Research Article

An LMI-Based Simple Robust Control for Load Sway Rejection of Rotary Cranes with Double-Pendulum Effect

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In the double-pendulum rotary crane case, the load sway properties become more complicated so that the difficulty of design and analysis of crane control system is increased. Moreover, the change of rope length not only affects the stability of the system, but also leads to the decline of control performance. In order to solve the foregoing problems, a simple model for controller design is obtained by linearizing and decoupling the complex nonlinear model of rotary crane. Then, a linear feedback controller which can furnish robust performance is presented. Finally, numerical simulations verify the effectiveness of the proposed method by comparing with a traditional approach.

1. Introduction

As a kind of handling robot, the crane systems have the advantages of less executive mechanism and simple structure; therefore, they have been widely used in various production departments [1]. However, load sways caused by the cart or boom motion not only damage goods, reduce production efficiency, but also cause accidents, even casualties. Hence, how to design a control method that can effectively suppress the load sways has been a hot and difficult point in both academic and industrial circles.

Until now, various approaches were applied to crane systems [2–26]. Nonetheless, when the hook mass or load shape cannot be ignored, the load swing presents two-stage swing. For this problem, open-loop methods such as input shaping technology [27–32], motion planning method [33, 34], were presented. On the other hand, closed-loop methods were also reported in [35, 36]. Furthermore, an input-shaping-based SIRMs fuzzy approach was reported in [37]. Ouyang et al. reported two novel sliding-mode based approaches and a simple LMI-based robust method [38–40]. A nonlinear quasi-PID scheme and an adaptive nonlinear controller were applied to overhead cranes in [41, 42].

In conclusion, although open-loop approaches were introduced in [27–34], accurate crane models were always needed. On the other hand, too complicated algorithms were adopted in [35–42] to achieve good robust or adaptive performance. Moreover, different from the load sways of double-pendulum overhead cranes which were dealt with in the most existing literatures [27–42], the double pendulums in the rotary crane system are both conical pendulums, and the system characteristics are more complex. Hence, even if the similar algorithm shown in [38] is adopted, the method for calculating the controller gain is different. Moreover, as far as we know, there are few related reports on the load sway suppression control of rotary cranes with double-pendulum effect.

In order to solve the foregoing problems, a simple model for controller design is obtained by linearizing and decoupling the complex nonlinear model of rotary crane. Next, a linear feedback controller which can furnish robust performance is presented. The controller gains are obtained by using synthesizing a pole placement and a optimal regulator problems [43]. Finally, numerical simulations verify the effectiveness of the proposed method by comparing with a traditional approach.

2. Rotary Crane Dynamics

Figure 1 shows the model of a rotary crane with double-pendulum effect [30]:
\[ \begin{align*}
(m_1 + m_2) \ddot{\theta}_1 + (m_1 + m_2) l_1^2 \dddot{\theta}_1 \\
+ m_2 l_2 (1 + \theta_1 \ddot{\theta}_3) \dot{\theta}_2 + m_2 l_2 \theta_1 \ddot{\theta}_3 \dot{\theta}_4 + (m_1 + m_2) \dot{\theta}_1
\cdot l_1 L (C_5 - \theta_5 S_5) \theta_5 - \left( (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 \dot{\theta}_4 \right) \\
- \left( (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 \dot{\theta}_4 \right) \theta_5 - (m_1 + m_2) l_1 L (S_5 + \theta_4 C_5) \theta_5^2 \\
- \left( (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 \dot{\theta}_4 \right) \theta_5 - (m_1 + m_2) l_1 LS_5 \theta_5^2 \\
+ (m_1 + m_2) l_1 g \theta_1 = 0
\end{align*} \]
\[
\begin{align*}
&+ m_2 l_1 L \dot{\theta}_1^2 - (m_1 + m_4) l_1 L \dot{\theta}_2^2 - m_2 l_2 L \dot{\theta}_5^2 \\
&- m_2 l_2 L \dot{\theta}_6^2 - (m_1 + m_2) l_1 L \dot{\theta}_2 \\
&+ 2 m_2 l_2 C \dot{\theta}_4 + (m_1 + m_2) l_1 \dot{\theta}_2 C \dot{\theta}_6 \\
&- g S_5 \left( \frac{1}{2} M_0 L + (m_1 + m_2) L - \frac{1}{2} M_1 L_1 \right) \\
&- \frac{1}{2} \left( (m_1 + m_2) L^2 + I_x - I_z \right) \sin 2 \dot{\theta}_5 \dot{\theta}_6 = \gamma_5 \\
&- (C_5 \dot{\theta}_5 + f_{51}) \\
&- \left( (m_1 + m_2) l_1^2 \dot{\theta}_3 + m_2 l_1 l_2 \dot{\theta}_4 \right) \dot{\theta}_1 + \left( (m_1 + m_2) l_1 L \dot{\theta}_5 \\
&+ m_2 l_1 l_2 \dot{\theta}_3 + (m_1 + m_2) l_1 \dot{\theta}_2 \right) \dot{\theta}_2 - (m_2 l_1 \dot{\theta}_2 \\
&+ m_2 l_1 \dot{\theta}_4) \dot{\theta}_3 + (m_1 + m_2) l_1 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_5 + m_2 l_2 \dot{\theta}_6 \right) \dot{\theta}_4 \\
&- \left( (m_1 + m_2) l_1 L \dot{\theta}_5 + m_2 l_2 L \dot{\theta}_5 \right) \dot{\theta}_5 \\
&+ \left( (m_1 + m_2) L^2 S_5^2 + (m_1 + m_2) l_1^2 \left( \dot{\theta}_1^2 + \dot{\theta}_2^2 \right) \\
&+ m_2 l_2 \left( \dot{\theta}_5^2 + \dot{\theta}_6^2 \right) + I_1 + I_2 S_5 + I_2 C_5 + I_6 \right) \dot{\theta}_6 \\
&+ (2 (m_1 + m_2) l_1 L \dot{\theta}_5 + 2 m_2 l_2 L \dot{\theta}_5) \dot{\theta}_5 \\
&+ 2 m_2 l_1 l_2 \left( \dot{\theta}_3 \dot{\theta}_4 + \dot{\theta}_2 \dot{\theta}_4 \right) + (m_1 + m_2) l_1 L \dot{\theta}_5 \dot{\theta}_5 \\
&+ 2 m_2 l_2 L \dot{\theta}_5 \dot{\theta}_6 \\
&+ \left( (m_1 + m_2) L^2 + I_x - I_z \right) \sin 2 \dot{\theta}_5 \dot{\theta}_6 = \gamma_6 \\
&- (C_6 \dot{\theta}_6 + f_{61})
\end{align*}
\]

\[
\begin{align*}
&- (m_1 + m_2) \dot{\theta}_3 + m_2 l_1 l_2 \dot{\theta}_4 \dot{\theta}_1 + (m_1 + m_2) l_1 L \dot{\theta}_5 \dot{\theta}_1 \\
&+ 2 m_2 l_1 l_2 \dot{\theta}_3 + (m_1 + m_2) l_1 \dot{\theta}_2 \dot{\theta}_2 - (m_2 l_1 \dot{\theta}_2 \dot{\theta}_2 \\
&+ m_2 l_1 \dot{\theta}_4) \dot{\theta}_3 + (m_1 + m_2) l_1 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_5 + m_2 l_2 \dot{\theta}_6 \dot{\theta}_4 \\
&- (m_1 + m_2) l_1 L \dot{\theta}_5 + m_2 l_2 L \dot{\theta}_5 \dot{\theta}_5 \\
&+ \left( (m_1 + m_2) L^2 S_5^2 + (m_1 + m_2) l_1^2 \left( \dot{\theta}_1^2 + \dot{\theta}_2^2 \right) \\
&+ m_2 l_2 \left( \dot{\theta}_5^2 + \dot{\theta}_6^2 \right) + I_1 + I_2 S_5 + I_2 C_5 + I_6 \right) \dot{\theta}_6 \\
&+ (2 (m_1 + m_2) l_1 L \dot{\theta}_5 + 2 m_2 l_2 L \dot{\theta}_5) \dot{\theta}_5 \\
&+ 2 m_2 l_1 l_2 \left( \dot{\theta}_3 \dot{\theta}_4 + \dot{\theta}_2 \dot{\theta}_4 \right) + (m_1 + m_2) l_1 L \dot{\theta}_5 \dot{\theta}_5 \\
&+ 2 m_2 l_2 L \dot{\theta}_5 \dot{\theta}_6 \\
&+ \left( (m_1 + m_2) L^2 + I_x - I_z \right) \sin 2 \dot{\theta}_5 \dot{\theta}_6 = \gamma_6 \\
&- (C_6 \dot{\theta}_6 + f_{61})
\end{align*}
\]

\[
\begin{align*}
\text{(5)}
\end{align*}
\]

\[
\begin{align*}
J_{k+4} \dot{\theta}_{k+4} + d_{k+4} &= y_{k+4}, \quad k = 1, 2
\end{align*}
\]

where \(d_{k+4}\) denotes the disturbance.

In order to compensate for the above disturbance, the disturbance observer as shown in Figure 2 is designed [43–48]. Then, we have

\[
\dot{\theta}_{k+4} = y_{k+4}
\]

where \(y_{k+4}\) is the control input for vertical and horizontal subsystem, respectively.

Equations (1)-(4) and (8) are represented as follows:

\[
\begin{align*}
M' \ddot{\Theta}' + K' \Theta' &= A_k B' v_{k+4}, \quad k = 1, 2 \\
M' &= \begin{bmatrix} 1 & m_2 l_2 \\ l_1 & (m_1 + m_2) l_1 \\ l_2 & 1 \end{bmatrix} \\
K' &= \begin{bmatrix} g & 0 \\ 0 & g \\ 0 & I_2 \end{bmatrix} \\
B' &= \begin{bmatrix} 1 \\ 0 \\ l_2 \end{bmatrix} \\
\Theta' &= \begin{bmatrix} \theta_k \\ \theta_{k+2} \end{bmatrix}
\end{align*}
\]

where \(A_1 = -L \cos \theta_{5f}\) and \(A_2 = -L \sin \theta_{5f}. \dot{\theta}_{5f}\) is the desired position of the vertical boom angle.
Then, (9) is represented as follows [49]:

\[
M'' \Psi + K'' \Psi = A_k B' \nu_{k+4}, \quad k = 1, 2
\]

\[
\Psi = \begin{bmatrix}
\psi_k \\
\psi_{k+2}
\end{bmatrix} = H^{-1} \begin{bmatrix}
\theta_k \\
\theta_{k+2}
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
1 - \frac{\omega_k^2 m_1 l_2}{g (m_1 + m_2)} & 1 - \frac{\omega_{k+2}^2 m_1 l_2}{g (m_1 + m_2)} \\
\end{bmatrix}
\]

\[
\omega_{k,k+2}^2 = \left( \frac{(m_1 + m_2) (l_1 + l_2) - \sqrt{(l_1 + l_2)^2 (m_1 + m_2)^2 - 4l_1 l_2 (m_1 + m_2)}}{2m_1 l_1 l_2} \right) g,
\]

where \(M'' = H^T M' H, K'' = H^T K' H,\) and \(B'' = H^T B'.\)

Then, we have

\[
\dot{X} = A'_k X + A_k b_k \nu_{k+4}, \quad k = 1, 2
\]

\[
A'_k = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-\omega_k^2 & 0 & 0 & 0 & 0 & 0 \\
0 & -\omega_{k+2}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
b_k = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{m_1 + m_2}{\delta} \\
\frac{-m_1 + m_2}{\delta} \\
\frac{1}{\delta}
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
\psi_k \\
\psi_{k+2} \\
\theta_k \\
\theta_{k+2}
\end{bmatrix}
\]

where \(\delta = \sqrt{(l_1 + l_2)^2 (m_1 + m_2)^2 + 2l_1 l_2 (m_2^2 - m_1^2)}\).

From the above equations, the crane system can be divided into two subsystems (i.e., vertical boom motion and horizontal boom motion subsystems). Therefore, we can design feasible and effective controllers for each subsystem, respectively.

**3. Controller Design Process**

3.1. Closed-Loop System Representation. The closed-loop control system for rotary crane is

\[
\dot{X} = \left( A'_k + A_k b_k k_{k+4}^T \right) X, \quad k = 1, 2
\]

\[
k_{k+4}^T = \begin{bmatrix}
k_{k(k+4),1} & k_{k(k+4),2} & k_{k(k+4),3} & k_{k(k+4),4} & k_{k(k+4),5} & k_{k(k+4),6}
\end{bmatrix}
\]

where \(k_{k(k+4),1}, k_{k(k+4),2}, k_{k(k+4),3}, k_{k(k+4),4}, k_{k(k+4),5}\), and \(k_{k(k+4),6}\) are feedback gains. These gains will be calculated via the LMI-based method in this study.

3.2. Pole Placement Problem by LMI. In linear control theory, if all eigenvalues of matrix \(A'_k + A_k b_k k_{k+4}^T\) lie in the left-half of a region shown in Figure 3, the system is stable. Inspired by the literature [43], the following formulations were suggested to solve the pole placement problem in this study:

\[
A'_k X_1 + X_1 A'_k^T + b'_k M_1 + M'_1 b_k^T + 2\alpha X_1 < 0
\]

\[
A'_k X_1 + X_1 A'_k^T + b'_k M_1 + M'_1 b_k^T + 2\beta X_1 > 0
\]

\[
\begin{bmatrix}
\sin \phi \left( A'_k X_1 + X_1 A'_k^T + b'_k M_1 + M'_1 b_k^T \right) \\
\cos \phi \left( A'_k X_1 + X_1 A'_k^T + b'_k M_1 + M'_1 b_k^T \right)
\end{bmatrix} < 0
\]

\[
X_1 > 0
\]

where \(M_1 = k_{k+4}^T X_1\) and \(b'_k = A_k b_k\).
3.3. Optimal Regulator Problem by LMI. This subsection proposes an LQR method to achieve the optimal control problem. Hence, the following cost function \( J \) is considered [43]:

\[
J = \frac{1}{2} \int_{0}^{\infty} \left( Q_{k+4} X_{k+4} + R_{k+4}^T V_{k+4} \right) T + (Q_{k+4} X_{k+4} + R_{k+4}^T V_{k+4}) dt
\]

where \( Q_{k+4} \geq 0 \) and \( R_{k+4} > 0 \).

Then, inspired by the method presented in [50], the above problem can be converted into the problem of finding \( X_{k} \) and \( V_{k+4} \), and hence, the following cost function

\[
\text{diag} J = 1 + (X_{k} A_{k}^T + M_{0}^T b_{k}^T) 0 0 -I < 0
\]

(15)

\[
\left[ \begin{array}{c}
Q_{k+4} X_{k+4} + R_{k+4}^T V_{k+4}
\end{array} \right] > 0
\]

where \( M_{0} = k_{k+4} X_{k} \).

3.4. Controller Design. In this study, we synthesize the above two problems to obtain the controller by setting \( X_{1} = X_{2} \) and \( M_{1} = M_{2} \) [43]:

\[
A_{k}^T X_{1} + X_{1} A_{k}^T + b_{k}^T M_{1} + M_{0}^T b_{k}^T + 2 \alpha X_{1} < 0
\]

(16)

\[
A_{k}^T X_{1} + X_{1} A_{k}^T + b_{k}^T M_{1} + M_{0}^T b_{k}^T + 2 \beta X_{1} > 0
\]

where \( M_{0} = k_{k+4} X_{k} \).

4. Results and Discussion

4.1. Simulation Conditions. In order to evaluate the control performance of the proposed controller, a cycloid-like motion trajectory was applied to the crane system [43]:

\[
\theta_{(k+4)d} = \begin{cases}
\left( \theta_{(k+4)f} - \theta_{(k+4)o} \right) \left( t - \frac{1}{2\pi} \sin \left( \frac{2\pi t}{t_{s}} \right) \right) + x_{(k+4)o} \quad t \in [0, t_{s}) \\
\theta_{(k+4)f} \quad t \in (t_{s}, t_{f}]
\end{cases}
\]

(17)

where \( \theta_{(k+4)f} \), \( x_{(k+4)o} \), \( t_{s} \), and \( t_{f} \) are the desired boom angle, the initial boom angle, the reaching time, and the finish time. These parameters were selected as \( \theta_{50} = 0[\text{deg}] \), \( \theta_{60} = 0[\text{deg}] \), \( \theta_{5f} = 30[\text{deg}] \), \( \theta_{6f} = 45[\text{deg}] \), \( t_{s} = 3[\text{s}] \), and \( t_{f} = 10[\text{s}] \).

Parameters as shown in Figure 3 were selected as 80, 1, and 30, respectively. We also set \( Q_{5} = \text{diag}[150, 15, 50, 15, 50, 50] \), \( Q_{6} = \text{diag}[50, 150, 50, 300, 150, 50] \), and \( R_{5} = R_{6} = 1 \) in (16). The controller gains for the decoupled linear system were obtained as follows by the method proposed in Section 3 with the rotary crane parameters shown in Table 1, when the rope length varies from 0.25[m] to 0.75[m].

\[
k_{5}^T = [-39.65 \quad 124.08 \quad 10.61 \quad -17.12 \quad -8.75 \quad 14.73]
\]

(18)

\[
k_{6}^T = [-61.92 \quad 210.78 \quad 10.77 \quad -27.01 \quad -12.99 \quad 14.45]
\]

(19)

Using the relationship shown in (10), the gains for the original rotary crane are

\[
k_{5}^T = [103.08 \quad -17.82 \quad 10.61 \quad 137.15 \quad -3.11 \quad 14.73]
\]

(20)

\[
k_{6}^T = [177.99 \quad -27.30 \quad 10.77 \quad 231.19 \quad -4.10 \quad 14.45]
\]

(21)

4.2. Stability Analysis. We conducted several numerical to explain the robust stability of the proposed control system when the rope length \( l_{1} \) varies from 0.25[m] to 0.75[m]. The initial poles of the vertical boom motion subsystem were set to \( p_{11} = -1.04[1/\text{s}] \), \( p_{21} = -1.45 + 9.07 j[1/\text{s}] \), \( p_{31} = -1.45 - 9.07 j[1/\text{s}] \), \( p_{41} = -4.80 + 6.92 j[1/\text{s}] \), \( p_{51} = -4.80 - 6.92 j[1/\text{s}] \), and \( p_{61} = -6.07[1/\text{s}] \), and the initial poles of the horizontal boom motion subsystem were set to \( p_{12} = -1.51 + 9.54 j[1/\text{s}] \), \( p_{22} = -1.51 - 9.45 j[1/\text{s}] \), \( p_{32} = -4.96 + 6.30 j[1/\text{s}] \), \( p_{42} = -4.96 - 6.30 j[1/\text{s}] \), \( p_{52} = -7.62 + 2.64 j[1/\text{s}] \), and \( p_{62} = -7.62 - 2.64 j[1/\text{s}] \). It is confirmed that the crane system can be stabilized by the controller from the simulation results in Figure 4.
Figure 3: Stable region.

(a) Original figure for vertical boom motion subsystem
(b) Enlarged view for vertical boom motion subsystem
(c) Original figure for horizontal boom motion subsystem
(d) Enlarged view for horizontal boom motion subsystem

Figure 4: Poles placement.
4.3. Simulation Results. We conducted several simulations based on the linear model under different rope length, firstly. The results for the vertical subsystem are shown in Figure 5. Although there are transient errors and slight overshoot in these figures, the vertical boom motion converges to the desired position rapidly. Meanwhile, there is almost no residual modal swaying vibration. On the other hand, the results for the horizontal subsystem are shown in Figure 6. Almost the same results were obtained as those in Figure 5. The results based on the original crane system for the terms of the load sways $\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$, the tracking error for vertical boom angle $e_5$, the tracking errors for horizontal boom angle $e_6$, the vertical boom angle $\theta_5$, the horizontal boom angle $\theta_6$, and the control inputs $v_5$ and $v_6$ are shown in Figure 7. Because of the effect of the ignored nonlinearity of the crane model, the residual
swaying angles are bigger than those in Figures 5 and 6.

For verifying that the presented controller also has robustness to the rigging length variation, we conducted simulations with the gains in (20) and (21) by set the \( l_2 \) as \( l_2 = 0.20 \text{[m]} \) and \( l_2 = 0.40 \text{[m]} \). By comparing the results in Figures 7 and 8, it is found that although the second sway angle of the horizontal direction \( \theta_4 \) increases with the increase of the rigging length, it still remains in a reasonable range.

4.4. Comparative Simulation. In order to confirm the proposed method can provide superior robust performance, some comparative simulations were conducted by a conventional state feedback controller. A dominant poles placement method was applied to calculate the controller gains which are as follows:

\[
k_5^{HT} = [-253.27 \ 6.42 \ 11.12 \ -292.64 \ -41.28 \ -47.10]
\]  

(22)

\[
k_6^{HT} = [-460.06 \ 14.78 \ 19.26 \ -499.43 \ -62.48 \ -81.58]
\]

(23)

The results are shown in Figure 9. Although the load sway angles reduced completely under condition \( l_1 = 0.25 \text{[m]} \) or \( l_1 = 0.50 \text{[m]} \), the load sway angles cannot be reduced by using gains in (22) and (23) under condition \( l_1 = 0.75 \text{[m]} \). Meanwhile, the quantitative analyses between the proposed method and the existing one are in Tables 2–4. It is indicated that the presented method can furnish preferable robust performance than the traditional one from these figures and tables.

5. Conclusion

In order to achieve the robust control for a rotary crane with double-pendulum effect, a simple model for controller design is obtained by linearizing and decoupling the complex
Table 2: Quantified results under condition $l_1 = 0.25$[m].

<table>
<thead>
<tr>
<th>Control method</th>
<th>$\varepsilon_{5\text{max}}$[deg]</th>
<th>$\varepsilon_{6\text{max}}$[deg]</th>
<th>$\theta_{1\text{max}}$[deg]</th>
<th>$\theta_{2\text{max}}$[deg]</th>
<th>$\theta_{3\text{max}}$[deg]</th>
<th>$\theta_{4\text{max}}$[deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed controller</td>
<td>3.25</td>
<td>3.84</td>
<td>2.20</td>
<td>1.70</td>
<td>2.49</td>
<td>2.14</td>
</tr>
<tr>
<td>Conventional controller</td>
<td>3.08</td>
<td>3.84</td>
<td>1.92</td>
<td>1.82</td>
<td>2.61</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Figure 7: Simulations under the original crane model with different rope length.
Table 3: Quantified results under condition $l_1 = 0.50$[m].

<table>
<thead>
<tr>
<th>Control method</th>
<th>$e_{5\text{max}}$ [deg]</th>
<th>$e_{6\text{max}}$ [deg]</th>
<th>$\theta_{1\text{max}}$ [deg]</th>
<th>$\theta_{2\text{max}}$ [deg]</th>
<th>$\theta_{3\text{max}}$ [deg]</th>
<th>$\theta_{4\text{max}}$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed controller</td>
<td>3.31</td>
<td>3.82</td>
<td>2.31</td>
<td>2.01</td>
<td>2.64</td>
<td>2.53</td>
</tr>
<tr>
<td>Conventional controller</td>
<td>3.12</td>
<td>3.82</td>
<td>2.10</td>
<td>2.10</td>
<td>2.75</td>
<td>2.94</td>
</tr>
</tbody>
</table>

Figure 8: Simulations under the original crane model with different rigging length.
Table 4: Quantified results under condition $l_1 = 0.75\,[\text{m}]$.

<table>
<thead>
<tr>
<th>Control method</th>
<th>$e_{\text{max}}^5,[\text{deg}]$</th>
<th>$e_{\text{max}}^6,[\text{deg}]$</th>
<th>$\theta_{\text{max}}^1,[\text{deg}]$</th>
<th>$\theta_{\text{max}}^2,[\text{deg}]$</th>
<th>$\theta_{\text{max}}^3,[\text{deg}]$</th>
<th>$\theta_{\text{max}}^4,[\text{deg}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed controller</td>
<td>3.36</td>
<td>3.81</td>
<td>2.40</td>
<td>2.31</td>
<td>2.76</td>
<td>2.81</td>
</tr>
<tr>
<td>Conventional controller</td>
<td>3.10</td>
<td>3.75</td>
<td>3.01</td>
<td>3.00</td>
<td>4.21</td>
<td>5.14</td>
</tr>
</tbody>
</table>

Figure 9: Conventional controller results.
nonlinear model of rotary crane. Then, a linear feedback controller which can furnish robust performance was presented. Finally, numerical simulations verified the effectiveness of the proposed method by comparing with a traditional approach.

Appendix

This section presents the derivation of the crane dynamics in (1)-(6). From Figure 1, the position of the hook with respect to the origin of coordinates is presented as follows:

\[
\begin{align*}
    x_1 &= (L \sin \theta_5 + l_1 \theta_1) \cos \theta_6 - l_1 \theta_2 \sin \theta_6 \\
    y_1 &= (L \sin \theta_5 + l_1 \theta_1) \sin \theta_6 + l_1 \theta_2 \cos \theta_6 \\
    z_1 &= L \cos \theta_5 - l_1 \cos \sqrt{\theta_1^2 + \theta_2^2} 
\end{align*}
\]  

(A.1)

Then, the position of the load with respect to the origin of coordinates is

\[
\begin{align*}
    x_2 &= (L \sin \theta_5 + l_1 \theta_1 + l_2 \theta_3) \cos \theta_6 \\
        &\quad - (l_1 \theta_2 + l_2 \theta_4) \sin \theta_6 \\
    y_2 &= (L \sin \theta_5 + l_1 \theta_1 + l_2 \theta_3) \sin \theta_6 \\
        &\quad - (l_1 \theta_2 + l_2 \theta_4) \cos \theta_6 \\
    z_2 &= L \cos \theta_5 - l_1 \cos \sqrt{\theta_1^2 + \theta_2^2} - l_2 \cos \sqrt{\theta_3^2 + \theta_4^2} 
\end{align*}
\]  

(A.2)

Taking the time derivation of (A.1) and (A.2), we have

\[
\begin{align*}
    \dot{x}_1 &= L \dot{\theta}_5 \cos \theta_5 \cos \theta_6 - L \dot{\theta}_6 \sin \theta_5 \sin \theta_6 \\
            &\quad + l_1 \dot{\theta}_1 \cos \theta_6 - l_1 \theta_1 \dot{\theta}_6 \sin \theta_6 - l_1 \theta_2 \dot{\theta}_6 \sin \theta_6 \\
            &\quad - l_1 \theta_2 \dot{\theta}_6 \cos \theta_6 \\
    \dot{y}_1 &= L \dot{\theta}_5 \cos \theta_5 \sin \theta_6 - L \dot{\theta}_6 \sin \theta_5 \cos \theta_6 \\
            &\quad + l_1 \dot{\theta}_1 \sin \theta_6 + l_1 \theta_1 \dot{\theta}_6 \cos \theta_6 + l_1 \theta_2 \dot{\theta}_6 \cos \theta_6 \\
            &\quad - l_1 \theta_2 \dot{\theta}_6 \sin \theta_6 \\
    \dot{z}_1 &= -L \dot{\theta}_5 \sin \theta_5 + l_1 \left( \theta_1 \dot{\theta}_1 + \theta_2 \dot{\theta}_2 \right) \\
\end{align*}
\]  

(A.3)

and

\[
\begin{align*}
    \dot{x}_2 &= L \dot{\theta}_5 \cos \theta_5 \cos \theta_6 - L \dot{\theta}_6 \sin \theta_5 \sin \theta_6 \\
            &\quad + l_1 \dot{\theta}_1 \cos \theta_6 - l_1 \theta_1 \dot{\theta}_6 \sin \theta_6 - l_1 \theta_2 \dot{\theta}_6 \sin \theta_6 \\
            &\quad - l_1 \theta_2 \dot{\theta}_6 \cos \theta_6 \\
    \dot{y}_2 &= L \dot{\theta}_5 \cos \theta_5 \sin \theta_6 - L \dot{\theta}_6 \sin \theta_5 \cos \theta_6 \\
            &\quad + l_1 \dot{\theta}_1 \sin \theta_6 + l_1 \theta_1 \dot{\theta}_6 \cos \theta_6 + l_1 \theta_2 \dot{\theta}_6 \cos \theta_6 \\
            &\quad - l_1 \theta_2 \dot{\theta}_6 \sin \theta_6 \\
    \dot{z}_2 &= -L \dot{\theta}_5 \sin \theta_5 + l_1 \left( \theta_1 \dot{\theta}_1 + \theta_2 \dot{\theta}_2 \right) \\
            &\quad + l_2 \left( \theta_3 \dot{\theta}_3 + \theta_4 \dot{\theta}_4 \right) \\
\end{align*}
\]  

(A.4)

Hence, the kinetic energy of the hook and the load \( T_1 \) is as follows:

\[
T_1 = \frac{1}{2} m_1 \left( x_1^2 + y_1^2 + z_1^2 \right) + \frac{1}{2} m_2 \left( x_2^2 + y_2^2 + z_2^2 \right)
\]
+ \frac{1}{2} m_1 l_2^2 (\dot{\theta}_3^2 + \dot{\theta}_4^2) + m_2 l_2^2 \dot{\theta}_3 \dot{\theta}_4 + m_2 l_2 \dot{\theta}_3 \dot{\theta}_5 + m_2 l_2 \dot{\theta}_3 \dot{\theta}_6 + \frac{1}{2} m_1 l_2^2 \dot{\theta}_5^2
\quad (A.5)

Next, the kinetic energy of the boom \( T_2 \) and the ballast \( T_3 \) can be presented as follows:

\[ T_2 = \frac{1}{2} \left( I_1 \dot{\theta}_1^2 \sin^2 \theta_5 + I_2 \dot{\theta}_5^2 + I_3 \dot{\theta}_6^2 \cos^2 \theta_5 \right) \quad (A.6) \]

\[ T_3 = \frac{1}{2} \dot{\theta}_6^2 \quad (A.7) \]

where \( I_1, I_2, \) and \( I_3 \) are the moment inertia of the boom in 3D space; \( I_6 \) is the moment inertia of the ballast. The total kinetic energy of the crane system is

\[ T = T_1 + T_2 + T_3 \quad (A.8) \]

The potential energy of the hook and load \( U_1 \), the boom \( U_2 \), and the ballast \( U_3 \) can be presented as follows, respectively:

\[ U_1 = m_1 g \left( L \cos \theta_5 - l_1 \cos \sqrt{\dot{\theta}_1^2 + \dot{\theta}_2^2} \right) \]
\[ + m_2 g \left( L \cos \theta_5 - l_2 \cos \sqrt{\dot{\theta}_3^2 + \dot{\theta}_4^2} \right) \]
\[ - l_2 \cos \sqrt{\dot{\theta}_3^2 + \dot{\theta}_4^2} \] \quad (A.9)

\[ U_2 = \frac{1}{2} M g L \cos \theta_5 \]

\[ U_3 = -\frac{1}{2} M_1 g L_1 \cos \theta_5 \]

Hence, the total potential energy of the rotary crane is

\[ U = U_1 + U_2 + U_3 \]
\[ = g \left( \frac{1}{2} M L + (m_1 + m_2) L - \frac{1}{2} M_1 L_1 \right) \cos \theta_5 \]
\[ - (m_1 + m_2) g L_1 \cos \sqrt{\dot{\theta}_1^2 + \dot{\theta}_2^2} \]
\[ - m_2 g l_2 \cos \sqrt{\dot{\theta}_3^2 + \dot{\theta}_4^2} \] \quad (A.10)

where \( g \) is the gravitational acceleration.

Lagrange's function \( L \) is presented as the difference between the kinetic energy \( T \) and the potential energy \( U \).

\[ L = T - U \quad (A.11) \]

Then, the double-pendulum rotary crane model can be derived by using Lagrange's equations of motion.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this article.

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