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Research Article

Capacity Allocation and Compensation in a Dual-Channel Supply Chain under Uncertain Environment

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The dual-channel supply chain is widely adopted by main manufacturers, potentially incurring channel conflicts between the traditional retail channel which is owned by the independent retailer and the online channel which is directly managed by the manufacturer. The purpose of this paper is to deal with the scenario where channel conflicts may arise under production capacity uncertainty, when the manufacturer tends to privilege the direct selling channel over the retail selling channel. To achieve the goal, this paper establishes a Stackelberg game model consisting of a manufacturer and a retailer, studies the scenario where the manufacturer satisfies the direct selling channel first in the presence of capacity uncertainty, employs the decision optimization and the backward induction method to find the optimal inventory decision in the direct selling channel and the optimal order quantity decision making in the retail selling channel, and designs a compensation mechanism aiming to coordinate the channel conflict in the decentralized decision-making process. Results show that the optimal decisions aiming to maximize the expected profit of each supply chain member are not able to maximize the expected profit of entire dual-channel supply chain. However, when the manufacturer compensates the retailer's profit loss based on the unsatisfied order and, in the meantime, adjusts the wholesale price to prevent the retailer which obtains the compensation from increasing order significantly, the compensation mechanism can coordinate the decision of each supply chain member, mitigate the channel conflict, maximize the expected profit of entire dual-channel supply chain, and achieve the Pareto improvement of supply chain members' expected profit in the decentralized decision-making process.

1. Introduction

With the rapid development of electronic commerce and the reduced cost of logistics, increasingly more manufacturers have started to sell their products through the direct channel in addition to their previous retailing channels. According to the introduction of Dan et al. [1], the survey from the New York Times has found that about 42% of famous manufacturers (e.g. Apple, Hewlett-Packard, Lenovo, etc.) have adopted the dual-channel supply chain structure to distribute their products. By launching the direct selling channel, one manufacturer can obtain more market share in product competition among manufacturers [2], better control the retail price of products with multichannel distribution [3], stimulate its independent retailers to improve the service level in the retail selling channel [4], encourage independent retailers to order more products under capacity uncertainty [5], and ultimately increase its own profits [6].

Despite many benefits that the dual-channel supply chain development brings to manufacturers and even to their independent retailers, the dual-channel channel management still faces huge challenges. For example, some

manufacturers tend to satisfy the direct selling channel first under production capacity uncertainty with an effort to support this channel. However, such a supply strategy may result in channel conflict between the direct selling channel and the retail selling channel and definitely cause complaints from independent retailers. eBizcuss, a big retailer in France, filed a lawsuit against Apple, alleging that its bankruptcy was a result of insufficient supply of commodities due to Apple's act of satisfying its own direct selling channel first [7]. As a result of this case, Autorite de la Concurrence, the antitrust administration in France searched the French office of Apple to assess whether Apple conducted unfair competition in France. In other countries, independent retailers also complain that Apple tends to unfairly favor its direct selling channel over the retail selling channel when the supply of new products is insufficient. Obviously, when the order from the retail selling channel is not adequately satisfied, it is of great significance to compensate independent retailers to mitigate the channel conflict and promote the sustainable development of dual-channel supply chain.

Different from the previous literature focusing on the channel conflict of dual-channel supply chain due to the price competition or the service competition, this paper concentrates on the channel conflict in the dual-channel supply chain under production capacity uncertainty when the manufacturer tends to privilege its direct selling channel over that of the retailer. This paper will establish a Stackelberg game model under the background that the direct selling channel tends to be satisfied first, solve the optimal inventory level in the direct selling channel and the optimal order quantity in the retail selling channel based on the method of decision optimization and backward induction, and reveal the incentive disorder issue experienced by dual-channel supply chain members during decision-making process under the decentralized decision making. At last, to eliminate channel conflict, this paper will design a compensation mechanism to compensate independent retailers in case their orders are not adequately supplied. It will also discuss the condition where the compensation mechanism maximizes the expected profits made by the supply chain and realize the Pareto improvement of the expected profits made by dual-channel supply chain members.

The remainder of this paper is organized as follows. Section 2 is the literature review. Section 3 introduces the benchmark model and basic assumptions. Section 4 analyzes the centralized decision-making scenario and the decentralized decision-making scenario, carries out sensitivity analysis, and discusses management insights in two scenarios. In Section 5, a compensation mechanism is proposed to coordinate the dual-channel supply chain. Section 6 presents the numerical results. Finally, the conclusion and future research topics are discussed in Section 7.

2. Literature Review

The proposed work is related to four streams of previous literature. The first stream defines the dual-channel supply chain and distinguishes it from the direct and reverse supply chain. The second stream explores the field exploiting the game model and, in particular, the Stackelberg model used in the dual-channel supply chain. The third stream characterizes channel conflicts between the direct selling channel and the retail selling channel. The last stream designs contracts to coordinate the dual-channel supply chain and aims to maximize the expected profit of dual-channel supply chain also in the decentralized decision-making process.

The dual-channel supply chain is quite prevalent nowadays. According to the definition of Tang et al. [8], the dual-channel supply chain typically includes the traditional retail channel which is owned by the independent retailer and the online channel which is directly managed by the manufacturer. Moreover, the manufacturer sells the same products in the direct selling channel as the retailer does in the retail selling channel. Some researchers also call such a structure as dual-channel distribution systems [9] or the hybrid-channel supply chain [10]. However, the dual-channel supply chain in this research is totally different from direct and reverse supply chains. According to the definition of Sellitto [11], the direct and reverse supply chain is a closed-loop supply chain, where the direct supply chain distributes raw materials or finished products to consumers, and the reverse supply chain aims to recover the remaining value or at least gives appropriate disposal to discarded raw materials or finished products by recycling, reuse, remanufacturing, or disposal.

A number of studies established the game model to construct the interaction between a manufacturer and a retailer and explored optimal decisions for each supply chain member in a dual-channel supply chain. Zhang and Ma [12] investigated two noncooperative dynamic game models, where the manufacturer acts as the leader and the retailer acts as the follower in the Stackelberg game model, and the manufacturer and retailer have equal bargaining power and thus make decisions simultaneously in the Nash game model. Zhang and Wang [13] established the dynamic price game model and proposed the dynamic price adjustment strategy to enhance the profit of each supply chain member. Dai et al. [14] adopted the Stackelberg game method to simulate the interaction between a manufacturer and a fair caring retailer and studied the equilibrium pricing strategy for each dual-channel supply chain members.

The introduction of direct selling channel inevitably results in channel conflicts between the direct selling channel and the retail selling channel in terms of price, service, and inventory decision making. Yan and Pei [15] constructed the consumer utility, assuming that consumers in the direct selling channel and in the retail selling channel have the same price sensitivity and characterized the price competition. Guo and Zhao [16] showed that channel conflicts arise if the introduction of direct selling channel depresses the retail price of products and then reduces retailers' profit. Xu et al. [17] found that a large number of consumers have heterogeneous preferences for the direct selling channel and the retail selling channel and constructed the channel conflict when the price comparison became the focus of consumers. Other scholars also conducted extensive study in price decision making [18–24], service decision making [25–27], and their coordination [28]. In the aspect of

inventory decision making, the literature concerning channel conflicts studied ordering issues in dual-channel supply chain. For example, Arya et al. [29] analyzed ordering decision making for supply chain members in a single-retailing channel and a dual-channel supply chain and explained the impact of manufacturer's own direct selling channel on the supply chain and its members by comparing the structures of the two supply chains. In the presence of capacity certainty and demand uncertainty, some scholars studied cannel conflict issues due to customers' behavior of switching between channels due to insufficient ordering. For example, Boyaci [30] discovered that as a result of the switching behavior of the customers due to insufficient supply, the total amount of stock of the dual-channel supply chain increased, thus leading to a higher risk of products remaining unsold. When it comes to capacity uncertainty, Xiao and Shi [31] studied the pricing and capacity allocation strategies in the dual-channel supply chain without giving a solution to that issue. This paper will propose a proper compensation mechanism to solve that issue from the perspective of dual-channel coordination.

In order to deal with the problems emerging during the development of dual-channel supply chain, scholars have proposed several forms of contract mechanism. For example, David and Adida [32] designed a package contract which combined benchmarking wholesale price with quantity discount to maximize the total profit of the dual-channel supply chain. In addition, they adopted the contract to coordinate a mixed-channel supply chain which consisted of several homogeneous retailers and direct selling channels. Zhu et al. [33] studied the order decision making of members in a dual-channel supply chain with a risk-neutral manufacturer and a risk-averse retailer, designed a risk-sharing contract which is a hybrid with multiple contracts, and found that the risk-sharing contract could perform steadily when the market circumstance varies. He et al. [24] studied a single-retailer single-vendor dual-channel supply chain model in which the vendor sells deteriorating products through its direct online channel and the indirect retail channel and developed a revenue sharing and two-part tariff contract to coordinate the supply chain. The above-mentioned contract mechanisms were initially designed to coordinate a dual-channel supply chain under capacity certainty. However, in the presence of capacity uncertainty, the decision-making process undergone by dual-channel supply chain members is much more complicated, and the coordination of the dual-channel supply chain is much harder. Therefore, the proper contract mechanisms need to be designed with full consideration of the impact of capacity uncertainty on optimal decisions as well as maximal profits made by supply chain members. In this way, the dual-channel supply chain will be able to maximize its expected profits, and the supply chain members will be able to realize Pareto improvement in their expected profits.

3. Model Description and Basic Assumptions

Consider a dual-channel supply chain which consists of a manufacturer and a retailer. The manufacturer provides seasonal (or periodical) products which are produced all at once before the selling season. The manufacturer sells the same products both through the retail channel owned by the retailer and through its self-owned direct channel. The manufacturer charges the wholesale price per unit product at w in the retail selling channel. w is determined by the manufacturer and the retailer based on their long-term agreement, thus being considered as an exogenous variable with reference to the literature [29].

Either breakdowns of equipment, undue supply of the upper-chain parts supplier, or an unstable new production line will lead to capacity uncertainty of the manufacturer. Therefore, assume the actual capacity is a random variable K, with its probability density function and cumulative distribution function, respectively, as $f(x)$ and $F(x)$. K belongs to $0\sim N$, and N is the maximum capacity. The manufacturer, as a leader in the Stackelberg game, first determines the inventory level Q_m of the direct selling channel. And next the retailer, as a follower in the Stackelberg game, determines the order quantity Q_r of the retail selling channel. Since N is a big enough positive number, we assume that $Q_m + Q_r < N$.

The manufacturer and the retailer are both risk neutral and complete rational, which means that they determine Q_m and Q_r based on the principle of maximizing expected profits. After receiving orders from the retailer, the manufacturer starts production and delivery. c_r is the unit cost at which it supplies the retail selling channel, and c_m is the unit cost at which the manufacturer supplies the direct selling channel. Both c_r and c_m include the manufacturer's unit production cost and delivery cost.

The quantity of products the direct selling channel and the retail selling channel receive equals the actual sales volume in either channel, which are referred to as q_m and q_r, respectively. If $Q_m + Q_r < K$, there is a surplus of the actual capacity. Under capacity surplus, $q_m = Q_m$ and $q_r = Q_r$. If $Q_m + Q_r \geq K$, there is a shortage of actual capacity. Under capacity shortage, the manufacturer satisfies the direct selling channel first. Therefore, if $Q_m + Q_r \geq K$ and $Q_m < K$, the predetermined inventory level of the direct selling channel is satisfied completely, while that of the retail selling channel is satisfied partially, that is, $q_m = Q_m$ and $q_r = K - Q_m$. If $Q_m + Q_r \geq K$ and $Q_m \geq K$, all products are distributed to the direct selling channel, leaving the retail selling channel no product, that is, $q_m = K$ and $q_r = 0$. Therefore, when the direct selling channel gets satisfied first, $E(q_m)$ and $E(q_r)$, the respective expected sales volume of direct channel and retail channel, are as follows:

$$E(q_m) = \int_0^{Q_m} x f(x)\,dx + \int_{Q_m}^{N} Q_m f(x)\,dx, \tag{1}$$

$$E(q_r) = \int_{Q_m}^{Q_r+Q_m} (x - Q_m) f(x)\,dx + \int_{Q_r+Q_m}^{N} Q_r f(x)\,dx. \tag{2}$$

Since the direct selling channel and the retail selling channel sell the same product, we assume that the two channels mainly compete on sales volume. A reverse

demand function is established with reference to the method provided by Zhao and Li [34]:

$$E(p_{\mathrm{m}}) = a_1 - E(q_{\mathrm{m}}) - \theta E(q_{\mathrm{r}}), \tag{3}$$

$$E(p_{\mathrm{r}}) = a_2 - E(q_{\mathrm{r}}) - \theta E(q_{\mathrm{m}}), \tag{4}$$

where $E(p_{\mathrm{m}})$ and $E(p_{\mathrm{r}})$ are the expected retail price in the direct and retail selling channels, respectively, a_1 and a_2 are the highest willingness of customers to pay in the two channels, respectively, and $\theta(0 < \theta < 1)$ is the cross sensitivity coefficient of expected sales volume, which measures how sensitive $E(p_{\mathrm{m}})$ or $E(p_{\mathrm{r}})$ is when the competitor's sales volume changes. In order to ensure that the retailer gains a positive profit by selling a unit product and to avoid selling beyond the agreed areas, we assume that $E(p_i) > w$, with $i = \mathrm{r}, \mathrm{m}$. The expected profits made by the manufacturer and the retailer are listed as follows:

$$\pi_{\mathrm{m}} = [a_1 - E(q_{\mathrm{m}}) - \theta E(q_{\mathrm{r}}) - c_{\mathrm{m}}]E(q_{\mathrm{m}}) + (w - c_{\mathrm{r}})E(q_{\mathrm{r}}), \tag{5}$$

$$\pi_{\mathrm{r}} = [a_2 - E(q_{\mathrm{r}}) - \theta E(q_{\mathrm{m}}) - w]E(q_{\mathrm{r}}). \tag{6}$$

For the convenience of reading, in this paper, the manufacturer and the retailer are referred to as subscripts m and r, respectively. Centralized decision situation, decentralized decision situation, and coordinated situation are referred to as superscripts i, d, and c. The optimal solution is referred to as superscript asterisk. A complete description of decision variables and model parameters is summarized in Table 1.

Table 1: Decision variables and parameters.

Decision variables	
Q_{r}	Retailer's order quantity decision in the retail selling channel
Q_{m}	Manufacturer's inventory level decision in the direct selling channel
Parameters	
K	Manufacturer's production capacity
$f(x)$	Probability density function of K
$F(x)$	Cumulative distribution function of K
N	Maximum value of K
w	Wholesale price per unit product in the retail selling channel
p_{r}, p_{m}	Retail price in the retail selling channel and the direct selling channel, respectively
a_1, a_2	Highest willingness of customers to pay in the direct selling channel and the retail selling channel, respectively
c_{r}, c_{m}	Unit cost at which the manufacturer supplies the retail selling channel and the direct selling channel, respectively
θ	Cross sensitivity coefficient of expected sales volume between the retail selling channel and the direct selling channel

4. Model Analysis

4.1. Centralized Decision Making. Under the centralized decision-making situation, the manufacturer and the retailer work together to pursue the maximization of expected profits of the whole supply chain. Therefore, assume the manufacturer and the retailer determine simultaneously Q_{m} and Q_{r}. The expected profit function of the dual-channel supply chain is as follows:

$$\begin{aligned}\Pi = {} & [a_1 - E(q_{\mathrm{m}}) - \theta E(q_{\mathrm{r}}) - c_{\mathrm{m}}]E(q_{\mathrm{m}}) \\ & + [a_2 - E(q_{\mathrm{r}}) - \theta E(q_{\mathrm{m}}) - c_{\mathrm{r}}]E(q_{\mathrm{r}}).\end{aligned} \tag{7}$$

Lemma 1 can be drawn through the way of solving the maximum of a multivariable function.

Lemma 1. *Under the centralized decision making, there is an optimal Q_m and an optimal Q_r, which will maximize the value of Π, the expected profits of whole supply chain.*

Proof. Taking the first derivative of equation (7) with respect to Q_{m} and Q_{r}, we get

$$\begin{aligned}\frac{\partial \Pi}{\partial Q_{\mathrm{m}}} &= A_1 \cdot \int_{Q_{\mathrm{m}}}^{N} f(x)dx - A_2 \cdot \int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx, \\ \frac{\partial \Pi}{\partial Q_{\mathrm{r}}} &= A_2 \cdot \int_{Q_{\mathrm{r}}+Q_{\mathrm{m}}}^{N} f(x)dx,\end{aligned} \tag{8}$$

where

$$A_1 = a_1 - 2E(q_{\mathrm{m}}) - 2\theta E(q_{\mathrm{r}}) - c_{\mathrm{m}}, \tag{9}$$

$$A_2 = a_2 - 2E(q_{\mathrm{r}}) - 2\theta E(q_{\mathrm{m}}) - c_{\mathrm{r}}. \tag{10}$$

Equation (7) obtains a maximum value at the stagnation point; Q_{m} and Q_{r} must satisfy $\partial\Pi/\partial Q_{\mathrm{m}} = 0$ and $\partial\Pi/\partial Q_{\mathrm{r}} = 0$. From $\partial\Pi/\partial Q_{\mathrm{m}} = 0$ and $\partial\Pi/\partial Q_{\mathrm{r}} = 0$, we get

$$a_1 - 2E(q_{\mathrm{m}}) - 2\theta E(q_{\mathrm{r}}) - c_{\mathrm{m}} = 0, \tag{11}$$

$$a_2 - 2E(q_{\mathrm{r}}) - 2\theta E(q_{\mathrm{m}}) - c_{\mathrm{r}} = 0. \tag{12}$$

Taking the second derivative of equation (7) with respect to Q_{m} and Q_{r}, we get

$$\frac{\partial^2 \Pi}{\partial Q_{\mathrm{m}}^2} = -A_1 \cdot f(Q_{\mathrm{m}}) - A_2 \cdot [f(Q_{\mathrm{r}} + Q_{\mathrm{m}}) - f(Q_{\mathrm{m}})] - A_3, \tag{13}$$

$$\frac{\partial^2 \Pi}{\partial Q_{\mathrm{r}}^2} = -A_2 - 2\left[\int_{Q_{\mathrm{r}}+Q_{\mathrm{m}}}^{N} f(x)dx\right]^2, \tag{14}$$

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial Q_{\mathrm{m}} \partial Q_{\mathrm{r}}} = \frac{\partial^2 \Pi}{\partial Q_{\mathrm{r}} \partial Q_{\mathrm{m}}} = -A_2 - 2\Bigg[\theta \int_{Q_{\mathrm{m}}}^{N} f(x)dx \\ - \int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx\Bigg] \int_{Q_{\mathrm{r}}+Q_{\mathrm{m}}}^{N} f(x)dx,\end{aligned} \tag{15}$$

where

$$A_3 = 2\left[\int_{Q_m}^{Q_r+Q_m} f(x)dx\right]^2 - 4\theta\int_{Q_m}^{N} f(x)dx\int_{Q_m}^{Q_r+Q_m} f(x)dx + 2\left[\int_{Q_m}^{N} f(x)dx\right]^2. \tag{16}$$

Bringing equations (9) and (10) into equations (13)–(15), we get

$$\frac{\partial^2\Pi}{\partial Q_m^2} < -2\left[\int_{Q_r+Q_m}^{N} f(x)dx\right]^2 < 0, \tag{17}$$

$$H = \begin{vmatrix} \dfrac{\partial^2\Pi}{\partial Q_m^2} & \dfrac{\partial^2\Pi}{\partial Q_m\partial Q_r} \\ \dfrac{\partial^2\Pi}{\partial Q_r\partial Q_m} & \dfrac{\partial^2\Pi}{\partial Q_r^2} \end{vmatrix} = 4\left(1-\theta^2\right) \cdot \left[\int_{Q_m}^{N} f(x)dx\int_{Q_r+Q_m}^{N} f(x)dx\right]^2 > 0. \tag{18}$$

Therefore, the stagnation point of Hessian matrix in equation (7) is "diagonally dominant," that is, there are optimal Q_m and Q_r which allow the continuous function Π to obtain a maximum value at the stagnation point. According to the solution method of the maximum value of the multivariate function, the point which allows the function to obtain the maximum value is likely to be within the definition domain or to be on the boundary of the definition domain. However, since this paper is aimed at a dual-channel supply chain where the direct selling channel and the retail selling channel coexist, the value on the boundary of the defined domain will make the dual-channel supply false. Therefore, according to the background of the issue in question in this paper, the maximum value of the function is obtained inside the domain, that is, there are optimal Q_m and Q_r which enable the continuous function to obtain the maximum value. Proof completed.

Based on Lemma 1, Proposition 1 will provide the conditions which should be met by the optimal Q_m and Q_r. Those conditions will also work as a benchmark of supply chain coordination under the decentralized decision making. □

Proposition 1. *Under the centralized decision making, in order to maximize the expected profits of the whole supply chain under capacity uncertainty, Q_m and Q_r must meet the following conditions:*

$$a_1 - 2A_4 - 2\theta A_5 - c_m = 0, \tag{19}$$

$$a_2 - 2A_5 - 2\theta A_4 - c_r = 0, \tag{20}$$

where

$$A_4 = \int_0^{Q_m} xf(x)\,dx + \int_{Q_m}^{N} Q_m f(x)\,dx,$$
$$A_5 = \int_{Q_m}^{Q_r+Q_m} (x-Q_m)f(x)\,dx + \int_{Q_r+Q_m}^{N} Q_r f(x)\,dx. \tag{21}$$

Proof. Bringing equations (1) and (2) into equations (9) and (10), we get equations (19) and (20). Proof completed. □

4.2. Decentralized Decision Making. Under the decentralized decision making, the manufacturer and the retailer will make decisions based on the principle of maximizing their respective expected profits. We will first use Lemma 2 to prove the existence of optimal Q_m and an optimal Q_r under the decentralized decision making.

Lemma 2. *Under the decentralized decision making, there is an optimal Q_m and an optimal Q_r, which will maximize the expected profits of each supply chain member.*

Proof. Taking the first derivative of π_r with respect to Q_r, we get

$$\frac{\partial\pi_r}{\partial Q_r} = \left[a_2 - 2E(q_r) - \theta E(q_m) - w^d\right]\int_{Q_r+Q_m}^{N} f(x)dx. \tag{22}$$

Let $\partial\pi_r/\partial Q_r = 0$; according to implicit function theorem, we get

$$\frac{\partial Q_r}{\partial Q_m} = -\frac{\partial^2\pi_r/\partial Q_r\partial Q_m}{\partial^2\pi_r/\partial Q_r^2} = \frac{2\int_{Q_m}^{Q_r+Q_m} f(x)dx - \theta\int_{Q_m}^{N} f(x)dx}{2\int_{Q_r+Q_m}^{N} f(x)dx}. \tag{23}$$

Taking the first derivative of π_m with respect to Q_m, we get

$$\frac{d\pi_m}{dQ_m} = \frac{\partial\pi_m}{\partial Q_m} + \frac{\partial\pi_m}{\partial Q_r}\cdot\frac{\partial Q_r}{\partial Q_m} = \frac{2A_6 - \theta\left[w^d - c_r - \theta E(q_m)\right]}{2}\int_{Q_m}^{N} f(x)dx, \tag{24}$$

where

$$A_6 = a_1 - 2E(q_m) - \theta E(q_r) - c_m. \tag{25}$$

Taking the second derivative of equation (7) with respect to Q_m and Q_r, we get

$$\frac{\partial^2\pi_r}{\partial Q_r^2} = -2\int_{Q_r+Q_m}^{N} f(x)dx,$$
$$\frac{\partial^2\pi_m}{\partial Q_m^2} = -\left(4-\theta^2\right)\left[\int_{Q_m}^{N} f(x)dx\right]. \tag{26}$$

In the Stackelberg game, the manufacturer determines Q_m first, and the retailer determines Q_r later. Since Q_m and Q_r are not determined simultaneously, π_r and π_m are not required to strictly jointly concave in Q_r and Q_m under the decentralized decision making. However, $\partial^2\pi_r/\partial Q_r^2 < 0$ ensures that π_r is strictly concave in Q_r. $\partial^2\pi_m/\partial Q_m^2 < 0$ ensures that π_m is strictly concave in Q_m. Proof completed.

Based on Lemma 2, the following Proposition 2 will present the condition where the optimal Q_m and Q_r must meet under the decentralized decision making. □

Proposition 2. *Under the decentralized decision making, in order to maximize the respective expected profits of the manufacturer and the retailer under capacity uncertainty, Q_m and Q_r must meet the following conditions:*

$$\begin{aligned} 2a_1 - \left(4-\theta^2\right)A_4 - 2\theta A_5 - 2c_{\mathrm{m}} - \theta\left(w^{\mathrm{d}} - c_{\mathrm{r}}\right) = 0, \\ a_2 - 2A_5 - \theta A_4 - w^{\mathrm{d}} = 0. \end{aligned} \tag{27}$$

Proof. Bringing equations (1), (2), and (23) into equations (22) and (24) and let $d\pi_{\mathrm{m}}/dQ_{\mathrm{m}} = 0$ and $\partial\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}} = 0$, we get equation (27). Proof completed.

Based on equation (23), we get Propositions 3 and 4 as follows. □

Proposition 3

$$\begin{aligned} &(1)\ \frac{\partial Q_{\mathrm{r}}^{\mathrm{d}^*}}{\partial Q_{\mathrm{m}}^{\mathrm{d}^*}} > 0, \quad \text{when } 0 < \theta < \frac{2\int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx}{\int_{Q_{\mathrm{m}}}^{N} f(x)dx}, \\ &(2)\ \frac{\partial Q_{\mathrm{r}}^{\mathrm{d}^*}}{\partial Q_{\mathrm{m}}^{\mathrm{d}^*}} < 0, \quad \text{when } \frac{2\int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx}{\int_{Q_{\mathrm{m}}}^{N} f(x)dx} < \theta < 1. \end{aligned} \tag{28}$$

Proposition 3 suggests that the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel increases as the manufacturer's optimal inventory level decision $Q_{\mathrm{m}}^{\mathrm{d}^*}$ in the direct selling channel increases, when the retail price p_{r} in the retail selling channel is not sensitive to changes in inventory level Q_{m} in the direct selling channel. It is unrealistic to increase sales volume and product prices in the retail selling channel simultaneously. Thus, in such a situation, the retailer enhances its profit mainly by increasing the sales volume instead of increasing the retail price in the retail selling channel, anticipating that the manufacturer will increase inventory level in the direct selling channel.

On the contrary, the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel decreases as the manufacturer's optimal inventory level decision $Q_{\mathrm{m}}^{\mathrm{d}^*}$ in the direct selling channel increases, when the retail price p_{r} in the retail selling channel is very sensitive to changes in inventory level Q_{m} in the direct selling channel. In such a situation, the retailer enhances its profit mainly by increasing the retail price instead of increasing the sales volume in the retail selling channel, anticipating that the manufacturer will increase inventory level in the direct selling channel.

Proposition 4

$$\begin{aligned} &(1)\ \frac{\partial Q_{\mathrm{r}}^{\mathrm{d}^*}}{\partial Q_{\mathrm{m}}^{\mathrm{d}^*}} > 0, \quad \text{when } Q_{\mathrm{m}} < N < N_1, \\ &\quad \text{where } 2\int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx - \theta\int_{Q_{\mathrm{m}}}^{N_1} f(x)dx = 0, \\ &(2)\ \frac{\partial Q_{\mathrm{r}}^{\mathrm{d}^*}}{\partial Q_{\mathrm{m}}^{\mathrm{d}^*}} < 0, \quad \text{when } N_1 < N < +\infty, \\ &\quad \text{where } 2\int_{Q_{\mathrm{m}}}^{Q_{\mathrm{r}}+Q_{\mathrm{m}}} f(x)dx - \theta\int_{Q_{\mathrm{m}}}^{N_1} f(x)dx = 0. \end{aligned} \tag{29}$$

Proposition 4 indicates that the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel increases as the manufacturer's optimal inventory level decision $Q_{\mathrm{m}}^{\mathrm{d}^*}$ in the direct selling channel increases, when the manufacturer's maximum production capacity N is limited (i.e., $Q_{\mathrm{m}} < N < N_1$). Possible explanations are as follows: when $N < N_1$, the risk that the supply is not adequate to the demand from two channels is very high. Thus, in such a situation, the retailer should try to get product as more as possible from the manufacturer, anticipating the manufacturer increase inventory level in the direct selling channel.

On the contrary, when $N_1 < N < +\infty$, the manufacturer's production capacity is infinite, and the retailer should never need to worry about its orders being cut by the manufacturer. In such a situation, the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel decreases as the manufacturer's optimal inventory level decision $Q_{\mathrm{m}}^{\mathrm{d}^*}$ in the direct selling channel increases. Moreover, the retailer enhances its profit mainly by increasing the retail price in the retail selling channel instead of increasing the sales volume in the retail selling channel, anticipating the manufacturer increase inventory level in the direct selling channel.

The following Proposition 5 will further explore the impact of maximum value of manufacturer's production capacity N on the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel and the manufacturer's optimal inventory level decision $Q_{\mathrm{m}}^{\mathrm{d}^*}$ in the direct selling channel.

Proposition 5. $(\partial Q_r^{d^*}/\partial N) < 0$, $(\partial Q_m^{d^*}/\partial N) < 0$.

Proof. Let $\partial\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}} = 0$; according to implicit function theorem, we get

$$\frac{\partial Q_{\mathrm{r}}}{\partial N} = -\frac{\partial^2\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}}\partial N}{\partial^2\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}}^2} = -\frac{\left(4-\theta^2\right)Q_{\mathrm{m}}f(N) + 2\theta Q_{\mathrm{r}}f(N)}{2\int_{Q_{\mathrm{r}}+Q_{\mathrm{m}}}^{N} f(x)dx} < 0. \tag{30}$$

Let $\partial\pi_{\mathrm{m}}/\partial Q_{\mathrm{m}} = 0$; according to implicit function theorem, we get

$$\frac{\partial Q_{\mathrm{m}}}{\partial N} = -\frac{\partial^2\pi_{\mathrm{m}}/\partial Q_{\mathrm{m}}\partial N}{\partial^2\pi_{\mathrm{m}}/\partial Q_{\mathrm{m}}^2} = -\frac{\left(4-\theta^2\right)Q_{\mathrm{m}}f(N) + 2\theta Q_{\mathrm{r}}f(N)}{\left(4-\theta^2\right)\left[\int_{Q_{\mathrm{m}}}^{N} f(x)dx\right]} < 0, \tag{31}$$

where $(\partial^2\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}}\partial N) = -2Q_{\mathrm{r}}f(N) - \theta Q_{\mathrm{m}}f(N) < 0$ and $(\partial^2\pi_{\mathrm{m}}/\partial Q_{\mathrm{m}}\partial N) = -(4-\theta^2)Q_{\mathrm{m}}f(N) - 2\theta Q_{\mathrm{r}}f(N) < 0$.

Proof completed. □

Proposition 5 indicates that both the retailer's optimal order quantity decision $Q_{\mathrm{r}}^{\mathrm{d}^*}$ in the retail selling channel and

the manufacturer's optimal inventory level decision $Q_m^{d^*}$ in the direct selling channel increase as the manufacturer's maximum production capacity N decreases, while decrease as the manufacturer's maximum production capacity N increases. Possible explanations are as follows: the risk that the supply does not meet the demand from two channels forces every member in the dual-channel supply chain to carry inventory as many as possible. A large production capacity is necessary for the manufacturer to construct an efficient dual-channel supply chain with a small number of inventories. Anticipating a large production capacity, the retailer does not need to worry about its orders being cut by the manufacturer and is willing to keep a good relationship with the manufacturer.

It requires the supply chain members to make the same decision when they are under the decentralized decision making as when they are under the centralized decision making to maximize the expected profits of the supply chain under the decentralized decision making. By solving equations (19), (20), and (27) in tandem, we get $w^d = \theta[\int_0^{Q_m} xf(x)\,dx + \int_{Q_m}^{N} Q_m f(x)\,dx] + c_r$ and $E(q_r) = \int_{Q_m}^{Q_r+Q_m} (x - Q_m) f(x)\,dx + \int_{Q_r+Q_m}^{N} Q_r f(x)\,dx = 0$. Obviously, under the decentralized decision making, the retailer will not accept the wholesale price which will get them 0 unit of product. Therefore, under the decentralized decision making, the wholesale price determined by the manufacturer and the retailer based on a long-term agreement will prevent the supply chain from maximizing its expected profits.

As mentioned above, when $w^d = \theta[\int_0^{Q_m} xf(x)\,dx + \int_{Q_m}^{N} Q_m f(x)\,dx] + c_r$, the expected profits of the supply chain can be maximized. As the leader of the supply chain, the manufacturer is motivated to increase the total profits of the supply chain [35–38]. However, if this argument is used by the manufacturer as a reason to enforce this wholesale price, the retailer will not buy it, thus leading to severe channel conflict between the retail and the direct selling channel. This is the reason why some retailers and their association (such as American Footwear Association and National Sporting Goods Association) protest against the direct selling channel and even force some manufactures (such as Estee Lauder) to only display rather than sell products in their direct selling channel [39–42].

In the light that the wholesale price determined by the manufacturer and the retailer based on a long-term agreement cannot maximize the expected profits of the dual-channel supply chain, the rest of this paper will suggest a compensation mechanism to coordinate the supply chain under capacity uncertainty from the perspective of the manufacturer and will analyze the conditions under which the retailer will accept the compensation mechanism.

5. A Compensation Mechanism to Coordinate the Supply Chain under Capacity Uncertainty

With the compensation mechanism, if the order from the retail selling channel cannot be satisfied or only gets partially satisfied due to the problems during production or delivery, the manufacturer will compensate the dissatisfied order at the price of $s\,(s > 0)$ per unit product so as to motivate the retailer to keep cooperating with it. In the meantime, in order to coordinate Q_m and Q_r when the retailer gets compensated, the manufacturer will adjust the wholesale price per unit product from w^d to w^c. After the manufacturer announces the compensation mechanism, if the retailer accepts it, the manufacturer, as the leader of Stackelberg game, will first determine Q_m, and next the retailer will determine Q_r. The manufacturer will then start production and delivery. With the compensation mechanism, the respective expected profits of the manufacturer and the retailer are as follows:

$$\pi_m = [a_1 - E(q_m) - \theta E(q_r) - c_m]E(q_m) + w^c E(q_r) - s \cdot A_7, \tag{32}$$

$$\pi_r = [a_2 - E(q_r) - \theta E(q_m) - w^c]E(q_r) + s \cdot A_7, \tag{33}$$

where

$$A_7 = \int_0^{Q_m} Q_r f(x)\,dx + \int_{Q_m}^{Q_r+Q_m} (Q_r + Q_m - x) f(x)\,dx. \tag{34}$$

Next, we will evaluate whether the compensation mechanism will maximize the expected profit of the dual-channel supply chain through Proposition 6.

Proposition 6. *The compensation mechanism requires contract parameter s and w^c to meet the following condition to maximize the expected profit of the dual-channel supply chain under capacity uncertainty:*

$$s = \frac{A_8 \cdot \int_{Q_m}^{N} f(x)dx \int_{Q_r+Q_m}^{N} f(x)dx}{\int_{Q_m}^{Q_r+Q_m} f(x)dx}, \tag{35}$$

$$w^c = \frac{A_8 \cdot \int_{Q_m}^{N} f(x)dx \int_0^{Q_r+Q_m} f(x)dx}{\int_{Q_m}^{Q_r+Q_m} f(x)dx} + \frac{A_9 \cdot \int_{Q_m}^{Q_r+Q_m} f(x)dx}{\int_{Q_m}^{Q_r+Q_m} f(x)dx}, \tag{36}$$

where

$$A_8 = a_1 - \frac{a_1 - c_m - (a_2 - c_r)\theta}{1 - \theta^2} - \theta\frac{a_2 - c_r - (a_1 - c_m)\theta}{2(1 - \theta^2)} - c_m,$$

$$A_9 = a_2 - \frac{a_2 - c_r - (a_1 - c_m)\theta}{1 - \theta^2} - \theta\frac{a_1 - c_m - (a_2 - c_r)\theta}{2(1 - \theta^2)}. \tag{37}$$

Proof. With the compensation mechanism, taking the first derivative of π_r with respect to Q_r, we get

$$\frac{\partial \pi_r}{\partial Q_r} = \left[a_2 - 2E(q_r) - \theta E(q_m) - w^c\right] \int_{Q_r+Q_m}^{N} f(x)dx + s\int_0^{Q_r+Q_m} f(x)dx. \tag{38}$$

Let $\partial \pi_r/\partial Q_r = 0$; according to implicit function theorem, we get

$$\frac{\partial Q_r}{\partial Q_m} = -\frac{\partial^2 \pi_r/\partial Q_r \partial Q_m}{\partial^2 \pi_r/\partial Q_r^2}, \tag{39}$$

where

$$\begin{aligned}\frac{\partial^2 \pi_r}{\partial Q_r \partial Q_m} &= -A_{10}\cdot f(Q_r+Q_m) - \left[\theta\int_{Q_m}^{N} f(x)dx - 2\int_{Q_m}^{Q_r+Q_m} f(x)dx\right]\int_{Q_r+Q_m}^{N} f(x)dx,\\ \frac{\partial^2 \pi_r}{\partial Q_r^2} &= -A_{10}\cdot f(Q_r+Q_m) - 2\left[\int_{Q_r+Q_m}^{N} f(x)dx\right]^2 < 0,\\ A_{10} &= a_2 - 2E(q_r) - \theta E(q_m) - w^c - s.\end{aligned} \tag{40}$$

Taking the first derivative of π_m with respect to Q_m, we get

$$\frac{d\pi_m}{dQ_m} = \frac{\partial \pi_m}{\partial Q_m} + \frac{\partial \pi_m}{\partial Q_r}\cdot\frac{\partial Q_r}{\partial Q_m}, \tag{41}$$

where

$$\begin{aligned}\frac{\partial \pi_m}{\partial Q_m} &= \left[a_1 - 2E(q_m) - \theta E(q_r) - c_m\right]\cdot\int_{Q_m}^{N} f(x)dx - \left[w^c - c_r - \theta E(q_m) + s\right]\int_{Q_m}^{Q_r+Q_m} f(x)dx,\\ \frac{\partial \pi_m}{\partial Q_r} &= \left[w^c - c_r - \theta E(q_m)\right]\int_{Q_r+Q_m}^{N} f(x)dx - s\int_0^{Q_r+Q_m} f(x)dx.\end{aligned} \tag{42}$$

From equation (12), we get $a_2 - 2E(q_r) - \theta E(q_m) = \theta E(q_m) + c_r$. Now, bringing equation (39) into equation (41), we get the optimal Q_m and Q_r which satisfy $(\partial \pi_r/\partial Q_r) = -(\partial \pi_m/\partial Q_r) = 0$ under the compensation mechanism. Then, by solving equations (11), (12), (38), and (41) we get equations (35) and (36). Proof completed. □

Proposition 6 shows that when contract parameter s and w^c meet the conditions stated in equations (35) and (36), the expected profits of the dual-channel supply chain can also be maximized even when supply chain members make decisions based on the principle of maximizing their own expected profits. In order to demonstrate the flexibility of the coordination mechanism, the following part will discuss how contract parameter s and w^c will change as θ changes through Corollary 1.

Corollary 1. *Within the compensation mechanism of the dual-channel supply chain, when θ grows, both s and w_1 decreases.*

Proof. From equations (35) and (36), we know that

$$\frac{\partial s}{\partial \theta} = -\frac{E(q_r^c)\int_{Q_m}^{N} f(x)dx\int_{Q_r+Q_m}^{N} f(x)dx}{\int_{Q_m}^{Q_r+Q_m} f(x)dx} < 0, \tag{43}$$

$$\frac{\partial w^c}{\partial \theta} = -\frac{E(q_r^c)\int_{Q_m}^{N} f(x)dx\int_0^{Q_r+Q_m} f(x)dx + E(q_m^0)\int_{Q_m}^{Q_r+Q_m} f(x)dx}{\int_{Q_m}^{Q_r+Q_m} f(x)dx} < 0. \tag{44}$$

Proof completed. □

Corollary 1 shows that when θ grows, the price competition between the retail selling channel and the direct selling channel gets stronger, which weakens the willingness of the retailer to order under the decentralized decision making. Therefore, in order to maximize the expected profit of the dual-channel supply chain, on the one hand, the manufacturer needs to offer lower wholesale price so as to encourage more order from the retailer, while on the other hand, the manufacturer needs to reduce the compensation due to supply shortage to prevent the retailer from expanding its order infinitely due to low wholesale price.

It is true that when parameters s and w_1 satisfy the conditions stated in equations (35) and (36), the expected profit of the supply chain under the centralized decision making equals that under the centralized decision making with the help of the compensation mechanism. However, only when the expected profits of all supply chain members are raised, the manufacturer will provide that compensation mechanism and the retailer will accept it. Therefore, Proposition 7 will further provide conditions which will facilitate the effective use of the compensation mechanism.

Proposition 7. *If the contract parameters w_1 and s allow $\pi_r^c > \pi_r^d$ and $\pi_m^c > \pi_m^d$, the compensation mechanism will realize the Pareto improvement of the expected profits of supply chain members under capacity uncertainty.*

Due to various factors and complicated situations considered in this paper, a numerical example is provided to vividly demonstrate the effectiveness of the compensation mechanism to coordinate the dual-channel supply chain and analyze the impact of different values of coordination parameters on expected profits of supply chain members.

6. Simulation

According to the information disclosed by Bank of America and Merrill Lynch about the cost and price of iPhone 6s 64 GB [43–46], assume that the unit product costs c_{m} and c_{r} at which the manufacturer supplies the direct and the retail selling channel, respectively, are both 234 dollars and assume that a_1 and a_2, the highest willingness to pay for one unit product by customers in the direct selling channel and the retail selling channel, respectively, are both 749 dollars. In addition, assume that the sensitivity coefficient θ of the retail price in either channel to its competitor's sales volume is 0.5 and assume that manufacturer's capacity K obeys uniform distribution $U(0, 1000)$. To facilitate reading, a list of parameter setting can be found in Table 2.

In order to demonstrate the effectiveness of the compensation mechanism which can coordinate the dual-channel supply chain, Figure 1 analyzes the change of expected profits of the supply chain members before and after the implementation of the compensation mechanism and evaluates whether the compensation mechanism can realize Pareto improvement of expected profits of the dual-channel supply chain members under capacity uncertainty.

By bringing the value of parameters in Table 2 into equation (27) and solving equation (27), we can get the expression $Q_{\mathrm{r}}^{\mathrm{d}}(w^{\mathrm{d}})$ and $Q_{\mathrm{m}}^{\mathrm{d}}(w^{\mathrm{d}})$ under the decentralized decision making. Bringing $Q_{\mathrm{r}}^{\mathrm{d}}(w^{\mathrm{d}})$ and $Q_{\mathrm{m}}^{\mathrm{d}}(w^{\mathrm{d}})$ into equations (5) and (6), we can get the manufacturer's optimal profit $\pi_{\mathrm{m}}^{\mathrm{d}^*}(w)$and the retailer's optimal profit $\pi_{\mathrm{r}}^{\mathrm{d}^*}(w^{\mathrm{d}})$. Similarly, bringing the value of parameters in Table 2 into equations (38) and (41) and solving $(\partial\pi_{\mathrm{m}}/\partial Q_{\mathrm{m}}) = 0$ and $(\partial\pi_{\mathrm{r}}/\partial Q_{\mathrm{r}}) = 0$, we can get the expression $Q_{\mathrm{r}}^{\mathrm{c}}(w^{\mathrm{d}})$ and $Q_{\mathrm{m}}^{\mathrm{c}}(w^{\mathrm{d}})$ under the coordination. Bringing $Q_{\mathrm{r}}^{\mathrm{c}}(w^{\mathrm{d}})$ and $Q_{\mathrm{m}}^{\mathrm{c}}(w^{\mathrm{d}})$ into equations (32) and (33), we can get the manufacturer's optimal profit $\pi_{\mathrm{m}}^{\mathrm{c}^*}(w^{\mathrm{d}})$ and the retailer's optimal profit $\pi_{\mathrm{r}}^{\mathrm{c}^*}(w^{\mathrm{d}})$.

By comparing $\pi_i^{\mathrm{c}}(w^{\mathrm{d}})$ with $\pi_i^{\mathrm{d}}(w^{\mathrm{d}})$ where $i = \mathrm{m}$ or r, we get the value of $\pi_{\mathrm{m}}^{\mathrm{c}} - \pi_{\mathrm{m}}^{\mathrm{d}}$ and $\pi_{\mathrm{r}}^{\mathrm{c}} - \pi_{\mathrm{r}}^{\mathrm{d}}$ as a response to the wholesale price per unit product w^{d}. Therefore, in Figure 1, the change of expected profits of supply chain members before and after coordination is influenced by the wholesale price w which is determined by the manufacturer and the retailer based on a long-term agreement. As an exogenous variable, w^{d} in Figure 1 obeys the uniform distribution which ranges between 300 dollars and 600 dollars. The value range of w^{d} in this section meets the following conditions. First, w^{d} is bigger than c_{r}, the unit product cost at which the manufacturer supplies the retail selling channel. Next, w^{d} is smaller than a_1, the highest willingness to pay for a unit product by customers in the retail selling channel. Finally, w^{d} ensures that the optimal inventory level of the direct selling channel and the optimal ordering quantity in the retail selling channel are positive.

Table 2: A list of parameter setting.

$f(x)$	0.001
N	1000
a_1, a_2	\$749
c_{r}, c_{m}	\$234
θ	0.5
w^{d}	U(\$300,\$600)

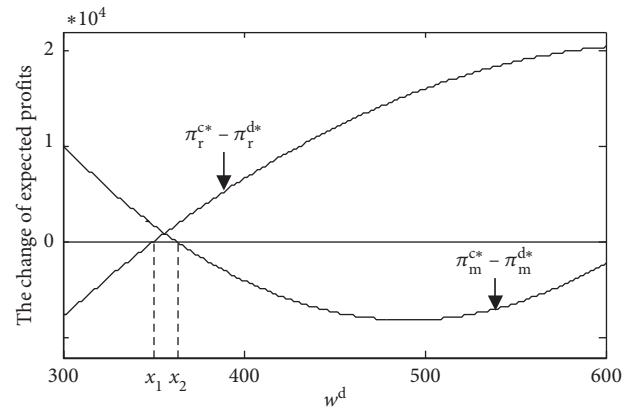

Figure 1: The change of expected profits of the manufacturer and the retailer before and after the implementation of the compensation mechanism.

In Figure 1, $\pi_j^{\mathrm{c}} - \pi_j^{\mathrm{d}} (j = \mathrm{r}, \mathrm{m})$ represents the relative change of the expected profits of the dual-channel supply chain members before and after the implementation of the compensation mechanism. As shown in Figure 1, when w^{d}, the wholesale price, which is determined by the manufacturer and the retailer based on a long-term agreement, is bigger than x_1 ($x_1 = 350.51$), implementation of the compensation mechanism can increase expected profits of the retailer. When w^{d} is smaller than x_2 ($x_2 = 364.03$), implementation of the compensation mechanism can increase expected profits of the manufacturer. It is only when the value of w^{d} is distributed between x_1 and x_2 can profits of the dual-channel supply chain be maximized and Pareto improvement of expected profits of supply chain members be realized.

Before the implementation of the compensation mechanism, the wholesale price w^{d} charged by the manufacturer to the retailer is determined by the manufacturer and the retailer based on a long-term agreement. After the implementation of the compensation mechanism, in order to coordinate the ordering decision of the retail selling channel, the wholesale price w^{c} charged by manufacturer to the retailer will definitely be different from w^{d}. Therefore, we will analyze through Figure 2 the change of the wholesale price before and after implementation of the compensation mechanism. In Figure 2, the wholesale price w^{c} under the coordination can be obtained by taking the value of $Q_{\mathrm{r}}^{\mathrm{c}}(w^{\mathrm{d}})$

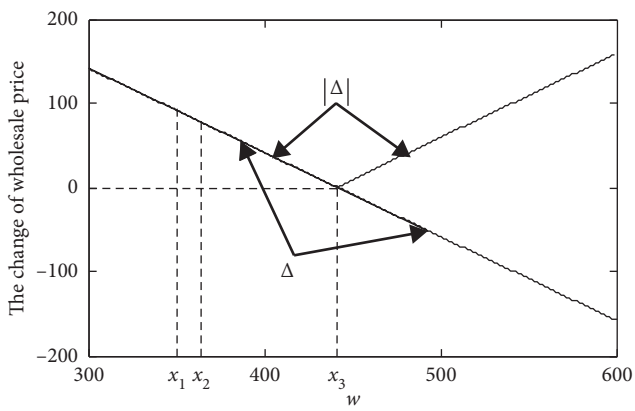

Figure 2: The wholesale price change before and after implementation of the compensation mechanism.

and $Q_m^c(w^d)$ in Figure 1 into equation (36) and can be expressed as a form of $w^c(w^d)$. By comparing the $w^c(w^d)$ with w^d, we get the value of $\Delta = w^c - w^d$ as a response of the wholesale price per unit product w^d.

In Figure 2, let $\Delta = w^c - w^d$, and there exists a threshold x_3 which equals 442.03. If the compensation mechanism only maximizes expected profits of the dual-channel supply chain, when the wholesale price under the decentralized decision making which was determined by the manufacturer and the retailer based on a long-term agreement is relatively small (that is, $w^d < x_3$), the manufacturer can raise wholesale price after implementing compensation mechanism to prevent the retailer from ordering too much under the decentralized decision making (that is, $\Delta > 0$); when the wholesale price under the decentralized decision making which was determined by the manufacturer and the retailer based on a long-term agreement is relatively big (that is, $w^d \geq x_3$), the manufacturer can reduce wholesale price after implementing compensation mechanism to encourage the retailer to order more under the decentralized decision making (that is, $\Delta \leq 0$). The retailer would rather accept a lower wholesale price after compensation mechanism is implemented than accept a higher price after that. However, when Figures 1 and 2 are combined, we can discover that manufacturer will make less profits in the situation where the wholesale price is reduced after implementation of the compensation mechanism (that is, $w^d \geq x_3$). Therefore, in this way, the compensation mechanism can only maximize expected profits of the dual-channel supply chain but cannot realize Pareto improvement of expected profits of supply chain members. However, in the situation where the wholesale price is increased after implementation of the compensation ;mechanism (that is, $x_1 < w^d < x_2$), due to less price competition between the direct and the retail selling channels, prices in both channels will rise, which will benefit both the manufacturer and the retailer. Therefore, the implementation of compensation system will not only maximize expected profits of the dual-channel supply chain but also realize Pareto improvement of expected profits of supply chain members.

7. Conclusions

Compared with the capacity certainty, the capacity uncertainty brings more complexity to ordering and inventory decision making for the dual-channel supply chain. Therefore, it is of great significance to study the decision making and coordination problems of the dual-channel supply chain members under the capacity uncertainty. This study contributes to optimize supply chain members' decision making, alleviate channel conflicts, and increase expected profits of both the entire supply chain and the supply chain members. Aiming at a dual-channel supply chain consisting of a manufacturer and a retailer, under a real situation where the direct selling channel gets satisfied first, this paper established a Stackelberg game model with the manufacturer as the leader. By analyzing double marginalization from a vertical perspective and channel conflict from a horizontal perspective, this paper obtained the optimal inventory level for the direct selling channel and the optimal ordering quantity for the retail selling channel under the centralized and decentralized decision making. Furthermore, it also designed a compensation mechanism to coordinate the dual-channel supply chain. The research shows that due to the double marginalization and channel conflict, the optimal decision of supply chain members under the centralized decision making is different from that under the decentralized decision making, which cannot maximize the expected profit of the dual-channel supply chain. However, if the manufacturer compensates the retailer's loss of expected profit due to unsatisfied order and adjusts the wholesale pricing strategy, the expected profit of the dual-channel supply chain can be maximized and the Pareto improvement of the expected profit of supply chain members can be realized.

This paper studied capacity uncertainty issues within a dual-channel supply chain considering information symmetry among supply chain members. It will be a future research topic to study the distribution, optimization, and coordination issues under capacity uncertainty and information asymmetry.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] B. Dan, G. Xu, and C. Liu, "Pricing policies in a dual-channel supply chain with retail services," *International Journal of Production Economics*, vol. 139, no. 1, pp. 312–320, 2012.

[2] I. Moon, S. P. Sarmah, and S. Saha, "The impact of online sales on centralised and decentralised dual-channel supply chains," *European Journal of Industrial Engineering*, vol. 12, no. 1, pp. 67–92, 2018.

[3] J. Zhou, R. Zhao, and W. Wang, "Pricing decision of a manufacturer in a dual-channel supply chain with asymmetric information," *European Journal of Operational Research*, vol. 278, no. 3, pp. 809–820, 2019.

[4] G. Li, L. Li, and J. Sun, "Pricing and service effort strategy in a dual-channel supply chain with showrooming effect," *Transportation Research Part E: Logistics and Transportation Review*, vol. 126, pp. 32–48, 2019.

[5] Q. Geng and S. Mallik, "Inventory competition and allocation in a multi-channel distribution system," *European Journal of Operational Research*, vol. 182, no. 2, pp. 704–729, 2007.

[6] P. Chen, B. Li, Y. Jiang, and P. Hou, "The impact of manufacturer's direct sales and cost information asymmetry in a dual-channel supply chain with a risk-averse retailer," *International Journal of Electronic Commerce*, vol. 21, no. 1, pp. 43–66, 2017.

[7] A. Leach, "France's biggest apple reseller shuts up shop," The Register, https://www.theregister.co.uk/2012/08/06/ebizcuss_closes_doors/, 2012.

[8] C. Tang, H. Yang, E. Cao, and K. K. Lai, "Channel competition and coordination of a dual-channel supply chain with demand and cost disruptions," *Applied Economics*, vol. 50, no. 46, pp. 4999–5016, 2018.

[9] Q. Lu and N. Liu, "Effects of e-commerce channel entry in a two-echelon supply chain: a comparative analysis of single- and dual-channel distribution systems," *International Journal of Production Economics*, vol. 165, pp. 100–111, 2015.

[10] X. D. Zhu, B. Y. Li, and Z. Wang, "A study on the manufacturing decision-making and optimization of hybrid-channel supply chain for original equipment manufacturer," *Advances in Production Engineering & Management*, vol. 12, no. 2, pp. 185–195, 2017.

[11] M. A. Sellitto, "Reverse logistics activities in three companies of the process industry," *Journal of Cleaner Production*, vol. 187, pp. 923–931, 2018.

[12] F. Zhang and J. Ma, "Research on the complex features about a dual-channel supply chain with a fair caring retailer," *Communications in Nonlinear Science and Numerical Simulation*, vol. 30, no. 1–3, pp. 151–167, 2016.

[13] F. Zhang and C. Wang, "Dynamic pricing strategy and coordination in a dual-channel supply chain considering service value," *Applied Mathematical Modelling: Simulation and Computation for Engineering and Environmental Systems*, vol. 54, pp. 722–742, 2018.

[14] L. Dai, X. Wang, X. Liu et al., "Pricing strategies in dual-channel supply chain with a fair caring retailer," *Complexity*, vol. 2019, Article ID 1484372, 23 pages, 2019.

[15] R. Yan and Z. Pei, "Retail services and firm profit in a dual-channel market," *Journal of Retailing and Consumer Services*, vol. 16, no. 4, pp. 306–314, 2009.

[16] Y. J. Guo and L. Zhao, "The conflict and coordination in dual channel based on e-market," *System Engineering Theory and Practice*, vol. 28, no. 9, pp. 59–66, 2008.

[17] Q. Xu, Z. Liu, and B. Shen, "The impact of price comparison service on pricing strategy in a dual-channel supply chain," *Mathematical Problems in Engineering*, vol. 2013, Article ID 613528, 13 pages.

[18] W. Yan, Y. Xiong, Z. Xiong, and N. Guo, "Bricks vs. clicks: which is better for marketing remanufactured products?," *European Journal of Operational Research*, vol. 242, no. 2, pp. 434–444, 2015.

[19] J. Chen, D. Wu, and P. Li, "Research on the pricing model of the dual-channel reverse supply chain considering logistics costs and consumers' awareness of sustainability based on regional differences," *Sustainability*, vol. 10, no. 7, p. 2229, 2018.

[20] W. Li, J. Chen, G. Liang, and B. Chen, "Money-back guarantee and personalized pricing in a Stackelberg manufacturer's dual-channel supply chain," *International Journal of Production Economics*, vol. 197, no. 1, pp. 84–98, 2018.

[21] J. Xu, Q. Qi, and Q. Bai, "Coordinating a dual-channel supply chain with price discount contracts under carbon emission capacity regulation," *Applied Mathematical Modelling*, vol. 56, no. 1, pp. 449–468, 2018.

[22] J. Heydari, K. Govindan, and A. Aslani, "Pricing and greening decisions in a three-tier dual channel supply chain," *International Journal of Production Economics*, 2018.

[23] M.-B. Jamali and M. Rasti-Barzoki, "A game theoretic approach for green and non-green product pricing in chain-to-chain competitive sustainable and regular dual-channel supply chains," *Journal of Cleaner Production*, vol. 170, no. 1, pp. 1029–1043, 2018.

[24] Y. He, H. Huang, and D. Li, "Inventory and pricing decisions for a dual-channel supply chain with deteriorating products," *Operational Research*, pp. 1–43, 2018.

[25] Y.-W. Zhou, J. Guo, and W. Zhou, "Pricing/service strategies for a dual-channel supply chain with free riding and service-cost sharing," *International Journal of Production Economics*, vol. 196, no. 1, pp. 198–210, 2018.

[26] J. Yang, J. Su, and L. Song, "Selection of manufacturing enterprise innovation design Project based on consumer's green preferences," *Sustainability*, vol. 11, no. 5, p. 1375, 2019.

[27] N. M. Modak and P. Kelle, "Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand," *European Journal of Operational Research*, vol. 272, no. 1, pp. 147–161, 2019.

[28] A. Ranjan and J. K. Jha, "Pricing and coordination strategies of a dual-channel supply chain considering green quality and sales effort," *Journal of Cleaner Production*, vol. 218, no. 1, pp. 409–424, 2019.

[29] A. Arya, B. Mittendorf, and D. E. M. Sappington, "The bright side of supplier encroachment," *Marketing Science*, vol. 26, no. 5, pp. 651–659, 2007.

[30] T. Boyaci, "Competitive stocking and coordination in a multiple-channel distribution system," *IIE Transactions*, vol. 37, no. 5, pp. 407–427, 2005.

[31] T. Xiao and J. Shi, "Pricing and supply priority in a dual-channel supply chain," *European Journal of Operational Research*, vol. 254, no. 3, pp. 813–823, 2016.

[32] A. David and E. Adida, "Competition and coordination in a two-channel supply chain," *Production and Operations Management*, vol. 24, no. 8, pp. 1358–1370, 2015.

[33] L. Zhu, X. Ren, C. Lee et al., "Coordination contracts in a dual-channel supply chain with a risk-averse retailer," *Sustainability*, vol. 9, no. 11, 2148 pages, 2017.
[34] D. Zhao and Z. Li, "The impact of manufacturer's encroachment and nonlinear production cost on retailer's information sharing decisions," *Annals of Operations Research*, vol. 264, no. 1, 2, pp. 499–539, 2018.
[35] S.-B. Tsai, Y. Xue, J. Zhang et al., "Models for forecasting growth trends in renewable energy," *Renewable and Sustainable Energy Reviews*, vol. 77, pp. 1169–1178, 2017.
[36] W. Liu, H.-B. Shi, Z. Zhang et al., "The development evaluation of economic zones in China," *International Journal of Environmental Research and Public Health*, vol. 15, no. 1, p. 56, 2018.
[37] S.-B. Tsai, J. Yu, L. Ma et al., "A study on solving the production process problems of the photovoltaic cell industry," *Renewable and Sustainable Energy Reviews*, vol. 82, pp. 3546–3553, 2018.
[38] B. Liu, T. Li, and S.-B. Tsai, "Low carbon strategy analysis of competing supply chains with different power structures," *Sustainability*, vol. 9, no. 5, p. 835, 2017.
[39] S.-B. Tsai, "Using the DEMATEL model to explore the job satisfaction of research and development professionals in China's photovoltaic cell industry," *Renewable and Sustainable Energy Reviews*, vol. 81, pp. 62–68, 2018.
[40] S.-B. Tsai, M.-F. Chien, Y. Xue et al., "Using the fuzzy DEMATEL to determine environmental performance: a case of printed circuit board industry in taiwan," *Plos One*, vol. 10, no. 6, Article ID e0129153, 2015.
[41] L. Fabisiak, "Web service usability analysis based on user preferences," *Journal of Organizational and End User Computing*, vol. 30, no. 4, pp. 1–13, 2018.
[42] A. Avdic, "Second order interactive end user development appropriation in the public sector," *Journal of Organizational and End User Computing*, vol. 30, no. 1, pp. 82–106, 2018.
[43] J. Su, C. Li, S.-B. Tsai, H. Lu, A. Liu, and Q. Chen, "A sustainable closed-loop supply chain decision mechanism in the electronic sector," *Sustainability*, vol. 10, no. 4, p. 1295, 2018.
[44] S.-B. Tsai, "Using grey models for forecasting China's growth trends in renewable energy consumption," *Clean Technologies and Environmental Policy*, vol. 18, no. 2, pp. 563–571, 2016.
[45] L. K. Huang, "A cultural model of online banking adoption," *Journal of Organizational and End User Computing*, vol. 29, no. 1, pp. 1–22, 2017.
[46] R. Matt, "Apple will pay about $234 for the parts in each iphone 6s," Business Insider, 2015, http://www.businessinsider.com/iphone-6s-component-costs-by-model-2015-9.