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*Research Article*

# Approach to Multicriteria Group Decision Making with Z-Numbers Based on TOPSIS and Power Aggregation Operators

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Decisions strongly rely on information, so the information must be reliable, yet most of the real-world information is imprecise and uncertain. The reliability of the information about decision analysis should be measured. Z-number, which incorporates a restraint of evaluation on investigated objects and the corresponding degree of confidence, is considered as a powerful tool to characterize this information. In this paper, we develop a novel approach based on TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method and the power aggregation operators for solving the multiple criteria group decision making (MCGDM) problem where the weight information for decision makers (DMs) and criteria is incomplete. In the MCGDM, the evaluation information made by DMs is represented in the form of linguistic terms and the following calculation is performed using Z-numbers. First, we establish an optimization model based on similarity measure to determine the weights of DMs and a linear programming model with partial weight information provided by DMs based on distance measure to determine the weights of criteria. Subsequently, decision matrices from all the DMs are aggregated into a comprehensive evaluation matrix utilizing the proposed ZWAPA operator or ZWGPA operator. Then, those considered alternatives are ranked in accordance with TOPSIS idea and the feature of Z-evaluation. Finally, a practical example about supplier selection is given to demonstrate the detailed implementation process of the proposed approach, and the feasibility and validity of the approach are verified by comparisons with some existing approaches.

## 1. Introduction

Since the fuzzy set theory was coined by Zadeh [1], it has been widely used in many application fields as a tool that is capable of capturing the certainty of information and depicting people's subjective thinking, especially decision-making problems with uncertain, imprecise even incomplete information. After that, the theory has been developing and type-2 fuzzy set [2], fuzzy multiset [3], intuitionistic fuzzy set [4–6], hesitant fuzzy set [7, 8] and their interval forms [9–11], various fuzzy numbers like triangular fuzzy number [12], trapezoidal fuzzy number [13], etc. appear successively. The classic fuzzy set and its generalizations and extensions successfully address lots of challenges from the complexity of practical problems but they do not take well into consideration the reliability of the information that is provided. The reliability of the evaluation is an important aspect in solving the decision making problem where it will affect the final outcome of the evaluation. Z-number, a novel fuzzy number with confidence degree, is pioneered by Zadeh [14] to overcome this limitation. Z-number is a 2-tuple fuzzy numbers that includes the restriction of the evaluation and the reliability of the judgment. We use the first fuzzy number to represent the uncertainty of evaluation and the second fuzzy number for a measure of certainty or other related concepts such as sureness, confidence, reliability, strength of truth, or probability for the first fuzzy number. Z-number has more ability to describe the knowledge of human since it considers the level of confidence of evaluators. Owing to the inclusion of the reliability factor, Z-numbers involve more uncertainties than fuzzy numbers. They can provide us with additional degree of freedom to represent the uncertainty and fuzziness of the real situation. It provides a new perspective for us in the decision making area. Therefore, using Z-numbers to model and process uncertainty of the reality is more reasonable and applicable than using fuzzy numbers.

Traditionally the given evaluations in multiple criteria decision making (MCDM) are real numbers. As the complexity of the problem we investigate, DMs' opinions are usually expressed only in a fairly vague manner where it cannot be measured numerically. Instead, they normally use terms like "poor," "fair," and "good" to express the judgment on an alternative, which is closer to people's perception. Then, the problem of how to compute with terms or words arises. It has gained enough attention by some researchers and they use typically fuzzy number to represent and compute linguistic terms which is able to seize the impreciseness of this information and handle subjective judgment of human well. In recent years, linguistic variables whose values are words or short sentences from natural or artificial languages have been researched extensively and various related fruits [14–21] are achieved. Because of the superiority of Z-number in expressing vague information, utilizing Z-number to characterize the issue with words is a new perspective. It is noted that the two components of Z-numbers are mostly described in natural language [14]. For DMs, the scale of the provided linguistic words is limited, so it is possible that two alternatives get the same semantic assessment under some criterion but small difference may exist between them. Z-number can distinguish this difference because experts' confidence degrees are different when giving the criterion values. The second component of Z-number is precisely capable of playing this role. At present the research involving Z-numbers mainly focuses on the decision making issue with linguistic variables. In most ill-defined decision environment, it is preferable for a DM to employ linguistic variables rather than real numbers when making assessments.

In decision making issue, the core is to determine the ranking order of alternatives. Hence choosing an effective method to rank Z-numbers gets very important when conducting decision making problem in Z-number information. Kang et al. [22] devised a method to transform a Z-number into a fuzzy number based on the fuzzy expectation of the second component of Z-number with Centroid Method, which highlights the importance of the first part in a Z-number compared to the second and reduces successfully a Z-number to a classical fuzzy number. Due to that, many researchers usually first converted Z-numbers to fuzzy numbers on the basis of the method provided in [22] and then implemented the related calculation on the fuzzy numbers that has been converted in the course of decision making. For example, authors in [23] first integrated the preference values given by DMs to a single value, then converted the integrated result (Z-number) to generalized fuzzy number, and finally calculated the spread and the fuzzy score value of standardized fuzzy number to attain ranking order of Z-numbers. Authors in [24] employed the same way to transform Z-numbers to fuzzy numbers before ordering Z-numbers, the whole course that rank Z-numbers was analogous to [23], and the research made lots of analysis and discussion on different Z-number combinations to investigate the effectiveness of the proposed ranking approach. Kang and his team focused on solving supplier selection problem by utilizing the methodology presented in that paper including two parts: one changed a Z-number to a traditional fuzzy number according to the fuzzy expectation; the other worked out the optimal priority weight for supplier selection with the improved genetic algorithm and the extended fuzzy AHP under Z-number environment in [25]. In the next few years, they developed a new uncertainty measure of fuzzy set considering the influence of fuzziness measure and the range (or cardinality) of the fuzzy set and proposed a method of generating Z-number based on the OWA weights using maximum entropy considering the attitude (preference) of the decision maker in [26, 27]. In 2008, they proposed a stable strategies analysis based on the utility of Z-number and apply it to the evolutionary games in [28]. The paper took full advantage of the structure of Z-numbers to simulate the procedure of humans competition and cooperation. The methodology extends the classical evolutionary games into linguistic-based ones, which is quite applicable in the real situations. Authors in [29] integrated a Jaccard similarity measure of Z-numbers to solve MCGDM problem. After linguistic values of DMs are converted to Z-numbers, the method used the idea from Kang et al. [22] to transform Z-numbers into trapezoidal fuzzy numbers (TpFNs) and then applied the Jaccard similarity measure between the expected intervals of TpFNs and the ideal alternative to attain the final decision. Authors in [30] put forward a multilayer methodology for ranking Z-numbers, which consists of two layers: converting Z-numbers to generalised TpFNs as the first layer and using CPS method based on centroid point and spread to rank the generalised fuzzy numbers that have been converted, as the second layer. The methodology is applicable to both positive and negative data values for alternatives and is considered as a generic decision making procedure. Different from those just mentioned literatures, Xiao [31] first converted Z-number to an interval-valued fuzzy set with footprint of uncertainty and then defuzzified the interval fuzzy sets as crisp numbers with the well-known K-M algorithm [32]. Authors in [33] converted directly Z-numbers to crisp numbers by performing the multiplication operation on the two parts of Z-numbers (represented by TFNs) where both TFNs were defuzzified to crisp numbers via the graded mean integration and then multiplied into a crisp number. There is no doubt that the approach seems a bit rough and the information of Z-numbers originally included is markedly reduced.

The above references always convert Z-numbers to classical fuzzy numbers or generalized fuzzy numbers in handling the linguistic data before performing the real decision. This way can result in the loss of some valuable Z-information and may give rise to the unreasonable phenomenon that two different Z-numbers may be converted to the identical fuzzy number. Some researchers have been aware of the drawback and made some improvements. Zamri et al. [34] applied the fuzzy TOPSIS to Z-numbers to handle uncertainty in the construction problem. In that paper, the distances of two parts in Z-evaluations from the PIS and the NIS were calculated, respectively, without any interaction. The structure of Z-numbers was not heavily damaged in the whole decision process such that less useful information of Z-numbers was lost. Peng and Wang [35] developed an innovative method to address MCGDM problems where the weight information for DMs and criteria is incompletely known by introducing hesitant uncertain linguistic Z-numbers (HULZNs). First it defined operations and distance of HULZNs by means of linguistic scale functions, then developed two power aggregation operators for HULZNs, and finally incorporated the proposed operators and the VIKOR model to solve the ERP system selection problem. The method system combined the advantages of Z-numbers and linguistic models with deep theory and rigorous logic. Jiang et al. [36] gave an improved method for ranking generalized fuzzy numbers, which took into consideration the weight of centroid points, degrees of fuzziness, and the spreads of fuzzy numbers. Subsequently, this method was extended for ranking Z-numbers and the final step of the procedures revised the score function of Z-numbers in view of different status of the two parts of Z-numbers. The methodology retains the fuzzy information of the reliability instead of converting the reliability part of a Z-number to a crisp number as many existing methods did and takes the full advantage that Z-number can express more vague information compared to other fuzzy numbers.

In general, there are not too many researches on Z-numbers so far and relevant achievements are limited. This paper tries to gain some breakthrough in this aspect on the basis of the aforementioned references. Suppose that there is a collection of Z-numbers $z_i = (A_i, R_i)$. $A_i$ is the main part of $z_i$ and $B_i$ is the subsidiary factor. $B_i$ can influence the ranking result but cannot decide the result, while $A_i$ can. Therefore, the weight of $A_i$ should be larger than $B_i$ when processing the decision making problem with Z-numbers. Besides, the characteristics of Z-number should attempt to be kept and the information of Z-numbers should be retained as much as possible. In view of these considerations, we will do some improved work and develop another MCGDM approach by applying the simple and practical TOPSIS method to the decision information characterized by Z-numbers where the weight information of DMs is entirely unknown and the weight for criteria is partly known. The decision procedures presented in consideration of various data weights manage to hold the previous amount of information. With these points, the remainder of this work is organized as follows: Section 2 briefly reviews some necessary concepts such as Z-number and triangular fuzzy number (TFN) and defines the operations of Z-numbers whose both parts are expressed by TFNs. Section 3 defines the distance of Z-numbers from the perspective of TFNs and shows the properties the distance measure possesses. Section 4 develops two power aggregation operators by extending the PA operator to Z-numbers and discusses comprehensively their properties. In Section 5, a MCGDM methodology in Z-information is proposed, which contains two phases: the first phase is to determine successively the weight vectors for DMs and criteria by establishing optimization models under the two circumstances that the prior information is completely unknown and incompletely unknown, respectively; the second makes the concrete decision procedures which combines the power aggregation operators towards Z-numbers and the TOPSIS method. Section 6 shows the decision-making course through a numerical example about supplier selection and conducts a careful comparison with the existing approaches to illustrate the validity and superiority of the suggested approach. Finally, the conclusions for the main points of this paper are drawn in Section 7.

## 2. Preliminaries

This section mainly introduces some basic concepts relating to Z-number, which will be needed in subsequent sections.

*Definition 1* (see [14]). A Z-numbers is an ordered pair of fuzzy numbers, denoted by $Z = (A, R)$, which is associated with a real-valued uncertain variable $X$. The first component of Z-number, $A$, is a restriction on the values that $X$ can take. The second component $R$ is a measure of reliability of the first component such as confidence, sureness, strength of belief, and probability or possibility.

A Z-number gives information about not only uncertain variable, but also the reliability of the information. The first component $A$ is the principal part of a Z-number while the second one $R$ is just an explanation for $A$. Typically, $A$ and $R$ are perception-based values which are described in natural language. For example, "travel time by car from Berkeley to San Francisco, about 30 min, usually," "degree of Robert's honesty, high, not sure," and "tomorrow's weather is warm, likely." For simplicity, these natural languages are presented by linguistic terms, that is, "30 min, usually", "high, not sure," and "warm, likely."

Due to the complexity of real problems, many domain experts give their opinions via fuzzy numbers. For instance, in a new product launch price forecast, one expert may give his opinion like this: the lowest price is 2 dollars, the most possible price may be 3 dollars, and the highest price will not exceed 4 dollars. Thus we can use a triangular fuzzy number (2, 3, 4) to express the expert's idea. Throughout this paper, the linguistic variables from the two components of Z-numbers, $A$ and $R$, are represented by triangular fuzzy numbers. Accordingly, the triangular fuzzy number is defined below.

*Definition 2* (see [12]). A triangular fuzzy number $A$ can be denoted by a triplet $(a_1, a_2, a_3)$, whose membership function $\mu_A(x)$ is determined as follows:

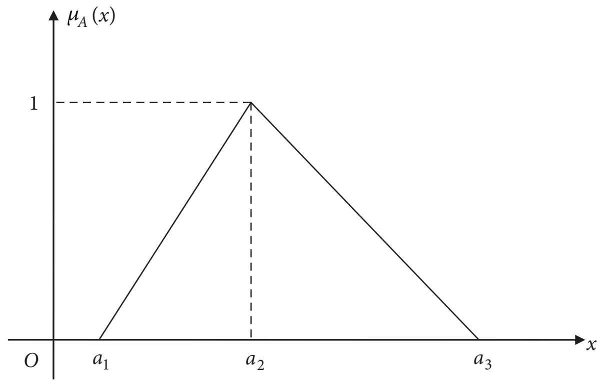


Figure 1: Triangular fuzzy number.

$$\mu_A(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ \dfrac{x - a_1}{a_2 - a_1}, & x \in [a_1, a_2] \\ \dfrac{a_3 - x}{a_3 - a_2}, & x \in [a_2, a_3] \\ 0, & x \in (a_3, +\infty) \end{cases} \tag{1}$$

For convenience, we signify the triangular fuzzy number with TFN for short in what follows. The TFN $A = (a_1, a_2, a_3)$ expressed by (1) can be shown in Figure 1.

Lacking of typical properties for Z-number forces us to simplify its representation. In order to take advantage of Z-number's trait and the relation of the two components in Z-number's structure, we introduce Kang et al.'s conversion thinking [22].

Let a Z-number $z = (A, R)$ and convert the reliability $R$ of $Z$ into a crisp value using Centroid Method

$$\alpha = \frac{\int x\mu_R(x)\,dx}{\int \mu_R(x)\,dx} \tag{2}$$

where $\int$ denotes an algebraic integration.

Since $R = (r_1, r_2, r_3)$, (6) becomes the following equation owing to the membership function $\mu_R(x)$ of a TFN:

$$\alpha = \frac{\int_{r_1}^{r_2} x\left((x - r_1)/(r_2 - r_1)\right)dx + \int_{r_2}^{r_3} x\left((r_3 - x)/(r_3 - r_2)\right)dx}{(1/2)(r_3 - r_1)\,r_2} \wedge 1 \tag{3}$$

Subsequently, take the centroid value $\alpha$ of the reliability $R$ as the weight of the restriction $A$ and add the weight from the second part $R$ to the first part $A$. Thereby the weighted Z-number can be written as $z^\alpha = \{\langle x, \mu_{A^\alpha}(x)\rangle \mid \mu_{A^\alpha}(x) = \alpha\mu_A(x),\ x \in X\}$.

Because both parts of Z-number are characterized by TFNs in the whole article, the partial operations of Z-numbers will follow those of TFNs. In light of the structure and meaning of Z-numbers, we define some operational laws concerning Z-numbers combining with the operations of TFNs.

*Definition 3.* Assume that $z_1 = ((a_1^l, a_1^m, a_1^u), (r_1^l, r_1^m, r_1^u))$, $z_2 = ((a_2^l, a_2^m, a_2^u), (r_2^l, r_2^m, r_2^u))$ are any two Z-numbers; then normalize their second components as

$z_1 = ((a_1^l, a_1^m, a_1^u), (r_1^l/r_1, r_1^m/r_1, r_1^u/r_1))$,

$z_2 = ((a_2^l, a_2^m, a_2^u), (r_2^l/r_2, r_2^m/r_2, r_2^u/r_2))$,

where $r_1 = r_1^l + r_1^m + r_1^u$, $r_2 = r_2^l + r_2^m + r_2^u$

(1) $z_1 + z_2 = ((\alpha_1 a_1^l + \alpha_2 a_2^l, \alpha_1 a_1^m + \alpha_2 a_2^m, \alpha_1 a_1^u + \alpha_2 a_2^u), (\max\{r_1^l/r_1, r_2^l/r_2\}, \max\{r_1^m/r_1, r_2^m/r_2\}, \max\{r_1^u/r_1, r_2^u/r_2\}))$;

(2) $z_1 z_2 = ((\alpha_1\alpha_2 a_1^l a_2^l, \alpha_1\alpha_2 a_1^m a_2^m, \alpha_1\alpha_2 a_1^u a_2^u), (\min\{r_1^l/r_1, r_2^l/r_2\}, \min\{r_1^m/r_1, r_2^m/r_2\}, \min\{r_1^u/r_1, r_2^u/r_2\}))$;

(3) $\lambda z_1 = ((\lambda\alpha_1 a_1^l, \lambda\alpha_1 a_1^m, \lambda\alpha_1 a_1^u), (r_1^l/r_1, r_1^m/r_1, r_1^u/r_1))$, $\lambda > 0$;

(4) $z_1^\lambda = (((\alpha_1 a_1^l)^\lambda, (\alpha_1 a_1^m)^\lambda, (\alpha_1 a_1^u)^\lambda), (r_1^l/r_1, r_1^m/r_1, r_1^u/r_1))$, $\lambda > 0$.

According to (2), $\alpha_i = \int x\mu_{R_i}(x)dx / \int \mu_{R_i}(x)dx \wedge 1$, $R_i = (r_i^l, r_i^m, r_i^u)$, $i = 1, 2$.

In Definition 3, the reliability measure of Z-numbers is unified in a scale of 0 to 1 in order to be compared easily by normalizing them. In the operations of the first components of Z-numbers, they are given their respective weights $\alpha$ from the degradation value of the corresponding second components by (2) in order that the supplementary role of the confidence component to the restriction component in a Z-number is highlighted. In Section 4, two power aggregation operators on Z-numbers are developed and construction of the operators will involve the operations of Definition 3. In the last application part of this paper, we will conduct a large amount of calculations where the four operation rules above will be used frequently.

Next, we give the ranking rule for Z-numbers used for decision-making.

*Definition 4.* Assume that $z_i = (A_i, R_i)$ are arbitrary two Z-numbers and $A_i, R_i$ are all normal fuzzy numbers, $i = 1, 2$.

If $A_1 \succ A_2$, then $z_1 \succ z_2$;

If $A_1 = A_2$, thus we have the following:

(i) $R_1 \succ R_2$, then $z_1 \succ z_2$;

(ii) $R_1 = R_2$, then $z_1 = z_2$.

For the preference orders of the fuzzy numbers $A_i, R_i (i = 1, 2)$, there have been varieties of existing methods to determine them.

## 3. Distance Measure of Z-Numbers

There have been numerous distance formulas for previous fuzzy numbers such as Hamming distance, Euclidean distance, and Hausdorff distance. With the aid of the concept of cross-entropy in information theory, we construct a new distance formula to measure the discrimination uncertain information of TFNs before defining the distance measure of Z-numbers.

*Definition 5.* Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be TFNs, used for describing the uncertain information of two objects; then their cross-entropy is defined by

$$CE(A, B) = \frac{1}{3}\sum_{i=1}^{3}\left|a_i \log_2 \frac{a_i}{(1/2)(a_i + b_i)}\right| \quad (4)$$

It is noticeable that (4) is meaningless if $a_i = 0$ or $b_i = 0$; thereupon we make the following extension.

*Remark 6.* For the cases $a_i = 0$ and $b_i \neq 0$, we reverse the position of $a_i$ and $b_i$ of (4); that is, turn $a_i \log_2(a_i/(1/2)(a_i+b_i))$ into $a_i \log_2(b_i/(1/2)(a_i + b_i))$, that is, $a_i$; if $a_i = 0$ and $b_i = 0$, naturally the distance between them should be 0; hence we take the cross entropy $a_i \log_2(a_i/(1/2)(a_i + b_i)) = 0$ at this point.

The cross entropy measure in fuzzy sets is able to discriminate effectively different fuzzy information, so it can be considered as an information distance. Despite its asymmetry of (4), it is necessary to symmetrize the cross-entropy so as to become a real distance measure as follows:

$$d(A, B) = \frac{1}{2}[CE(A, B) + CE(B, A)] \quad (5)$$

Clearly, the distance measure of (5) satisfies the three basic properties that ordinary distance measure in fuzzy environment has (see Theorem 7)

**Theorem 7.** *Let $A, B$ be two arbitrary TFNs and $A = (a_1, a_2, a_3), B = (b_1, b_2, b_3)$; then*

*(1) $d(A, B) = d(B, A)$;*
*(2) $d(A, B) \geqslant 0$;*
*(3) $d(A, B) = 0 \Longleftrightarrow A = B$.*

*Proof.* Items (1) and (2) apparently hold, so we primarily prove item (3).

$d(A, B) = 0 \Longleftrightarrow |a_i \log_2(a_i/(1/2)(a_i + b_i))| = 0$ and $|b_i \log_2(b_i/(1/2)(a_i + b_i))| = 0$, $i = 1, 2, 3$. Let $f(x) = x\log_2(x/(1/2)(x + a))$ ($a$ is a constant); then $f(x)$ is a monotonic function. Thus if $a_i \log_2(a_i/(1/2)(a_i + b_i)) = 0$ or $b_i \log_2(b_i/(1/2)(a_i+b_i)) = 0$, the solution is unique. Apparently the only solution is $a_i = b_i, i = 1, 2, 3$; that is, $A = B$. □

In this paper, both components of Z-number are characterized by TFNs, so the distance between Z-numbers can be broken down into the distance for the two TFNs. Thus, we can give the distance formula for Z-numbers as follows.

*Definition 8.* For two arbitrary Z-numbers $z_1 = (A_1, R_1)$, $z_2 = (A_2, R_2)$, where $A_i = (a_i^l, a_i^m, a_i^u), R_i = (r_i^l, r_i^m, r_i^u), i = 1, 2$, then

$$d(z_1, z_2) = \frac{1}{3}\left[d(A_1, A_2) + d(R_1, R_2) + d\left(A_1^{\alpha_1}, A_2^{\alpha_2}\right)\right] \quad (6)$$

where $\alpha_i = \int x\mu_{R_i}(x)dx / \int \mu_{R_i}(x)dx \wedge 1, A_i^{\alpha_i} = (\alpha_i a_i^l, \alpha_i a_i^m, \alpha_i a_i^u), i = 1, 2.$

From (5) and (6), we easily know that $d(\cdot, \cdot)$ satisfies the following properties.

If $z_1, z_2$ are two Z-numbers, then

(1) $d(z_1, z_2) \geqslant 0$;
(2) $d(z_1, z_2) = d(z_2, z_1)$;
(3) $d(z_1, z_2) = 0 \Longleftrightarrow z_1 = z_2$.

Through the distance measure between Z-numbers in Definition 8, we notice that the influence factor of the first component of Z-number is higher than the second.

## 4. Power Aggregation Operator for Z-Numbers

Power average (PA) operator was pioneered by Yager [37] in 2001. As an effective data aggregation tool, it allows input values to support each other and provide more versatility. This section manages to apply PA operator in the situation where the input arguments are Z-numbers and derive two aggregation operators with desirable properties based on the PA operator.

*Definition 9.* PA operator is a mapping: $\mathbb{R}^n \longrightarrow \mathbb{R}$ that is given by the following $n$-variate function:

$$PA(a_1, a_2, \ldots, a_n) = \frac{\sum_{i=1}^{n}(1 + T(a_i))a_i}{\sum_{i=1}^{n}(1 + T(a_i))} \quad (7)$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} \sup(a_i, a_j)$ and $\sup(a_i, a_j)$ is called the support for $a_i$ from $a_j$, which satisfies the following three properties:

(1) $\sup(a_i, a_j) \in [0, 1]$;
(2) $\sup(a_i, a_j) = \sup(a_j, a_i)$;
(3) $\sup(a_i, a_j) \geqslant \sup(a_s, a_t)$, if $|a_i - a_j| < |a_s - a_t|$.

Motivated by the PA operator and the geometric average operator (GA), Xu and Yager [38] developed a power geometric (PG) operator as follows.

*Definition 10.* PG operator is a mapping: $\mathbb{R}^n \longrightarrow \mathbb{R}$ that is given by the following $n$-variate function:

$$PG(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} a_i^{(1+T(a_i))/\sum_{i=1}^{n}(1+T(a_i))} \quad (8)$$

where $T(a_i)$ is the same as that of Definition 9.

Combining the PA operator and the weighted arithmetic average (WAA) operator, we can deduce a weighted arithmetic power average operator with Z-numbers ZWAPA.

*Definition 11.* Let $z_1, z_2, \ldots, z_n$ be a collection of Z-numbers, let $\Omega$ be the family of all Z-numbers, and $w_i$ corresponds to the weight of $z_i$, with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$; then the ZWAPA operator is this mapping: $\Omega^n \longrightarrow \Omega$, which is defined as

$$ZWAPA(z_1, z_2, \ldots, z_n) = \frac{\sum_{i=1}^{n} w_i(1 + T(z_i))z_i}{\sum_{i=1}^{n} w_i(1 + T(z_i))} \quad (9)$$

where $T(z_i) = \sum_{j=1, j\neq i}^{n} w_j \sup(z_i, z_j)$ and $\sup(z_i, z_j)$ is the support for $z_i$ from $z_j$ with the following conditions:

(1) $\sup(z_i, z_j) \geqslant 0$;
(2) $\sup(z_i, z_j) = \sup(z_j, z_i)$;
(3) $\sup(z_i, z_j) \geqslant \sup(z_s, z_t)$, if $d(z_i, z_j) < d(z_s, z_t)$, where $d(\cdot,\cdot)$ is the distance measure between Z-numbers.

From Definition 11, it can be deduced that the support measure $\sup(\cdot,\cdot)$ can be used to measure the proximity of a preference value in the form of Z-number provided by a DM to another. The closer the two preferences $z_i$ and $z_j$, the smaller the distance of both, and the more they support each other.

It is evident that the aggregated result using ZWAPA operator is still a Z-number. Moreover, it can be easily proved that ZWAPA operator has the following properties.

**Theorem 12.** *Let $z_1, z_2, \dots, z_n$ be a collection of Z-numbers; if $\sup(z_i, z_j) = k$ for all $i \neq j$, then*

$$ZWAPA(z_1, z_2, \dots, z_n) = \sum_{i=1}^{n} w_i z_i \tag{10}$$

*where k is a constant.*

Theorem 12 implies that when all the supports between Z-numbers are equal, the ZWAPA operator reduces to the WAA operator.

**Theorem 13** (commutativity). *Let $z_1, z_2, \dots, z_n$ be a collection of Z-numbers and let $z_1', z_2', \dots, z_n'$ be any permutation of $z_1, z_2, \dots, z_n$. If the weights $w_1, w_2, \dots, w_n$ are irrelevant to the position of arguments, then*

$$ZWAPA(z_1, z_2, \dots, z_n) = ZWAPA(z_1', z_2', \dots, z_n') \tag{11}$$

**Theorem 14** (idempotency). *Let $z_i (i = 1, 2, \dots, n)$ be a set of Z-numbers; if $z_i = z$ for all i, then*

$$ZWAPA(z_1, z_2, \dots, z_n) = z \tag{12}$$

**Theorem 15** (boundedness). *Let $z_1, z_2, \dots, z_n$ be a set of Z-numbers; then*

$$z_{min} \leqslant ZWAPA(z_1, z_2, \dots, z_n) \leqslant z_{max} \tag{13}$$

*where $z_{min} = (A_{min}, R_{min})$, $A_{min} = (a_{min}^l, a_{min}^m, a_{min}^u)$, $R_{min} = (r_{min}^l, r_{min}^m, r_{min}^u)$ and $z_{max} = (A_{max}, R_{max})$, $A_{max} = (a_{max}^l, a_{max}^m, a_{max}^u)$, $R_{max} = (r_{max}^l, r_{max}^m, r_{max}^u)$.*

Combining PA operator with the weighted geometric average (WGA) operator, we can obtain a weighted geometric power average operator in Z-numbers case ZWGPA.

*Definition 16.* Let $z_1, z_2, \dots, z_n$ be a collection of Z-numbers, let $\Omega$ be the set of all Z-numbers, and $w_i$ is the weight of $z_i$, with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$; then the ZWGPA operator is a mapping: $\Omega^n \longrightarrow \Omega$, which is expressed as

$$ZWGPA(z_1, z_2, \dots, z_n) = \prod_{i=1}^{n} z_i^{w_i(1+T(z_i))/\sum_{i=1}^{n} w_i(1+T(z_i))} \tag{14}$$

where $T(z_i)$ is the same as the counterpart of Definition 11.

In compliance with the operations of Z-numbers and Definition 16, the resulting value determined by ZWGPA operator is still a Z-number.

Being similar to ZWAPA operator, ZWGPA operator also possesses the following properties.

**Theorem 17.** *Let $z_1, z_2, \dots, z_n$ be a collection of Z-numbers; if $\sup(z_i, z_j) = k$ for all $i \neq j$, then*

$$ZWGPA(z_1, z_2, \dots, z_n) = \sum_{i=1}^{n} z_i^{w_i} \tag{15}$$

*where k is a constant.*

Theorem 17 indicates that when all the support degrees are equal, the ZWGPA operator reduces to the WGA operator. Furthermore, ZWGPA operator has also the three properties: commutativity, idempotency, and boundedness.

Obviously, the ZWAPA and ZWGPA operators are two nonlinear weighted aggregation tools, where the weight $w_i(1 + T(z_i))/\sum_{i=1}^{n} w_i(1 + T(z_i))$ (not the weight $w_i$ of $z_i$, $i = 1, 2, \dots, n$) depends upon the input arguments and allows these values being aggregated to support and reinforce each other, which are able to retain the input information well and take fully into account the interrelation among the inputs.

## 5. A MCGDM Method with Z-Numbers

This section will build a framework for group decision making under Z-number environment; that is, DMs' evaluation values are expressed with Z-numbers. This process contains the determination of weights of DMs and criteria by establishing optimization models and the ranking of all the alternatives by means of the thought of TOPSIS.

Before presenting elaborate operation, we need to draw the outline of MCGDM problem. Suppose alternatives set $O = \{o_1, o_2, \dots, o_m\}$, criteria set $C = \{c_1, c_2, \dots, c_n\}$, whose weight vector is $\boldsymbol{w} = (w_1, w_2, \dots, w_n)^{\mathrm{T}}$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and decision-makers set $D = \{d_1, d_2, \dots, d_q\}$, whose weight vector is $\boldsymbol{\omega} = (\omega_1, \omega_2, \dots, \omega_q)^{\mathrm{T}}$ with $\omega_k \in [0,1]$ and $\sum_{k=1}^{q} \omega_k = 1$. Generally the set $C$ is divided into two types, $C_b$ and $C_c$, where $C_b$ denotes the set of benefit criteria and $C_c$ represents the set of cost criteria; besides, $C_b \cap C_c = \varnothing$ and $C_b \cup C_c = C$. Furthermore, the evaluation information of $o_i (i = 1, 2, \dots, m)$ with respect to $c_j (j = 1, 2, \dots, n)$ is denoted by $z_{ij}^k$. It is a Z-number and transformed by the semantic information of the $k$th DM $d_k (k = 1, 2, \dots, q)$, with $z_{ij}^k = (A_{ij}^k, R_{ij}^k)$. Eventually, the decision matrix $Z^k = (z_{ij}^k)_{m \times n}$ for the DM $d_k$ is produced.

*5.1. Weight-Determining Method for DMs and Criteria in MCGDM.* The determination of DMs' weights and criteria is an important research topic in MCGDM because they will have vital impacts on the final decision. Different DMs possess different knowledge backgrounds and professional degrees, so they cannot be assigned optional weights or given equal weight. Moreover, an alternative or object is evaluated via multiple criterion indexes and these criteria play different roles in the final decision. Therefore, they should also be given different weights. When weights information is completely unknown or partly unknown, we need to excavate fully clues from the decision matrices provided by DMs.

In previous studies, DMs' weights are subjectively given on the basis of experience with some subjective randomness. In this research, an objective approach for working out DMs' weights will be adopted. Similarity measure, a means that distinguishes diverse specialization degrees of DMs for decision-making problems, will be utilized under Z-number information. If the overall similarity of evaluation values in the decision matrix $Z^k = (z_{ij}^k)_{m\times n}$ given by the $k$th DM $d_k$ is greater than the overall similarity of the decision matrix $Z^l = (z_{ij}^l)_{m\times n}$ from the $l$th DM $d_l$, indicating that $d_k$ provides less inconsistent and conflicting decision information than $d_l$ among DMs and plays a relatively important role in decision-making process, then $d_k$ should be endowed bigger weight than $d_l$. In contrast, decision values a DM provides are figured out as smaller overall similarity; then this DM will be endowed smaller weight.

This article does not put forward similarity measure for Z-numbers but distance measure has been suggested. In compliance with the relation between distance and similarity, we can transform distance measure into similarity measure by using

$$s\left(z_{ij}^k, z_{ij}^l\right) = \max_{i,j,k,l}\left\{d\left(z_{ij}^k, z_{ij}^l\right)\right\} - d\left(z_{ij}^k, z_{ij}^l\right) \tag{16}$$

According to the analysis above, the assignment principle of DMs' weight vector $\boldsymbol{\omega}$ is to make the total similarity that DMs' evaluations form reaches the maximum. Motivated by Xu's work [39], we establish the following programming model to obtain the weights of DMs:

$$\text{(M-1)}\begin{cases}\max S(\boldsymbol{\omega}) = \displaystyle\sum_{k=1}^{q}\left(\frac{1}{mnq}\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)\right)\omega_k \\ \text{s.t. } \omega_k \geqslant 0,\ \displaystyle\sum_{k=1}^{q}\omega_k^2 = 1,\ k = 1,2,\ldots,q\end{cases} \tag{17}$$

To solve the above model, we construct Lagrange function

$$\begin{aligned}L(\boldsymbol{\omega},\xi) = {} & \sum_{k=1}^{q}\left(\frac{1}{mnq}\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)\right)\omega_k \\ & + \frac{\xi}{2}\left(\sum_{k=1}^{q}\omega_k^2 - 1\right)\end{aligned} \tag{18}$$

where $\xi$ is a Lagrange multiplier. Let all the partial derivatives of the function $L$ be 0; we have

$$\begin{aligned}\frac{\partial L}{\omega_j} &= \frac{1}{mnq}\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right) + \xi\omega_k = 0 \\ \frac{\partial L}{\xi} &= \frac{1}{2}\left(\sum_{k=1}^{q}\omega_k^2 - 1\right) = 0\end{aligned} \tag{19}$$

By solving the above equations, we can get

$$\omega_k = \frac{\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)}{\sqrt{\sum_{k=1}^{q}\left(\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)\right)^2}} \tag{20}$$

Then normalize $\omega_k$ as $\varpi_k$ to ensure that $\varpi_k$ satisfies $0 \leqslant \varpi_k \leqslant 1$ and $\sum_{k=1}^{q}\varpi_k = 1$. Now (20) is standardized as

$$\varpi_k = \frac{\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)}{\sum_{k=1}^{q}\sum_{l=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m} s\left(z_{ij}^k, z_{ij}^l\right)}, \quad k = 1,2,\ldots,q \tag{21}$$

It is (21) that is weight coefficient of DMs. By inspecting the formula structure, we can find out that the bigger the overall similarity degree that a DM corresponds to, the bigger the weighting value he or she can be endowed. Hence, the result decided by (21) accords with the aforementioned requirement about the weights assignment.

When it comes to criterion weights, after aggregating preference information of all the DMs by utilizing aggregation operators, we apply the well-known maximizing deviation method [40] to Z-numbers and still establish an optimization model to acquire weights of criteria from the aggregated matrix $Z = (z_{ij})_{m\times n}$. If there are marked differences between preference values of alternatives under a criterion $c_j$, then $c_j$ has a strong power that distinguishes distinct alternatives and the criterion is considered relatively important in choosing the best alternative. It will account for a relatively high weight. On the contrary, if there are more similar performance values under one criterion, this criterion will be assigned a small weight. In this paper, the deviation between any two alternatives' preference values is measured by the distance measure $d(\cdot,\cdot)$ of Z-numbers. In this way, the optimal criterion weight distribution $\boldsymbol{w}$ should make the total deviation of all alternatives with respect to all criteria maximized. However, the information about criterion weights is not entirely unknown in many actual circumstances. Evaluators usually have a subjective judgment or limitation on weight coefficients of criteria but the given weighting information is often inadequate and imprecise, and partial weights only fall within the range set by them out of the complexity of practical situation. In this case, we are supposed to combine the subjective weighting method and the objective weighting method.

Assume that the set of known weighting information is denoted by $W$. We construct another programming model with some constraints utilizing the maximizing deviation idea of the reference [39]:

$$(\text{M-2}) \begin{cases} \max D(\boldsymbol{w}) = \sum_{j=1}^{n} w_j \sum_{i=1}^{m} \sum_{l=1}^{m} d\left(z_{ij}, z_{lj}\right) \\ \text{s.t. } \boldsymbol{w} \in W,\ w_j \geqslant 0,\ \sum_{j=1}^{n} w_j = 1,\ j = 1, 2, \ldots, n \end{cases} \tag{22}$$

Table 1: Linguistic terms and the corresponding TFNs for restriction.

| Linguistic terms | TFNs |
|---|---|
| Very Low (VL) | (0, 0, 1) |
| Low (L) | (0, 1, 3) |
| Medium Low (ML) | (1, 3, 5) |
| Medium (M) | (3, 5, 7) |
| Medium High (MH) | (5, 7, 9) |
| High (H) | (7, 9, 10) |
| Very High (VH) | (9, 10, 10) |

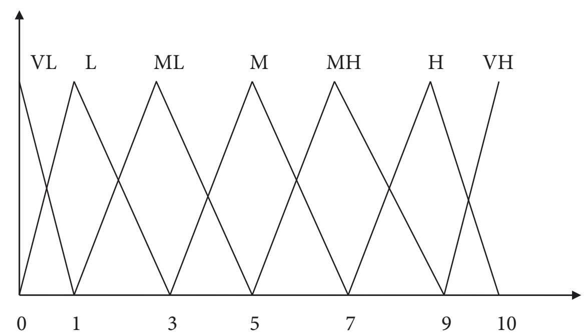

Figure 2: Various triangular fuzzy representations for restriction.

Table 2: Linguistic terms and the corresponding TFNs for reliability.

| Linguistic terms | TFNs |
|---|---|
| Not Sure (NS) | (0, 0, 1) |
| Not Very Sure (NVS) | (1, 3, 5) |
| Sure (S) | (5, 7, 9) |
| Very Sure (VS) | (9, 10, 10) |

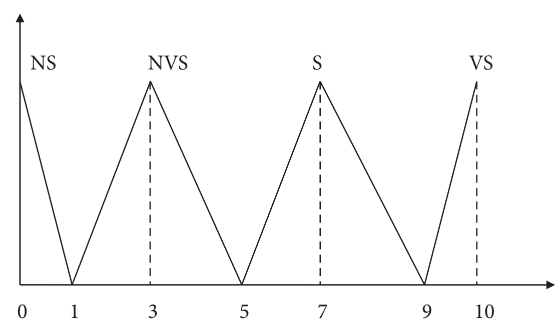

Figure 3: Various triangular fuzzy representations for reliability.

where $W$ is a set of constraint conditions that criterion weights $\boldsymbol{w}$ must satisfy in terms of the real situation.

Model (M-2) is a linear programming model. We can use the specialized LINGO software to work out the optimal solution of the model (M-2).

*5.2. Approach to MCGDM with Z-Number Information.* In this research, the evaluations made by DMs take the form of linguistic variables and are represented by TFNs. The experts are asked to specify ratings for alternatives over evaluation factors using seven linguistic values varying from “Very Low” to “Very High” as restriction part and the linguistic scale ranging from “Not Sure” to “Very Sure” as reliability part. The transformations between linguistic values and TFNs about two parts of each evaluation are shown in Tables 1 and 2, respectively. The corresponding two scales of TFNs are presented graphically in Figures 2 and 3, respectively. By referring to linguistic variables of Tables 1 and 2, DMs give their preference ratings on alternatives under all criteria. Each preference involves two components: the restriction rating from Table 1 and the matching reliability measure using the short sentences of Table 2.

Based on the above-mentioned knowledge preparation and the evaluation information given by DMs for decision problem, we integrate the suggested power aggregation operators and the popular TOPSIS method to develop a MCGDM approach in Z-number context. TOPSIS, proposed by Hwang and Yoon [41], is a kind of method to solve MADM problems, which aims at choosing the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). TOPSIS is simple in the operation and swift in the calculation. It is able to make full use of the information in the original data, reduce effectively the information loss, and improve the precision in the final results. In addition, it is suitable for both small sample data and large sample. Therefore, the method is widely used for tackling the ranking problems in real situations. In this paper, the main procedures are demonstrated in Figure 4 including 3 stages, and the detailed process is presented below.

*Step 1* (construct decision matrixes). According to the transformation between words and TFNs in Tables 1 and 2, all

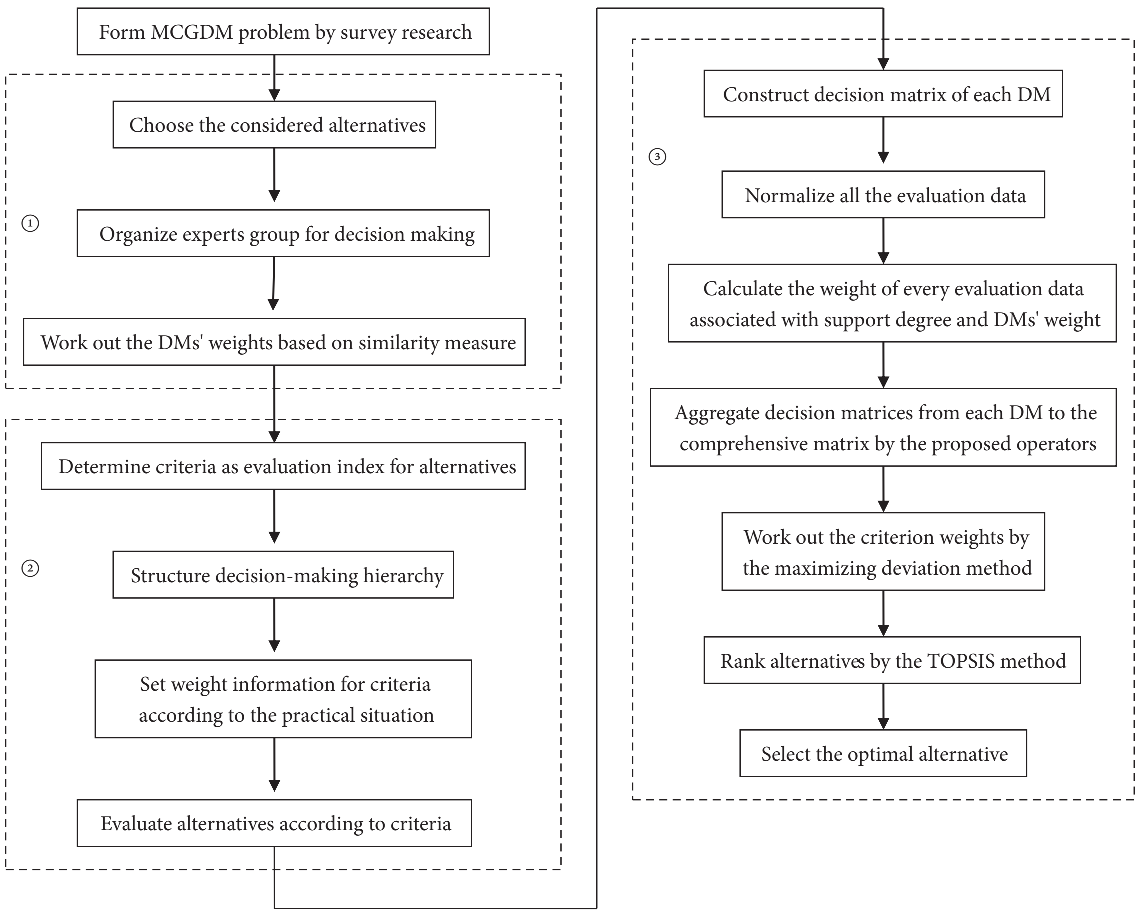


Figure 4: The flow diagram of the proposed approach for MCGDM. ① Decision preparation; ② Decision implementation; ③ Decision operation.

the evaluations provided by DMs are converted into Z-numbers and a set of decision matrixes $Z^k = (z_{ij}^k)_{m\times n}$ are obtained, where the preference value $z_{ij}^k = (A_{ij}^k, R_{ij}^k)$ denotes the numerical evaluation (including the restriction $A_{ij}^k$ and reliability $R_{ij}^k$) of the alternative $o_i (i = 1, 2, \dots, m)$ with respect to the criterion $c_j (j = 1, 2, \dots, n)$, provided by the $k$th DM $d_k (k = 1, 2, \dots, q)$.

*Step 2* (normalize the evaluation information). In order to eliminate the influence of different dimensions and the difference in priority that different types of criteria may bring, we normalize all preference values to the same magnitude grade and make all the research objects comparable. Concretely, let $Z^k = (z_{ij}^k)_{m\times n}$ be normalized as $\widetilde{Z}^k = (\tilde{z}_{ij}^k)_{m\times n}$, where $\tilde{z}_{ij}^k = (\widetilde{A}_{ij}^k, R_{ij}^k)$,

$$\widetilde{A}_{ij}^k = \begin{cases} \left( \dfrac{a_{ij}^{kl}}{\max_i a_{ij}^{ku}}, \dfrac{a_{ij}^{km}}{\max_i a_{ij}^{km}}, \dfrac{a_{ij}^{ku}}{\max_i a_{ij}^{kl}} \right), & c_j \in C_b \\ \left( \dfrac{\min_i a_{ij}^{kl}}{a_{ij}^{ku}}, \dfrac{\min_i a_{ij}^{km}}{a_{ij}^{km}}, \dfrac{\min_i a_{ij}^{ku}}{a_{ij}^{kl}} \right), & c_j \in C_c \end{cases} \tag{23}$$

and $R_{ij}^k$ is unchanged.

*Step 3* (calculate the supports). Set

$$\sup\left(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l\right) = \max_{i,j,k,l} \left\{ d\left(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l\right) \right\} - d\left(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l\right) \tag{24}$$

where $d(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l)(i = 1, 2, \dots, m;\ j = 1, 2, \dots, n;\ k, l = 1, 2, \dots, q)$ can be computed by (6). Obviously, the support degree $\sup(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l)$ completely satisfies support conditions (1)-(3) listed in Definition 11.

*Step 4* (calculate the comprehensive support with DMs' weights and weighting coefficient for each performance value). Utilize (21) to acquire DMs' weights $\varpi_1, \varpi_2, \dots, \varpi_q$ and calculate the comprehensive weighted support degree $T(\tilde{z}_{ij}^k)$ of $\tilde{z}_{ij}^k$ by the other performance value $\tilde{z}_{ij}^l (l \neq k)$

$$T\left(\tilde{z}_{ij}^k\right) = \sum_{l=1, l\neq k}^{q} \varpi_l \sup\left(\tilde{z}_{ij}^k, \tilde{z}_{ij}^l\right), \quad k = 1, 2, \dots, q \tag{25}$$

and compute the weights associated with the performance value $\tilde{z}_{ij}^k$

$$\xi_{ij}^k = \frac{\varpi_k \left(1 + T\left(\tilde{z}_{ij}^k\right)\right)}{\sum_{k=1}^{q} \varpi_k \left(1 + T\left(\tilde{z}_{ij}^k\right)\right)} \tag{26}$$

where $\xi_{ij}^k \geqslant 0, \sum_{k=1}^{q} \xi_{ij}^k = 1$.

*Step 5* (aggregate the evaluation values for all DMs). Utilize the ZWAPA operator (9) or ZWGPA operator (14) to aggregate all the individual decision matrix $\widetilde{Z}^k = (\widetilde{z}_{ij}^k)_{m\times n}$ as the collective decision matrix $\widetilde{Z} = (\widetilde{z}_{ij})_{m\times n}$. That is, let $\xi_{ij}^k$ be weight of $\widetilde{z}_{ij}^k$ $(k = 1,2,3)$ in Step 4 and utilize the WAA or WGA operator to aggregate decision evaluations from all DMs.

$$\begin{aligned}\widetilde{z}_{ij} &= ZWAPA\left(\widetilde{z}_{ij}^1, \widetilde{z}_{ij}^2, \ldots, \widetilde{z}_{ij}^q\right)\\ &= \frac{\sum_{k=1}^q \varpi_k\left(1+T\left(\widetilde{z}_{ij}^k\right)\right)\widetilde{z}_{ij}^k}{\sum_{k=1}^q \varpi_k\left(1+T\left(\widetilde{z}_{ij}^k\right)\right)} = \sum_{k=1}^q \xi_{ij}^k \widetilde{z}_{ij}^k\\ \widetilde{z}_{ij} &= ZWGPA\left(\widetilde{z}_{ij}^1, \widetilde{z}_{ij}^2, \ldots, \widetilde{z}_{ij}^q\right)\\ &= \prod_{k=1}^q \left(\widetilde{z}_{ij}^k\right)^{\varpi_k(1+T(\widetilde{z}_{ij}^k))/\sum_{k=1}^q \varpi_k(1+T(\widetilde{z}_{ij}^k))} = \prod_{k=1}^q \left(\widetilde{z}_{ij}^k\right)^{\xi_{ij}^k}\end{aligned} \tag{27}$$

*Step 6* (compute the criterion weights). Based on the integrated decision matrix $\widetilde{Z}$, construct the model (M-2) in terms of a practical problem and obtain the weight vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ of criteria by solving the (M-2).

*Step 7* (determine the PIS $o^+$ and the NIS $o^-$ for alternatives on the integrated matrix $\widetilde{Z}$). The PIS $o^+$, numerically expressed by $\widetilde{z}^+$, here $\widetilde{z}^+ = \{\widetilde{z}_1^+, \widetilde{z}_2^+, \ldots, \widetilde{z}_n^+\}$, where

$$\begin{aligned}\widetilde{z}_j^+ &= \left(\widetilde{A}_j^+, \widetilde{R}_j^+\right) = \left(\max_i \widetilde{A}_{ij}, \max_i \widetilde{R}_{ij}\right)\\ &= \left(\left(\max_i \widetilde{a}_{ij}^l, \max_i \widetilde{a}_{ij}^m, \max_i \widetilde{a}_{ij}^u\right),\right.\\ &\quad \left.\left(\max_i \widetilde{r}_{ij}^l, \max_i \widetilde{r}_{ij}^m, \max_i \widetilde{r}_{ij}^u\right)\right), \quad j = 1,2,\ldots,n\end{aligned} \tag{28}$$

The NIS $o^-$, expressed by $\widetilde{z}^-$, here $\widetilde{z}^- = \{\widetilde{z}_1^-, \widetilde{z}_2^-, \ldots, \widetilde{z}_n^-\}$, where

$$\begin{aligned}\widetilde{z}_j^- &= \left(\widetilde{A}_j^-, \widetilde{R}_j^-\right) = \left(\min_i \widetilde{A}_{ij}, \min_i \widetilde{R}_{ij}\right)\\ &= \left(\left(\min_i \widetilde{a}_{ij}^l, \min_i \widetilde{a}_{ij}^m, \min_i \widetilde{a}_{ij}^u\right),\right.\\ &\quad \left.\left(\min_i \widetilde{r}_{ij}^l, \min_i \widetilde{r}_{ij}^m, \min_i \widetilde{r}_{ij}^u\right)\right), \quad j = 1,2,\ldots,n\end{aligned} \tag{29}$$

*Step 8* (compute the relative closeness degree). According to the distance formula Eq. (5), compute the weighted distance $D_i^+$ between the restraint components $\widetilde{A}_{i\cdot}$ of the alternative $o_i$ and the corresponding PIS $\widetilde{A}_j^+$ of $o^+$

$$D_i^+ = \sum_{j=1}^n w_j d\left(\widetilde{A}_{ij}, \widetilde{A}_j^+\right), \quad i = 1,2,\ldots,m \tag{30}$$

and the weighted distance $D_i^-$ associated with the alternative $o_i$

$$D_i^- = \sum_{j=1}^n w_j d\left(\widetilde{A}_{ij}, \widetilde{A}_j^-\right), \quad i = 1,2,\ldots,m \tag{31}$$

Analogously we can get the weighted distance $d_i^+$ and $d_i^-$ about the confidence component $\widetilde{R}_{i\cdot}$ of the alternative $o_i$ from the PIS $\widetilde{R}_j^+$ and the NIS $\widetilde{R}_j^-$, respectively.

$$d_i^+ = \sum_{j=1}^n w_j d\left(\widetilde{R}_{ij}, \widetilde{R}_j^+\right), \quad i = 1,2,\ldots,m \tag{32}$$

$$d_i^- = \sum_{j=1}^n w_j d\left(\widetilde{R}_{ij}, \widetilde{R}_j^-\right), \quad i = 1,2,\ldots,m \tag{33}$$

and then calculate the relative closeness of the alternative $o_i$ to the ideal alternative $o^+$, in form of $(C_i^1, C_i^2)$, which is a pair of numbers derived from the two components of Z-numbers.

$$\left(C_i^1, C_i^2\right) = \left(\frac{D_i^-}{D_i^+ + D_i^-}, \frac{d_i^-}{d_i^+ + d_i^-}\right), \quad i = 1,2,\ldots,m \tag{34}$$

*Step 9* (determine the ranking order of all the alternatives). First we define such a priority relation between any two pairs $(C_\alpha^1, C_\alpha^2)$ and $(C_\beta^1, C_\beta^2)$.

If $C_\alpha^1 < C_\beta^1$, then $(C_\alpha^1, C_\alpha^2) < (C_\beta^1, C_\beta^2)$.

If $C_\alpha^1 = C_\beta^1$, then we have the following:(i) if $C_\alpha^2 = C_\beta^2$, then $(C_\alpha^1, C_\alpha^2) = (C_\beta^1, C_\beta^2)$; (ii) if $C_\alpha^2 < C_\beta^2$, then $(C_\alpha^1, C_\alpha^2) < (C_\beta^1, C_\beta^2)$.

Rank all alternatives $o_i (i = 1,2,\ldots,m)$ through the above ordering rule on $(C_i^1, C_i^2)$. Notice that the first part of each pair of closeness coefficients plays a leading role when compared with each other. A desirable alternative should be as close to the PIS and as far from the NIS, as possible. As a result, its two closeness coefficients in the pair will be as large as possible. Therefore, we can use the closeness coefficients to prioritize alternatives and choose the best one. The larger the two closeness degrees, the better the corresponding alternative. The best alternative is the one with the greatest relative closeness to the ideal solution.

## 6. Application in Supplier Selection

The selection of suppliers is an important issue as company's face in the business management where raw materials and components represent a significant percentage of the total product cost. The importance of supplier selection has increased also from outsourcing initiatives in which companies rely more on suppliers to improve the quality of their products, to reduce their costs, or to focus on a specific part of their operations. Thus, supplier selection constitutes a strategic decision [42]. Numerous companies take this work as the one of key tasks in outside cooperative trade. Therefore, how to select a satisfying supplier is crucial. Recently researches involving this issue have triggered enough attention and have obtained fruitful achievements. In this section, we apply the suggested algorithm in this background and make some simple comparisons and discussion against several existing decision-making approaches after deriving consequences of this problem.

This case study is about the supplier selection problem concerning an automobile manufacturing company. Due to its activity, the company needs a certain amount of raw

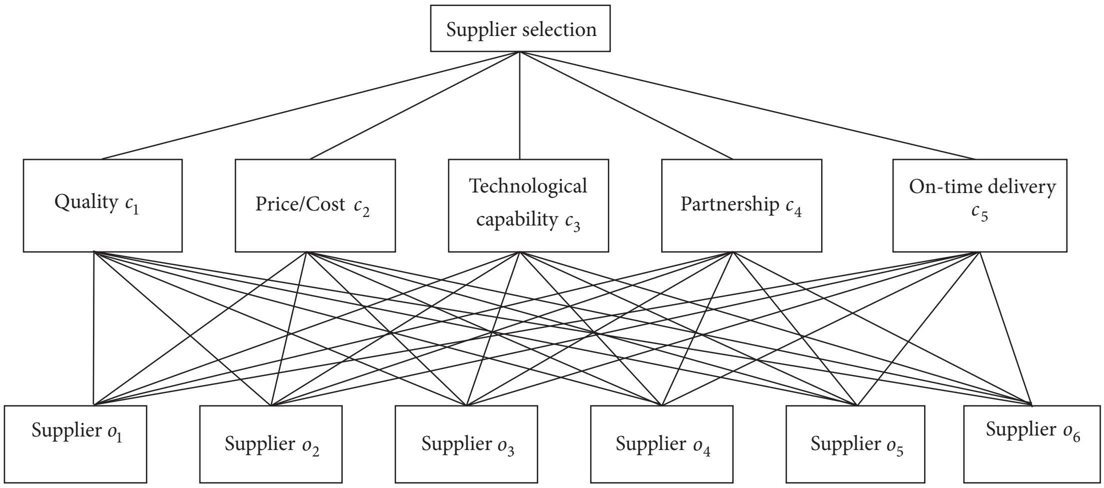


FIGURE 5: Hierarchical structure for the supplier selection.

TABLE 3: All evaluations made by $d_1$.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $o_1$ | (VL,NVS) | (ML,VS) | (MH,S) | (H,NVS) | (MH,NS) |
| $o_2$ | (L,S) | (MH,NVS) | (M,NS) | (H,VS) | (M,NVS) |
| $o_3$ | (MH,NVS) | (ML,S) | (H,VS) | (M,NVS) | (VL,NVS) |
| $o_4$ | (H,S) | (H,NS) | (MH,VS) | (ML,NS) | (VH,S) |
| $o_5$ | (M,NVS) | (M,VS) | (MH,NS) | (H,NS) | (VH,S) |
| $o_6$ | (ML,NS) | (ML,NS) | (H,S) | (H,NVS) | (L,S) |

TABLE 4: All evaluations made by $d_2$.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $o_1$ | (M,NVS) | (ML,NS) | (MH,S) | (MH,NVS) | (H,VS) |
| $o_2$ | (MH,S) | (M,S) | (MH,NVS) | (H,NS) | (M,VS) |
| $o_3$ | (MH,NS) | (M,VS) | (H,NVS) | (M,NVS) | (MH,NVS) |
| $o_4$ | (MH,NVS) | (L,NVS) | (VH,NS) | (M,NS) | (ML,NS) |
| $o_5$ | (MH,VS) | (VH,NS) | (M,VS) | (H,NS) | (VH,NS) |
| $o_6$ | (H,NS) | (H,NVS) | (ML,NVS) | (H,S) | (MH,VS) |

materials such as iron, wire, and tire and has to coordinate with a number of suppliers. Through collecting plenty of information on material producing companies, there are six qualified suppliers being able to supply the required raw materials, denoted by $\{o_1, o_2, \ldots, o_6\}$. To promote stable and healthy development of the manufacturing company, it is very necessary to use a reasonable algorithm for selecting trustworthy supplier(s). For this purpose, the company invites some experts and organizes a professional team to evaluate performances of the potential suppliers by the consideration of various factors. The team consists of three experts (DMs), denoted by $\{d_1, d_2, d_3\}$. They define five criteria to evaluate these alternatives including $c_1$ quality, $c_2$ price/cost, $c_3$ technological capability, $c_4$ partnership, and $c_5$ on-time delivery. After a heated discussion, partial information regarding the weights of criteria is provided as $W = \{w_1 \geqslant w_2, w_1 + w_2 < 0.5, w_1 - w_2 \leqslant w_3 \leqslant w_1 + w_2, 1.5w_2 \geqslant w_5, w_3 > 0.15, w_4 \leqslant w_5, 1.5w_4 \geqslant w_5\}$ like the format S.H. Kim, B.S. Ahn, and Z.S. Xu previously mentioned in [43, 44]. Further, each DM gives their respective evaluative ratings of suppliers with respect to those setting criteria by employing linguistic terms of Table 1 (for restriction) and Table 2 (for reliability). It is noted that each criterion value given by experts is actually the combination of two linguistic ratings. The entire evaluations made by the three DMs are listed in Tables 3–5, and the hierarchical structure of this problem is displayed in Figure 5.

*6.1. Illustration of the Proposed Approach.* Now we take advantage of the proposed approach and implement its steps step by step to make the final choice of suppliers.

*Step 1.* By the conversion relation of words and TFNs in Tables 1 and 2, the evaluation information provided by $d_1, d_2, d_3$ is turned into three numerical decision matrices $Z^1 = (z_{ij}^1)_{6\times5}, Z^2 = (z_{ij}^2)_{6\times5}, Z^3 = (z_{ij}^3)_{6\times5}$.

Table 5: All evaluations made by $d_3$.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $o_1$ | (MH,S) | (M,S) | (H,NVS) | (M,NVS) | (M,S) |
| $o_2$ | (H,NVS) | (MH,NVS) | (H,NS) | (M,NS) | (ML,NVS) |
| $o_3$ | (M,VS) | (ML,NS) | (H,S) | (MH,S) | (MH,NS) |
| $o_4$ | (MH,VS) | (VL,NS) | (VH,S) | (MH,S) | (M,NS) |
| $o_5$ | (H,NVS) | (VH,NVS) | (ML,NS) | (H,VS) | (VH,NVS) |
| $o_6$ | (MH,S) | (H,NS) | (ML,NS) | (VH,NVS) | (H,NS) |

*Step 2.* Evidently, only $c_2$ is a cost-type index among all the criteria and the rest are benefit index. In the light of normalization formula Eq. (23), let all the evaluation values of $Z^1, Z^2, Z^3$ become $\widetilde{Z}^1, \widetilde{Z}^2, \widetilde{Z}^3$, where $\widetilde{Z}^k = (\widetilde{z}^k_{ij})_{6\times 5}, k = 1, 2, 3$.

*Step 3.* By (24), compute the support degrees between any two normalized matrices $\widetilde{Z}^1, \widetilde{Z}^2, \widetilde{Z}^3$ and obtain three support matrices between diverse pairwise decision matrices as follows:

$$\sup\left(\widetilde{Z}^1, \widetilde{Z}^2\right) = \begin{pmatrix} 0.9938 & 0.4572 & 0.9965 & 0.9974 & 0.6772 \\ 0.9920 & 0.9834 & 0.7083 & 0.5212 & 0.9890 \\ 0.6154 & 0.9529 & 0.9883 & 1.0000 & 0.9953 \\ 0.9832 & 0.2837 & 0.5704 & 0.8585 & 0.8462 \\ 0.9753 & 0.9124 & 0.6135 & 1.0000 & 0.5454 \\ 0.6042 & 0.0677 & 0.9935 & 0.9978 & 0.9764 \end{pmatrix}$$

$$\sup\left(\widetilde{Z}^2, \widetilde{Z}^3\right) = \begin{pmatrix} 0.9964 & 0.4932 & 0.9919 & 0.9963 & 0.9931 \\ 0.9972 & 0.9965 & 0.5839 & 0.6774 & 0.9963 \\ 0.6053 & 0.8599 & 0.9998 & 0.9940 & 0.6711 \\ 0.9968 & 0.4452 & 0.5305 & 0.7129 & 0.8409 \\ 0.9967 & 0.9257 & 0.8483 & 0.5275 & 0.5366 \\ 0.5244 & 0.9624 & 0.8527 & 0.9971 & 0.5846 \end{pmatrix} \tag{35}$$

$$\sup\left(\widetilde{Z}^1, \widetilde{Z}^3\right) = \begin{pmatrix} 0.9936 & 0.9811 & 0.9976 & 0.9931 & 0.6624 \\ 0.9924 & 0.9953 & 0.8260 & 0.7447 & 0.9962 \\ 0.9963 & 0.8383 & 0.9855 & 0.9988 & 0.6656 \\ 0.9942 & 0.8015 & 0.9862 & 0.8184 & 0.9551 \\ 0.9965 & 0.9916 & 0.6647 & 0.5323 & 0.9998 \\ 0.8061 & 0.0000 & 0.8572 & 0.9982 & 0.5874 \end{pmatrix}$$

*Step 4.* Utilize (21) to figure out DMs' weights $\varpi_1 = 0.3358, \varpi_2 = 0.3284, \varpi_3 = 0.3358$ and by (25) calculate all the comprehensive support $T(\widetilde{z}^1_{ij}), T(\widetilde{z}^2_{ij}), T(\widetilde{z}^3_{ij})$ with the weights. Finally figure out the weight $\xi^k_{ij}$ in relation to $\widetilde{z}^k_{ij}$ by (26) and get the following three weighting matrixes $\xi^k = (\xi^k_{ij})_{6\times 5}, k = 1, 2, 3$:

$$\xi^1 = \begin{pmatrix} 0.3364 & 0.3492 & 0.3339 & 0.3366 & 0.3201 \\ 0.3359 & 0.3441 & 0.3461 & 0.3354 & 0.3367 \\ 0.3466 & 0.3389 & 0.3406 & 0.3410 & 0.3452 \\ 0.3372 & 0.3417 & 0.3424 & 0.3422 & 0.3404 \\ 0.3358 & 0.3352 & 0.3259 & 0.3488 & 0.3485 \\ 0.3473 & 0.2893 & 0.3389 & 0.3390 & 0.3458 \end{pmatrix}$$

$$\xi^2 = \begin{pmatrix} 0.3273 & 0.3014 & 0.3278 & 0.3268 & 0.3357 \\ 0.3275 & 0.3218 & 0.3204 & 0.3206 & 0.3275 \\ 0.3172 & 0.3315 & 0.3272 & 0.3275 & 0.3361 \\ 0.3287 & 0.3021 & 0.3125 & 0.3257 & 0.3214 \\ 0.3277 & 0.3250 & 0.3321 & 0.3402 & 0.3043 \\ 0.3220 & 0.3592 & 0.3322 & 0.3277 & 0.3375 \end{pmatrix} \tag{36}$$

$$\xi^3 = \begin{pmatrix} 0.3371 & 0.3507 & 0.3362 & 0.3364 & 0.3451 \\ 0.3371 & 0.3328 & 0.3358 & 0.3465 & 0.3347 \\ 0.3463 & 0.3322 & 0.3379 & 0.3353 & 0.3213 \\ 0.3355 & 0.3542 & 0.3484 & 0.3331 & 0.3404 \\ 0.3364 & 0.3381 & 0.3441 & 0.3122 & 0.3482 \\ 0.3411 & 0.3615 & 0.3301 & 0.3366 & 0.3244 \end{pmatrix}$$

*Step 5.* Aggregate the evaluation information provided by all the DMs using ZWGPA operator and get the comprehensive evaluation matrix $\widetilde{Z}$ (see Table 6).

*Step 6.* Use (6) to compute all the distances between Z-numbers of the aggregated matrix $\widetilde{Z}$. According to the partly known criteria information discussed by experts, construct the model (M-2) as

Table 6: Comprehensive evaluative matrix $\widetilde{Z}$ for suppliers.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $o_1$ | ((0.0526, 0.0778, 0.5759), (5, 7, 9)) | ((0.0112, 0.0821, 1.6643), (9, 10, 10)) | ((0.4733, 0.7104, 1.0385), (5, 7, 9)) | ((0.4711, 0.7294, 1.1307), (1, 3, 5)) | ((0.4690, 0.6230, 0.9493), (9, 10, 10)) |
| $o_2$ | ((0.0694, 0.4509, 0.9209), (5, 7, 9)) | ((0.0062, 0.0507, 0.5824), (5, 7, 9)) | ((0.4672, 0.7028, 1.0365), (1, 3, 5)) | ((0.5323, 0.7984, 1.1617), (9, 10, 10)) | ((0.2072, 0.4206, 0.6951), (9, 10, 10)) |
| $o_3$ | ((0.4188, 0.7022, 1.1834), (9, 10, 10)) | ((0.0093, 0.0915, 1.7343), (9, 10, 10)) | ((0.6994, 0.9311, 1.2207), (9, 10, 10)) | ((0.3602, 0.6013, 1.0000), (5, 7, 9)) | ((0.0583, 0.0749, 0.4712), (1, 3, 5)) |
| $o_4$ | ((0.5589, 0.8564, 1.3315), (5, 7, 9)) | ((0.0155, 0.6866, 2.6278), (1, 3, 5)) | ((0.7347, 0.9093, 1.1771), (9, 10, 10)) | ((0.2343, 0.5033, 0.8920), (5, 7, 9)) | ((0.3064, 0.5525, 0.7904), (5, 7, 9)) |
| $o_5$ | ((0.4803, 0.7557, 1.2343), (9, 10, 10)) | ((0.0059, 0.0406, 0.3967), (9, 10, 10)) | ((0.2425, 0.4949, 0.8202), (9, 10, 10)) | ((0.6993, 0.9675, 1.3314), (9, 10, 10)) | ((0.8995, 1.0000, 1.1220), (5, 7, 9)) |
| $o_6$ | ((0.3183, 0.6331, 1.120), (5, 7, 9)) | ((0.0042, 0.0397, 0.5859), (1, 3, 5)) | ((0.1933, 0.4489, 0.7745), (5, 7, 9)) | ((0.7558, 1.0000, 1.3205), (5, 7, 9)) | ((0.0640, 0.3857, 0.7136), (9, 10, 10)) |

$$\max \quad D(\boldsymbol{w}) = 0.2412w_1 + 0.0723w_2 + 0.1125w_3 + 0.1538w_4 + 0.2861w_5$$

$$\text{s.t.} \quad \begin{cases} w_1 \geqslant w_2 \\ w_1 + w_2 < 0.5 \\ w_1 - w_2 \leqslant w_3 \leqslant w_1 + w_2 \\ 1.5w_2 \geqslant w_5 \\ w_3 > 0.15 \\ w_4 \leqslant w_5 \\ 1.5w_4 \geqslant w_5 \\ \sum_{j=1}^{5} w_j = 1 \end{cases} \tag{37}$$

Solve this model by LINGO software and get the optimal weight vector of criteria $\boldsymbol{w} = (0.31, 0.16, 0.15, 0.16, 0.23)^{\mathrm{T}}$.

*Step 7.* From the collective decision matrix $\widetilde{Z}$, in compliance with (28) and (29), find the positive and negative ideal performance values concerning each criterion and constitute the PIS $\widetilde{z}^{+} = \{((0.5589, 0.8564, 1.3315), (9, 10, 10)), ((0.0155, 0.6866, 2.6278), (9, 10, 10)), ((0.7347, 0.9311, 1.2207), (9, 10, 10)), ((0.7558, 1.0000, 1.3314), (9, 10, 10)), ((0.8995, 1.0000, 1.1220), (9, 10, 10))\}$, the NIS $\widetilde{z}^{-} = \{((0.0526, 0.0778, 0.5759), (5, 7, 9)), ((0.0042, 0.0397, 0.3967), (1, 3, 5)), ((0.1933, 0.4489, 0.7745), (1, 3, 5)), ((0.2343, 0.5033, 0.8920), (1, 3, 5)), ((0.0583, 0.0749, 0.4712), (1, 3, 5))\}$.

*Step 8.* In accordance with (30)–(33), calculate, respectively, the distance $D_i^{+}, D_i^{-}, d_i^{+}, d_i^{-}$ about all the suppliers:

$$\begin{aligned} D_1^{+} &= 0.2352, \\ D_2^{+} &= 0.2527, \\ D_3^{+} &= 0.1788, \\ D_4^{+} &= 0.1163, \\ D_5^{+} &= 0.1201, \\ D_6^{+} &= 0.2004; \\ D_1^{-} &= 0.2335, \\ D_2^{-} &= 0.1934, \\ D_3^{-} &= 0.2926, \\ D_4^{-} &= 1.4011, \\ D_5^{-} &= 0.4045, \\ D_6^{-} &= 0.2332; \\ d_1^{+} &= 1.2388, \\ d_2^{+} &= 1.2271, \\ d_3^{+} &= 0.9303, \\ d_4^{+} &= 1.6459, \\ d_5^{+} &= 0.3902, \\ d_6^{+} &= 1.5102; \\ d_1^{-} &= 2.8569, \\ d_2^{-} &= 2.8914, \\ d_3^{-} &= 3.0674, \\ d_4^{-} &= 2.2448, \\ d_5^{-} &= 4.2687, \\ d_6^{-} &= 2.4489. \end{aligned} \tag{38}$$

Then work out the relative closeness pair $(C_i^1, C_i^2)$ of the supplier $o_i (i = 1, 2, \ldots, m)$ by (34)

$$\begin{aligned} \left(C_1^1, C_1^2\right) &= (0.5003, 0.6975), \\ \left(C_2^1, C_2^2\right) &= (0.4335, 0.7021), \\ \left(C_3^1, C_3^2\right) &= (0.6207, 0.7673), \\ \left(C_4^1, C_4^2\right) &= (0.8234, 0.5770), \\ \left(C_5^1, C_5^2\right) &= (0.7711, 0.9162), \\ \left(C_6^1, C_6^2\right) &= (0.5378, 0.6185). \end{aligned} \tag{39}$$

*Step 9.* With all the closeness pairs for suppliers in Step 8, in compliance with the rule used for comparing different two-tuple closenesses that the two large closenesses indicate a good performance for an alternative we easily get the ranking order of the considered suppliers by the first values of the six pairs of closeness degrees: $o_4 \succ o_5 \succ o_3 \succ o_6 \succ o_1 \succ o_2$. Thus the most desirable supplier is $o_4$ with the largest closeness value.

In Step 5, if we adopt the ZWAPA operator to integrate different DMs' evaluation information and the other steps remain unchanged until Step 8, the eventual relative closeness values of all the suppliers become

$$\begin{aligned} \left(C_1^1, C_1^2\right) &= (0.5242, 0.6975), \\ \left(C_2^1, C_2^2\right) &= (0.4944, 0.7021), \\ \left(C_3^1, C_3^2\right) &= (0.5513, 0.7673), \end{aligned}$$

$$\begin{aligned}\left(C_4^1, C_4^2\right) &= (0.5641, 0.5770)\,,\\ \left(C_5^1, C_5^2\right) &= (0.5539, 0.9162)\,,\\ \left(C_6^1, C_6^2\right) &= (0.5539, 0.6185)\,.\end{aligned} \tag{40}$$

Therefore we have $C_4^1 > C_5^1 = C_6^1 > C_3^1 > C_1^1 > C_2^1$; obviously the priority order of these suppliers can be all determined except that of $z_5$ and $z_6$ (in fact, the two closenesses are slight different, but they are equal when they kept four decimals). Owing to $C_5^2 > C_6^2$, $o_5 \succ o_6$ through the ranking rule for alternatives in Step 9. Then, the global priority is $o_4 \succ o_5 \succ o_6 \succ o_3 \succ o_1 \succ o_2$. It follows that though the eventual priority order for suppliers changes a little (just the position of $o_3$ and $o_6$ is exchanged), the best supplier is still $o_4$, which does not affect the selection of the global suppliers.

*6.2. Comparative Analysis to Existing Approaches.* In order to verify the feasibility and validity of the recommended approach, we conduct a comparative analysis with some existing approaches for the illustrative example described above.

In the process of decision making with Z-numbers, many researchers try to defuzzify the preference values expressed by Z-numbers such as [23, 24, 33]. They use classic defuzzified method to handle two components of Z-numbers because the two components are denoted by classic fuzzy numbers and then convert the whole Z-numbers to crisp numbers or construct the score function for each object even average them when ordering Z-numbers. However, these ways of dealing with Z-numbers inevitably involve the loss and distortion of original information. If we use the method system in [23, 24], the order of the suppliers will be $o_2 \succ o_5 \succ o_6 \succ o_1 \succ o_4 \succ o_3$; if we use the methodology in [33], the priority order of the suppliers is $o_3 \succ o_5 \succ o_4 \succ o_6 \succ o_1 \succ o_2$. It can be noted that different defuzzified methods will get different ranking outcomes of alternatives. Sometimes it is difficult to identify their advantages and disadvantages toward specific problem among these methods. For the illustrative example in this paper, we can find that the supplier $o_4$ is the best through carefully inspecting evaluation ratings of 6 suppliers provided by 3 experts, while the results derived from those methods as in [23, 24, 33] are not. Additionally, $o_3$ is superior to $o_1$ by comparing their respective preference rating and the matching confidence level but the ranking order in [23, 24] is converse. Therefore, simple defuzzified way to tackle Z-numbers is not very convincing. Jiang et al. [36] overcome the drawback of these literature and present an improved method for ranking Z-numbers where they construct a pair of score functions about $A$ and $R$ (suppose $z = (A, R)$) instead of transforming $z$ into a fuzzy number by converting $B$ to a crisp number. The method reduces the loss of useful decision information to some extent but it regards $A$ and $R$ as two independent components and uses the same means to tackle the two aspects of information. However, we know that there is a certain relationship between $A$ and $R$; $R$ is the measure of the confidence extent that $A$ is given, so the two parts of Z-numbers cannot be considered separately and their interrelationship should be reflected in the decision-making process.

Recently Zamri et al. [34] and Mohamad et al. [29] developed a fuzzy TOPSIS method to solve MCGDM problems with Z-numbers, which are relatively close to our approach. So we might as well conduct a comparison in the choice of those suppliers. In [34], both original evaluation information and experts' weights are simply averaged and then the average evaluation values are endowed the average weights. That leads to a challenge that all the information related to experts fails to be used fully. The relative closeness in the last step, a binary structure, is composed of the closeness coefficients of two components of Z-numbers to the PIS and the NIS, respectively. However, the way of ranking alternatives is to calculate and compare the means of the two closeness coefficients in the relative closeness for an alternative. As we know, the first component is more important than the second one in a Z-number. Apparently it is unreasonable that the final outcomes are obtained by the simple unweighted average of the two parts; in other words, two components of Z-numbers are deemed as equally important. Authors in [29] transform directly Z-numbers into trapezoidal fuzzy numbers (TpFNs) and use the TpFNs to depict the uncertainty of objects instead of the corresponding Z-numbers, which will lead to the damage of the feature of Z-numbers. More importantly, it converts a TpFN to an expected interval number as a vector when constructing Jaccard similarity measure of two vectors but it is not reasonable that interval numbers are seen as vectors. So it is not very accurate that the paper utilizes the similarity measure to measure the proximity between each alternative and the ideal one in order to rank the alternatives. If we adopt the decision-making methods in [29, 34] in the supplier selection, the final ranking results are $o_3 \succ o_1 \succ o_4 \succ o_2 \succ o_5 \succ o_6$ and $o_4 \succ o_5 \succ o_6 \succ o_2 \succ o_3 \succ o_1$, respectively. We easily notice that the supplier $o_4$ is not the most favoured one in the first preference orders and the second ranking is similar to ours, but the poorest supplier should be $o_2$ not $o_1$. Hence our approach has some advantages over those methods mentioned above at least.

(1) In the treatment of Z-numbers, we take the difference of the first component and the second one into account. When the distance measure between Z-numbers is defined, the first component plays a larger portion than the second one because the latter is merely a measure of reliability of the former. When the priority of alternatives is compared, the binary relative closeness degrees play, respectively, different roles. The first number, stemmed from the first component of Z-numbers, is the main force.

(2) The distance measure under Z-number context that we structure is derived from the cross-entropy of information science, which has higher ability that discriminates different uncertain information than those conventional distance forms due to its specific construction. In the condition of high uncertainty depicted by Z-numbers, the constructed distance formula can measure relatively accurate discrimination measurement. Based on this distance measure, we utilize the maximizing deviation method to determine the criteria weights with DMs' subjective information about

criteria, and the assignment of DMs' weights is not subjective but a programming model is established in light of the principle that all the DMs' evaluations on alternatives get the most similar while DMs' weights are given subjectively in many relevant articles. Moreover we put forward ZWAPA and ZWGPA operators to aggregate each expert evaluation information. In the aggregation process, all the performance values of each alternative are endowed different weights associated with DMs' weights and the supports, which adequately considers the interaction among various kinds of evaluation information and grasp effectively the characteristic of each alternative under each criterion. This is a more versatile data aggregation tool that allows DMs' evaluation information supports and reinforces each other. It can reduce the information loss, which always happens in the process of information aggregation.

(3) In the whole process of decision making, the structure of Z-numbers is retained well; namely, the information denoted by Z-numbers is not defuzzified into ordinary fuzzy numbers or crisp numbers, which holds the information of Z-numbers and keeps the advantage of Z-numbers in describing the uncertainty of things. When we take advantage of the TOPSIS method to prioritize Z-numbers, the PIS and NIS still keep the feature of Z-number unchanged so that alternatives can be compared in terms of the ranking rule of Z-structure. Those objects that are compared are not integrated into crisp numbers but two tuples originated from the two components of Z-numbers; consequently the information of Z-numbers included is less lost, which makes the ranking result of alternatives more persuasive.

In addition, the classic TOPSIS method is extended to the situation with Z-numbers, which avoids using the extended operators to integrate multiple criterion values for alternatives and enhances the operability of the decision-making algorithm and thereby reduces the amount of calculation and the loss and distortion of the original information so that the eventual calculated results are more reliable and convincing.

## 7. Conclusions

The reliability of information is an important issue in decision making. However, the reliability of the knowledge from experts is not efficiently taken into consideration in the past decades. After the notion of Z-number was introduced by Zadeh in 2011, a door has been opened to handle this issue. Z-number has more capability of describing uncertain information with both restraint and reliability, which is paid much attention recently. This paper puts forward a methodology for MCGDM where all the decision evaluations are expressed by Z-numbers. First, the four operations about Z-numbers are defined according to the trait of Z-numbers and the operational laws of TFNs. In view of different status of the two components of a Z-number, we transform the second component into a real number through Centroid Method as the weight of the first component. Consequently the distance between Z-numbers includes two parts: the distance between the first components and the distance between the weighted first components. Next, the efficient PA operator is generalized to integrate the information depicted by Z-numbers. As a result, the ZWAPA operator and the ZWGPA operator are presented, and they are proved to be the analogous properties to the PA operator and the PG operator, respectively. Based on these theoretical preparations, a MCGDM method can be set up consisting mainly of two stages: weight-determining stage and decision procedure-conducting stage. The determination of weights is divided into two parts: one for DMs and the other for criteria. Being different from most literature, DMs' weights are not given subjectively but an objective method is established in this article based on the consensus that the allocation of experts' weights should make the evaluations on alternatives they make be as similar as possible. The weights of criteria are gained based on the thought that the weight of a criterion can maximize the difference of evaluation values under this criterion. Through the two principles, we can build two programming models to obtain the corresponding weight vectors. In the second stage, we formally give decision procedures inspired by PA aggregation thought in [37] and TOPSIS theory in [41] towards the MCGDM problem with Z-numbers. When the evaluation information from different experts is aggregated, the weight of each evaluation value is constructed based on the support measure in which the mutual support of these evaluation values is taken into account sufficiently. When ranking alternatives (the performances of these alternatives are described by Z-numbers), we do not distort the basic structure of Z-numbers; namely, every Z-number is not converted to a crisp value whether it was the PISs and NISs or the final relative closenesses. Meanwhile, we also give the rule that uses the binary closeness degrees to determine the priority of alternatives in consideration of the different importance of the two components in a Z-number. Last, this article shows a course to solve one application example step by step according to the proposed operation procedures and verifies the practicability and availability of our method system by comparing it against other methods in depth.

## Data Availability

The text data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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## References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning I," *Information Sciences*, vol. 8, pp. 199–249, 1975.

[3] R. R. Yager, "On the theory of bags," *International Journal of General Systems: Methodology, Applications, Education*, vol. 13, no. 1, pp. 23–37, 1987.

[4] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[5] K. T. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 33, no. 1, pp. 37–45, 1989.

[6] L. Fei, H. Wang, L. Chen et al., "A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators," *Iranian Journal of Fuzzy Systems*, vol. 16, no. 3, pp. 113–126, 2019.

[7] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 1378–1382, Jeju-do, Republic of Korea, August 2009.

[8] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.

[9] D. Wu and J. M. Mendel, "Uncertainty measures for interval type-2 fuzzy sets," *Information Sciences*, vol. 177, no. 23, pp. 5378–5393, 2007.

[10] K. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[11] J.-Q. Wang, J.-T. Wu, H.-Y. Zhang, and X.-H. Chen, "Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems," *Information Sciences*, vol. 288, pp. 55–72, 2014.

[12] D. Dubois and H. Prade, "Operations on fuzzy numbers," *International Journal of Systems Science*, vol. 9, no. 6, pp. 613–626, 1978.

[13] S. Abbasbandy and T. Hajjari, "A new approach for ranking of trapezoidal fuzzy numbers," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 413–419, 2009.

[14] L. A. Zadeh, "A note on Z-numbers," *Information Sciences*, vol. 181, no. 14, pp. 2923–2932, 2011.

[15] Y.-H. Chen, T.-C. Wang, and C.-Y. Wu, "Multi-criteria decision making with fuzzy linguistic preference relations," *Applied Mathematical Modelling*, vol. 35, no. 3, pp. 1322–1330, 2011.

[16] R. M. Rodríguez, Á. Labella, and L. Martínez, "An overview on fuzzy modelling of complex linguistic preferences in decision making," *International Journal of Computational Intelligence Systems*, vol. 9, pp. 81–94, 2016.

[17] Z. S. Xu, "A method based on linguistic aggregation operators for group decision making with linguistic preference relations," *Information Sciences*, vol. 166, no. 1–4, pp. 19–30, 2004.

[18] Z. S. Xu, "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," *Information Sciences*, vol. 168, no. 1–4, pp. 171–184, 2004.

[19] G. W. Wei, X. F. Zhao, R. Lin, and H. J. Wang, "Uncertain linguistic Bonferroni mean operators and their application to multiple attribute decision making," *Applied Mathematical Modelling*, vol. 37, no. 7, pp. 5277–5285, 2013.

[20] J. H. Park, M. G. Gwak, and Y. C. Kwun, "Uncertain linguistic harmonic mean operators and their applications to multiple attribute group decision making," *Computing*, vol. 93, no. 1, pp. 47–64, 2011.

[21] Z.-P. Tian, J. Wang, J.-Q. Wang, and H.-Y. Zhang, "A likelihood-based qualitative flexible approach with hesitant fuzzy linguistic information," *Cognitive Computation*, vol. 8, no. 4, pp. 670–683, 2016.

[22] B. Y. Kang, D. Wei, Y. Li, and Y. Deng, "A method of converting Z-number to classical fuzzy number," *Journal of Information and Computational Science*, vol. 9, no. 3, pp. 703–709, 2012.

[23] D. Mohamada, S. A. Shaharanib, and N. H. Kamisc, "A Z-number-based decision making procedure with ranking fuzzy numbers," in *Proceedings of the International Conference on Quantitative Sciences and Its Applications*, pp. 160–166, 2014.

[24] D. Mohamada, S. A. Shaharanib, and N. H. Kamisc, "Ordering of Z-numbers," in *Proceedings of the 24th National Symposium on Mathematical Sciences*, pp. 1–6, 2017.

[25] B. Y. Kang, Y. Hu, Y. Deng, and D. Zhou, "A new methodology of multicriteria decision-making in supplier selection based on Z-Numbers," *Mathematical Problems in Engineering*, vol. 2016, Article ID 8475987, 17 pages, 2016.

[26] B. Y. Kang, Y. Deng, K. Hewage et al., "A method of measuring uncertainty for Z-number," *IEEE Transactions on Fuzzy Systems*, vol. 27, pp. 731–738, 2019.

[27] B. Y. Kang, Y. Deng, K. Hewage, and R. Sadiq, "Generating Z-number based on OWA weights using maximum entropy," *International Journal of Intelligent Systems*, vol. 33, no. 8, pp. 1745–1755, 2018.

[28] B. Y. Kang, G. Chhipi-Shrestha, Y. Deng, K. Hewage, and R. Sadiq, "Stable strategies analysis based on the utility of Z-number in the evolutionary games," *Applied Mathematics Computation*, vol. 324, pp. 202–217, 2018.

[29] D. Mohamad and S. Z. Ibrahim, "Decision making procedure based on jaccard similarity measure with Z-numbers," *Pertanika Journal of Science & Technology*, vol. 25, no. 2, pp. 561–574, 2017.

[30] A. S. Abu Bakar and A. Gegov, "Multi-layer decision methodology for ranking Z-numbers," *International Journal of Computational Intelligence Systems*, vol. 8, no. 2, pp. 395–406, 2015.

[31] Z.-Q. Xiao, "Application of Z-numbers in multi-criteria decision making," in *Proceedings of the International Conference on Informative and Cybernetics for Computational Social Systems (ICCSS '14)*, pp. 91–95, IEEE, Qingdao, China, October 2014.

[32] N. N. Karnik and J. M. Mendel, "Centroid of a type-2 fuzzy set," *Information Sciences*, vol. 132, no. 1–4, pp. 195–220, 2001.

[33] B. Y. Kang, D. Wei, Y. Li, and Y. Deng, "Decision making using Z-numbers under uncertain environment," *Journal of Computational Information Systems*, vol. 8, no. 7, pp. 2807–2814, 2012.

[34] N. Zamri, F. Ahmad, A. N. M. Rose et al., "A fuzzy TOPSIS with Z-numbers approach for evaluation on accident at the construction site," in *Recent Advances on Soft Computing and Data Mining*, pp. 41–50, 2017.

[35] H. G. Peng and J. Q Wang, "Hesitant uncertain linguistic Z-numbers and their application in multi-criteria group decision-making problems," *International Journal of Fuzzy Systems*, vol. 19, pp. 1300–1316, 2017.

[36] W. Jiang, C. Xie, Y. Luo, and Y. Tang, "Ranking Z-numbers with an improved ranking method for generalized fuzzy numbers," *Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology*, vol. 32, no. 3, pp. 1931–1943, 2017.

[37] R. R. Yager, "The power average operator," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 31, no. 6, pp. 724–731, 2001.

[38] Z. S. Xu and R. R. Yager, "Power-geometric operators and their use in group decision making," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 1, pp. 94–105, 2010.

[39] Z. S. Xu, *Uncertain Multiple Attribute Decision Making: Methods and Applications*, Springer, Beijing, China, 2004.

[40] Y. M. Wang, "Using the method of maximizing deviations to make decision for multi-indices," *Journal of Systems Engineering and Electronics*, vol. 7, pp. 24–26, 1998.

[41] C. L. Hwang and K. Yoon, *Multiple Attribute Decision Making: Methods and Applications*, vol. 186, Springer, Berlin, Germany, 1981.

[42] S. Hamdan and A. Cheaitou, "Supplier selection and order allocation with green criteria: an MCDM and multi-objective optimization approach," *Computers & Operations Research*, vol. 81, pp. 282–304, 2017.

[43] S. H. Kim and B. S. Ahn, "Interactive group decision making procedure under incomplete information," *European Journal of Operational Research*, vol. 116, no. 3, pp. 498–507, 1999.

[44] Z. S. Xu, "Intuitionistic preference relations and their application in group decision making," *Information Sciences*, vol. 177, no. 11, pp. 2363–2379, 2007.