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Research Article

Joint Pricing and Inventory Replenishment Decisions with Returns and Expediting under Reference Price Effects

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This paper considers a single-item joint pricing and inventory replenishment problem under reference price effects in consecutive T periods. Demands in consecutive periods are sensitive to price and reference price with general demand distribution. At the end of each period, after the demand realization, a firm can return excess stocks to a supplier or place an expediting order to reduce the loss by shortage. Unfilled demands are fully backlogged. In order to maximize the total expected discounted profit with reference price effects the optimal pricing and inventory replenishment policies for regular order and the inventory adjustment decisions for returning/expediting are derived. The optimal replenishment policy for regular order is a base-stock policy, the optimal pricing policy is a base-stock-list-price policy, and the optimal policy for returning/expediting inventory adjustment follows a dual-threshold policy. Furthermore, the analysis of the operational impacts (from the perspective of adding returning/expediting and reference price effects, respectively) is researched. Numerical results also show that considering both returning/expediting and reference price effects is more profitable than considering only one of them.

1. Introduction

Reference price, as the cognitive price of customers, is formed through the customers' repeated purchasing experiences. Reference price was first derived from the adaptation level [1], and its definition was still vague. Since then, prospects theory [2] and behavioral sciences [3] elaborated the reference price in detail; they indicated that customers will remember past prices when they repeat the purchase of the commodity. Especially, the growing information transparency with the advent of the Internet has made it more convenient for customers to learn the historical price information of a commodity. Customers often develop their own "fair price" which is named as the reference price after observing past prices of a commodity. If the current sales price is lower (higher) than the reference price, customers think that price is a gain (loss) price. Hence they are more likely (less inclined) to make the purchase. This phenomenon is usually referred to as the reference price effects. Customers are called loss averse (loss neutral) if customers perceived losses are more sensitive than their perceived gains. Otherwise, they are called loss seeking. Many firms are aware of the importance of this effect and take it into account to maximize their profits. For example, Alibaba who is the largest Internet company in China and is also the second largest Internet company in the world sold 213.5 billion yuan during "double 11" in 2018, which exceeded the record of 168.2 billion yuan in 2017. In addition, the number of express delivery during the "double 11" period exceeded 1 billion. These show that more and more customers are willing to choose to add the desired commodities to the "shopping cart" firstly and wait until the "double 11" day to pay for them at a promotion (lower) price. Actually, this phenomenon of "double 11" is a typical strategic consumer behavior. When people encounter the commodity they need they will no longer impulsively buy it immediately. Instead, they will wait the sales price reasonably comparing to the reference price before purchasing. There are similar examples in the market which are sales for electronic, clothing, and other tidal commodities [4]. Hence, the reference price effect has an important impact on demand and therefore becomes an indispensable part of firms' decision making.

With the economic globalization and the increasingly fierce market competition, improving supply chain performance becomes more crucial for firms and the revenue management theory in combining dynamic pricing and inventory control methods have significant effects on the improving of supply chain performance. Therefore, in making decisions, many firms not only consider the impact of reference price effects on pricing strategies, but also the impact on inventory strategies simultaneously, such as Amazon, Dell, and Wal-Mart [5]. Besides, the existence of strategic consumers makes demand difficult to accurately estimate. In order to match supplies with demands in a cost-effective way, firms require their suppliers to provide more flexibility for the replenishment process, such as the opportunities of return and expediting. One typical example is the eHub system launched by Cisco in 2001 [6]. It is a trading e-marketplace that provides a platform for the planning and executing tasks across the company's extended manufacturing supply chain. By eHub, Cisco connects with its suppliers to build up a flexible/agile supply channel, where Cisco is allowed to return the excess stock and to place expediting orders. In this way, Cisco can reduce the waste in inventory and increase the speed to response to customers' needs. Similar operational practice is also observed in Toyota and Motorola [7, 8]. As a result, the impact of flexibility for the replenishment on the operational efficiency for firms is also a very noteworthy aspect. It is precisely because the reference price and strategy customers have great effects on firms' pricing and ordering operations; therefore, it is essential to investigate the joint pricing and flexible ordering strategies with considering the effects of the reference price.

As a matter of fact, the inventory strategies (including regular and returning/expediting inventory) and reference price interact with each other in a sales period. So it is necessary to study how the reference price affects both the regular and returning/expediting replenishment policies. However, to the best of our knowledge, the reference price effect has not been considered in the study of flexibility for the replenishment in recent years (see, for example, [9–13]). Besides, most recent research on coordinating pricing and inventory control problem with reference price effects has focused on the ordering policy for regular inventory (see, for example, [14–17]). Little attention has been paid to the flexibility for the replenishment, such as returns and expediting [9]. These motivate us to do the explore in this aspect in our paper.

The strategy customers' behavior has great impact on the joint decisions on pricing and inventory by demand. In a single-item, periodic-review model, demands in consecutive periods are price and reference price sensitive random variables. We study the joint decision problem on determining pricing and inventory control strategies with returns and expediting under reference price effects. When introducing returning/expediting decisions, the following technical problems will arise: (1) whether the supermodularity of the value function can be guaranteed when the decision variable of returning/expediting is added; (2) whether the regular replenishment strategy established in previous literature, such as Güler et al. [14, 15] and Chen et al. [17], is still the base-stock policy; (3) how the reference price impacts the returning/expediting decision making; (4) do firms benefit from the simultaneous consideration of returning/expediting and reference price. To answer these research questions, this paper develops a dynamic programming model to find the optimal dynamic policies that determine the pricing and inventory replenishment for regular order and inventory adjustment decisions for returning/expediting adjustment under reference price effects in each period so that the total expected discounted profit is maximized. For a very general stochastic demand function, we show that the optimal inventory replenishment and optimal pricing policies for regular order are base-stock policy and base-stock-list-price policy, respectively. The optimal policy for returning/expediting inventory adjustment follows a dual-threshold policy. The preservation of supermodularity of value function enables us to discuss the effects of reference price on returning/expediting. We further study the operational impacts of returning/expediting under reference price effects by comparing with the model proposed by Chen et al. [17] and Zhu [9], respectively.

The remainder of this paper is organized as follows. Section 2 reviews related literature. We present the finite period model with stochastic dynamic programming in Section 3 and characterize the optimal policies in Section 4. The infinite planning horizon problem is discussed in Section 5. Section 6 investigates the operational impacts from the perspective of adding returning/expediting and reference price effects, respectively. Numerical results are represented in Section 7. Section 8 concludes our paper.

2. Literature Review

The work is mainly related to two streams of literature: (i) inventory models with supply flexibility and (ii) joint pricing and inventory control under reference price effects.

In the literature on inventory models with supply flexibility, the earliest studies on this stream can be traced back to the late 1980s and the early 1990s. Eppen and Iyer [18] study a special form of the quantity flexibility contract, which allows the retailer to return a portion of its purchase to the supplier. Henig et al. [19] consider a minimum ordering quantity contract under which the firm decides whether to order the prefixed contract amount or order more than this amount at the beginning of each period, but an incremental cost will be charged for the excess amount ordered. They show that the optimal policy is a modified order-up-to policy under the assumption that the demands are independent and identically distributed. Further, the optimal policies are threshold type. Tsay and Lovejoy [20] extend the replenishment decision problems to a three-stage setting where the authors use a heuristic approach to transform the original stochastic problem into the deterministic problem that can be solved more easily. Sethi et al. [21] study the impact of forecast quality and the level of flexibility by quantity flexibility contract on the ordering decisions. Ben-Tal et al. [22] apply robust optimization to analyze the replenishment decisions under the quantity flexibility contract. Feinberg and Lewis [23] consider a broader problem, where in addition to increasing inventory or disposing of it, the manager can borrow or store some inventory for one period. They show that the optimal inventory policies depend on four thresholds. Yin and Rajaram [24] consider the emergency ordering model and prove the optimality of state-dependent (s, S, p) policy for a class of Markovian demand. Lian and Deshmukh [25] use a frozen fence to restrict any change in the next one or more periods, penalty costs to discourage excessive modifications in other periods, and price discount to boost advanced orders. Fu et al. [26] analyze the effects of regular replenishment and expediting on the lead time from the viewpoint of inventory cost minimization. Chen et al. [27] use heuristic algorithms to derive the optimality of complex inventory systems based on the emergency ordering model of cost-price changes. Zhu [9] studies the pricing and inventory strategy with return and expediting and shows that the optimal inventory policy is a modified base-stock policy, the optimal pricing policy is a modified base-stock-list-price policy, and the optimal policy for inventory adjustment follows a dual-threshold policy. Fu et al. [10] consider the newsvendor problem with multiple options of expediting. Zhou and Chao [11] consider a periodic-review inventory system with regular and expedited supply modes, they show that the optimal inventory policy is determined by two state-independent thresholds, one for each supply mode, and the optimal price follows a list-price policy. Roni et al. [12] develop a stochastic inventory model based on a hybrid inventory policy with both regular and emergency orders responding to regular and surge demands. Li et al. [13] study a quantity flexibility contract that the retailer commits an amount of quantity of newly developed commodities, and in return the manufacturer allows the retailer to adjust the order quantities of the commitment quantities based on the inventory balance status and the likely customer demand. They show that, with this arrangement, both parties can attain maximum profit under the concept of the synergy effect. This stream of literature analyzes the impacts of the flexibility for replenishment process on firm's operation, but they do not take the customers' behavior into consideration. Even if Cachon and Swinney [28, 29] study the impact of customers' strategic behavior on quick response, they just pay attention to price adjustment or enhance system design. A more complete literature review of this line of research is provided in recent paper by Yao and Minner [30].

As an important factor affecting customers' purchase decision, reference price has received much attention from researchers. Researches on reference price effects mainly focus on pricing strategy. Krishnamurthi et al. [31] study the impact of reference price effects on brand selection and purchase quantity and show that customers have the characteristics of brand loyalty under symmetrical reference prices, while it does not appear such characteristics under the asymmetric reference price effects. Greenleaf [32] first analyzes the firm's pricing strategy with reference price effects and explains how the reference price effects affect the promotion decision of a firm during a period; it is concluded that firm's pricing decision when considering the reference price effects will increase the firm's profits. Some recent works explore how pricing strategies should account for the reference price effects, for example, see Kopalle et al. [33]; Fibich et al. [34, 35]; Popescu and Wu [36]; Nasiry and Popescu [37]; Chen et al. [38]; Hu et al. [39]; Wang (2016) and the references therein. Arslan and Kachani [40] and Mazumdar et al. [41] provide reviews of dynamic pricing model with reference price effects. However, there are few studies that consider the coordination of pricing and inventory control under reference price effects. This stream of research starts from Gimpl-Heersink [42, 43], who proves the optimality of the base-stock-list-price for single-period and two-period model when the customers are loss neutral. However, the optimality of the base-stock-list-price is stricter for the multiperiod setting. Urban [44] analyzes a single-period joint pricing and inventory model with symmetric and asymmetric reference price effects and shows that the consideration of reference price has a substantial impact on the firm's profitability. Even if the single-period profit model function is nonconcave, Zhang [45] uses a class of transformation techniques to prove the optimality of the base-stock-list-price. It is further proved that when the planning horizon is infinite, the optimal reference price trajectories converge to a steady state in both the loss neutral and loss averse cases. Taudes and Rudloff [46] provide an application of the two-period model from Gimpl-Heersink [42, 43] to electronic commodities. Güler [47] studies the joint pricing and inventory model of a single commodity under periodic review and investigates the impact of reference price on the firm's average profit via the perspective of numerical analysis. Güler et al. [14] extend the model of Gimpl-Heersink [42, 43] to the concave demand function, and they address the nonconcavity of the revenue function by combining the transformation technique proposed by Zhang [45] and the inverse demand function. The optimality of the state-dependent order-up-to strategy is proved for the transformed concave revenue function model. Güler et al. [15] use the safety stock as a decision variable to characterize the steady state solution to the problem when the planning horizon is infinite. Wu et al. [16] studied the optimal dynamic pricing and inventory strategy when strategic customers choose the purchase time dynamically based on historical and current prices of the commodity. Chen et al. [17] introduce a new type of concave transform technique to ensure the profit function to be concave by using the preservation property of supermodularity in parameter optimization problems with nonlattice structure proposed by Chen et al. [48] and then prove the optimality of the base-stock-list-price strategy. This stream of literature captures the reference price effects on joint pricing and inventory control, but it cannot take the flexibility for replenishment process into consideration. For other related works in this stream of research, interested readers may refer to the review by Ren and Huang [49].

Although either the dynamic pricing and inventory strategy or the supply flexibility strategy is well developed by these papers, few of them delve into the discussion of joint pricing and inventory decision with supply flexibility under the reference price effects. This paper considers the joint pricing and inventory (including regular inventory and returning/expediting inventory) control problem with the reference price effects. Since the inventory of a commodity can influence the customer's reference price and the reference price has a significant impact on the customer's purchasing behavior. Actually, the inventory strategy and reference price interact with each other, especially within a sales period, so it is necessary to study the reference price effects' impact on both regular and returning/expediting replenishment policies. This gives reason for us to investigate the joint strategy of both the pricing and inventory with the opportunity of returns and expediting under effects of reference price.

3. Model Description

Consider a single-item, periodic-review inventory problem in a finite planning horizon with T $(1 \le T \le \infty)$ periods. The demand in period t is denoted by D_t, and $\{D_t \mid t = 1, 2, \ldots, T\}$ are nonnegative random variables. Similar to Güler et al. [15] and Chen et al. [17], the demand in period t is given by

$$D_t\left(p_t, r_t, \epsilon_t\right) = \beta_t d_t\left(p_t, r_t\right) + \epsilon_t, \tag{1}$$

where $d_t(p_t, r_t)$ is the average demand which is a function of the sales price per unit item, denoted by p_t, and the reference price r_t in period t, β_t and ϵ_t are random variables of which means are 1 and 0, respectively, and independent of p_t and r_t. This demand function is so general that both the additive and multiplicative demand models are its special cases.

The average demand function is given by $d_t(p_t, r_t) = \mu_t(p_t) + R_t(r_t - p_t, r_t)$, where $\mu_t(p_t) = d_t(p_t, p_t)$ is the base demand and $R_t(r_t - p_t, r_t) = \eta^+ \max\{r_t - p_t, 0\} + \eta^- \min\{r_t - p_t, 0\}$ is the reference price effects on demand [1]. The nonnegative parameters η^+ and η^- measure the demand sensitivities to gain and loss of the reference cost to the sales price, respectively. Such as if $\eta^+ \le \eta^-$ the demand is loss averse, if $\eta^+ = \eta^-$ the demand is loss neutral, and if $\eta^+ \ge \eta^-$ the demand is loss seeking. For more information about $R_t(r_t - p_t, r_t)$, refer to Güler [14, 15] and the references therein. Furthermore, the average demand function has some properties as follows.

Assumption 1. The average demand function $d_t(p_t, r_t)$ is concave, bounded, nonnegative, and continuous, strictly decreasing in p_t and increasing in r_t for $t = 1, 2, \ldots, T$.

It is worth mentioning that the concave hypothesis of the average demand function $d_t(p_t, r_t)$ is discussed in Güler et al. [14, 15] where customers being loss neutral or loss averse are presented in some cases. Hence, the customers in our model are loss neutral or loss averse. Moreover, with $p_t(d_t, r_t)$ being the inverse function of the average demand $d_t(p_t, r_t)$, Assumption 1 implies that $p_t(d_t, r_t)$ is strictly decreasing in d_t and increasing in r_t in every period t (referring to Proposition 1, [15]). So determining the price p_t is equivalent to determining the average demand d_t. Accordingly in the follow-up discussion, we will focus on finding the optimal average demand d_t in period t. Hence, we assume that the feasible region of the average demand d_t in period t is $[\underline{d}_t, \overline{d}_t]$ with $\underline{d}_t \le \overline{d}_t$, $\underline{d}_t \ge 0$, and $\overline{d}_t < +\infty$.

Now the cost structure of the inventory system is introduced as follows. The sales price per unit item p_t has a greatest lower bound $\underline{p}$ and a greatest upper bound $\overline{p}$ with $\underline{p} \le p_t \le \overline{p}$, which are independent of their period t. The reference price of the next period (period $t+1$) depends on the reference price and sales price in the current period (period t), of which modeling by the evolution of the reference price is the exponential smoothing model [14, 15, 17, 38, 42, 43], i.e.,

$$r_{t+1} = \alpha r_t + (1 - \alpha) p_t, \tag{2}$$

where α $(0 \le \alpha < 1)$ is the memory parameter. This evolution shows that the reference price is generated by exponentially weighting historical prices. The larger the memory parameter α, the longer the memory of the historical prices. If α is high, then customers have a long memory and past price effect is larger. If α is small, then current price has a greater effect than the past on the reference price. The initial reference price is given by $r_1 \in [\underline{p}, \overline{p}]$, and hence all r_t belong to the interval. Furthermore, there is purchasing cost c_t per unit item in period t for regular order which is smaller than the greatest lower bound of the sale price $\underline{p}$, i.e., $\underline{p} \ge c_t$. In period t we have other costs listed as follows:

- v_t = the purchasing cost per unit item for the expediting order;
- s_t = the rebate per unit return;
- h_t = the holding cost per unit item;
- b_t = the penalty cost per unit backorder.

To avoid a trivial solution, the following inequalities are satisfied $b_t > v_t > c_t > s_t \ge 0$, where $c_t > s_t$ guarantees that the firm has no incentive to make profit by ordering too much at the beginning of the period, $v_t > c_t$ indicates that the expediting order incurs a higher cost, and $b_t > v_t$ induce the firm to place the expediting order only in emergence case.

The sequence of events is as follows with all leadtimes are zero. First, at the beginning of the tth period $(t = 1, 2, \ldots, T)$, referring to the initial inventory level x_t and current reference price r_t the regular order q_t is placed. The order arrives immediately. So the inventory level after regular ordering is $y_t = x_t + q_t$. Second, after the demand realizes by the end of the tth period inventory manager either places an expediting order or returns the excess stock based on the current reference price r_t. The expediting order will be delivered with an expediting shipping mode so that it can also be used to satisfy the demand in the current period. The quantity of the expediting order or the return is denoted by z_t. If z_t is positive, the inventory manager will return commodities back, while a negative z_t incurs an expediting order. Further, due to the supply or return capacity constraints, the regular replenishment is limited by Q_t and the expediting replenishment or return is not more than M_t $(M_t \ge 0)$. So there may be the unsatisfied demand which is backordered after expediting replenishment. Otherwise, due to the capacity limitation of return, the firm may still have some surplus inventory. Third, all costs and revenue are incurred.

Given the initial inventory x_t and reference price r_t in period t $(t = 1, 2, \ldots, T)$. $V_t(x_t, r_t)$ represents the maximization expected profit sum from period t onward and

$J_t(y_t, d_t, r_t)$ is the expected sum of the profit from t period. Then, the inventory cost control problem can be formulated as a stochastic dynamic programming and the Bellman equations for the inventory cost control can be written as follows:

$$V_t(x_t, r_t) = \max_{x_t \le y_t \le x_t + Q_t, \underline{d}_t \le d_t \le \overline{d}_t} J_t(y_t, d_t, r_t) + c_t x_t, \tag{3}$$

where

$$\begin{aligned} J_t(y_t, d_t, r_t) = \mathsf{E}\Bigg[& d_t \cdot p_t(d_t, r_t) - c_t y_t \\ & + \max_{-M_t \le z_t \le (w_t)^+} H_t(z_t, w_t, r_{t+1}) \Bigg], \end{aligned} \tag{4}$$

$y_t = x_t + q_t$ is the inventory level after placing the regular order, $w_t = y_t - D_t$ is the ending inventory level before returning/expediting in period t, and E denotes the averaging operator. The first term on the right-hand side of (4) is the revenue in period t and $c_t q_t$ in the second term $c_t y_t = c_t(x_t + q_t)$ is the purchasing cost of the regular order;

$$\begin{aligned} H_t(z_t, w_t, r_{t+1}) = {} & s_t(z_t)^+ - v_t(-z_t)^+ - h_t(w_t - z_t)^+ \\ & - b_t(z_t - w_t)^+ \\ & + \gamma V_{t+1}(w_t - z_t, r_{t+1}), \end{aligned} \tag{5}$$

where $(a)^+ = \max\{a, 0\}$, $\gamma \in [0, 1)$ is the discount factor and the first term at the right-hand side is return rebate and the second is the cost for the expediting order. The third and the fourth are the holding cost and the backorder penalty cost, respectively. The last term is the profit function for the next period.

Moreover, the terminal value of the inventory is given by $V_{T+1}(x_{T+1}, r_{T+1}) = 0$, which means that there is no value left after the planning horizon ends at period T.

Furthermore, similar to Güler et al. [15], we make the following assumption.

Assumption 2. $p_t(d_t, r_t)$ which is the inverse function of the average demand $d_t(p_t, r_t)$ is supermodular in (d_t, r_t) and the revenue function $d_t \cdot p_t(d_t, r_t)$ is joint concave in (d_t, r_t).

Mention Assumption 2; according to Theorem 6 in Güler et al. [15] the revenue function $d_t \cdot p_t(d_t, r_t)$ is supermodular in (d_t, r_t).

4. Optimal Policy and Its Analysis

In this section, the optimal decision variables are characterized including the ordering variables for regular and returning/expediting as well as pricing strategies in the inventory system. Firstly for any given x_t and r_t in period t, in order to prove the uniqueness of the optimal decision the concavities of H_t and J_t are important and needed.

Theorem 3. *For $t = 1, 2, \dots, T$, we have the following:*

(i) *$H_t(z_t, w_t, r_{t+1})$ is joint concave in (z_t, w_t, r_{t+1}) and supermodular in (z_t, w_t);*

(ii) *$J_t(y_t, d_t, r_t)$ is joint concave in (y_t, d_t, r_t) and $V_t(x_t, r_t)$ is joint concave in (x_t, r_t);*

(iii) *$V_t(x_t, r_t)$ is increasing in x_t for a given r_t;*

(iv) *$V_t(x_t, r_t)$ is supermodular in (x_t, r_t).*

Proof. See Appendix. □

Define

$$\begin{aligned} & \left(y_t^{*}(x_t, r_t), d_t^{*}(x_t, r_t)\right) \\ & \quad = \arg \max_{x_t \le y_t \le x_t + Q_t, \underline{d}_t \le d_t \le \overline{d}_t} J_t(y_t, d_t, r_t), \end{aligned} \tag{6}$$

then $y_t^*(x_t, r_t) = x_t + q_t^*(x_t, r_t)$ and $p_t^*(x_t, r_t) = p_t(d_t^*(x_t, r_t), r_t)$. Note that $y_t^*(x_t, r_t)$, $d_t^*(x_t, r_t)$, and $q_t^*(x_t, r_t)$ are the optimal decisions in period t when the initial inventory is x_t and current reference price is r_t. Since $J_t(y_t, d_t, r_t)$ is joint concave in (y_t, d_t, r_t), $V_t(x_t, r_t)$ can be obtained through maximizing $J_t(y_t, d_t, r_t)$ sequentially, i.e.,

$$V_t(x_t, r_t) = \max_{x_t \le y_t \le x_t + Q_t} \left[-c_t y_t + G_t(y_t, r_t)\right] + c_t x_t, \tag{7}$$

where

$$\begin{aligned} G_t(y_t, r_t) = \max_{\underline{d}_t \le d_t \le \overline{d}_t} \mathsf{E}\Bigg[& d_t \cdot p_t(d_t, r_t) \\ & + \max_{-M_t \le z_t \le (y_t - \beta_t d_t - \epsilon_t)^+} H_t(z_t, y_t - \beta_t d_t - \epsilon_t, \alpha r_t \\ & + (1 - \alpha) p_t(d_t, r_t)) \Bigg]. \end{aligned} \tag{8}$$

The next theorem states that $G_t(y_t, r_t)$ is concave in (y_t, r_t); $y_t^*(x_t, r_t)$ and $d_t^*(x_t, r_t)$ are increasing in initial inventory level x_t, while $q_t^*(x_t, r_t)$ and $p_t^*(x_t, r_t)$ are decreasing in x_t. $z_t^*(w_t, r_{t+1})$ is increasing in the ending inventory level w_t before returning/expediting in period t.

Theorem 4. *For $t = 1, 2, \dots, T$, we have the following:*

(i) *$G_t(y_t, r_t)$ is joint concave in (y_t, r_t);*

(ii) *$y_t^*(x_t, r_t)$ is increasing in x_t and $q_t^*(x_t, r_t)$ is decreasing in x_t;*

(iii) *$d_t^*(x_t, r_t)$ is increasing in x_t and $p_t^*(x_t, r_t)$ is decreasing in x_t;*

(iv) *$z_t^*(w_t, r_{t+1})$ is increasing in w_t, where*

$$z_t^*(w_t, r_{t+1}) = \arg \max_{-M_t \le z_t \le (w_t)^+} H_t(z_t, w_t, r_{t+1}). \tag{9}$$

Proof. See Appendix. □

Based on Theorems 3 and 4, we can characterize the optimal inventory replenishment, pricing policies for the regular order via the theorem below, which shows that the optimal inventory replenishment policy follows a base-stock policy and the pricing policy follows a base-stock-list-price policy. Here, $\overline{y}_t$ and $\overline{p}_t$ are the base-stock level and list price in period t, respectively.

Theorem 5. *For $t = 1, 2, \ldots, T$, the optimal regular replenishment policy for $y_t^*(x_t, r_t)$ is given by*

$$y_t^*(x_t, r_t) = \begin{cases} x_t + Q_t, & \overline{y}_t(r_t) > x_t + Q_t, \\ \overline{y}_t(r_t), & x_t < \overline{y}_t(r_t) \le x_t + Q_t, \\ x_t, & \overline{y}_t(r_t) \le x_t, \end{cases} \quad (10)$$

where $\overline{y}_t(r_t)$ is given by

$$\overline{y}_t(r_t) = \arg\max_{y_t} [-c_t y_t + G_t(y_t, r_t)]. \quad (11)$$

Thus, the optimal order quantity $q_t^*(x_t, r_t)$ is given by

$$q_t^*(x_t, r_t) = \begin{cases} Q_t, & \overline{y}_t(r_t) > x_t + Q_t, \\ \overline{y}_t(r_t) - x_t, & x_t \le \overline{y}_t(r_t) \le x_t + Q_t, \\ 0, & \overline{y}_t(r_t) < x_t. \end{cases} \quad (12)$$

Furthermore, the optimal pricing policy for $p_t^*(x_t, r_t)$ is given by

$$p_t^*(x_t, r_t) = \begin{cases} > \overline{p}_t(r_t), & \overline{y}_t(r_t) > x_t + Q_t, \\ = \overline{p}_t(r_t), & x_t \le \overline{y}_t(r_t) \le x_t + Q_t, \\ < \overline{p}_t(r_t), & \overline{y}_t(r_t) < x_t, \end{cases} \quad (13)$$

where $\overline{p}_t(r_t) = p_t(\overline{d}_t(r_t), r_t)$ and $\overline{d}_t(r_t)$ is given by

$$\overline{d}_t(r_t) = \arg\max_{\underline{d}_t \le d_t \le \overline{d}_t} \Big[d_t \cdot p_t(d_t, r_t) + \max_{-M_t \le z_t \le (\overline{y}_t - \beta_t d_t - \epsilon_t)^+} H_t(z_t, \overline{y}_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t)) \Big]. \quad (14)$$

Proof. Firstly, we have shown that $G_t(y_t, r_t)$ is concave in (y_t, r_t) in Theorem 4. Because of the concavity of $G_t(y_t, r_t)$, it is clear that (10) holds. The optimal order quantity $q_t^*(x_t, r_t)$ follows from $q_t^*(x_t, r_t) = y_t^*(x_t, r_t) - x_t$.

Secondly, when $x_t \le \overline{y}_t(r_t) \le x_t + Q_t$, by (4), the optimal mean demand is given by (14). Since $p_t(d_t, r_t)$ is strictly decreasing in d_t, the corresponding optimal price is uniquely given by $p_t(\overline{d}_t, r_t)$. Together with (iii) of Theorem 4, since $p_t^*(x_t, r_t)$ is decreasing in x_t, we thus get (13). □

Let $\theta_t = w_t - z_t$, where θ_t represents the inventory level after returning/expediting. The next theorem characterizes the optimal inventory adjustment policy for returning/expediting which follows a dual-threshold policy.

Theorem 6. *For $t = 1, 2, \ldots, T$, the optimal inventory level $\theta_t^*(x_t, r_t)$ after returning/expediting is given by the following two cases.*

Case I ($y_t^*(x_t, r_t) \ge D_t$). We have

$$\theta_t^*(x_t, r_t) = \begin{cases} \min\{\theta_t^l, y_t^*(x_t, r_t) - D_t + M_t\}, & \theta_t^l > y_t^*(x_t, r_t) - D_t, \\ y_t^*(x_t, r_t) - D_t, & \theta_t^l \le y_t^*(x_t, r_t) - D_t \le \theta_t^m, \\ (\theta_t^m)^+, & \theta_t^m < y_t^*(x_t, r_t) - D_t, \end{cases} \quad (15)$$

where $\theta_t^m(x_t, r_t) = \arg\max_{\theta_t}\{-(s_t + h_t)\theta_t + \gamma V_{t+1}(\theta_t, r_{t+1}(x_t, r_t))\}$ and $\theta_t^l(x_t, r_t) = \arg\max_{\theta_t}\{-(v_t + h_t)\theta_t + \gamma V_{t+1}(\theta_t, r_{t+1}(x_t, r_t))\}$ with $\theta_t^m > \theta_t^l$.

Case II ($y_t^*(x_t, r_t) < D_t$). We have

$$\theta_t^*(x_t, r_t) = \begin{cases} \min\{\theta_t^l, y_t^*(x_t, r_t) - D_t + M_t\}, & \theta_t^l > 0, \\ \min\{0, y_t^*(x_t, r_t) - D_t + M_t\}, & \theta_t^l \le 0. \end{cases} \quad (16)$$

Therefore, the optimal returning/expediting quantity $z_t^*(x_t, r_t)$ is given by the following two cases.

Case I ($y_t^*(x_t, r_t) \ge D_t$). We have

$$z_t^*(x_t, r_t) = \begin{cases} \max\{-M_t, y_t^*(x_t, r_t) - D_t - \theta_t^l\}, & \theta_t^l > y_t^*(x_t, r_t) - D_t, \\ 0, & \theta_t^l \le y_t^*(x_t, r_t) - D_t \le \theta_t^m, \\ y_t^*(x_t, r_t) - D_t - (\theta_t^m)^+, & \theta_t^m < y_t^*(x_t, r_t) - D_t. \end{cases} \quad (17)$$

Case II ($y_t^*(x_t, r_t) < D_t$). We have

$$z_t^*(x_t, r_t) = \begin{cases} \max\left\{-M_t, y_t^*(x_t, r_t) - D_t - \theta_t^l\right\}, & \theta_t^l > 0, \\ \max\left\{-M_t, y_t^*(x_t, r_t) - D_t\right\}, & \theta_t^l \le 0. \end{cases} \quad (18)$$

Proof. The optimal inventory level $\theta_t^*(x_t, r_t)$ after returning/expediting given by (15) and (16) is exactly the same as the proof related to θ_t for Theorem 3, while $z_t^*(x_t, r_t)$ is obtained by the definition of $\theta_t = w_t - z_t$. □

Follows from Theorems 5 and 6, we can get the following results which demonstrate how the optimal list price $\overline{p}_t(r_t)$, the optimal mean demand $\overline{d}_t(r_t)$, the optimal base-stock level $\overline{y}_t(r_t)$, the optimal regular order quantity $q_t^*(x_t, r_t)$, and the optimal returning/expediting quantity $z_t^*(x_t, r_t)$ depend on the current reference price r_t. Moreover, we also give the change characteristics of profit-to-go function $V_t(x_t, r_t)$ with the current reference price r_t.

Theorem 7. *For $t = 1, 2, \ldots, T$, we have the following:*

(i) *The optimal mean demand $\overline{d}_t(r_t)$ and the optimal list price $\overline{p}_t(r_t)$ are increasing in r_t.*

(ii) *The optimal base-stock level $\overline{y}_t(r_t)$ and the optimal regular order quantity $q_t^*(x_t, r_t)$ are increasing in r_t.*

(iii) *The optimal returning/expediting quantity $z_t^*(x_t, r_t)$ is decreasing in r_t.*

(iv) *The optimal profit-to-go function $V_t^*(x_t, r_t)$ is increasing in r_t.*

Proof. (i) From the proof of (iii) in Theorem 4, we see that the function in the right-hand side of (14) is supermodularity in (d_t, r_t). Thus, the monotonicity of $\overline{d}_t(r_t)$ in r_t can be obtained which follows from Theorem 2.2.8 in Simchi-Levi et al. [50]. Since the list price $\overline{p}_t(r_t) = p_t(\overline{d}_t(r_t), r_t)$ at present is only related to r_t, therefore, $\overline{p}_t(r_t)$ is increasing in r_t which follows from Assumption 1 in Section 2 and Proposition 1 in Güler et al. [15].

(ii) Following from the proof of (iv) in Theorem 3, we see that the function in the right-hand side in (11) is supermodular in (y_t, r_t). Hence, $\overline{y}_t(r_t)$ is increasing in r_t according to Theorem 2.2.8 in Simchi-Levi et al. [50]. In addition, according to (12), $q_t^*(x_t, r_t)$ is increasing in r_t.

(iii) Following from the supermodularity of $V_{t+1}(\theta_t, r_{t+1})$ in Theorem 3 (iv) and the definition of θ_t^l and θ_t^m, $-(s_t + h_t)\theta_t + \gamma V_{t+1}(\theta_t, r_{t+1})$ and $-(v_t + h_t)\theta_t + \gamma V_{t+1}(\theta_t, r_{t+1})$ are supermodular in (θ_t, r_{t+1}). Thus, θ_t^l and θ_t^m are increasing in r_{t+1} by applying Theorem 2.2.8 in Simchi-Levi et al. [50]. Then $\theta_t^*(x_t, r_{t+1})$ is increasing in r_{t+1} by (15) and (16). In addition, since r_{t+1} is increasing in r_t, we thus have $\theta_t^*(x_t, r_t) = \theta_t^*(x_t, r_{t+1}(x_t, r_t))$ increases with r_t. This, together with $\theta_t = w_t - z_t$, yields the result.

(iv) We prove $V_t(x_t, r_t)$ is increasing in r_t inductively. Let us define $(y_t^*(x_t, r_t), d_t^*(x_t, r_t)) = \arg\max_{x_t \le y_t \le x_t + Q_t, \underline{d}_t \le d_t \le \overline{d}_t} J_t(y_t, d_t, r_t)$ for a given r_t. In order to show that $V_T(x_T, r_T)$ is increasing in r_T, one needs to show the following two cases.

Case I ($x_T \le y_T^*(x_T, r_T)$). The optimal solution is $(y_T^*(x_T, r_T), d_T^*(x_T, r_T))$. Without changing the optimal pair, if r_t is increased by an amount of $\delta > 0$, the costs in (4) remain the same except for revenue term since $p_T(d_T, r_T)$ is increasing in r_T. Because the terminal value $V_{T+1}(x_{T+1}, r_{T+1}) = 0$, the optimal solution for the new state, namely, $r_T + \delta$, will be larger than or equal to the current solution, that is,

$$\begin{aligned} V_T(x_T, r_T) &= J_T(y_T^*(x_T, r_T), d_T^*(x_T, r_T), r_T) \\ &\le J_T(y_T^*(x_T, r_T), d_T^*(x_T, r_T), r_T + \delta) \\ &\le V_T(x_T, r_T + \delta). \end{aligned} \quad (19)$$

Case II. ($x_T > y_T^*(x_T, r_T)$). The optimal solution is $(x_T, d_T^*(x_T, r_T))$. If r_T is increased by $\delta > 0$ while the solution remains the same, the argument in Case I remains valid. Hence,

$$\begin{aligned} V_T(x_T, r_T) &= J_T(x_T, d_T^*(x_T, r_T), r_T) \\ &\le J_T(x_T, d_T^*(x_T, r_T), r_T + \delta) \\ &\le V_T(x_T, r_T + \delta). \end{aligned} \quad (20)$$

Assume that the result holds for $t = k + 1$, i.e., $V_{t+1}(x_{t+1}, r_{t+1})$ is increasing in r_{t+1}. Next, we need to show that the results are still true for $t = k$.

$V_t(x_t, r_t)$ is increasing in r_t which can be shown with an additional argument to the case of $V_T(x_T, r_T)$. The terms in the profit-to-go function (3) are shown to be increasing with r_t except for the last term $\gamma V_{t+1}(w_t - z_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$. Since $p_t(d_t, r_t)$ increases with r_t, so $\alpha r_t + (1-\alpha) p_t(d_t, r_t)$ increases with r_t. The arguments for $V_t(x_t, r_t)$ still remain valid since $V_{t+1}(x_{t+1}, r_{t+1})$ is increasing in r_{t+1} by the induction hypothesis. This completes the proof. □

5. The Infinite Planning Horizon Problem

In this section, we extend above results to the infinite planning horizon case. All the cost and revenue parameters as well as demand distribution are stationary. In the analysis of infinite horizon models it is necessary to have the one-period reward uniformly nonpositive so that the results in negative dynamic programming can be applied. Since the original problem has no such property, we subtract a constant

$$\Lambda = \max_{\underline{p} \le p_t, r_t \le \overline{p}} p_t \cdot d_t(p_t, r_t), \quad (21)$$

which is assumed to be finite from the original one period expected revenue (for $t \ge 1$). We then obtain the transformed profit-to-go function $\overline{V}_t(x_t, r_t)$ for the finite horizon problem from the original profit-to-go function $V_t(x_t, r_t)$:

$$\overline{V}_t(x_t, r_t) = V_t(x_t, r_t) - \frac{\Lambda(1-\gamma^t)}{1-\gamma}, \quad (22)$$

and

$$\overline{J}_t(y_t,d_t,r_t) = J_t(y_t,d_t,r_t) - \frac{\Lambda(1-\gamma^t)}{1-\gamma}. \tag{23}$$

Thus, the profit-to-go function in each period for the transformed model is nonpositive with $\overline{V}_{T+1}(x_{T+1}, r_{T+1}) = 0$. So the optimal profit-to-go function of the infinite horizon problem $\overline{V}(x,r)$ satisfies the following equations (e.g., Proposition 3.1.1, [51]):

$$\overline{V}(x,r) = \max_{x\le y\le x+Q, \underline{d}\le d\le \overline{d}} \overline{J}(y,d,r) + cx, \tag{24}$$

where

$$\begin{aligned}
\overline{J}(y,d,r) &= \mathsf{E}\Big[d\cdot p(d,r) - cy \\
&\quad + \max_{-M\le z\le (y-\beta d-\epsilon)^+} \overline{H}(z,w,\alpha r + (1-\alpha)p(d,r))\Big], \\
\overline{H}(z,w,\xi) &= s(z)^+ - v(-z)^+ - h(w-z)^+ - b(z \\
&\quad - w)^+ - \Lambda + \gamma\overline{V}(w-z,\xi), \\
w &= y - \beta d - \epsilon, \\
\xi &= \alpha r + (1-\alpha)p(d,r).
\end{aligned} \tag{25}$$

In the following, we present the relationship between $\overline{V}(x,r)$ and $\overline{V}_t(x_t,r_t)$, $\overline{J}(y,d,r)$ and $\overline{J}_t(y_t,d_t,r_t)$ as well as those of the original problem.

Theorem 8. *(i)*

$$\begin{aligned}
\overline{V}(x,r) &= \lim_{t\to\infty}\overline{V}_t(x_t,r_t), \\
V(x,r) &= \lim_{t\to\infty}V_t(x_t,r_t), \\
\overline{J}(y,d,r) &= \lim_{t\to\infty}\overline{J}_t(y_t,d_t,r_t), \\
J(y,d,r) &= \lim_{t\to\infty}J_t(y_t,d_t,r_t); \text{ and} \\
\overline{V}(x,r) &= V(x,r) - \frac{\Lambda}{1-\gamma}, \\
\overline{J}(y,d,r) &= J(y,d,r) - \frac{\Lambda}{1-\gamma}.
\end{aligned} \tag{26}$$

(ii) V and J satisfy the following optimality equation:

$$V(x,r) = \max_{x\le y\le x+Q, \underline{d}\le d\le \overline{d}} J(y,d,r) + cx, \tag{27}$$

where

$$\begin{aligned}
J(y,d,r) &= \mathsf{E}\Big[d\cdot p(d,r) - cy \\
&\quad + \max_{-M\le z\le (y-\beta d-\epsilon)^+} H(z,w,\alpha r + (1-\alpha)p(d,r))\Big], \\
H(z,w,\xi) &= s(z)^+ - v(-z)^+ - h(w-z)^+ - b(z \\
&\quad - w)^+ + \gamma V(w-z,\xi), \\
w &= y - \beta d - \epsilon, \\
\xi &= \alpha r + (1-\alpha)p(d,r).
\end{aligned} \tag{28}$$

(iii) $V(x,r)$ is concave in (x,r) and is increasing in x for a given r; $J(y,d,r)$ is concave in (y,d,r).

Proof. (i) and (ii) follow from Theorem 3 in Section 3 and Proposition 3.1.7 in Bertsekas [51]. For (iii), $V(x,r)$ and $J(y,d,r)$ inherit the properties of $V_t(x_t,r_t)$ and $J_t(y_t,d_t,r_t)$. □

From Propositions 3.1.3 and 3.1.7 in Bertsekas [51], there exists a stationary optimal policy for such a negative dynamic programming. The results discussed in Section 4 are presented for the infinite horizon problem via the following theorem.

Theorem 9. *(i) The stationary policies for $p^*(x,r), q^*(x,r)$ and $z^*(x,r)(z^*(w,\xi))$ are optimal.*

(ii) $p^*(x,r)$ and $q^*(x,r)$ are decreasing in x; $z^*(w,\xi)$ is increasing in w.

(iii) $V(x,r)$ is increasing in r, the base-stock level $\overline{y}(r)$ and the list price $\overline{p}(r)$ are increasing in r, and $z^*(x,r)$ is decreasing in r.

(iv) The stationary optimal policy for $q^*(x,r)$ is a base-stock type, the policy for $p^*(x,r)$ is a base-stock-list-price type, and the policy for $z^*(x,r)$ is a dual-threshold type.

6. Operational Impacts of Returning/Expediting under Reference Price Effects

To illustrate the effectiveness of our model, in this section we investigate the operational impacts from the following two aspects. On the one hand, we consider the impact of adding returning/expediting on joint pricing and inventory with reference price effects. On the other hand, we consider the impact of adding reference price effects on joint pricing and inventory with returning/expediting.

6.1. Operational Impact of Adding Returning/Expediting. We first consider the case where the firm introduces returning/expediting and study the impact on pricing and inventory control decisions with reference price effects as well as the firm's expected profit by comparing ours with that of Chen et al. [17, 38]. For simplicity of notation, we call their model CHSZ model. Although the CHSZ model considers the reference price effects, it does not take into account the returning/expediting after demand realization. To distinguish the CHSZ model from ours, we use the superscript c to signify the notation for the CHSZ model.

Since the CHSZ model is a special case of our model, i.e., $z_t = 0$. Consequently, the corresponding optimal equation is

$$V_t^c(x_t, r_t) = \max_{x_t \le y_t^c \le x_t + Q_t, \underline{d}_t \le d_t^c \le \overline{d}_t} J_t^c(y_t^c, d_t^c, r_t) + c_t x_t, \tag{29}$$

where

$$\begin{aligned} J_t^c(y_t^c, d_t^c, r_t) &= \mathsf{E}\big[d_t^c \cdot p(d_t^c, r_t) - c_t y_t^c \\ &\quad - h_t(y_t^c - \beta_t d_t^c - \epsilon_t)^+ - b_t(\beta_t d_t^c + \epsilon_t - y_t^c)^+ \\ &\quad + \gamma V_{t+1}^c(y_t^c - \beta_t d_t^c - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t^c, r_t))\big]. \end{aligned} \tag{30}$$

The first result states that our model always yields an expected profit no less than the CHSZ model.

Theorem 10. *After the returning/expediting is introduced, the optimal profit-to-go function satisfies $V_t(x_t, r_t) \ge V_t^c(x_t, r_t)$ for $t = 1, 2, \ldots, T$.*

Proof. This follows directly from the observation that the CHSZ model is a special case of our model, i.e., $z_t(x_t, r_t) = 0$, $t = 1, 2, \ldots, T$. □

In what follows, we continue to discuss the operational impacts on the price, replenishment, and adjustment policy. The following theorem summarizes the relationships between the optimal policies with and without returning/expediting.

Theorem 11. *After the returning/expediting is introduced, the optimal policy parameters satisfy, for $t = 1, 2, \ldots, T$,*

(i) $z_t^* \le 0$ *iff* $q_t^*(x_t, r_t) \le q_t^{c*}(x_t, r_t)$, $p_t^*(x_t, r_t) \le p_t^{c*}(x_t, r_t)$, $\overline{y}_t \le \overline{y}_t^c$ *and* $\overline{d}_t \ge \overline{d}_t^c$;

(ii) $z_t^* > 0$ *iff* $q_t^*(x_t, r_t) > q_t^{c*}(x_t, r_t)$, $p_t^*(x_t, r_t) > p_t^{c*}(x_t, r_t)$, $\overline{y}_t > \overline{y}_t^c$ *and* $\overline{d}_t < \overline{d}_t^c$.

Proof. We show the statement is true for $z_t^* \le 0$. Then, the statement for $z_t^* > 0$ can be shown in the similar way.

On the one hand, if $z_t^* \le 0$, we can rewrite (8) as

$$\begin{aligned} G_t(y_t, r_t) &= \max_{\underline{d}_t \le d_t \le \overline{d}_t} \big[d_t \cdot p_t(d_t, r_t) + v_t z_t^* \mathsf{E} h_t(y_t \\ &\quad - \beta_t d_t - \epsilon_t - z_t^*)^+ - \mathsf{E} b_t(\beta_t d_t + \epsilon_t + z_t^* - y_t)^+ \\ &\quad + \gamma \mathsf{E} V_{t+1}(y_t - \beta_t d_t - \epsilon_t - z_t^*, \alpha r_t \\ &\quad + (1-\alpha) p_t(d_t, r_t))\big]. \end{aligned} \tag{31}$$

Then $G_t(y_t, r_t)$ can be seen as a function of y_t, z_t^*, and r_t. Because $G_t(y_t, r_t)$ is supermodular in (y_t, z_t^*) by the concavity of V_{t+1}, so $\overline{y}_t$ is increasing in z_t^* which follows from Theorem 2.2.8 in Simchi-Levi et al. [50]. Since $q_t^*(x_t, r_t)$ is increasing in $\overline{y}_t$ by (12) and the optimal solution of the CHSZ model can be treated as a feasible solution of our model with $z_t^* = 0$, we thus have $q_t^*(x_t, r_t) \le q_t^{c*}(x_t, r_t)$ and $p_t^*(x_t, r_t) \le p_t^{c*}(x_t, r_t)$ for $z_t^* \le 0$.

Next, since $J_t(y_t, d_t, r_t)$ is joint concave in (y_t, d_t, r_t) and z_t^* is given, we can rewrite (3) as

$$\begin{aligned} V_t(y_t, r_t) &= \max_{\underline{d}_t \le d_t \le \overline{d}_t} \big[d_t \cdot p_t(d_t, r_t) + v_t z_t^* \\ &\quad + \widetilde{G}_t(\beta_t d_t + z_t^* + \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))\big], \end{aligned} \tag{32}$$

where

$$\begin{aligned} &\widetilde{G}_t(\beta_t d_t + z_t^* + \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t)) \\ &\quad = \max_{x_t \le y_t \le x_t + Q_t} \big\{-c_t y_t - \mathsf{E} h_t(y_t - \beta_t d_t - z_t^* - \epsilon_t)^+ \\ &\quad - \mathsf{E} b_t(\beta_t d_t + z_t^* + \epsilon_t - y_t)^+ + \gamma \mathsf{E} V_{t+1}(y_t - \beta_t d_t \\ &\quad - z_t^* - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))\big\}. \end{aligned} \tag{33}$$

To show $\overline{d}_t$ is decreasing in z_t^*, it is necessary to prove the submodularity of $\widetilde{G}_t(\beta_t d_t + z_t^* + \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ in (d_t, z_t^*). Because the first three terms are obviously submodular, we only need the submodularity of $V_{t+1}(y_t - \beta_t d_t - z_t^* - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ in (d_t, z_t^*). Fix ϵ_t and r_t, and consider an arbitrary pair of (d_t^1, d_t^2) with $d_t^1 > d_t^2$ and any pair $((z_t^*)', (z_t^*)'')$ with $(z_t^*)' > (z_t^*)''$. Let $(\tau_1, r_{t+1}^1) = (\beta d_t^1 + (z_t^*)' + \epsilon_t, r_{t+1}^1)$, $(\tau_2, r_{t+1}^1) = (\beta d_t^1 + (z_t^*)'' + \epsilon_t, r_{t+1}^1)$, $(\tau_3, r_{t+1}^2) = (\beta d_t^2 + (z_t^*)' + \epsilon_t, r_{t+1}^2)$, $(\tau_4, r_{t+1}^2) = (\beta d_t^2 + (z_t^*)'' + \epsilon_t, r_{t+1}^2)$, where $r_{t+1}^1 = \alpha r_t + (1-\alpha) p_t(d_t^1, r_t)$ and $r_{t+1}^2 = \alpha r_t + (1-\alpha) p(d_t^2, r_t)$. It is clear that $r_{t+1}^1 < r_{t+1}^2, \tau_1 > \tau_3 > \tau_4$ and thus $y_t - \tau_1 < y_t - \tau_3 < y_t - \tau_4$. Then we have

$$\begin{aligned} &V_{t+1}(y_t - \tau_1, r_{t+1}^1) - V_{t+1}(y_t - \tau_3, r_{t+1}^2) \\ &\quad \le V_{t+1}(y_t - \tau_3, r_{t+1}^1) - V_{t+1}(y_t - \tau_3, r_{t+1}^2) \\ &\quad \le V_{t+1}(y_t - \tau_4, r_{t+1}^1) - V_{t+1}(y_t - \tau_4, r_{t+1}^2) \\ &\quad \le V_{t+1}(y_t - \tau_4 + \beta(d_t^1 - d_t^2), r_{t+1}^1) \\ &\qquad - V_{t+1}(y_t - \tau_4, r_{t+1}^2) \\ &\quad = V_{t+1}(y_t - \tau_2, r_{t+1}^1) - V_{t+1}(y_t - \tau_4, r_{t+1}^2), \end{aligned} \tag{34}$$

where the first and the third inequalities follows from Theorems 5 (iii) and 1 (iii), respectively. The second inequality follows from the supermodularity of $V_{t+1}(x_{t+1}, r_{t+1})$ by Theorem 3 (iv), and thus the difference $V_{t+1}(x_{t+1}, r_{t+1}^2) - V_{t+1}(x_{t+1}, r_{t+1}^1)$ is increasing in x_{t+1} for any $\xi_2 > \xi_1$. Therefore, we conclude that $V_{t+1}(y_t - \beta_t d_t - z_t^* - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ has decrease difference in z_t^* for any $d_t^1 > d_t^2$, which implies that $V_{t+1}(y_t - \beta_t d_t - z_t^* - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ is submodular in (d_t, z_t^*). Hence, $\overline{d}_t$ is decreasing in z_t^*.

By (13), since $p_t^*(x_t, r_t)$ is decreasing in $\overline{d}_t$, so $p_t^*(x_t, r_t)$ is increasing in z_t^*. Because the optimal solution of the CHSZ model can be treated as a feasible solution of our model with $z_t^* = 0$, we thus have $\overline{d}_t \ge \overline{d}_t^c$ and $p_t^*(x_t, r_t) \le p_t^{c*}(x_t, r_t)$.

On the other hand, according to the above analysis, we have shown that $\overline{y}_t$ is increasing in z_t^*, $q_t^*(x_t, r_t)$ is increasing in z_t^*, $\overline{d}_t$ is decreasing in z_t^*, and $p_t^*(x_t, r_t)$ is increasing in z_t^*. Therefore, if $q_t^*(x_t, r_t) \le q_t^{c*}(x_t, r_t)$, $p_t^*(x_t, r_t) \le p_t^{c*}(x_t, r_t)$, $\overline{y}_t \le \overline{y}_t^c$, and $\overline{d}_t \ge \overline{d}_t^c$, we have $z_t^* \le 0$ since the optimal solution of the CHSZ model can be treated as a feasible solution of our model with $z_t^* = 0$. This completes the proof. □

We offer the following interpretation of Theorem 11. Part (i) of this theorem is obvious: when the firm has the opportunity of expediting, the firm has the new option when needed to raise the inventory level; hence, it can reduce the base-stock level from the regular order, so the order quantity will also decrease. The optimal list price in our system is lower than that of CHSZ model. As a result, the mean demand will increase. Part (ii) is exactly the opposite situation.

6.2. Operational Impact of Adding Reference Price Effects. We next analyze the operational impact of reference price effects on joint pricing and inventory with returning/expediting by comparing ours with that of Zhu [9]. For simplicity of notation, we call their model ZS model. Although the ZS model considers returning/expedition after the demand is realized, it does not take the reference price effects into consideration. To distinguish the ZS model from ours, we use the superscript f to signify the notation for the ZS model. The following is the main results on the impact of adding the reference price effects.

Theorem 12. *After the reference price effects is considered, the optimal profit-to-go function and optimal policy parameters satisfy, for $t = 1, 2, \ldots, T$,*

(i) $V_t(x_t, r_t) \ge V_t^f(x_t)$;

(ii) $\overline{y}_t \ge \overline{y}_t^f$;

(iii) $q_t^*(x_t, r_t) \ge q_t^{f*}(x_t)$;

(iv) $\overline{p}_t \ge \overline{p}_t^f$;

(v) $\overline{d}_t \ge \overline{d}_t^f$;

(vi) $z_t^*(x_t, r_t) \le z_t^{f*}(x_t)$.

Proof. Since the ZS model is a special case of our model, i.e., $r_t = 0$, $t = 1, 2, \ldots, T$. This, together with Theorem 7 implies these results. □

This theorem can be intuitively illustrated as follows. When the firm does not consider the customers' reference price, the ordering strategy would be relatively conservative. However, when the firm takes the customers' reference price into consideration, with the increase of customers' reference price, the mean demand will increase, the order quantity will also increase and so is the inventory level, and the firm's price will rise, so the profit increases. Furthermore, for part (vi), when $z_t \le 0$, the expediting quantity will increase compared with that without considering the reference price effects due to the high demand. When $z_t > 0$, the returning quantity will decrease compared with that without considering the reference price effects. This is because the ordering decision which consider the reference price effects are more precise than that of without considering the reference price effects.

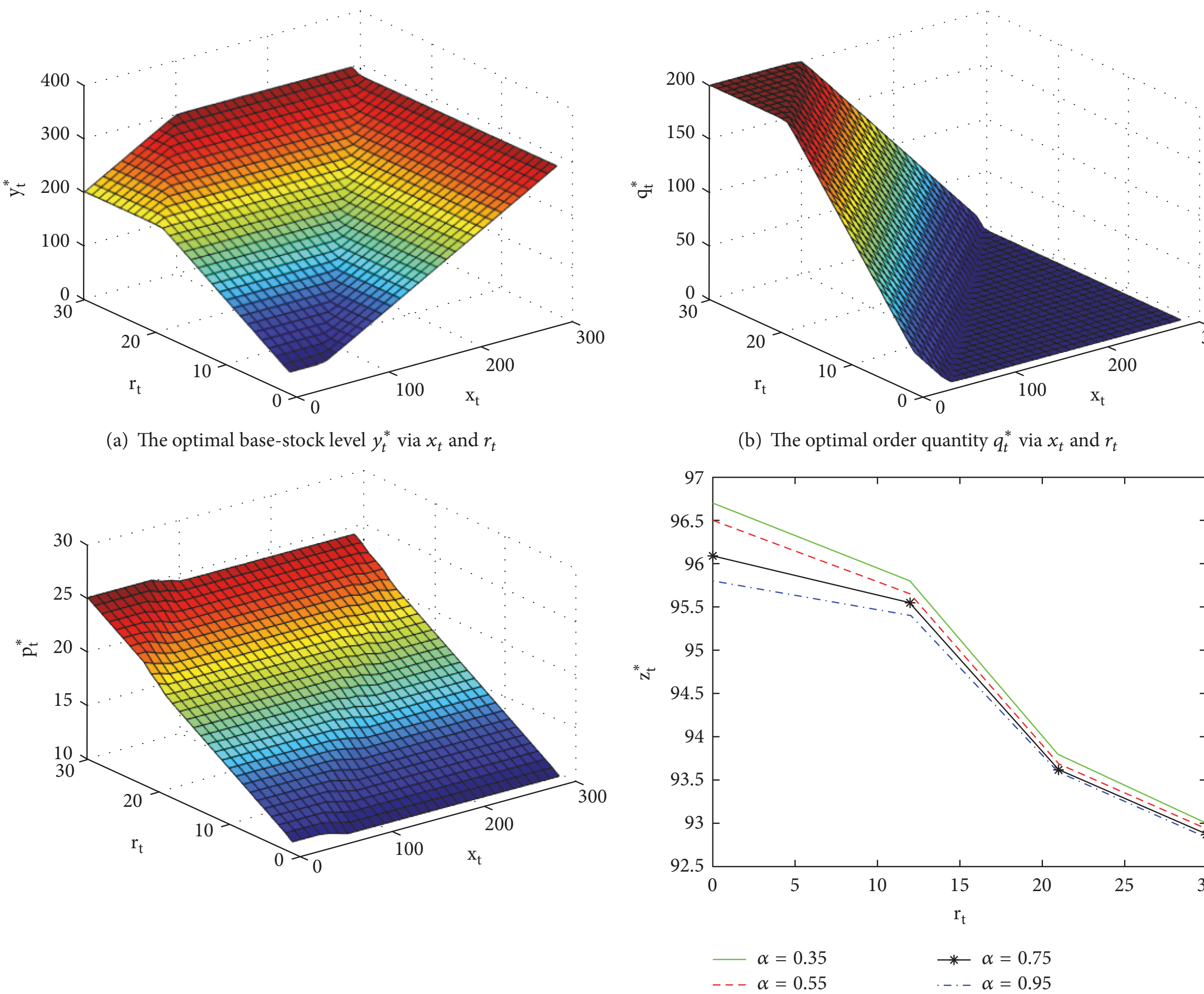
7. Numerical Analysis

In this section, we proceed several numerical experiments used to illustrate the following two aspects. Firstly, verify the accuracy of the conclusions of this paper. Secondly, analyze the operational impacts on firm's profit by adding returning/expediting and reference price effects via comparing ours with those of CHSZ and ZS model.

We consider a system with planning horizon $T = 4$. Suppose the following stationary parameter values: $c_t = 10$, $s_t = 9$, $v_t = 12$, $h_t = 1.5$, $b_t = 15$, $\eta^+ = 1.5$, $\eta^- = 2.5$, $\gamma = 0.95$, $\alpha = 0.5$, $Q_t = 200$, and $M_t = 50$. The mean demand function is assumed to be additive, i.e., $d_t(p_t, r_t) = 200 - 15p_t + 1.5\max\{r_t - p_t, 0\} + 2.5\min\{r_t - p_t, 0\}$. Moreover, we assume that β_t ~Uniform$[0.5, 1.5]$ and ϵ_t ~ Uniform$[-0.5, 0.5]$. All experiments below are performed in MATLAB R2014b on a laptop with an Intel(R) Core (TM) i5-7200U central processing unit CPU (2.50 GHz, 2.70GHz) and 8.0 GB of RAM running 64-bit Windows 10 Enterprise.

Figures 1(a)–1(c) give the trends of optimal regular base-stock level y_t^*, optimal regular order quantity q_t^*, and optimal price p_t^* with initial inventory x_t and reference price r_t, respectively, which are consistent with Theorems 4 and 7. This indicates that, with the increase of customers' reference price, customers valuation of commodities will increase; the firm will increase the price and raise its inventory level (order quantity) for regular supply to gain more profit. Therefore, reference price has a positive effect on optimal price, optimal inventory level, and optimal order quantity. Figure 1(d) illustrates the impact of current reference price r_t on optimal expediting or returning quantity z_t^*, which is also consistent with Theorem 7. Figure 1(d) indicates that when the customers' reference price is high, the customers' valuation of the commodities will increase, so the firm will increase the expediting replenishment order quantity (or reduce the quantity of returning) to meet the increasing demand. In addition, it is shown from Figure 1(d) that the optimal returning or expediting quantity z_t^* decreases with memory parameter α; this suggests that the firm should decrease the expediting replenishment order quantity (or increase the quantity of returning) to save the holding cost when consumers have short-term memories of previous commodities' prices or are less loyal to its commodities.

Figure 2 provides the comparison of optimal profit among ours, CHSZ, and ZS model. We set β_t ~Uniform$[0.5, 1.5]$, and let ϵ_t follow the uniform distribution on $[-0.5, 0.5]$, normal distribution with mean 0, and standard deviation 25, respectively. Figure 2 shows that when ϵ_t takes these two different distributions, even if the firm can either benefit from returning/expediting or benefit from reference price effects. However, considering both returning/expediting and reference price effects will bring more profit than just considering one of the two aspects.



(a) The optimal base-stock level y_t^* via x_t and r_t

(b) The optimal order quantity q_t^* via x_t and r_t

(c) The optimal price p_t^* via x_t and r_t

(d) The optimal expediting/returning quantity z_t^* via r_t

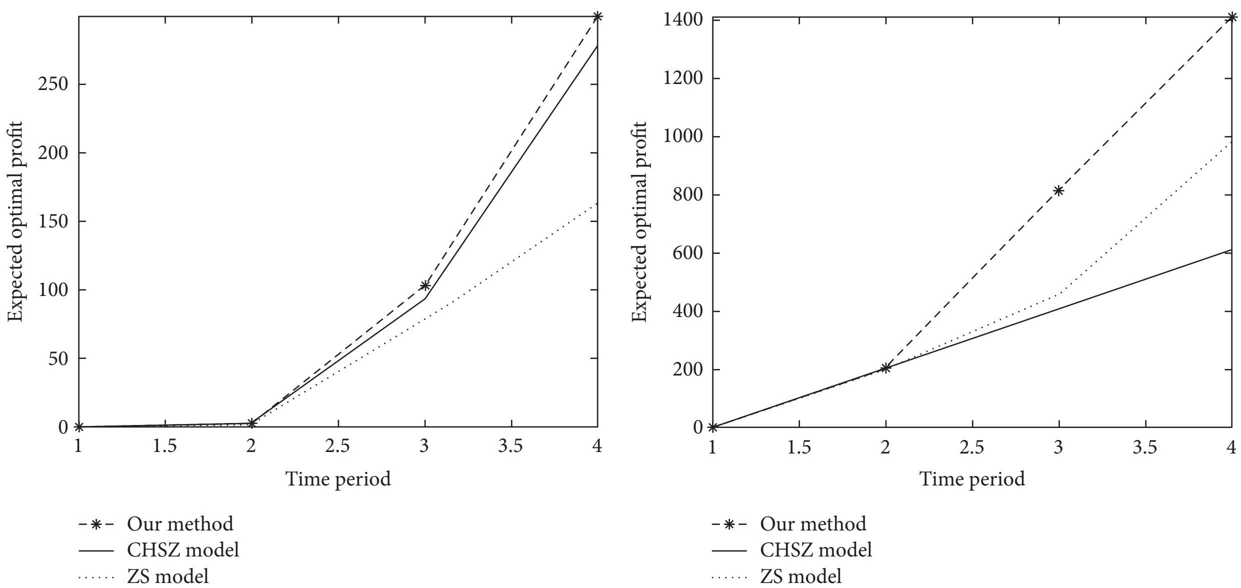
FIGURE 1: The optimal regular and expediting/returning decisions under reference price effects.

8. Conclusion

Our research complements the existing research stream in coordinating pricing and inventory replenishment decisions from two aspects. On the one hand, we consider the inventory planning decisions for returns and expediting. On the other hand, we consider the influence of the customers' behavior (i.e., customers' reference price) on the joint pricing and inventory replenishment decisions.

In this paper, we investigate a single-item periodic-review finite horizon joint pricing and inventory replenishment problem with returns and expediting under reference price effects. Demand in each period is random and sensitive to price and reference price. At the beginning of each period, the firm first observes the current inventory level and simultaneously decides the unit selling price and the quantity of the regular order for the current period based on the current reference price. At the end of each period, after the demand is realized, a firm can return excess stocks to a supplier. Or, if there are stockouts, the firm can place an expediting order at the supplier to reduce the amount of shortage. Unfilled demands are fully backlogged. For a very general stochastic demand function, our research shows that the optimal replenishment policy for regular order is a base-stock policy, the optimal pricing policy is a base-stock-list-price policy, and the optimal policy for returning/expediting inventory adjustment follows a dual-threshold policy. We further analyze the operational effects of returns and expediting under reference price effects by comparing ours with that of Chen et al. [17] and Zhu [9], respectively. Numerical results also demonstrate that considering both returning/expediting and reference price effects will bring more benefits than considering only one of them.

Though this paper has identified the effects of reference price on dynamic pricing and ordering for regular and returning/expediting decisions, there are still some shortcomings that can be investigated in the future. First, this paper analyzes the pricing and order decisions of a single firm



(a) β_t ~Uniform[0.5, 1.5], ϵ_t ~Uniform[−0.5, 0.5]

(b) β_t ~Uniform[0.5, 1.5], $\epsilon_t \sim N(0, 25^2)$

Figure 2: The comparison of optimal profit among ours, CHSZ, and ZS model.

under reference price effects and unaware of the influence of reference price effects on suppliers. An interesting future research topic is to examine the pricing and inventory decisions for suppliers, and to design an appropriate coordination mechanism so that a win-win outcome for both parties can be obtained. Second, in our study, the customers reference price can be observed by firms. However, the information on customers reference price is difficult to get in reality. Thus, demand learning can be incorporated into formulating pricing and inventory strategy in the presence of the reference price effects. Third, with the rapid development of information technology centered on the mobile Internet, customers purchase patterns are also diversified. In this case, how to study the reference price of customers on firms pricing and inventory decisions is also one of interesting and meaningful research directions in the future.

Appendix

Proof of Theorem 3. (i), (ii), and (iii) of this theorem will be proved together by induction. Starting from $t = T$, it is obvious that $H_T(z_T, w_T, r_{T+1})$ is joint concave in (z_T, w_T, r_{T+1}). Thus, $J_T(y_T, d_T, r_T)$ is joint concave in (y_T, d_T, r_T). Similar to the proof of Lemma 1 in Zhu [9], we have that

$$\begin{aligned} &\max_{-M_T \le z_T \le (w_T)^+} H_T(z_T, w_T, r_{T+1}) \\ &\quad = \max_{-M_T \le z_T} H_T(z_T, w_T, r_{T+1}). \end{aligned} \tag{A.1}$$

Because (A.1) is true, we can optimize z_T based on the set $\{z_T \mid -M_T \le z_T\}$. Since $H_T(z_T, w_T, r_{T+1})$ is joint concave on convex set $\{z \mid -M_T \le z_T\}$ and concavity is preserved by maximization, combination with Assumption 2 yields the concavity of $J_T(y_T, d_T, r_T)$ in (y_T, d_T, r_T). Then $V_T(x_T, r_T)$ is concave in (x_T, r_T) since maximization preserves concavity.

Next, we will show that $V_T(x_T, r_T)$ is increasing in x_T by proving $\partial V_T(x_T, r_T)/\partial x_T \ge 0$ for a given r_T. Since $J_T(y_T, d_T, r_T)$ is joint concave in (y_T, d_T, r_T), we can rewrite $V_T(x_T, r_T)$ as

$$\begin{aligned} V_T(x_T, r_T) = &\max_{x_T \le y_T \le x_T + Q_T} [-c_T y_T + G_T(y_T, r_T)] \\ &+ c_T x_T, \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} G_T(y_T, r_T) = &\max_{\underline{d}_T \le d_T \le \overline{d}_T} \mathsf{E}\Big[d_T \cdot p_T(d_T, r_T) \\ &+ \max_{-M_T \le z_T} H_T(z_T, y_T - \beta_T d_T - \epsilon_T, \alpha r_T \\ &+ (1-\alpha)\, p_T(d_T, r_T))\Big], \end{aligned} \tag{A.3}$$

Denote $\overline{y}_T = \arg\max_{y_T}[-c_T y_T + G_T(y_T, r_T)]$. Following from the concavity of V_T, it is clear that $\partial V_T(x_T, r_T)/\partial x_T \ge c_T$ for $x_T \le \overline{y}_T$. For $x_T > \overline{y}_T$, we have $\partial V_T(x_T, r_T)/\partial x_T = \partial G_T(x_T, r_T)/\partial x_T \ge \partial G_T(x_T, r_T)/\partial x_T|_{x_T \to +\infty}$. Besides, we have

$$\begin{aligned} \left.\frac{\partial G_T(x_T, r_T)}{\partial x_T}\right|_{x_T \to +\infty} = &\lim_{x_T \to +\infty} \{s_T \Phi_T(x_T) \\ &+ v_T[\Phi_T(x_T + M_T) - \Phi_T(x_T)] \\ &+ b_T[1 - \Phi_T(x_T + M_T)]\} = s_T \ge 0. \end{aligned} \tag{A.4}$$

where Φ_T is the cumulative distribution function of D_T. Thus, we obtain $\partial V_T(x_T, r_T)/\partial x_T \ge 0$ for a given r_T.

Assume that the results hold for $t = k + 1$. Next, we need to show that the results are still true for $t = k$.

By induction, since $V_{k+1}(x_{k+1}, r_{k+1})$ is concave in (x_{k+1}, r_{k+1}), $H_k(z_k, w_k, r_{k+1})$ is joint concave in (z_k, w_k, r_{k+1}). By definition, we have

$$\begin{aligned} H_k(z_k, w_k, r_{k+1}) = {} & s_k(z_k)^+ - v_k(-z_k)^+ \\ & - h_k(w_k - z_k)^+ - b_k(z_k - w_k)^+ \\ & + \gamma V_{k+1}(w_k - z_k, r_{k+1}). \end{aligned} \quad \text{(A.5)}$$

Now, we intend to show that

$$\begin{aligned} & \max_{-M_k \le z_k \le (w_k)^+} H_k(z_k, w_k, r_{k+1}) \\ & \quad = \max_{-M_k \le z_k} H_k(z_k, w_k, r_{k+1}), \end{aligned} \quad \text{(A.6)}$$

and we consider the following two cases: $w_k \ge 0$ and $w_k < 0$.

Case I ($w_k \ge 0$). By (4), we have $-M_k \le z_k \le w_k$. For $z_k > w_k$, we have

$$\begin{aligned} & H_k(z_k, w_k, r_{k+1}) \\ & \quad = b_k w_k \\ & \quad\quad + \max_{z_k \ge w_k} \left[-(b_k - s_k) z_k + \gamma V_{k+1}(w_k - z_k, r_{k+1}) \right]. \end{aligned} \quad \text{(A.7)}$$

Since $b_k > s_k$ and $V_{k+1}(x_{k+1}, r_{k+1})/\partial x_{k+1} \ge 0$ for a fixed r_{k+1}, it is clear that $H_k(z_k, w_k, r_{k+1})$ is decrease in z_k for $z_k > w_k$. Then, we get $H_k(z_k, w_k, r_{k+1}) \le H_k(w_k, w_k, r_{k+1})$ for $z_k > w_k$. Thus, (A.6) is true.

Case II ($w_k < 0$). By (4), we have $-M_k \le z_k \le 0$. For $z_k > 0$, we have

$$\begin{aligned} & H_k(z_k, w_k, r_{k+1}) \\ & \quad = b_k w_k \\ & \quad\quad + \max_{z_k \ge 0} \left[-(b_k - s_k) z_k + \gamma V_{k+1}(w_k - z_k, r_{k+1}) \right]. \end{aligned} \quad \text{(A.8)}$$

By following the similar argument like that of Case I, it is clear that $H_k(z_k, w_k, r_{k+1}) \le H_k(0, w_k, r_{k+1})$ for $z_k > 0$. Thus, (A.6) is still true.

Because (A.6) is true, we can optimize z_k based on the set $\{z_k \mid -M_k \le z_k\}$. Since $H_k(z_k, w_k, r_{k+1})$ is joint concave on the convex set $\{z_k \mid -M_k \le z_k\}$ and concavity is preserved by maximization, then $J_k(y_k, d_k, r_k)$ is joint concave in (y_k, d_k, r_k). Therefore, we obtain that $V_k(x_k, r_k)$ is concave in (x_k, r_k).

Next, we will show that $\partial V_k(x_k, r_k)/\partial x_k \ge 0$ for a given r_k. Since $J_k(y_k, d_k, r_k)$ is joint concave in (y_k, d_k, r_k), we can rewrite $V_k(x_k, r_k)$ as

$$\begin{aligned} V_k(x_k, r_k) = {} & \max_{x_k \le y_k \le x_k + Q_k} \left[-c_k y_k + G_k(y_k, r_k) \right] \\ & + c_k x_k, \end{aligned} \quad \text{(A.9)}$$

where

$$\begin{aligned} G_k(y_k, r_k) = {} & \max_{\underline{d}_k \le d_k \le \overline{d}_k} \mathsf{E} \Big[d_k \cdot p_k(d_k, r_k) \\ & + \max_{-M_k \le z_k \le (y_k - \beta_k d_k - \epsilon_k)^+} H_k(z_k, y_k - \beta_k d_k - \epsilon_k, \alpha r_k \\ & + (1 - \alpha) p_k(d_k, r_k)) \Big], \end{aligned} \quad \text{(A.10)}$$

Denote $\overline{y}_k = \arg\max_{y_k} [-c_k y_k + G_k(y_k, r_k)]$. Because of the concavity of V_k, it is clear that $\partial V_k(x_k, r_k)/\partial x_k \ge c_k$ for $x_k \le \overline{y}_k$. For $x_k > \overline{y}_k$, we have $\partial V_k(x_k, r_k)/\partial x_k = \partial G_k(x_k, r_k)/\partial x_k \ge \partial G_k(x_k, r_k)/\partial x_k|_{x_k \to +\infty}$.

Now, we need to show that $G_k(x_k, r_k)/\partial x_k|_{x_k \to +\infty} \ge 0$. Because $G_k(x_k, r_k)$ depends on the maximum of $H_k(z_k, w_k, r_{k+1})$, we need to firstly analyze the maximum of $H_k(z_k, w_k, r_{k+1})$. Then, conditioning on the value of w_k, we consider two cases: $w_k \ge 0$ and $w_k < 0$. To brief notation, we denote $\theta_k = w_k - z_k$. To optimize z_k which is equivalent to optimize θ_k, we rewrite

$$\begin{aligned} H_k(\theta_k, w_k, r_{k+1}) = {} & s_k(w_k - \theta_k)^+ - v_k(\theta_k - w_k)^+ \\ & - h_k(\theta_k)^+ - b_k(-\theta_k)^+ \\ & + \gamma V_{k+1}(\theta_k, r_{k+1}). \end{aligned} \quad \text{(A.11)}$$

Define

$$\begin{aligned} \theta_k^l(x_k, r_k) & = \theta_k^l(r_{k+1}) \\ & = \arg\max_{\theta_k} \left\{ -(v_k + h_k)\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1}) \right\}, \\ \theta_k^m(x_k, r_k) & = \theta_k^m(r_{k+1}) \\ & = \arg\max_{\theta_k} \left\{ -(s_k + h_k)\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1}) \right\}. \end{aligned} \quad \text{(A.12)}$$

$\theta_k^l(r_{k+1}) \le \theta_k^m(r_{k+1})$ because of the concavity of V_{k+1} and $v_k > s_k$.

Case I ($w_k \ge 0$). By (4), since $-M_k \le z_k \le w_k$, we have $0 \le \theta_k \le w_k + M_k$. Substituting θ_k into (5), we define

$$\begin{aligned} Y_k^{(1)} = {} & s_k \mathrm{w}_k \\ & + \max_{\theta_k \le w_k} \left\{ -(s_k + h_k)\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1}) \right\}, \end{aligned} \quad \text{(A.13)}$$

and

$$\begin{aligned} W_k^{(1)} = {} & v_k w_k \\ & + \max_{\theta_k \ge w_k} \left\{ -(v_k + h_k)\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1}) \right\}. \end{aligned} \quad \text{(A.14)}$$

According to the above analysis, $\theta_k^*(r_{k+1})$ depends on the relationship between two thresholds ($\theta_k^l(r_{k+1})$ and $\theta_k^m(r_{k+1})$) and w_k. In other words, we need to consider three different subcases.

Case I-a ($w_k < \theta_k^l(r_{k+1})$). We have

$$\begin{aligned} W_k^{(1)} &= v_k w_k - (v_k + h_k)\,\theta_k^l(r_{k+1}) \\ &\quad + \gamma V_{k+1}\left(\theta_k^l(r_{k+1}), r_{k+1}\right) \\ &\geq -h_k w_k + \gamma V_{k+1}(w_k, r_{k+1}) = Y_k^{(1)}, \end{aligned} \tag{A.15}$$

where the inequality is true since $\theta_k^l(r_{k+1})$ yields the maximal $W_k^{(1)}$. So, it is profitable to increase the inventory level from w_k to $\theta_k^l(r_{k+1})$. However, since the amount of the expediting order is limited by M_k, we have $\theta_k^*(r_{k+1}) = \min\{\theta_k^l(r_{k+1}), w_k + M_k\}$. Note that $H_k(w_k + M_k, w_k, r_{k+1}) \geq H_k(w_k, w_k, r_{k+1})$ if $\theta_k^l(r_{k+1}) \geq w_k + M_k$.

Case I-b ($\theta_k^l(r_{k+1}) \leq w_k \leq \theta_k^m(r_{k+1})$). It is straightforward to verify that $W_k^{(1)} = -h_k w_k + \gamma V_{k+1}(w_k, r_{k+1}) = Y_k^{(1)}$. Consequently, the firm has no incentive to make adjustment, i.e., $\theta_k^*(r_{k+1}) = w_k$.

Case I-c ($\theta_k^m(r_{k+1}) < w_k$). We have

$$\begin{aligned} W_k^{(1)} &= -h_k w_k + \gamma V_{k+1}(w_k, r_{k+1}) \\ &\leq s_k w_k - (s_k + h_k)\,\theta_k^m(r_{k+1}) \\ &\quad + \gamma V_{k+1}\left(\theta_k^m(r_{k+1}), r_{k+1}\right) = Y_k^{(1)}, \end{aligned} \tag{A.16}$$

where the inequality is true since $\theta_k^m(r_{k+1})$ yields the maximal $Y_k^{(1)}$. So, it is profitable to reduce the inventory level to $\theta_k^m(r_{k+1})$. However, since $\theta_k \geq 0$, we have $\theta_k^*(r_{k+1}) = (\theta_k^m(r_{k+1}))^+$, which means that the firm cannot return more stock than he or she has.

Case II ($w_k < 0$). By (4), since $-M_k \leq z_k \leq 0$, we have $w_k \leq \theta_k \leq w_k + M_k$. Substituting θ_k into (5), we have

$$\begin{aligned} Y_k^{(2)} &= v_k w_k \\ &\quad + \max_{\theta_k \leq 0}\left[(b_k - v_k)\,\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1})\right], \end{aligned} \tag{A.17}$$

and

$$\begin{aligned} W_k^{(2)} &= v_k w_k \\ &\quad + \max_{\theta_k \geq 0}\left[-(v_k + b_k)\,\theta_k + \gamma V_{k+1}(\theta_k, r_{k+1})\right]. \end{aligned} \tag{A.18}$$

Since $b_k > v_k$ and $\partial V_{k+1}(x_{k+1}, r_{k+1})/\partial x_{k+1} \geq s_{k+1}$ for given r_{k+1}, we have $Y_k^{(2)} = H_k(0, w_k, r_{k+1}) = v_t w_k + \gamma V_{k+1}(0, r_{k+1})$. According to the above analysis, $\theta_k^*(r_{k+1})$ depends on the relationship between $\theta_k^l(r_{k+1})$ and w_k. In other words, we need to consider two different subcases.

Case II-a ($\theta_k^l(r_{k+1}) > 0$). We have

$$\begin{aligned} W_k^{(2)} &= v_k w_k - (v_k + h_k)\,\theta_k^l(r_{k+1}) \\ &\quad + \gamma V_{k+1}\left(\theta_k^l(r_{k+1}), r_{k+1}\right) \\ &\geq v_k w + \gamma V_{k+1}(0, r_{k+1}) = Y_k^{(2)}, \end{aligned} \tag{A.19}$$

where the inequality is true since $\theta_k^l(r_{k+1})$ yields the maximal $W_k^{(2)}$. So, it is profitable to increase the inventory from w_k to $\theta_k^l(r_{k+1})$. However, since the amount of the expediting order is limited by M_k, we have $\theta_k^*(r_{k+1}) = \min\{\theta_k^l(r_{k+1}), w_k + M_k\}$. Note that if $w_k \leq -M_k$, $\theta_k^*(r_{k+1}) = w_k + M_k$.

Case II-b ($\theta_k^l(r_{k+1}) \leq 0$). It is straightforward to show that $W_k^{(2)} = v_k w_k + \gamma V_{k+1}(0, r_{k+1}) = Y_k^{(2)}$. Consequently, it is optimal to fulfill all the backorders. However, since the amount of the expediting order is limited by M_k, we have $\theta_k^*(r_{k+1}) = \min\{0, w_k + M_k\}$.

Then, for $x_k \longrightarrow +\infty$, we have $x_k - D_k \geq 0$, i.e., $w_k \geq 0$. Thus, $\partial G_k(x_k, r_k)/\partial x_k|_{x_k \longrightarrow +\infty}$ depends on the analysis in Case I; we have

$$\left.\frac{\partial G_k(x_k, r_k)}{\partial x_k}\right|_{x_k \longrightarrow +\infty} = s_k \geq 0, \tag{A.20}$$

because of the concavity, $\partial V_k(x_k, r_k)/\partial x_k \geq \partial G_k(x_k, r_k)/\partial x_k|_{x_k \longrightarrow +\infty}$. Then $\partial V_k(x_k, r_k)/\partial x_k \geq 0$. Therefore, $V_k(x_k, r_k)$ is increasing in x_k for a given r_k.

Finally, we show the supermodularity of $H_t(z_t, w_t, r_{t+1})$ in (z_t, w_t). From (5), we only need to prove the supermodularity of $V_{t+1}(w_t - z_t, r_{t+1})$ and the submodularity of $h_t(w_t - z_t)^+ + b_t(z_t - w_t)^+$ in (z_t, w_t). Because the concavity of $V_{t+1}(w_t - z_t, r_{t+1})$, it is clear that $V_{t+1}(w_t - z_t, r_{t+1})$ is supermodular in (z_t, w_t). In addition, the submodularity of $h_t(w_t - z_t)^+ + b_t(z_t - w_t)^+$ follows from the convexity of $h_t(x)^+ + b_t(x)^+$. Thus, $H_t(z_t, w_t, r_{t+1})$ is supermodular in (z_t, w_t).

(iv) The supermodularity of $V_t(x_t, r_t)$ in (x_t, r_t) is also proved by induction. Starting from $t = T$, it is obvious that $V_{T+1}(x_{T+1}, r_{T+1})$ is supermodular in (x_{T+1}, r_{T+1}). Thus $J_T(y_T, d_T, r_T)$ is supermodular in (x_T, r_T) since the terms in $J_T(y_T, d_T, r_T)$ are independent of x_T. Following the maximization preserves supermodularity which yields the supermodularity of $V_T(x_T, r_T)$ in (x_T, r_T).

Assume that the result holds for $t = k + 1$. Next, we need to show that the result is still true for $t = k$. Since $J_k(y_k, d_k, r_k)$ is independent of x_k, we only need to proof the supermodularity of $J_k(y_k, d_k, r_k)$ in (y_k, r_k), (d_k, r_k), and (y_k, d_k), which is equivalent to the supermodularity of $J_k(y_k, x_k, r_k)$ in (y_k, x_k, r_k), (d_k, x_k, r_k), and (y_k, d_k, x_k).

First, we proof the supermodularity of $J_k(y_k, d_k, r_k)$ in (y_k, r_k). The terms in $J_k(y_k, d_k, r_k)$ either depends on y_k or r_k or is a constant with respect to y_k and r_k except for $V_{k+1}(y_k - \beta_k d_k - \epsilon_k - z_k, \alpha r_k + (1-\alpha)p_k)$, so it suffices to show the supermodularity of $V_{k+1}(y_k - \beta_k d_k - \epsilon_k - z_k, \alpha r_k + (1-\alpha)p_k)$ in (y_k, r_k).

Consider arbitrary pair (y_k^1, y_k^2) and (r_k^1, r_k^2) with $y_k^1 > y_k^2$ and $r_k^1 > r_k^2$. Fix ϵ_k; let

$$\begin{aligned} (\tau_1, \xi_1) &= \left(y_k^1 - \beta_k d_k\left(p_k, r_k^1\right) - \epsilon_k, \xi_1\right), \\ (\tau_2, \xi_2) &= \left(y_k^1 - \beta_k d_k\left(p_k, r_k^2\right) - \epsilon_k, \xi_2\right) \\ (\tau_3, \xi_1) &= \left(y_k^2 - \beta_k d_k\left(p_k, r_k^1\right) - \epsilon_k, \xi_1\right), \\ (\tau_4, \xi_2) &= \left(y_k^2 - \beta_k d_k\left(p_k, r_k^2\right) - \epsilon_k, \xi_2\right) \end{aligned} \tag{A.21}$$

where $\xi_1 = \alpha r_k^1 + (1-\alpha) p_k(d_k, r_k^1)$; $\xi_2 = \alpha r_k^2 + (1-\alpha) p_k(d_k, r_k^2)$. Then we obviously have $\xi_1 > \xi_2$, $\tau_3 < \tau_4$. Thus, we have

$$\begin{aligned} & V_{k+1}(\tau_1, \xi_1) - V_{k+1}(\tau_3, \xi_1) \\ & \quad = V_{k+1}\left(\tau_3 + \left(y_k^1 - y_k^2\right), \xi_1\right) - V_{k+1}(\tau_3, \xi_1) \\ & \quad \geq V_{k+1}\left(\tau_4 + \left(y_k^1 - y_k^2\right), \xi_1\right) - V_{k+1}(\tau_4, \xi_1) \\ & \quad = V_{k+1}(\tau_2, \xi_1) - V_{k+1}(\tau_4, \xi_1) \\ & \quad \geq V_{k+1}(\tau_2, \xi_2) - V_{k+1}(\tau_4, \xi_2), \end{aligned} \tag{A.22}$$

where the first inequality follows from the concavity of V_{k+1} and the second inequality follows from the supermodularity of $V_{k+1}(\tau, \xi)$ in (τ, ξ) by induction assumption, which implies that $V_{k+1}(y_k^1 - \beta_k d_k - \epsilon_k - z_k, \alpha r_k + (1-\alpha) p_k) - V_{k+1}(y_k^2 - \beta_k d_k - \epsilon_k - z_k, \alpha r_k + (1-\alpha) p_k)$ is increasing in r_k. We thus get the supermodularity of $V_{k+1}(y_k - \beta_k d_k - \epsilon_k - z_k, \alpha r_k + (1-\alpha) p_k)$ in (y_k, r_k). Consequently, $J_k(y_k, d_k, r_k)$ is supermodular in (y_k, r_k).

Second, the supermodularity of $J_k(y_k, d_k, r_k)$ in (d_k, r_k) is similar to that of Theorem 6 in Güler et al. [15].

Third, the first two terms in (4) are supermodular in (y_k, d_k) by Assumption 2. Furthermore, because $H_t(z_t, w_t, r_{t+1})$ is supermodular in (y_k, d_k) by the concavity of H. Thus, $J_k(y_k, d_k, r_k)$ is supermodular in (y_k, d_k).

In summary, $J_k(y_k, d_k, r_k)$ is supermodular in (x_k, r_k). Therefore, $V_k(x_k, r_k)$ is supermodular in (x_k, r_k). This completes the proof. □

Proof of Theorem 4. (i) From (8), according to Assumption 2 and Theorem 3, $d_t \cdot p_t(d_t, r_t)$ and $H_t(z_t, w_t, r_{t+1})$ are concave, together with the concavity preserves by maximization yield the concavity of $G_t(y_t, r_t)$ in (y_t, r_t).

(ii) By the concavity of $H_t(z_t, w_t, r_{t+1})$ and supermodularity of $d_t \cdot p_t(d_t, r_t)$, we have that $-c_t y_t + G_t(y_t, r_t)$ is supermodular in (x_t, y_t) and $\{(x_t, y_t, r_t) \mid x_t \leq y_t \leq x_t + Q_t, \underline{p} \leq r_t \leq \overline{p}\}$ is a lattice in (x_t, y_t, r_t); thus $y_t^*(x_t, r_t)$ is increasing in x_t by Theorem 2.2.8 in Simchi-Levi et al. [50].

Note that $q_t^*(x_t, r_t)$ is given by

$$q_t^*(x_t, r_t) = \arg\max_{0 \leq q_t \leq Q_t} \{-c_t q_t + G_t(x_t + q_t, r_t)\}. \tag{A.23}$$

By the concavity of G_t, $-c_t y_t + G_t(x_t + q_t, r_t)$ is submodular in (x_t, q_t), because $\{(x_t, q_t, r_t) \mid 0 \leq q_t \leq Q_t, \underline{p} \leq r_t \leq \overline{p}\}$ is a lattice in (x_t, q_t, r_t), we obtain that $q_t^*(x_t, r_t)$ is decreasing in x_t.

(iii) Since $J_t(y_t, d_t, r_t)$ is joint concave in (y_t, d_t, r_t), we first rewrite (3) as

$$\begin{aligned} V_t(x_t, r_t) = & \max_{\underline{d}_t \leq d_t \leq \overline{d}_t} \mathsf{E}\left\{d_t \cdot p_t(d_t, r_t) + c_t x_t \right. \\ & + \max_{x_t \leq y_t \leq x_t + Q_t} \left[-c_t y_t \right. \\ & \left.\left. + \widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))\right]\right\}, \end{aligned} \tag{A.24}$$

where

$$\begin{aligned} & \widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t)) \\ & \quad = \max_{-M_t \leq z_t} H_t(z_t, y_t - \beta_t d_t - \epsilon_t, \alpha r_t \\ & \quad + (1-\alpha) p_t(d_t, r_t)). \end{aligned} \tag{A.25}$$

To see $d_t^*(x_t, r_t)$ is increasing in x_t, we only need to verify the supermodularity of function $d_t \cdot p_t(d_t, r_t) + c_t x_t + \max_{x_t \leq y_t \leq x_t + Q_t}[-c_t y_t + \widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))]$ in (x_t, d_t). Because the first two terms are supermodular according to Assumption 2, it suffices to show the supermodularity of

$$\begin{aligned} g_t(x_t, d_t, r_t) = & \max_{x_t \leq y_t \leq x_t + Q_t} \mathsf{E}\left[-c_t y_t \right. \\ & \left. + \widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))\right], \end{aligned} \tag{A.26}$$

in (x_t, d_t). Follows from the fact that V_t and $\alpha r_t + (1-\alpha) p_t(d_t, r_t)$ are increasing in r_t by Theorem 7 and Assumption 1, respectively, we have that $H_t(z_t, y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ is increasing in r_t. This, together with the concavity of $\widetilde{G}_t$, we can show the supermodularity of $\widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ in (d_t, r_t) similarly to the proof of Theorem 6 in Güler et al. [15]. Therefore, $-c_t y_t + \widetilde{G}_t(y_t - \beta_t d_t - \epsilon_t, \alpha r_t + (1-\alpha) p_t(d_t, r_t))$ is supermodular in (x_t, d_t, r_t) since it is independent of x_t. Taking expectation over β_t and ϵ_t preserves supermodularity and $\{(x_t, y_t, d_t, r_t) \mid x_t \leq y_t \leq x_t + Q_t, \underline{p} \leq r_t \leq \overline{p}\}$ is a lattice in (x_t, y_t, d_t, r_t), then $g_t(x_t, d_t, r_t)$ is supermodular in (x_t, d_t). By Theorem 2.2.8 in Simchi-Levi et al. [50], $d_t^*(x_t, r_t)$ is increasing in x_t. Finally, since $p_t(d_t, r_t)$ is decreasing in d_t, then $p_t^*(x_t, r_t)$ is decreasing in x_t.

(iv) Since $H_t(z_t, w_t, r_{t+1})$ is supermodular in (z_t, w_t) by Theorem 3 and $\{(z_t, w_t, r_{t+1}) \mid -M_t \leq z_t \leq (w_t)^+, \underline{p} \leq r_{t+1} \leq \overline{p}\}$ is a lattice in (z_t, w_t, r_{t+1}), we obtain that $z_t^*(w_t, r_{t+1})$ is increasing in w_t by Theorem 2.2.8 in Simchi-Levi et al. [50]. □

Data Availability

We did not use data in our research but programmed our results by using MATLAB 2014 to simulate the results of our research. If necessary, we can provide the Matlab source code.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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