\tau}^{t_k} z^T(\theta)e^{\chi_j(t-\theta)}Rz(\theta)\,d\theta \\ &\quad - \int_{t_k}^{t} z^T(\theta)e^{\chi_i(t-\theta)}Rz(\theta)\,d\theta \\ &\quad + \chi_j\int_{t-\tau}^{t_k}(s-t+\tau)z^T(\theta)e^{\chi_j(t-\theta)}Rz(\theta)\,d\theta \\ &\quad + \chi_i\int_{t_k}^{t}(s-t+\tau)z^T(\theta)e^{\chi_i(t-\theta)}Rz(\theta)\,d\theta\end{aligned} \tag{18}$$

Suppose that the system is switched from the jth mode to the ith mode at instant t_k, and in the ith mode for $t \in [t_k, t_{k+1})$, one has

$$\begin{aligned}&\dot{V}_i(t) + \alpha V_i(t) \\ &\quad = 2x^T(t)Q_iz(t) + \tau z^T(t)Rz(t) + x^T(t)\alpha Q_ix(t) \\ &\quad - \int_{t-\tau}^{t_k} z^T(\theta)e^{\beta(t-\theta)}Rz(\theta)\,d\theta \\ &\quad - \int_{t_k}^{t} z^T(\theta)e^{-\alpha(t-\theta)}Rz(\theta)\,d\theta \\ &\quad + \beta\int_{t-\tau}^{t_k}(s-t+\tau)z^T(\theta)e^{\beta(t-\theta)}Rz(\theta)\,d\theta \\ &\quad + \alpha\int_{t-\tau}^{t_k}(s-t+\tau)z^T(\theta)e^{\beta(t-\theta)}Rz(\theta)\,d\theta\end{aligned} \tag{19}$$

For $t_k \in [t-\tau, t]$, the quadrature estimation inequality is established as follows:

$$\begin{aligned}&\int_{t-\tau}^{t_k}(s-t+\tau)Y(s)\,ds \\ &\quad \le q\frac{t_k - t + \tau}{\tau}\int_{t-\tau}^{t}(s-t+\tau)Y(s)\,ds\end{aligned} \tag{20}$$

where $q = e^{q_0(1+p)\tau/\lambda_{min}(R)}$ and $Y(s) = z^T(s)e^{\beta(t-s)}Rz(s)$. In order to obtain inequality (20), we take the derivative of $Y(s)$:

$$\begin{aligned}\dot{Y}(s) &= \Big[\big(A_j + B_jK_j\big)z(s) \\ &\quad + \big(1-\dot{\tau}_j(s)\big)D_jz\big(s-\tau_j(s)\big) + \dot{f}_j(x(s))\Big]^T \\ &\quad \cdot e^{\beta(t-s)}Rz(s) + \beta e^{\beta(t-s)}z^T(s)Rz(s) + z^T(s) \\ &\quad \cdot e^{\beta(t-s)}R\Big[\big(A_j + B_jK_j\big)z(s) \\ &\quad + \big(1-\dot{\tau}_j(s)\big)D_jz\big(s-\tau_j(s)\big) + \dot{f}_j(x(s))\Big] \\ &\le \Big[\big(A_j + B_jK_j\big)z(s) \\ &\quad + \big(1-\dot{\tau}_j(s)\big)D_jz\big(s-\tau_j(s)\big) + M_jz(s)\Big]^T \\ &\quad \cdot e^{\beta(t-s)}Rz(s) + \beta e^{\beta(t-s)}z^T(s)Rz(s) + z^T(s) \\ &\quad \cdot e^{\beta(t-s)}R\Big[\big(A_j + B_jK_j\big)z(s) \\ &\quad + \big(1-\dot{\tau}_j(s)\big)D_jz\big(s-\tau_j(s)\big) + M_jz(s)\Big] = \eta^T(s) \\ &\quad \cdot e^{\beta(t-s)}\Lambda_j\eta(s)\end{aligned} \tag{21}$$

where $\Lambda_j = \begin{bmatrix}\varrho_{11j} & (1-\dot{\tau}(s))D_jR \\ * & 0\end{bmatrix}$ and $\eta(s) = [z(s)\ z(s-\tau_j(s))]^T$.

Since $0 \le \dot{\tau}_j(s) = 1 - \sum_{k=1}^{2}\theta_k\delta_k$, where $0 \le \theta_k \le 1$, we can get $\dot{Y}(s) \ge -q_0\|\eta(s)\|^2$. Since $\|z(s)\|^2 \le (Y(s)/\lambda_{min}(R))$, we can obtain

$$\dot{Y}(s) \ge -\frac{q_0(1+p)}{\lambda_{min}(R)}Y(s) \tag{22}$$

Taking $\zeta \in [t-\tau, t_k]$, we can get $\ln(Y(t)/Y(\zeta)) \ge -(q_0(1+p)/\lambda_{min}(R))(t-\zeta) \ge -(q_0(1+p)/\lambda_{min}(R)) = \ln(1/q)$. Then one has

$$Y(\zeta) \le qY(t) \tag{23}$$

For $t_k \in [t-\tau, t]$, (23) can help us further establish the estimation inequality (20).

During $[t-\tau, t_k)$, the unstabilizable subsystem is active. According to (20), we can get

$$\begin{aligned}&(\alpha+\beta)\int_{t-\tau}^{t_k}(s-t+\tau)z^T(\theta)e^{\beta(t-\theta)}Rz(\theta)\,d\theta \\ &\quad \le (\alpha+\beta)\frac{t_k - t + \tau}{\tau}q\tau\int_{t-\tau}^{t}z^T(\theta)e^{\beta\tau}Rz(\theta)\,d\theta \\ &\quad \le (\alpha-\alpha^*)q\tau\int_{t-\tau}^{t}z^T(\theta)e^{\beta\tau}Rz(\theta)\,d\theta\end{aligned} \tag{24}$$

We have $\|f_i(x(t)) - f_i(0)\| \le M_i\|x(t)\|$ from Assumption 1. Then we can further obtain

$$\begin{aligned}&[f_i(x(t)) - f_i(0)]^T[f_i(x(t)) - f_i(0)] \\ &\quad \le x^T(t)M_i^TM_ix(t)\end{aligned} \tag{25}$$

Notice that

$$\begin{aligned} &-\int_{t-\tau}^{t_k} z^T(\theta) e^{\beta(t-\theta)} Rz(\theta)\, d\theta \\ &\quad -\int_{t_k}^{t} z^T(\theta) e^{-\alpha(t-\theta)} Rz(\theta)\, d\theta \\ &\le -\int_{t-\tau}^{t} z^T(\theta) e^{-\alpha\tau} Rz(\theta)\, d\theta \end{aligned} \tag{26}$$

For matrices U_i, T_i, taking (24), (25), and (26) into (19), one obtains

$$\begin{aligned} &\dot{V}_i(t) + \alpha V_i(t) + \Gamma(t) \\ &\quad \le 2x^T(t) Q_i z(t) + \tau z^T(t) Rz(t) + x^T(t) \alpha Q_i x(t) \\ &\qquad - \int_{t-\tau}^{t} z^T(s) e^{-\alpha\tau} Rz(s)\, ds \\ &\qquad + (\alpha - \alpha^*) q\tau \int_{t-\tau}^{t} z^T(\theta) e^{\beta\tau} Rz(\theta)\, d\theta \\ &\qquad + x^T(t) M_i^T M_i x(t) \\ &\qquad - [f_i(x(t)) - f_i(0)]^T [f_i(x(t)) - f_i(0)] \\ &\qquad + x^T(t) Q_i x(t) - \gamma^2 \omega^T(t) \omega(t) \\ &\quad \le 2\left[x^T(t), z^T(t)\right] \begin{bmatrix} Q_i & U_i \\ 0 & T_i \end{bmatrix} \times \begin{bmatrix} z(t) \\ \vartheta_i \end{bmatrix} \\ &\qquad + \tau z^T(t) Rz(t) + x^T(t) \alpha Q_i x(t) \\ &\qquad - \int_{t-\tau(t)}^{t} z^T(s) \left[e^{-\alpha\tau} - (\alpha - \alpha^*) q\tau e^{\beta\tau}\right] Rz(s)\, ds \\ &\qquad + x^T(t) M_i^T M_i x(t) \\ &\qquad - [f_i(x(t)) - f_i(0)]^T [f_i(x(t)) - f_i(0)] \\ &\qquad + x^T(t) Q_i x(t) - \gamma^2 \omega^T(t) \omega(t) \end{aligned} \tag{27}$$

where $\vartheta_i = -z(t) + (A_i + B_i K_i + D_i)x(t) - D_i \int_{t-\tau(t)}^{t} z(s)ds + f_i(x(t)) + \omega(t)$.

Let $\varsigma(t,s) = [x^T(t), z^T(t), z^T(s), (f_i(x(t)) - f_i(0))^T, \omega^T(t)]^T$; then we have

$$\begin{aligned} \dot{V}_i(t) + \alpha V_i(t) + \Gamma(t) &\le \frac{1}{\tau_i(t)} \\ &\quad \times \int_{t-\tau_i(t)}^{t} \varsigma^T(t,s)\, \Xi_i \varsigma(t,s)\, ds \end{aligned} \tag{28}$$

where Ξ_i are obtained by performing the congruent transformation with the matrix $\mathrm{diag}\{I, I, \tau_i(t), I, I\}$ to both sides of (13). Then we can get $\dot{V}_i(t) + \alpha V_i(t) + \Gamma(t) \le 0$. We can further get $V_i(t) \le e^{-\alpha(t-t_0)} V_i(t_0) - \int_{t_0}^{t} e^{-\alpha(t-s)} \Gamma(s) ds$.

Case 2. $t_{k+1} - t_k > \tau$.

In this case, two intervals, $[t_k, t_k + \tau)$ and $[t_k + \tau, t)$, should be considered. When $t \in [t_k, t_k + \tau)$, the proof is similar to Case 1. When $t \in [t_k + \tau, t)$, $V_i(t)$ can be shown in (12). Notice that there is no need for the time interval decomposition of the integral, so condition (13) can satisfy it. We omit it.

Similarly, if the system satisfies condition (14), it is not hard to obtain $V_j(t) \le e^{\beta(t-t_0)} V_j(t_0) - \int_{t_0}^{t} e^{\beta(t-s)} \Gamma(s) ds$ with the similar proof above.

The proof is completed. □

Remark 5. Case 1 is a typical case for switched systems with mixed modes, which is essentially different from [15]. In [15], only the case where $t_{k+1} - t_k > \tau$ is considered. This has strict constraints on switching rules and time delays. In this paper, conditions (13)–(15) are the added parameter constraints compared with [15], which can make the Lyapunov-Krasovskii functional exponentially decay whether it is Case 1 or Case 2. There is no need for the design of the switching rules and time delays to avoid Case 1 occurring. So it can reduce the constraints on switching rules and time delays.

Remark 6. From Lemma 4, we can get that if conditions (13)–(15) hold, (16) and (17) will be guaranteed. So if $\omega(t) \equiv 0$, we have $V_i(t) \le e^{-\alpha(t-t_0)} V_i(t_0)$ for $\sigma(t) = i \in \underline{N}$ and $V_j(t) \le e^{\beta(t-t_0)} V_j(t_0)$ for $\sigma(t) = j \in \Omega \setminus \underline{N}$. Noticing that $\alpha > 0$ and $\beta > 0$, we can say that the exponential decay rate for the stable subsystem is $-\alpha$ and the exponential divergence rate for the unstable subsystem is β.

Lemma 4 achieves the exponential decay and divergence of Lyapunov-Krasovskii functional (10). Here we analyze the H_∞ control of switched systems (5), which contains stabilizable subsystems and unstabilizable subsystems.

Next we will give the switching H_∞ control conditions for the switched delay system (5).

Theorem 7. *For given constants $\alpha > 0, \beta > 0$, if there exist symmetric positive definite matrices Q_l and R, any matrices U_l, T_l, and positive parameters q, q_0, and τ such that (13)–(15) are satisfied and if the switching signal meets the activation rate (9) and the average dwell-time condition*

$$N_\sigma(t_0, t) \le N_0 + \frac{t - t_0}{T_\alpha}, \quad T_\alpha > T_\alpha^* = \frac{\ln \kappa\varepsilon}{\alpha^*} > \frac{\ln \kappa\varepsilon}{\alpha}, \tag{29}$$

the H_∞ performance index (3) will be achieved, where $\varepsilon = e^{(\alpha+\beta)\tau/2}$ and $\kappa = \inf\{\nu \mid Q_i \le \nu Q_j\}$.

Proof. We suppose that $V_i(t)$ is activated during $[t_{2k}, t_{2k+1})$ and $V_j(t)$ is activated during $[t_{2k+1}, t_{2k+2})$.

First, one will prove the stability of system (5) when $\omega(t) \equiv 0$.

Then one can find from Lemma 4 that

$$\begin{aligned} &V_{\sigma(t)}(t) \\ &\quad \le \begin{cases} e^{-\alpha(t-t_{2k})} V_i(t_{2k}), & \text{if } t_{2k} \le t < t_{2k+1} \\ e^{\beta(t-t_{2k+1})} V_j(t_{2k+1}), & \text{if } t_{2k+1} \le t < t_{2k+2} \end{cases} \end{aligned} \tag{30}$$

Then one has

$$V_i(t) \le \kappa V_j(t), \quad V_j(t) \le e^{(\alpha+\beta)\tau}\kappa V_i(t) \tag{31}$$

If the system is switched from mode j to i, we define $\nu = \kappa$. If the system switched from mode i to mode j, we define $\nu = \kappa e^{(\alpha+\beta)\tau}$. Then we can get

$$V_{\sigma(t_i)}(t_i) \le \nu V_{\sigma(t_i^-)}(t_i^-), \quad i = 1, 2, \ldots, \tag{32}$$

and taking $N_u(t_0, t)$ as total switching times from the unstabilizable subsystems to the stabilizable subsystems during $[t_0, t)$, one can get

$$\begin{aligned} V_{\sigma(t)}(t) &\le e^{\chi_{\sigma(t_k)}(t-t_k)}\nu V_{\sigma(t_k^-)}(t_k^-) \\ &\le \kappa^{N_\sigma(t_0,t)} e^{(\alpha+\beta)\tau N_u(t_0,t)} \\ &\quad \times e^{-\alpha(t-t_0-T_u(t_0,t))+\beta T_u(t_0,t)} V_{\sigma(t_0)}(t_0) \end{aligned} \tag{33}$$

It can be seen that $N_u(t_0, t) \le (N_\sigma(t_0, t) + 1)/2$. Because $N_\sigma(t_0, t) \le N_0 + ((t - t_0)/T_\alpha)$, we can further get

$$\begin{aligned} &V_{\sigma(t)}(t) \\ &\le e^{N_\sigma(t_0,t)\ln\kappa} e^{(\alpha+\beta)\tau((N_\sigma(t_0,t)+1)/2)} \\ &\quad \times e^{[-\alpha(1-T_u(t_0,t)/(t-t_0))+\beta(T_u(t_0,t)/(t-t_0))](t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\le e^{(\alpha+\beta)\tau/2} e^{N_\sigma(t_0,t)(\ln\kappa+(\alpha+\beta)\tau/2)} \\ &\quad \times e^{[-\alpha(1-\Delta_u)+\beta\Delta_u](t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\le \varepsilon^{N_0+1}\kappa^{N_0} e^{[\ln\kappa\varepsilon-\alpha+(\alpha+\beta)\Delta_u](t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\le \varepsilon^{N_0+1}\kappa^{N_0} e^{-2\lambda(t-t_0)} V_{\sigma(t_0)}(t_0) \end{aligned} \tag{34}$$

where $\lambda = -(1/2)[(\ln\kappa\varepsilon/T_\alpha) - \alpha + (\alpha+\beta)\Delta_u]$.

Referring to (8), we can obtain

$$a\|x(t)\|^2 \le V_{\sigma(t)}(t), \quad V_{\sigma(t_0)}(t_0) \le b\left\|x_{t_0}\right\|^2 \tag{35}$$

where $a = \min_{l\in\underline{N}}\lambda_{\min}(Q_l)$ and $b = \max_{l\in\underline{N}}\lambda_{\max(Q_l)} + (\tau^2/2)\lambda_{\max}(R)$.

Therefore, one has $\|x(t)\|^2 \le (1/a)V_{\sigma(t)}(t) \le (b\varepsilon^{N_0+1}\kappa^{N_0}/a)e^{-2\lambda(t-t_0)}\|x_{t_0}\|^2$. So we have $\|x(t)\| \le \sqrt{(b\varepsilon^{N_0+1}\kappa^{N_0}/a)}e^{-\lambda(t-t_0)}\|x_{t_0}\|$. It can be seen that $\lambda > 0$. So the stability of system (5) is guaranteed when $\omega(t) \equiv 0$.

Next, one will prove the system achieves the performance index (3) when $\omega(t) \ne 0$.

We can find from Lemma 4 that

$$V_{\sigma(t)}(t) \le \begin{cases} e^{-\alpha(t-t_{2k})}V_i(t_{2k}) - \int_{t_{2l}}^{t} e^{-\alpha(t-s)}\Gamma(s)\,ds, \\ \text{if } t_{2k} \le t < t_{2k+1} \\ e^{\beta(t-t_{2k+1})}V_j(t_{2k+1}) - \int_{t_{2l+1}}^{t} e^{\beta(t-s)}\Gamma(s)\,ds, \\ \text{if } t_{2k+1} \le t < t_{2k+2} \end{cases} \tag{36}$$

Then we can get

$$\begin{aligned} V_{\sigma(t)}(t) &\le e^{\chi_{\sigma(t_k)}(t-t_k)}\nu V_{\sigma(t_k^-)}(t_k^-) - \nu\int_{t_l}^{t} e^{\chi_{\sigma(t)}(t-s)}\Gamma(s)\,ds \\ &\le \kappa^{N_\sigma(t_0,t)} e^{(\alpha+\beta)\tau N_u(t_0,t)} \\ &\quad \times e^{-\alpha(t-t_0-T_u(t_0,t))+\beta T_u(t_0,t)} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} e^{(\alpha+\beta)\tau N_u(s,t)-\alpha(t-s-T_u(s,t))+\beta T_u(s,t)+N_\sigma(s,t)\ln\mu}\Gamma(s)\,ds \end{aligned} \tag{37}$$

Because $N_\sigma(t_0, t) \le N_0 + ((t - t_0)/T_\alpha)$, it yields

$$\begin{aligned} V_{\sigma(t)}(t) &\le e^{N_\sigma(t_0,t)\ln\kappa} e^{(\alpha+\beta)\tau((N_\sigma(t_0,t)+1)/2)} \\ &\quad \times e^{[-\alpha(1-T_u(t_0,t)/(t-t_0))+\beta(T_u(t_0,t)/(t-t_0))](t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} e^{N_\sigma(s,t)\ln\kappa} e^{(\alpha+\beta)\tau((N_\sigma(s,t)+1)/2)} \\ &\quad \times e^{[-\alpha(1-T_u(s,t)/(t-s))+\beta(T_u(s,t)/(t-s))](t-s)}\Gamma(s)\,ds \\ &\le e^{(\alpha+\beta)\tau/2} e^{N_\sigma(t_0,t)(\ln\kappa+(\alpha+\beta)\tau/2)} \\ &\quad \times e^{[-\alpha(1-\Delta_u)+\beta\Delta_u](t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} e^{(\alpha+\beta)\tau/2} e^{N_\sigma(s,t)(\ln\kappa+(\alpha+\beta)\tau/2)} \\ &\quad \times e^{[-\alpha(1-\Delta_u)+\beta\Delta_u](t-s)}\Gamma(s)\,ds \\ &\le \varepsilon^{N_\sigma(t_0,t)+1}\kappa^{N_\sigma(t_0,t)} e^{-\alpha(t-t_0)} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} \varepsilon^{N_\sigma(s,t)+1}\kappa^{N_\sigma(t_0,t)} e^{-\alpha(t-s)}\Gamma(s)\,ds \end{aligned} \tag{38}$$

Then

$$\begin{aligned} \frac{1}{\varepsilon}V_{\sigma(t)}(t) &\le e^{-\alpha(t-t_0)+N_\sigma(t_0,t)\ln\kappa\varepsilon} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} e^{-\alpha(t-s)+N_\sigma(s,t)\ln\kappa\varepsilon}\Gamma(s)\,ds \end{aligned} \tag{39}$$

Let $t_0 = 0$; and multiplying $e^{-N_\sigma(0,t)\ln\kappa\varepsilon}$ on both sides of (39), we can obtain

$$\begin{aligned} &\int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\kappa\varepsilon} x^T(s)Qx(s)\,ds \\ &\quad \le e^{-\alpha(t-t_0)-N_\sigma(0,t_0)} V_{\sigma(t_0)}(t_0) \\ &\quad + \gamma^2\int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\kappa\varepsilon}\omega^T(s)\omega(s)\,ds \end{aligned} \tag{40}$$

Because $N_\sigma(0, s) \le N_0 + s/T_\alpha$, $N_0 > 0$, and $T_\alpha > \ln\kappa\varepsilon/\alpha$, we obtain

$$N_\sigma(0, s)\ln\kappa\varepsilon \le N_0\ln\kappa\varepsilon + s\alpha \tag{41}$$

Under the zero initial condition, taking (41) into (40), we have

$$\int_0^t e^{-\alpha t} x^T(s) Q x(s)\, ds \le \gamma^2 \int_0^t e^{-\alpha(t-s)} \omega^T(s)\, \omega(s)\, ds \quad (42)$$

Integrating (42), one has

$$\begin{aligned} &\int_0^\infty \int_0^t e^{-\alpha t} x^T(s) Q x(s)\, ds\, dt \\ &= \int_0^\infty \int_s^\infty e^{-\alpha t} x^T(s) Q x(s)\, dt\, ds \\ &= \frac{1}{\alpha} \int_0^\infty e^{-\alpha s} x^T(s) Q x(s)\, ds \end{aligned} \quad (43)$$

and

$$\begin{aligned} &\gamma^2 \int_0^\infty \int_0^t e^{-\alpha(t-s)} \omega^T(s)\, \omega(s)\, ds\, dt \\ &= \gamma^2 \int_0^\infty \int_s^\infty e^{-\alpha(t-s)} \omega^T(s)\, \omega(s)\, dt\, ds \\ &= \frac{\gamma^2}{\alpha} \int_0^\infty \omega^T(s)\, \omega(s)\, ds \end{aligned} \quad (44)$$

So we can obtain $\int_0^\infty e^{-\alpha s} x^T(s) Q x(s) ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s) ds$.

The proof is completed. □

Remark 8. Lemma 4 is proposed to handle the effects of mixed modes, where the parameter constraints (13)–(15) are utilized to guarantee the decay of the Lyapunov-Krasovskii functionals. Theorem 7 is proposed to achieve the H_∞ performance based on Lemma 4, which is different from [15]. In [15], the mixed modes are not taken into account, which cannot achieve the stability of the system merely with average dwell-time approach and unstable subsystems activation rate.

Remark 9. Traditionally, as for the systems with time-varying delays, the variable delay terms are usually used in the Lyapunov-Krasovskii functional. In this paper, instead of the variable delay terms, in Lyapunov-Krasovskii functional (8), the upper bound of time-varying delay is used. Then the exponential decay of functional (8) can be guaranteed by Lemma 4. The merits of the Lyapunov-Krasovskii functional not containing variable delay terms can be summarized as two points. On one hand, the Lyapunov-Krasovskii functional (8) does not contain the variable delay terms, so the number of constaint conditions is reduced. On the other hand, the Lyapunov-Krasovskii functional (8) can be converted to single integral forms (9). Then it is convenient to be analyzed by dividing it into two segments based on the mixed modes.

4. Simulation Examples

In this section, numerical examples are introduced to demonstrate the theoretical results.

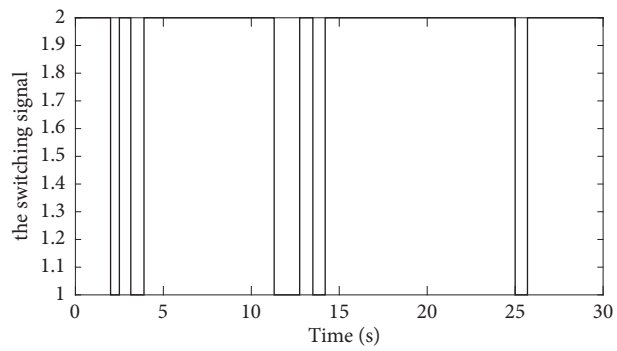

Figure 1: The switching signal.

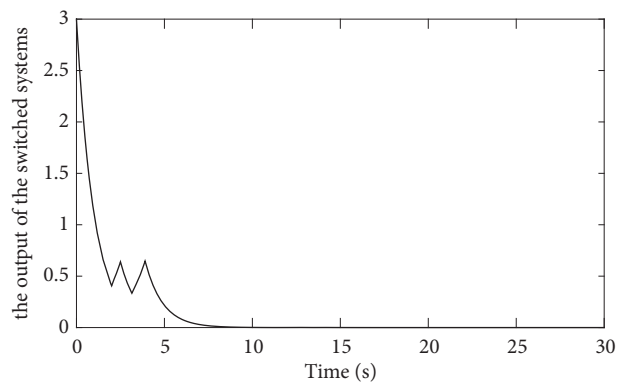

Figure 2: Output of the switched systems.

Example 1. Consider the switched system (6), where $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}$, $B_1 = \begin{bmatrix} -0.2 & 0.4 \\ 1 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} -0.6 & -0.2 \\ 0.2 & -0.4 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0.25 & 0.15 \\ 0.25 & -0.5 \end{bmatrix}$, $D_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix}$, $\tau(t) = 0.5 + 0.5 \sin t$, $\tau = 1$, $\gamma = 1$, $f_1(x(t)) = \begin{bmatrix} 0.1 \cos 0.1x_1 \\ 0.1 \cos 0.1x_2 \end{bmatrix}$, $f_2(x(t)) = \begin{bmatrix} 0.2 \cos 0.1x_1 \\ 0.2 \cos 0.1x_2 \end{bmatrix}$, $f_1(0) = [0.1 \;\; 0.1]^T \ne 0$, $f_2(0) = [0.2 \;\; 0.2]^T \ne 0$, $C_1 = C_2 = [1 \;\; 0]$, $M_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, and $M_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$. Note that $\lambda(A_1) = 1, -1$ and $\lambda(A_2) = -1, -5$; then A_1 is unstable with open loop and A_2 is stable. External disturbance is as follows:

$$\omega(t) = \begin{cases} 0.1 \sin t, & 0 \le t < 2 \\ 0, & t > 2 \end{cases} \quad (45)$$

We select $\alpha = 0.16$ and $\beta = 1.5$. By solving LMI (13)–(15), we can get $Q_1 = \begin{bmatrix} 0.9132 & 0.5370 \\ 0.5370 & 1.0028 \end{bmatrix}$, $Q_2 = \begin{bmatrix} 4.0186 & -0.9375 \\ -0.9375 & 3.8104 \end{bmatrix}$, $R = \begin{bmatrix} 1.2785 & 0.3905 \\ 0.3905 & 1.4072 \end{bmatrix}$, $K_1 = 0$, and $K_2 = \begin{bmatrix} 5.3184 & -1.7940 \\ 3.7704 & 9.2840 \end{bmatrix}$.

We can get $\kappa = 1.2603$ and $T_\alpha^* = 16.8205$ according to equation (9) and condition (29). Then $\Delta_u \le 0.1206$. Because the mixed modes are taken into account, this paper is different from the case of [15]. The simulation results are shown in Figures 1 and 2.

From Figures 1 and 2, we can obtain that the output of the system is exponentially stable under the switching of the stabilizable and unstabilizable subsystems. It also has good transient performance. It should be pointed out that if the system is not satisfied with conditions (13)–(15), the asymptotical stability will not be guaranteed. This will be verified in Example 2.

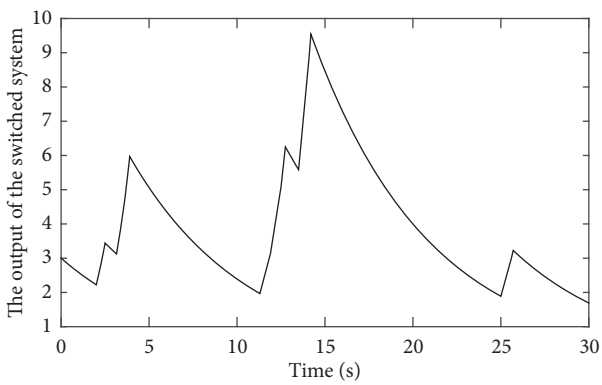

Figure 3: Contrast for the output of the switched systems of [15].

Example 2. We use the control method of [15] for the switched system (5), which does not take the mixed modes into account. It means that the system is not satisfied with conditions (13)–(15). So the control strategy is merely based on average dwell time and unstable subsystems activation rate.

We select that A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, $\tau(t)$, $f_1(x(t))$, $f_2(x(t))$, M_1, M_2, $\omega(t)$, α, and β are the same as Example 1. We select $K_1 = \begin{bmatrix} -1.25 & 0.5 \\ 1 & 2 \end{bmatrix}$ and $K_2 = \begin{bmatrix} 0.5 & -0.2 \\ 0.2 & 0.8 \end{bmatrix}$. It is obvious that the parameters are not satisfied with conditions (14) and (15). This means that the effects of mixed modes are not handled. We can further get $\kappa = 1.2603$ and $T_\alpha^* = 16.8205$ according to equation (9) and condition (29). The switching signal is the same as Figure 1. Then we can obtain the contrast simulation result listed as Figure 3.

As Figure 3 shows, the output is not stable. So we can say that if the parameters of the switched systems are not satisfied with the constraints of the mixed modes, the exponential stability for such system will not be guaranteed. It further proves the importance of mixed modes on such systems.

5. Conclusion

This paper addressed the H_∞ control for the mixed-mode based switched delay systems. The two cases of the H_∞ control analysis of switched delay systems were considered, and the sufficient conditions for the exponential decay of the Lyapunov functionals were developed. Then the combination of the average dwell-time approach and the subsystem activation rate was utilized to achieve the H_∞ performance for the switched system with both stabilizable subsystems and unstabilizable subsystems. Finally, numerical examples and contrast simulation were given to show the effectiveness of the theoretical results.

Data Availability

The data source of the system model comes from [13]: "Stability of Switched Nonlinear Time-Delay Systems with Stable and Unstable Subsystems."

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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