Research Article

Three-Way Decisions with Single-Valued Neutrosophic Decision Theory Rough Sets Based on Grey Relational Analysis

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The single-valued neutrosophic set (SVNS) can not only depict imperfect information in the real decision system but also handle undetermined and inconformity information flexibly and effectively. Three-way decisions (3WDs) are often used as an effective method to deal with uncertainties, but the conditional probability is given by the decision maker subjectively, which makes the decision result too subjective. This paper proposes a novel model based on 3WDs to settle the multiattribute decision-making (MADM) problems, where the attribute values are described by SVNS, and the attribute weights are entirely unknown. At first, we build a single-valued neutrosophic decision theory rough set (SVNDTRS) model based on Bayesian decision process. Then, we use the analytic hierarchy process (AHP) approach to calculate the subjective weight of each attribute, the information entropy to obtain the attribute’s objective weight, and the minimum total deviation approach to determine the combined weight of the attributes. After obtaining the standard weight, the grey relational analysis (GRA) method is utilized to calculate the grey correlation closeness with the ideal solution, and the conditional probability is estimated by it. In addition, we develop a decision-making method in view of the ideal solution of 3WDs with the SVNS. This approach not only considers the lowest cost but also gives a corresponding semantic explanation for the decision result of each alternative, which can supplement the decision results of GRA. At last, we illustrate the feasibility and effectiveness of 3WDs through an example of supplier selection and compare it with other methods to verify the advantages of our approach.

1. Introduction

Multiattribute decision making (MADM) is more and more momentous for modern decision science. Its essence is to use the existing decision information to sort and optimize a limited number of alternatives in a certain way. Due to the complexity and unpredictability of the external environment, the ambiguity of the object itself, the limitations of human cognition, and the subjectivity of the decision maker, decision makers usually need to provide preference information through various types of attribute values. Since Zadeh [1] introduced the concepts of the fuzzy sets (FSs), the FSs have been widely studied. Atanassov [2] put forward the intuitionistic fuzzy sets (IFs) by adding the nonmembership degree based on the traditional FSs. IFs consist of membership degree and nonmembership degree, and they can more easily express fuzzy information and have been rapidly developed and widely used since they were introduced. However, the degree of hesitation in IFSs cannot be defined separately. Therefore, even if IFSs can effectively describe imperfect information, they are less flexible when dealing with uncertain and inconformity information. Then, the clearly quantified neutrosophic sets (NSs) can describe the value of the proposition between true and false, which was initially proposed by Smarandache [3]. NSs are made up of membership degree, hesitancy degree, and nonmembership degree. In addition, Wang et al. [4] proposed a subcategory of NS called single-valued neutrosophic sets (SVNSs) and discussed its related rules and properties. The trait of SVNS is that membership degree, hesitancy degree, and nonmembership degree are mutually independent; all three are between 0 and 1, and
the sum of them is between 0 and 3. Deli and Şubaş [5] developed a sorting method and extended it into MADM problems. Wang et al. [6, 7, 8] introduced a MADM approach in view of Maclaurin symmetric mean (MSM) operator and TODIM for SVNS. Sodenkamp et al. [9] used SVNS to process independent multisource underdetermined measurements, which affected the dependability of expert evaluation in MADM problems.

In many actual MADM problems, owing to the uncertainty or imperfect of information, it is difficult to adopt a method that only accepts and rejects these two decisions. Through the expansion of the two decisions, Yao [10, 11] proposed three-way decisions (3WDs) involving acceptance, rejection, and delayed decision making [12, 13, 14]. The principle of the 3WDs is derived from the probability rough set. In light of the positive, boundary, and negative domains of the probabilistic rough set, the 3WD model including acceptance decision, delayed decision, and rejection decision is established. So far, 3WDs are widely used in some areas such as influenza emergency management, granular computing, enterprise evaluation, and social networking [13, 15, 16, 17, 18]. At the same time, many theoretical results have been achieved in the study of 3WDs. For example, Zhang et al. [19] considered a new risk measurement function by utility theory and derived a 3WD model with DTRS. Sun et al. [20] introduced a decision-theoretic rough fuzzy set model with linguistic information based on 3WDs and applied it to MADMs. Zhang et al. [21] proposed a dynamic 3WD model and proved the model is practicable and valid. According to the TODIM method, Hu et al. [22] constructed a new 3WD model and demonstrated its application in online diagnosis and medical selection.

In the 3WD model, the loss function (LF) is a key parameter. The scholars have studied a great deal of 3WD rules based on LFs of diverse forms, such as interval number [23], IFSs [24], intuitionistic uncertain linguistic variables [25], dual hesitant Fss [26], and Pythagorean FSSs [27]. SVNS can handle uncertain, incomplete, and inconsistent information more flexibly. To this end, we use the SVNS to express the LF in this paper. Furthermore, how to determine the conditional probability is also the key to the 3WD method. The conditional probability in many references is subjectively given by the decision makers, which makes the decision results too subjective. Therefore, we use the GRA method to calculate conditional probability. The goal and motivation of this paper are (1) to extend 3WDs to the environment of SVNS, using SVNS to represent the LF in 3WDs; (2) to propose the SVNDTRS model and explore its properties; and (3) to use the GRA method to calculate conditional probability in 3WDs. The proposed method extends the use environment of 3WDs and provides a new idea for the determination of conditional probability in 3WDs.

The remainder of this paper is arranged as follows. In Section 2, we briefly review the basics of the NSs and SVNSs. In Section 3, we propose a method to determine the combination weight of attributes. In Section 4, we propose a single-valued neutrosophic decision theory rough set (SVNDTRS) model and its propositions. In Section 5, we estimate the conditional probability of 3WDs based on the GRA method and presented a MADM method to deal with SVNSs based on the SVNDTRS. In Section 6, we use a numerical example to demonstrate the availability, and other methods are compared and analyzed. In Section 7, we reach the conclusion.

2. SVNS

In this section, we introduce some basic concepts of the NSs and the SVNSs.

Definition 1 [3]. Let \( X \) be an object set, and the common elements of \( X \) are represented by \( x \). A NS of \( X \) consists of \( T^+ (x), I^+ (x), F^+ (x) \). \( \bar{Y} \) can be represented as \( Y = \{ T^+ (x), I^+ (x), F^+ (x) \mid x \in X \} \), where \( T^+ (x), I^+ (x), F^+ (x) \) represent the membership degree, hesitancy degree, and nonmembership degree, respectively. \( T^+ (x), I^+ (x), F^+ (x) \in [0, 1] \) and satisfies \( 0 \leq T^+ (x) + I^+ (x) + F^+ (x) \leq 3 \).

Definition 2 [4]. Let \( X \) be an object set, and the common elements of \( X \) are represented by \( x \). A SVNS \( \bar{Y} \) of \( X \) consists of \( T^+ (x), I^+ (x), F^+ (x) \). \( \bar{Y} \) can be represented as \( Y = \{ T^+ (x), I^+ (x), F^+ (x) \mid x \in X \} \), where \( T^+ (x), I^+ (x), F^+ (x) \) represent the membership degree, hesitancy degree, and nonmembership degree, respectively. \( T^+ (x), I^+ (x), F^+ (x) \in [0, 1] \) and satisfies \( 0 \leq T^+ (x) + I^+ (x) + F^+ (x) \leq 3 \). Let \( (T^+ (x), I^+ (x), F^+ (x)) \) be the single-valued neutrosophic number (SVNN) and abbreviated as \( x = (T_x, I_x, F_x) \).

Definition 3 [31]. For any two SVNNs \( x_m = (T_m, I_m, F_m) \) and \( x_n = (T_n, I_n, F_n) \), the related operations are defined as follows:

\[
x_m + x_n = (T_m + T_n - T_m T_n, I_m I_n, F_m F_n),
\]

\[
x_m \times x_n = (T_m T_n I_m + I_n - I_m I_n, F_m + F_n - F_m F_n),
\]

\[
kx_m = (1 - (1 - T_m)^k, (I_m)^k, (F_m)^k), \quad k > 0
\]

\[
(x_m)^k = ((T_m)^k, (1 - (1 - I_m))^k, (1 - (1 - F_m))^k), \quad k > 0.
\]
Table 1: Loss functions.

<table>
<thead>
<tr>
<th>( \Gamma (P) )</th>
<th>( \neg \Gamma (N) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_p )</td>
<td>( \lambda_{pp} = (T_{pp}, I_{pp}, F_{pp}) )</td>
</tr>
<tr>
<td>( a_b )</td>
<td>( \lambda_{bn} = (T_{bn}, I_{bn}, F_{bn}) )</td>
</tr>
<tr>
<td>( a_n )</td>
<td>( \lambda_{nn} = (T_{nn}, I_{nn}, F_{nn}) )</td>
</tr>
</tbody>
</table>

Definition 4 [32]. The complement set of a SVNS \( \tilde{\Gamma} \) is \( \tilde{\Gamma} \), which is defined by

\[
\begin{align*}
T_{\tilde{\Gamma}} (x) &= F_{\Gamma} (x); \\
I_{\tilde{\Gamma}} (x) &= 1 - I_{\Gamma} (x); \\
F_{\tilde{\Gamma}} (x) &= T_{\Gamma} (x).
\end{align*}
\]

Definition 5 [31]. Let \( x = (T, I, F) \) be a SVNN; the cosine similarity \( S(x) \) of \( x \) is described as follows:

\[
S(x) = \frac{T(x)}{\sqrt{T(x) + I(x) + F(x)}}
\]

Definition 6 [31]. For any two SVNNs \( x_m = (T_m, I_m, F_m) \) and \( x_n = (T_n, I_n, F_n) \), if \( S(x_m) \leq S(x_n) \), then \( x_m \preceq x_n \).

Definition 7 [32]. Let \( x_m = (T_m, I_m, F_m) \) and \( x_n = (T_n, I_n, F_n) \) be any two SVNNs; the Hamming distance between two SVNNs \( x_m \) and \( x_n \) is described as follows:

\[
d(x_m, x_n) = |T_m - T_n| + |I_m - I_n| + |F_m - F_n|.
\]

The normalized Hamming distance between two SVNNs \( x_m \) and \( x_n \) is described as follows:

\[
d_N (x_m, x_n) = \frac{1}{3} \left( |T_m - T_n| + |I_m - I_n| + |F_m - F_n| \right),
\]

where \( 0 \leq d_N (x_m, x_n) \leq 3, 0 \leq d_N (x_m, x_n) \leq 1 \).

3. Basic Model of SVNDTRS

In this section, we introduce a model of SVNDTRS based on 3WDs. At first, we use SVNN to build a LF matrix. The loss of different decision schemes under different state variables is illustrated in Table 1.

For the Bayesian decision [33, 34], the decision-making process is described by the state set and action set. The state set is described by \( \Omega = \{ \Gamma, \neg \Gamma \} \) and indicates whether the element \( x \) is in \( \Gamma \). Among them, \( \Gamma \) is the set of objects for correct classification and \( \neg \Gamma \) is the set of objects for wrong classification. This action set is represented by \( A = \{ a_p, a_b, a_n \} \), where \( a_p \), \( a_b \), and \( a_n \) signify the decision actions that divide an object \( x \) into positive, boundary, and negative domains. The positive domain signifies that \( x \) belongs to \( \Gamma \), that is, the accept event objects. The negative domain signifies that \( x \) does not belong to \( \Gamma \), that is, the reject event objects. The boundary domain signifies whether the uncertainty \( x \) belongs to \( \Gamma \), that is, it does not promise or delay the decision event objects. In addition, the parameters \( \lambda_{mn} (m = P, B, N; n = P, N) \) describe the LFs. \( \lambda_{pp} \) and \( \lambda_{bn} \) signify the costs of correct classification and error classification of the object \( x \) in accepted decision; \( \lambda_{bp} \) and \( \lambda_{bn} \) signify the costs of correct classification and error classification of the object \( x \) in delayed decision; and \( \lambda_{np} \) and \( \lambda_{nn} \) signify the costs of correct classification and error classification of the object \( x \) in rejected decision.

\[
\begin{align*}
T_{pp} &\leq T_{bp} < T_{np}, \quad (6) \\
I_{np} &\leq I_{bp} < I_{pp}, \quad (7) \\
F_{np} &\leq F_{bp} < F_{pp}, \quad (8) \\
T_{nn} &
\leq T_{bn} < T_{pn}, \quad (9) \\
I_{pn} &\leq I_{bn} < I_{nn}, \quad (10) \\
F_{pn} &\leq F_{bn} < F_{nn}. \quad (11)
\end{align*}
\]

Moreover, new constraints should be added as follows:

\[
\begin{align*}
0 &\leq T_{pp} + I_{pp} + F_{pp} \leq 3, \\
0 &\leq T_{bp} + I_{bp} + F_{bp} \leq 3, \\
0 &\leq T_{np} + I_{np} + F_{np} \leq 3, \\
0 &\leq T_{bn} + I_{bn} + F_{bn} \leq 3, \\
0 &\leq T_{nn} + I_{nn} + F_{nn} \leq 3.
\end{align*}
\]

According to references [35, 36], (6)–(11) are the prerequisites for SVNDTRS. By taking advantage of the conclusions of Definitions 5-6, the following proposition can be depicted.

Proposition 1. According to conditions (6)–(11), we can get the following relationship:

\[
\begin{align*}
\lambda_{pp} &\leq \lambda_{bp} < \lambda_{np}, \\
\lambda_{nn} &\leq \lambda_{bn} < \lambda_{pp}.
\end{align*}
\]

For the matter \( x \) belonging to \( \Gamma \), the loss caused by demarcating it into positive domain will not be greater than the loss of dividing it into boundary domain. The loss of both is not more than that caused by dividing it into negative domain. Similarly, for the matter \( x \) which belongs to \( \neg \Gamma \), the loss caused by falling it into negative domain will not be greater than that caused by bringing it to boundary domain, both of them are less than that caused by brings it to positive domain.

Let \( P(\Gamma \mid [x]) \) be the conditional probability that \( x \) belongs to \( \Gamma \), and \( P(\neg \Gamma \mid [x]) \) is the conditional probability that \( x \) belongs to \( \neg \Gamma \). Therefore, we can get \( P(\Gamma \mid [x]) + P(\neg \Gamma \mid [x]) = 1 \). By using the Bayesian risk decision theory
[24, 33, 34], for the matter $x$, the expected losses $R(a_j | [x]) (j = P, B, N)$ from taking different operations are indicated as follows:

$$R(a_p | [x]) = \lambda_{pp}P(\Gamma | [x]) \oplus \lambda_{pq}P(\neg \Gamma | [x]),$$

$$R(a_b | [x]) = \lambda_{bp}P(\Gamma | [x]) \oplus \lambda_{bn}P(\neg \Gamma | [x]),$$

$$R(a_n | [x]) = (1 - (1 - T_{pp})P(\Gamma | [x]) - (1 - T_{pp})P(\neg \Gamma | [x])), \quad \Theta \left(1 - (1 - T_{pq})P(\Gamma | [x]) - (1 - T_{pq})P(\neg \Gamma | [x]) \right),$$

(17)

$$R(a_n | [x]) = (1 - (1 - T_{BN})P(\Gamma | [x]) - (1 - T_{BN})P(\neg \Gamma | [x]), \quad \Theta \left(1 - (1 - T_{BP})P(\Gamma | [x]) - (1 - T_{BP})P(\neg \Gamma | [x]) \right),$$

(18)

$$R(a_n | [x]) = (1 - (1 - T_{NP})P(\Gamma | [x]) - (1 - T_{NP})P(\neg \Gamma | [x]), \quad \Theta \left(1 - (1 - T_{NP})P(\Gamma | [x]) - (1 - T_{NP})P(\neg \Gamma | [x]) \right).$$

(19)

For the expected losses of (17)–(19), we can get the following propositions.

**Proposition 2.** In accordance with the algorithms of SVNNs, the expected losses $R(a_j | [x]) (j = P, B, N)$ can be recoupled as follows:

$$R(a_p | [x]) = (1 - (1 - T_{pp})P(\Gamma | [x]) - (1 - T_{pp})P(\neg \Gamma | [x]), \quad \Theta \left(1 - (1 - T_{pp})P(\Gamma | [x]) - (1 - T_{pp})P(\neg \Gamma | [x]) \right),$$

(20)

Similarly, the expected losses $R(a_b | [x])$ and $R(a_n | [x])$ can be proved.

**Proposition 3.** Assuming that $P(C | [x])$ is an invariant constant with $P(\Gamma | [x]) + P(\neg \Gamma | [x]) = 1$, $T_{IN} < T_{IP}$ is a nonmonotonically increasing as $T_{IP}$ and $T_{IN} (i = P, B, N)$ increase.

Proof. Let $x_{IP} = (T_{IP}, I_{IP}, F_{IP})$ and $x_{IP} = (T_{IP}, I_{IP}, F_{IP})$ be any two SVNNs, and $T_{IP} < T_{IP}$, $I_{IP} < I_{IP}$, and $F_{IP} < F_{IP}$. Let $x_{IN} = (T_{IN}, I_{IN}, F_{IN})$ and $x_{IN} = (T_{IN}, I_{IN}, F_{IN})$ be any two SVNNs, and $T_{IN} < T_{IN}$, $I_{IN} < I_{IN}$, and $F_{IN} < F_{IN}$.

Since $T_{IP} < T_{IP}$ and $T_{IN} < T_{IN}$, we have $1 - T_{IP} > 1 - T_{IP}$ and $1 - T_{IN} > 1 - T_{IN}$. Then, we obtain $(1 - T_{IP})P(\Gamma | [x]) > (1 - T_{IP})P(\Gamma | [x])$, $T_{IN} > T_{IN}$, and $T_{IN} > T_{IN}$.

Furthermore, we have $(1 - T_{IP})P(\Gamma | [x]) > (1 - T_{IP})P(\Gamma | [x])$, $T_{IN} > T_{IN}$, and $T_{IN} > T_{IN}$.

Finally, we have $1 - (1 - T_{IP})P(\Gamma | [x]) > (1 - T_{IP})P(\Gamma | [x]), and $T_{IN} < T_{IN}$, $I_{IN} < I_{IN}$, and $F_{IN} < F_{IN}$.

**Proposition 4.** Assuming that $P(\Gamma | [x])$ is an invariant constant with $P(\Gamma | [x]) + P(\neg \Gamma | [x]) = 1$, $I_{IP} < I_{IP}$ is a nonmonotonically increasing as $I_{IP}$ and $I_{IP} (i = P, B, N)$ increase.
Proof. Since \( I_{IP,1} < I_{IP,2} \) and \( I_{IN,1} < I_{IN,2} \), we have \( I_{IP,1}^{P(T[x])} < I_{IP,2}^{P(T[x])} \) and \( I_{IN,1}^{P(T[x])} < I_{IN,2}^{P(T[x])} \); then, we obtain \( I_{IP,1}^{P(T[x])} I_{IN,1}^{P(T[x])} < I_{IP,2}^{P(T[x])} I_{IN,2}^{P(T[x])} \).

Proposition 5. Assuming that \( P(T[x]) \) is an invariant constant with \( P(T[x]) + P(-T[x]) = 1 \), \( F_{IP} \) and \( F_{IN} \) is nonmonotonically increasing as \( F_{IP} \) and \( F_{IN} \) increase.

Proof. Since \( F_{IP,1} < F_{IP,2} \) and \( F_{IN,1} < F_{IN,2} \), we have \( F_{IP,1}^{P(T[x])} < F_{IP,2}^{P(T[x])} \) and \( F_{IN,1}^{P(T[x])} < F_{IN,2}^{P(T[x])} \); then, we obtain \( F_{IP,1}^{P(T[x])} F_{IN,1}^{P(T[x])} < F_{IP,2}^{P(T[x])} F_{IN,2}^{P(T[x])} \).

4. Establishing the Weights

In fact, in most MADM processes, due to the pressure of time, the limitation of expertise, and the lack of data in problem areas, we often encounter weights of each attribute that are unknown or partly known. Because many existing methods are not suitable for solving such problems, two types of methods are generally used to determine weights. One is subjective weighting, such as the point estimation method and AHP method; the other is objective weighting, such as the deviation maximimization method and entropy method [28]. The subjective weight method gives the subjective weight according to the individual preferences of decision makers. While applying the objective weighting method, the subjective judgment of the decision maker is neglected, although the data are fully utilized. Therefore, these two approaches have limitations. This paper synthesized these two methods and proposed an overall merit model based on the AHP method and entropy method to optimize the integration weights.

4.1. AHP Method. Suppose that the complementary judgment matrix (the scale is 0.1–0.9) given by the decision maker for each attribute in the solution set is \( Q = (q_{jk})_{mn} \):

\[
Q = \begin{bmatrix}
q_{11} & q_{12} & \cdots & q_{1n} \\
q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
q_{m1} & q_{m2} & \cdots & q_{mn}
\end{bmatrix}
\] (23)

The judgment matrix is constructed by using the 0.1–0.9 five-scale method proposed by Du [37]. The scale values of \( q_{jk} \) are shown in Table 2.

The properties are shown as follows:

\[
\begin{align*}
(P) & : \text{If } R(a_p | x) \leq R(a_b | x) \text{ and } R(a_p | x) \leq R(a_N | x), \text{ then } x \in \text{POS}(\Gamma), \\
(B) & : \text{If } R(a_b | x) \leq R(a_p | x) \text{ and } R(a_b | x) \leq R(a_N | x), \text{ then } x \in \text{BND}(\Gamma), \\
(N) & : \text{If } R(a_N | x) \leq R(a_p | x) \text{ and } R(a_N | x) \leq R(a_b | x), \text{ then } x \in \text{NEG}(\Gamma).
\end{align*}
\] (22)

We have

\[
q_{jk} > 0,
\]

\[
q_{jk} + q_{kj} = 1,
\]

\[
q_{kk} = 0.5.
\] (24)

Then, we get the subjective weights \( \mu_j (j = 1, 2, \ldots, n) \) by the following formulas [8]:

\[
\mu_j = \frac{\sum_{k=1}^{n} q_{jk} + (n/2) - 1}{n(n - 1)}.
\] (25)

Then, we get the subjective weight vector \( \mu = (\mu_1, \mu_2, \ldots, \mu_n)^T \) of attribute \( C_j (j = 1, 2, \ldots, n) \) with \( \mu_j \geq 0 \) and \( \sum_{j=1}^{n} \mu_j = 1 \). Of course, (25) is right only when the complementary judgment matrix has consistency.

4.2. Entropy Method. The entropy method can dynamically mine the effective information provided by data, and its objective way of empowerment can measure the importance of evaluation index. It determines the corresponding weight by calculating the size of the entropy measure [29].

The entropy measure [29] of a SVNS \( \tilde{Y} = \{ T_{\tilde{Y}}(x_i), F_{\tilde{Y}}(x_i) \mid x_i \in X \} \) is

\[
E_j(\tilde{Y}) = 1 - \frac{\sum_{i=1}^{m} (T_{\tilde{Y}}(x_i) + F_{\tilde{Y}}(x_i))|I_{\tilde{Y}}(x_i) - E_j(x_i)|}{m}
\] (26)

The properties are as follows [29]:

(1) \( E_j(\tilde{Y}) = 0 \) if \( \tilde{Y} \) is a clear set and \( I_{\tilde{Y}}(x_i) = 0, \forall x_i \in X \)
(2) \( E_j(\tilde{Y}) = 1 \) if \( \{ T_{\tilde{Y}}(x_i), I_{\tilde{Y}}(x_i), F_{\tilde{Y}}(x_i) \} = (0.5, 0.5, 0.5), \forall x_i \in X \)
(3) \( E_j(\tilde{Y}) \geq E_j(\tilde{Y}) \) if \( \tilde{Y} \) is more deterministic than \( \tilde{Y} \), i.e., \( T_{\tilde{Y}}(x_i) + F_{\tilde{Y}}(x_i) \leq T_{\tilde{Y}}(x_i) + F_{\tilde{Y}}(x_i) \) and \( |I_{\tilde{Y}}(x_i) - I_{\tilde{Y}}(x_i)| \leq |I_{\tilde{Y}}(x_i) - I_{\tilde{Y}}(x_i)| \)
(4) \( E_j(\tilde{Y}) = E_j(\tilde{Y}), \forall x_i \in X \)
4.3 Determining the Combination Weight. Attribute weight is of great significance to MADM, which directly affects the accuracy of decision making. In order to make an accurate and scientific decision, it is necessary to consider the subjective preferences of decision makers and strive to reduce the subjective arbitrariness of weights. Besides, the objective information of decision objects is fully utilized to achieve the unity of subjectivity and objectivity. Therefore, it is of great significance to combine the subjective and objective weights reasonably to form a combination weight. We use the method of minimum total deviation [28] to determine the combination weight. The core idea of the optimal combination weight model is that the weight deviation obtained by various weighting methods should be as small as possible [38, 39]. To this end, the following basic models can be established:

\[
E_j = 1 - \frac{\sum_{i=1}^{m} (T_{ij}(x_i) + F_{ij}(x_i)) |I_{ij}(x_i) - I_{ij}^*(x_i)|}{m},
\]
\[
i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.
\]

The entropy value for \( C_j \) can be denoted as

\[
E_j = 1 - \frac{\sum_{i=1}^{m} (T_{ij}(x_i) + F_{ij}(x_i)) |I_{ij}(x_i) - I_{ij}^*(x_i)|}{m},
\]
\[
i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.
\]

Then, we get the objective weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) of attribute \( C_j (j = 1, 2, \ldots, n) \) with \( \omega_j \geq 0 \) and \( \sum_{j=1}^{n} \omega_j = 1 \).

The entropy weight [28] \( \omega_j \) is represented by

\[
\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)} = \frac{\sum_{i=1}^{m} (T_{ij}(x_i) + F_{ij}(x_i)) |I_{ij}(x_i) - I_{ij}^*(x_i)|}{\sum_{j=1}^{n} \left( \sum_{i=1}^{m} (T_{ij}(x_i) + F_{ij}(x_i)) |I_{ij}(x_i) - I_{ij}^*(x_i)| \right)},
\]
\[
j = 1, 2, \ldots, n.
\]

The Lagrange multiplier function is constructed by solving the model:

\[
L(\omega_j, \eta) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ \left( w_j - \omega_j \right) S(\bar{Y}_{ij}) \right\}^2 + \left\{ \left( w_j - \mu_j \right) S(\bar{Y}_{ij}) \right\}^2 + 2\eta \left( \sum_{j=1}^{n} w_j - 1 \right),
\]

and we have

\[
\frac{\partial L(\omega_j, \eta)}{\partial \omega_j} = \sum_{i=1}^{m} 2 \left[ 2 \omega_j - \left( \mu_j + \omega_j \right) \right] S(\bar{Y}_{ij})^2 + 2\eta = 0,
\]
\[
\frac{\partial L(\omega_j, \eta)}{\partial \eta} = 2 \left( \sum_{j=1}^{n} w_j - 1 \right) = 0.
\]

Then, we obtain

\[
\omega_j = \frac{\sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 - 2}{\sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 - 2} \cdot \frac{1}{n},
\]

\[
\eta = \frac{1}{n} \left( \sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 \right) - 2.
\]

Then, we have

\[
\eta = \frac{1}{n} \left( \sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 \right) - 2,
\]

\[
\omega_j = \frac{\sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 - 2}{\sum_{i=1}^{m} \left( \mu_j + \omega_j \right) \sum_{i=1}^{m} S(\bar{Y}_{ij})^2 - 2} \cdot \frac{1}{n}.
\]
Then we get the weight vector $W = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of attribute $C_j (j = 1, 2, \ldots, n)$ with $\omega_j \geq 0$ and $\sum_{j=1}^{n} \omega_j = 1$.

5. Decision Analysis for 3WDs with SVNDTRS

In the existing 3WDs models, the LF is an important parameter. How to use the appropriate information form to represent the LF is very important. In addition to describing imperfect information in real decision-making system, SVNs can handle uncertain and inconformity information flexibly and effectively. Furthermore, how to determine the conditional probability is also the key to 3WDs. The conditional probability in many documents is subjectively given by the decision makers, making the decision results too subjective. The GRA method provides a unique perspective for the assessment of conditional probability. According to the above presentation, we constructed a new SVNDTRS model based on the GRA method. In this section, we mainly describe the decision rules of the SVNDTRS model and how to determine the conditional probability by the GRA method.

5.1. Basic Rules of SVNDTRSs

In light of the results of Definitions 5-6 and (P)-(N), we give the decision rules (P1)-(N1) as follows:

(P1): if $S(R(a_p | [x])) \leq S(R(a_B | [x]))$ and $S(R(a_p | [x])) \leq S(R(a_N | [x]))$, then $x \in \text{POS}(\Gamma)$,

(B1): if $S(R(a_B | [x])) \leq S(R(a_p | [x]))$ and $S(R(a_B | [x])) \leq S(R(a_N | [x]))$, then $x \in \text{BND}(\Gamma)$,

(N1): if $S(R(a_N | [x])) \leq S(R(a_p | [x]))$ and $S(R(a_N | [x])) \leq S(R(a_B | [x]))$, then $x \in \text{NEG}(\Gamma)$,

where the similar degree $S(R(a_i | [x])) (i = P, B, N)$ of the expected losses is calculated as

$$S(R(a_i | [x])) = \frac{T(R(a_i | [x]))}{T(R(a_i | [x]))}$$

5.2. Computing Conditional Probability of SVNDTRS with GRA Method

Let $\bar{Y} = (\bar{Y}_{ij})_{mn} = (T_{ij}, I_{ij}, F_{ij})_{mn}$ be a SVNS-based decision matrix, where $T_{ij}, I_{ij},$ and $F_{ij}$ represent the membership degree, hesitancy degree, and nonmembership degree of evaluation for the attribute $C_j$ with respect to the alternative $A_i$.

Firstly, we determine the relative RNNIS $X^+$ and the RNNIS $X^-$. The RNNIS $X^+$ can be defined as

$$X^+ = \{x_1^+, x_2^+, \ldots, x_n^+\},$$

where $x_i^+ = (T_i^+, I_i^+, F_i^+) = (\max_{i \in \text{slsm}} T_{ij}, \min_{i \in \text{slsm}} I_{ij}, \min_{i \in \text{slsm}} F_{ij})$ for $j = 1, 2, \ldots, n$.

And the RNNIS $X^-$ can be defined as

$$X^- = \{x_1^-, x_2^-, \ldots, x_n^-\},$$

where $x_i^- = (T_i^-, I_i^-, F_i^-) = (\min_{i \in \text{slsm}} T_{ij}, \min_{i \in \text{slsm}} I_{ij}, \min_{i \in \text{slsm}} F_{ij})$ for $j = 1, 2, \ldots, n$. The grey relational coefficient (GRC) between $x_i$ and the RNNIS $X^+$ on the $j$–th attribute is

$$g_{ij}^+ = \frac{\min_{i \in \text{slsm}} \min_{i \in \text{slsm}} \Delta_{ij}^s + \xi \max_{i \in \text{slsm}} \max_{i \in \text{slsm}} \Delta_{ij}^s}{\Delta_{ij}^s + \xi \max_{i \in \text{slsm}} \max_{i \in \text{slsm}} \Delta_{ij}^s},$$

where $\Delta_{ij}^s = d_N(x_i, x_j^s)$, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$, $\xi = 0.5$. 
The GRC between $x_i$ and the RNNIS $X^+\text{ }$is
\[
G^+_i = \sum_{j=1}^{n} w_j g^+_i, \quad i = 1, 2, \ldots, m. \tag{43}
\]

The GRC between $x_i$ and the RNNIS $X^-$ on the $j$ - th attribute is
\[
G^-_i = \frac{\min_{1 \leq j \leq m} \min_{1 \leq j \leq n} \Delta^-_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^-_{ij}}{\Delta^-_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^-_{ij}}, \tag{44}
\]
where $\Delta^-_{ij} = d_N(x_{ij}, x^+_i) \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n, \xi = 0.5$.

The GRC between $x_i$ and the RNNIS $X^-$ is
\[
G^-_i = \sum_{j=1}^{n} w_j g^-_i, \quad i = 1, 2, \ldots, m. \tag{45}
\]

Compute the neutrosophic relative degree (NRRD) $H_i$:
\[
H_i = \frac{G^+_{i}}{G^+_{i} + G^-_{i}} = \sum_{j=1}^{n} w_j \left(\frac{\min_{1 \leq j \leq m} \min_{1 \leq j \leq n} \Delta^+_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^+_{ij}}{\Delta^+_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^+_{ij}}\right)
\sum_{j=1}^{n} w_j \left(\frac{\min_{1 \leq j \leq m} \min_{1 \leq j \leq n} \Delta^-_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^-_{ij}}{\Delta^-_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^-_{ij}}\right) + \sum_{j=1}^{n} w_j \left(\frac{\min_{1 \leq j \leq m} \min_{1 \leq j \leq n} \Delta^+_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^+_{ij}}{\Delta^+_{ij} + \xi \max_{1 \leq j \leq m} \max_{1 \leq j \leq n} \Delta^+_{ij}}\right).
\]

where \( \bar{Y}_{ij} \) is the preference value given by the decision maker
for the alternatives \( A_i \in A (i = 1, 2, \ldots, m) \) to the attribute
\( C_j \in C (j = 1, 2, \ldots, n) \). Based on these necessary conditions, the decision results are required.

Next, we use the SVNDTRS model to solve this MADM problem. The method comprises the following procedures:

Step 1: according to the nonempty object sets and nonempty attribute sets, establish the LF matrix.

Step 2: with Section 4, determine the weight \( W = (w_1, w_2, \ldots, w_n)^T \) of all attributes.

Step 2-1: according to the complementary judgment matrix given by experts, the subjective weight of attributes is calculated by formula (25):
\[
\mu_j = \frac{\sum_{k=1}^{n} q_{jk} + (n/2) - 1}{n(n-1)}. \tag{48}
\]

Step 2-2: according to the entropy method, the objective weight of attributes is computed by formula (28):
\[
\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)} = \frac{\sum_{i=1}^{m} \left(T_{ij}(x_i) + F_{ij}(x_i)\right)|I_{ij}(x_i) - I_{ij}(x_i)|}{\sum_{j=1}^{n} \left(T_{ij}(x_i) + F_{ij}(x_i)\right)|I_{ij}(x_i) - I_{ij}(x_i)|}. \tag{49}
\]

Step 2-3: based on the optimal combination weighting model with minimum total deviation, the combination weights of attributes are calculated by formula (32):
\[
\omega_j = \left(\frac{(\mu_j + \omega_j)\sum_{i=1}^{m} (S(\bar{Y}_{ij}))}{2} - \left(\frac{\sum_{j=1}^{m} (\mu_j + \omega_j)\sum_{i=1}^{n} (S(\bar{Y}_{ij}))}{2}\right)^2 - 2\right) / (n). \tag{50}
\]
Step 3: for (40) and (41), identify the RNPIS $X^+$ and the RNNIS $X^-$. 

Step 4: in light of (46), calculate the NRRD of $x_i$ to the RNPIS $X^+$, expressed as $H_i$. Furthermore, reckon the value of conditional probability of the $x_i$ as $P(\Gamma | x_i) = H_i$, ($i = 1, 2, \ldots, m$). 

Step 5: on the basis of the LFs and the conditional probability, use (37)–(39) to compute cosine similarity $S(R(a_i | x))$ ($i = P, B, N$) of the expected losses. 

Step 6: in line with the decision rules (P1)-(N1), further determine the decision results of each alternative.

6. A Numerical Example

A company is preparing to expand its production scale to cope with the complexity of the market and the diversity of demand. At present, it is planning to select long-term suppliers from six suppliers. For core enterprises, how to select suppliers scientifically and reasonably plays a crucial role in cost control, supply chain stability, and risk management. The six suppliers form the sets $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$. The company considers the four attributes of the supplier in terms of quality, production capacity, after-sales service, and management ability. The four attributes constitute the attribute sets $C = \{C_1, C_2, C_3, C_4\}$, and the weight of each attribute is unknown. The six alternatives $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ are appraised by the decision maker under the four attributes $C = \{C_1, C_2, C_3, C_4\}$, and the decision matrices $Y = (\bar{Y}_{ij})_{6 \times 4}$ are built, as listed in Table 3.

<table>
<thead>
<tr>
<th>Table 3: SVNS decision matrix.</th>
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<tbody>
<tr>
<td>$C_1$</td>
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<tr>
<td>$A_1$</td>
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<td>$A_2$</td>
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<td>$A_3$</td>
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<td>$A_4$</td>
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<tr>
<td>$A_5$</td>
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<tr>
<td>$A_6$</td>
</tr>
</tbody>
</table>

Step 2-3: based on the optimal combination weighting model with minimum total deviation, the combination weights of attributes are calculated by formula (32):

$$\omega = (0.2250, 0.2773, 0.2425, 0.2552)^T.$$ (52)

Step 3: for (40) and (41), identify the RNPIS $X^+$ and the RNNIS $X^-$. 

$$X^+ = [(0.7, 0.0, 0.0, 0.1), (0.9, 0.0, 0.1), (0.7, 0.1, 0.1), (0.7, 0.1, 0.1)],$$

$$X^- = [(0.4, 0.3, 0.4), (0.4, 0.3, 0.4), (0.3, 0.2, 0.5), (0.5, 0.2, 0.3)].$$ (53)

Step 4: on the basis of (46), we compute the NRRD of $x_i$ with respect to the RNPIS $X^+$, expressed as $H_i$. Furthermore, we reckon the value of conditional probability of $x_i$ as $P(\Gamma | x_i) = H_i$, ($i = 1, 2, \ldots, 6$). The results are shown in Table 4.

Step 5: on the basis of the LFs and the conditional probability, we can calculate cosine similarity of the expected losses by using (37)–(39). The results are listed in Table 5.

In order to more intuitively express the 3D results of each alternative, we present the results of Table 5 in Figure 1.

Step 6: according to the decision rules (P1)-(N1), we can further determine the result of each alternative: $A_k \in \text{POS}(\Gamma)$, $A_3, A_5, A_6 \in \text{BND}(\Gamma)$, and $A_1 \in \text{NEG}(\Gamma)$. From the above results, we can see that $A_6$ should be invested, $A_1$ should not be invested, and $A_3, A_5, A_6$ and $A_2$ whether or not should be invested should be further studied.

6.2. Comparative Analysis. For thoroughly testing the availability and significance of our development method, two methods are adopted for comparative analysis, including the grey relation analysis method based on Biswas et al. [32] and the extended TOPSIS method proposed by Zhang and Wu [40]. Furthermore, we apply these two methods to the same example as in this paper.

(1) The decision result based on the GRA method proposed by Biswas et al. [32].

Utilizing the GRA method, the GRC and the NRRD are indicated in Table 6.
From Table 4, we can get $H_6 > H_5 > H_1 > H_4 > H_3 > H_2 > H_1$, so $A_6 > A_5 > A_3 > A_4 > A_2 > A_1$. Thus the best alternative is $A_6$.

(2) The decision result based on the extended TOPSIS method proposed by Zhang and Wu [40].

The weights of attributes are determined by maximizing deviation method as follows:

$$\omega = [0.2561, 0.3069, 0.2709, 0.1661]^T.$$  \hspace{1cm} (54)

We can get the relative closeness $RC_i$ of each alternative to the RNPIS $X^2$ (Table 7).

From Table 7, we can get $RC_6 > RC_4 > RC_5 > RC_2 > RC_3 > RC_1$, so $A_6 > A_4 > A_5 > A_4 > A_2 > A_1$. Thus the best option is $A_6$.

From the above results, we can see that the GRA method proposed by Biswas et al. [32] has the same ranking results as that of our proposed method and has the same best option $A_6$. In this way, we can demonstrate the effectiveness of our proposed method. Nevertheless, the decision-making method in [32] is the attribute weight obtained by the entropy method, without considering the subjective preferences of decision makers. In this paper, the objective weight is obtained by the entropy method, while the subjective weight is obtained by the AHP method, and the combination weight is obtained based on the method of minimum total deviation. Moreover, the GRA method based on [32] has a drawback, which can only give the ranking. The decision results based on SVNDTRS model not only gives the ranking but also provides the relevant semantic explanation for the selection of each alternative, which is more scientific and flexible than the GRA method [32].
Comparing the ranking obtained by the MADM method in [40] and the proposed method, we get the same best choice \( A_0 \), but the positions of alternatives \( A_1 \) and \( A_4 \) are not the same. Among the rankings obtained in this paper, alternative \( A_3 \) is better than alternative \( A_5 \). But in [40], alternative \( A_4 \) is better than alternative \( A_5 \). The reason for this inconsistency is that the attribute weights of the two methods are different. In [40], attribute weights are obtained by maximizing deviation, ignoring the subjective preferences of decision makers, and \( C_3 \) is the most important attribute. This paper combines the subjective weights obtained by the AHP method and finds \( C_1 \) as the most important attribute. This paper not only considers the subjective preferences of decision makers but also takes full advantage of the evaluative information of decision-making objects, reduces the subjective randomness of weight, and achieves the unity of subjective and objective. Therefore, the ranking of our proposed method is more reasonable.

In a word, the 3WD method based on the SVNNDTRS model proposed in this paper has the following advantages: (1) It not only considers the subjective preferences of decision makers but also makes the best of objective information of decision objects to realize the unity of subjectivity and objectivity. (2) Using the GRA method to obtain the conditional probability of 3WDs provides a new viewpoint for obtaining the conditional probability of 3WDs. (3) It not only gives the ranking results of each alternative but also provides the corresponding semantic explanation for the selection of alternatives.

7. Conclusion

In this paper, we extended 3WDs to the environment of SVNNS and used SVNNS to express the evaluation values and LFs given by decision makers. We also proposed a method to obtain the attribute weights. We applied the AHP method to get the subjective weight of attributes and entropy method to obtain the objective weight and established the combination weight of attributes based on the principle of minimum total deviation. It not only considers the subjective preferences of decision makers but also takes most advantage of the evaluative information of decision makers to realize the unity of subjectivity and objectivity. Based on this, we proposed a SVNNDTRS model based on the GRA method. The NRRD calculated by the GRA method is defined as the conditional probability of 3WDs. Finally, we extended the proposed method to a numerical example of supplier selection and compared it with other methods to demonstrate the effectiveness of our proposed method. We presented a new idea for the determination of conditional probability in 3WDs and a new solution for the LF expressed by SVNNS in 3WDs. In future, we will study the application of 3WDs to settle the MADM problem composed of fuzzy information.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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