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Research Article

Motion Reliability Analysis of the Delta Parallel Robot considering Mechanism Errors

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Delta parallel robots are widely used in assembly detection, packaging sorting, precision positioning, and other fields. With the widespread use of robots, people have increasing requirements for motion accuracy and reliability. This paper considers the influence of various mechanism errors on the motion accuracy and analyzes the motion reliability of the mechanism. Firstly, we establish a kinematic model of the robot and obtain the relationship between the position of the end effector and the structural parameters based on the improved D–H transform rule. Secondly, an error model considering the dimension error, the error of revolute joint clearance, driving error, and the error of spherical joint clearance is established. Finally, taking an actual robot as an example, the comprehensive influence of mechanism errors on motion accuracy and reliability in different directions is quantitatively analyzed. It is shown that the driving error is a key factor determining the motion accuracy and reliability. The influence of mechanism errors on motion reliability is different in different directions. The influence of mechanism errors on reliability is small in the vertical direction, while it is great in the horizontal direction. Therefore, we should strictly control the mechanism errors, especially the driving angle, to ensure the motion accuracy and reliability. This research has significance for error compensation, motion reliability analysis, and reliability prediction in robots, and the conclusions can be extended to similar mechanisms.

1. Introduction

Compared with series robots, parallel robots have the advantages of large carrying capacity, good motion performance, fast moving speed, etc. [1–4]. Delta parallel robots [5] are widely used in assembly detection, packaging, aerospace, medical devices, precision positioning, and other fields. People's demand for accuracy in parallel robots is also increasing. The motion accuracy of parallel robots is a key problem in their design and manufacturing [6, 7]. Northeastern University professor Sun Zhili [8] and his team made a thorough research on error modeling of parallel robot and analyzed the motion accuracy. Popponen and Arai [9] proposed an accuracy analysis method for linear parallel mechanisms based on position and clearance errors of hinges. In 2009, Chebbi et al. [10] studied the 3-UPU parallel robot and constructed a model of the mechanism. Mohan [11] presented the end effector pose error modeling and motion accuracy analysis of a planar 2PRP–PPR parallel manipulator with an unsymmetrical (U-shaped) fixed base. The error model was established based on the screw theory with considerations of configuration (geometrical) errors. Based on a novel six-degree-of-freedom (DOF) parallel manipulator mechanism, Fu [12] studied the forward and inverse kinematics. The paper addressed the kinematic accuracy problem of the proposed new parallel robot with three legs due to the location of the U joint errors, clearance, and driving errors. Chouaibi's paper showed that the orientation error variation depends essentially on the parallelogram configuration of the passive legs out of its plane [13].

Aiming at the error analysis and motion reliability analysis of the Delta parallel robot, currently, most studies mainly consider the influence of dimension error of the components. Some studies consider the clearance errors, but the main consideration is the error of revolute joint

Table 1: Homogeneous transformation matrix between local coordinate systems.

Local coordinate system	Variable θ_i	Homogeneous transformation matrix
$(xyz)_1$	θ_{i1}	${}^{O}\mathbf{T}_1 = \mathbf{R}(y, \alpha_i)\mathbf{M}(x, r_1)\mathbf{R}(z, -\theta_{i1})$
$(xyz)_2$	θ_{i2}	${}^{1}\mathbf{T}_2 = \mathbf{R}(x, a)\mathbf{M}(z, \theta_{i1})\mathbf{R}(z, -\theta_{i2})$
$(xyz)_3$	θ_{i3}	${}^{2}\mathbf{T}_3 = \mathbf{R}(y, \theta_{i3})$
$(xyz)_4$	*None*	${}^{3}\mathbf{T}_4 = \mathbf{M}(x, c)\mathbf{R}(y, -\theta_{i3})\mathbf{R}(z, \theta_{i2})$
$(xyz)_5$	θ_4	${}^{4}\mathbf{T}_5 = \mathbf{M}(x, -r_2)\mathbf{R}(y, -\alpha_i)\mathbf{R}(y, \theta_4)$
$(xyz)_6$	θ_5	${}^{5}\mathbf{T}_6 = \mathbf{M}(y, -d)\mathbf{R}(x, \theta_5)$

clearance in rotating structures. Because parallel mechanisms are used in high-speed situations, the error of spherical joint clearance will reduce the kinematic accuracy and aggravate wear and affect the motion stability of mechanisms. In addition, the existing research mainly considers the individual influence of each error on the kinematic accuracy of the mechanism. The influence on the motion error and reliability of the mechanism in different directions is not considered under the comprehensive effect of various errors.

According to the above situation, the kinematic model of the robot is established by the D–H matrix transformation method. An error model considering dimension error, the error of revolute joint clearance, driving error, and the error of spherical joint clearance is established. The influence of each error source on the motion error is analyzed. In addition, based on the above analysis, the comprehensive influence of mechanism errors on motion accuracy and reliability in different directions is analyzed. A reliability analysis provides an important theoretical basis for improving the kinematic performance of the mechanism and is of great significance to robot mechanism optimization [14].

2. The Coordinate System of the Delta Parallel Mechanism

The structural model of the Delta parallel robot is shown in Figure 1, and the kinematic chain of the Delta parallel robot is shown in Figure 2.

The global coordinate system O − XYZ is located at the center point of the fixed platform at point O. The x axis points to the OA_1, the y axis is perpendicular to the platform surface, and the negative direction of the y axis is the gravity direction of the moving platform.

The position of the rotating pair on the fixed platform and the moving platform is arranged in a triangle, of which the circumradius is, respectively, r_1 and r_2.

The length of the driving arm A_iB_i is a, and the length of the parallelogram mechanism (the driven arm B_iC_i) is c. The length of PN of the mechanism is d (PN is the distance between the center point P of the moving platform and the origin N of the local coordinate system). α_i is the angle between the OA_i and the global coordinate x axis, which is shown in the diagram. When the corresponding kinematic

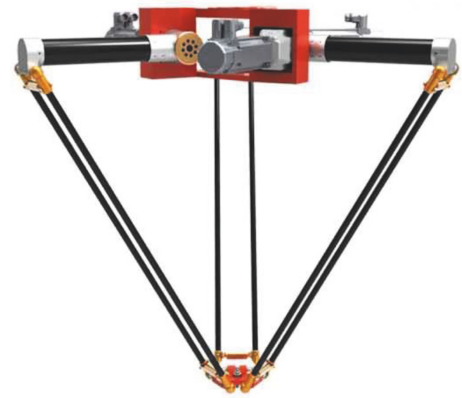

Figure 1: The structural model of the robot.

chain i is 1, 2, 3, α_i is 0°, 120°, and 240°, respectively. The input angle (driving angle) is θ_{i1}, θ_4, θ_5, and the rotation angle of driven arm is θ_{i2}. The swing angle of driven arm is θ_{i3}. Point N is the origin of $(xyz)_6$ in the local coordinate system. Point T is the working position.

3. Modeling of Mechanism Position Error

According to the structural characteristics of the delta parallel robot, it can be seen that the sources of the position error of the mechanism are mainly in the following categories.

(1) Dimension error.

(2) The error of revolute joint clearance.

(3) Driving error.

(4) The error of spherical joint clearance.

3.1. Considering Dimension Error. According to the homogeneous transformation rule, we can get the homogeneous transformation matrix between local coordinate systems, as shown in Table 1.

Therefore, the homogeneous transformation matrix ${}^{O}\mathbf{T}_6$ of the coordinate system $(xyz)_6$ relative to the global coordinate system $(XYZ)_O$ can be expressed in two ways as follows:

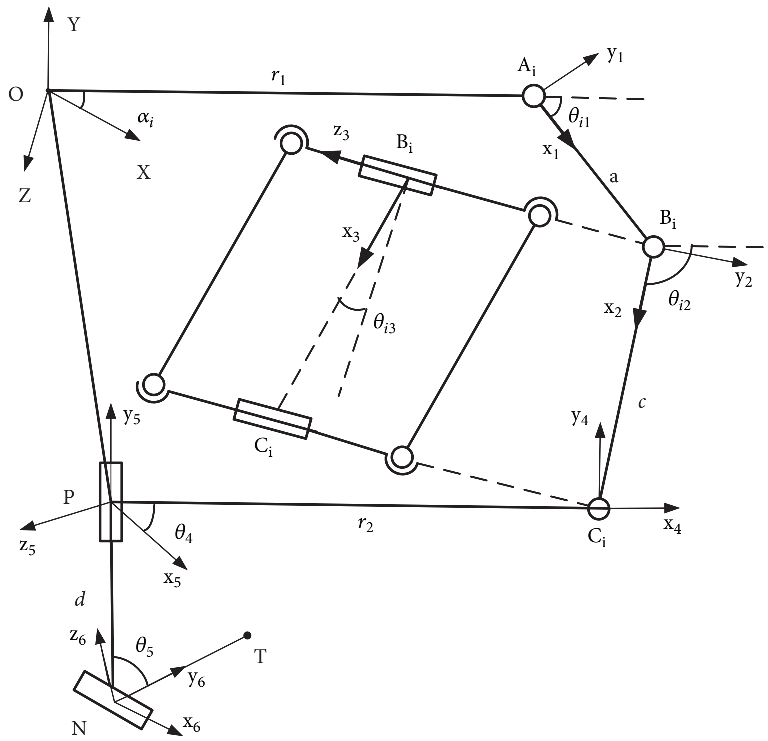

Figure 2: Motion diagram of kinematic chain.

$$
{}^{\mathrm{O}}\mathbf{T}_6 = \prod_{k=1}^{6} {}^{k-1}\mathbf{T}_k
$$

$$
= \begin{bmatrix} \cos\theta_4 & \sin\theta_5\sin\theta_4 & \cos\theta_5\sin\theta_4 & a\ \cos\alpha_i\cos\theta_{i1} + c\left(\cos\alpha_i\cos\theta_{i2}\cos\theta_{i3} - \sin\alpha_i\sin\theta_{i3}\right) + \left(r_1 - r_2\right)\cos\alpha_i \\ 0 & \cos\theta_5 & -\sin\theta_5 & -d - c\sin\theta_{i2}\cos\theta_{i3} - a\ \sin\theta_{i1} \\ -\sin\theta_4 & \sin\theta_5\cos\theta_4 & \cos\theta_5\cos\theta_4 & -a\ \sin\alpha_i\cos\theta_{i1} - c\left(\cos\alpha_i\sin\theta_{i3} + \sin\alpha_i\cos\theta_{i2}\cos\theta_{i3}\right) + \left(r_2 - r_1\right)\sin\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)
$$

$$
{}^{\mathrm{O}}\mathbf{T}_6 = \begin{bmatrix} \cos\theta_y & \sin\theta_y\cos\theta_x & \sin\theta_y\cos\theta_x & x \\ 0 & \cos\theta_x & -\sin\theta_x & y \\ -\sin\theta_y & \sin\theta_x\cos\theta_y & \cos\theta_x\cos\theta_y & z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2)
$$

where $\theta_x$and θ_y are the angles of the rotating mechanism around the x axis and the y axis of the global coordinate system, respectively. It is easy to get $\theta_x = \theta_5$, $\theta_y = \theta_4$.

Let (1) and (2) be transformed as follows: $\mathbf{R}(y, \alpha_i)^{-1}{}^{\mathrm{O}}\mathbf{T}_6 R(x, \theta_5)^{-1}\mathbf{M}(y, -d)^{-1}\mathbf{R}(y, \theta_4)^{-1}\mathbf{R}(y, -\alpha_i)^{-1}$; we can get

$$
\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & x\cos\alpha_i - z\sin\alpha_i \\ 0 & 1 & 0 & d + y \\ 0 & 0 & 1 & x\sin\alpha_i + z\cos\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)
$$

$$
\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & r_1 - r_2 + c\cos\theta_{i2}\cos\theta_{i3} + a\cos\theta_{i1} \\ 0 & 1 & 0 & -c\sin\theta_{i2}\cos\theta_{i3} - a\sin\theta_{i1} \\ 0 & 0 & 1 & -c\sin\theta_{i3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)
$$

From (3) and (4), we can obtain

$$
\begin{aligned} x &= -c\sin\theta_{i3}\sin\alpha_i + r_1\cos\alpha_i - r_2\cos\alpha_i \\ &\quad + a\cos\theta_{i1}\cos\alpha_i + c\cos\theta_{i2}\cos\theta_{i3}\cos\alpha_i \\ y &= -a\sin\theta_{i1} - c\sin\theta_{i2}\cos\theta_{i3} - d \end{aligned}
$$

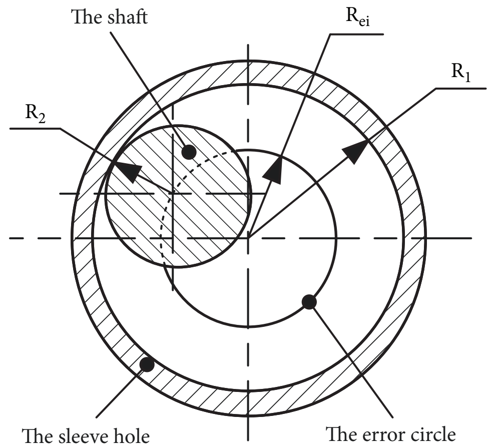

FIGURE 3: Model of revolute joint.

$$z = -c\sin\theta_{i3}\cos\alpha_i - r_1\sin\alpha_i + r_2\sin\alpha_i - a\cos\theta_{i1}\sin\alpha_i - c\cos\theta_{i2}\cos\theta_{i3}\sin\alpha_i. \quad (5)$$

Equation (5) is the relationship between the end output position and the structural parameters in the ideal case. In fact, the parameters of each structure have certain deviations due to the influence of various factors.

The Taylor series expansion is carried out on the upper form. The expression is carried out as follows:

$$\begin{aligned}\Delta x^{o} &= \sum_{i=1}^{3}\big(\cos\alpha_i\Delta r_{i1} - \cos\alpha_i\Delta r_{i2} + \cos\theta_{i1}\cos\alpha_i\Delta a_i \\ &\quad + (\cos\theta_{i2}\cos\theta_{i3}\cos\alpha_i - \sin\theta_{i3}\sin\alpha_i)\Delta c_i \\ &\quad + (-r_{i1}\sin\alpha_i + r_{i2}\sin\alpha_i - a_i\cos\theta_{i1}\sin\alpha_i \\ &\quad - c_i\cos\theta_{i2}\cos\theta_{i3}\sin\alpha_i - c_i\sin\theta_{i3}\cos\alpha_i)\Delta\alpha_i\big) \\ \Delta y^{o} &= -\sum_{i=1}^{3}(\sin\theta_{i1}\Delta a_i + \sin\theta_{i2}\cos\theta_{i3}\Delta c_i + \Delta d) \qquad (6) \\ \Delta z^{o} &= \sum_{i=1}^{3}\big(-\sin\alpha_i\Delta r_{i1} + \sin\alpha_i\Delta r_{i2} - \cos\theta_{i1}\sin\alpha_i\Delta a_i \\ &\quad - (\cos\theta_{i2}\cos\theta_{i3}\sin\alpha_i + \sin\theta_{i3}\cos\alpha_i)\Delta c_i \\ &\quad + (-r_{i1}\cos\alpha_i + r_{i2}\cos\alpha_i - a_i\cos\theta_{i1}\cos\alpha_i \\ &\quad - c_i\cos\theta_{i2}\cos\theta_{i3}\cos\alpha_i + c_i\sin\theta_{i3}\sin\alpha_i)\Delta\alpha_i\big).\end{aligned}$$

The relation between the input error and the output error of the mechanism is obtained from (6).

According to the above equation, the error sources are Δr_{i1}, Δr_{i2}, Δa_i, Δc_i, $\Delta\alpha_i$, etc. Moreover, there are also errors in revolute joints, driving errors, and errors in spherical joints.

3.2. Considering the Error of Revolute Joint Clearance. The existence of clearances in these joints is inevitable due to machining tolerances, wear, and material deformation. The revolute joint of the Delta robot is composed of a sleeve hole and a shaft. The error of the revolute joint is the difference between the radii of the two components; that is, the clearance error is the radial error of the revolute joint. Therefore, the planar model is used to describe the clearance of the revolute joint, as shown in Figure 3.

The error circle is the circle with a radius of R_{ei}. From the model of revolute joint, we can get

$$R_{ei} = R_1 - R_2, \quad (7)$$

where R_1 and R_2 are radii of the sleeve hole and the shaft.

This paper only considers the clearance error between the revolute joint of the platform and the driving arm. According to the effective length theory [15], the clearance error R_{ei} of the revolute joint is equivalent to the change in length of the driving arm Δa. Namely, $R_{ei} = \Delta a$. Therefore, the error caused by the revolute joint is as follows:

$$\begin{aligned}\Delta x^{R} &= \sum_{i=1}^{3}\cos\theta_{i1}\cos\alpha_i R_{ei} \\ \Delta y^{R} &= -\sum_{i=1}^{3}\sin\theta_{i1} R_{ei} \qquad (8) \\ \Delta z^{R} &= -\sum_{i=1}^{3}\cos\theta_{i1}\sin\alpha_i R_{ei}.\end{aligned}$$

3.3. Considering Driving Error. The input angle θ_{i1} is the driving angle of the mechanism at any moment. The first-order Taylor series expansion of (5) is at θ_{i1}. Therefore, the error caused by driving error is as follows:

$$\begin{aligned}\Delta x^{D} &= \sum_{i=1}^{3} - a\sin\theta_{i1}\cos\alpha_i\Delta\theta_{i1} \\ \Delta y^{D} &= \sum_{i=1}^{3} - a\cos\theta_{i1}\Delta\theta_{i1} \qquad (9) \\ \Delta z^{D} &= \sum_{i=1}^{3} a\sin\theta_{i1}\sin\alpha_i\Delta\theta_{i1}.\end{aligned}$$

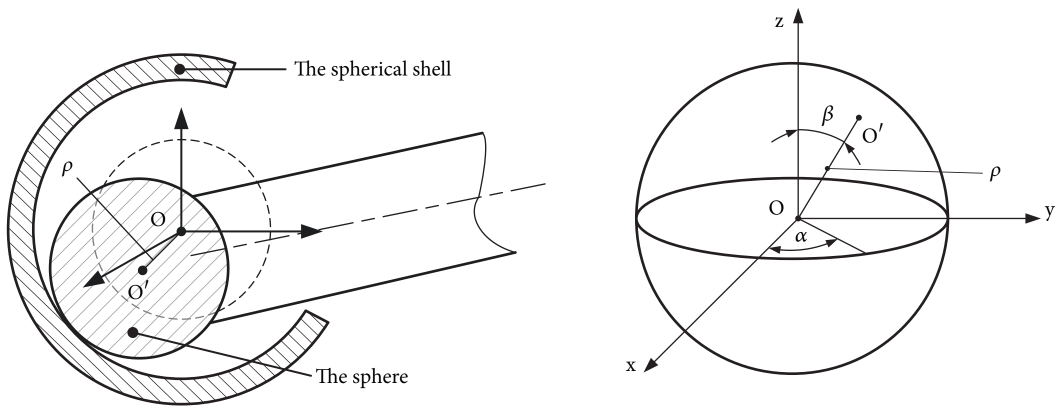

Figure 4: Model of spherical joint.

3.4. Considering the Error of Spherical Joint Clearance. The structure of spherical joint is composed of spherical shell and sphere. The structure of the spherical joint is shown in Figure 4. When the robot works, the sphere moves in the spherical shell, and the motion model at this time is a spatial model. Ideally, the spherical center of the sphere is located at point O. Because the clearance is inevitable, the spherical center is actually located at point O′. In two cases, the distance between the center of the sphere and the origin is the clearance of the spherical joint.

The error of the spherical joint affects the output error of the Delta parallel robot. The spherical joint constrains the translational freedom in three directions, while it does not limit the rotational degree of freedom. Therefore, the error caused by the clearance between the spherical joint is only three translational errors [16]. Namely, the errors in the x, y, and z directions are Δx, Δy, and Δz, respectively.

When the clearance error of the spherical joint is ρ, the angle between the projection in the xoy plane of ρ and the x axis is α, and the angle between the clearance direction and the z axis is β. The errors of the spherical joint in the x, y, and z directions are as follows:

$$\begin{aligned} \Delta x^{\mathrm{Q}} &= \rho \sin\beta\cos\alpha \\ \Delta y^{\mathrm{Q}} &= \rho \sin\beta\sin\alpha \\ \Delta z^{\mathrm{Q}} &= \rho\cos\beta. \end{aligned} \tag{10}$$

According to the structural diagram of the Delta parallel mechanism, it is known that the clearance ρ of spherical joint is equivalent to the change in length of the parallelogram mechanism (driven arm) rod Δc. Namely, $\rho = \Delta c$. Therefore, the position error of the end effector caused by spherical joint is equivalent to the output error considering Δc. So it can be obtained:

$$\begin{aligned} \Delta x^{\mathrm{Q}} &= \sum_{i=1}^{3} \left(\cos\theta_{i2}\cos\theta_{i3}\cos\alpha_i - \sin\theta_{i3}\sin\alpha_i\right)\rho \\ \Delta y^{\mathrm{Q}} &= -\sum_{i=1}^{3} \sin\theta_{i2}\cos\theta_{i3}\rho \\ \Delta z^{\mathrm{Q}} &= -\sum_{i=1}^{3} \left(\cos\theta_{i2}\cos\theta_{i3}\sin\alpha_i + \sin\theta_{i3}\cos\alpha_i\right)\rho \end{aligned} \tag{11}$$

From the above analysis, the total position error of the end effector in the x, y, and z directions is the superposition of the four kinds of errors above at any time. So it can be obtained:

$$\Delta q = \Delta q^{\mathrm{O}} + \Delta q^{\mathrm{R}} + \Delta q^{\mathrm{D}} + \Delta q^{\mathrm{Q}} = \sum_{i=1}^{n} K q_{si} \Delta s_i, \quad q = x, y, z \tag{12}$$

In (12), Kq_{si} is the error transfer coefficient of Δs_i.

The value of Kq_{si} is usually determined by the posture and structural parameters of the mechanism together. The value of Kq_{si} can quantitatively reflect the degree of the influence of error Δs_i on the motion accuracy of the robot.

4. Motion Reliability Analysis of the Mechanism

Due to the existence of random factors such as tolerance and driving, the value of the error in the error model is not a fixed single value but is distributed randomly. Motion reliability analysis is to calculate the probability that the output error of the end effector falls within the allowable range. Namely, that is to obtain the motion reliability of the mechanism.

The size distribution in the mechanical manufacturing industry generally conforms to the normal distribution [17]. By referring to the relevant literature [18], it is normalized and verified that the joint clearance obeys the normal distribution.

The error of the rods, the error of revolute joint clearance and the spherical joint clearance, the angle error, etc. belong to the dimension errors. Therefore, each error source of the mechanism can be seen as a normal distribution. It is easy to know that the position error of the end effector also obeys the normal distribution in the x, y, and z directions at any time.

The expressions of mean value and standard deviation are as follows:

$$\mu_s = \sum_{i=1}^{n} K q_{si} \mu_{si} \tag{13}$$

$$\sigma_s = \sqrt{D_s} = \sqrt{\sum_{i=1}^{n} K q^2{}_{si} \sigma^2{}_{si}}, \tag{14}$$

where μ_{si} and σ_{si} are the mean value and standard deviation of each original input error.

By statistical knowledge and by analogy with the theory of stress intensity distribution interference and reliability engineering, the allowable error distribution is similar to the intensity distribution, and the actual error distribution is similar to the stress distribution. The actual error distribution obeys the normal distribution $N \sim (\mu_s, \sigma_s{}^2)$.

In most cases, the random errors affecting the motion accuracy of the mechanism are independent or weakly correlated, and they generally have a normal distribution.

When calculating the reliability of a mechanism's motion accuracy, there are many random error components that need to be considered. Their mean and standard deviations are $\mu_1, \mu_2, \ldots, \mu_n$ and$\sigma_1, \sigma_2, \ldots, \sigma_n$, respectively. According to the operation rules of normal distribution and variance, the mean and standard deviation of the synthesized output error are

$$\mu_s = \mu_1 + \mu_2 + \cdots + \mu_n \tag{15}$$

$$\sigma_s = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \cdots + \sigma_n{}^2}. \tag{16}$$

When the allowable error is ε, $\varepsilon \in [\varepsilon_1, \varepsilon_2]$; then the reliability is

$$R = P \quad (\varepsilon_1 < \Delta q < \varepsilon_2) \tag{17}$$

$$\begin{aligned} R &= P\ (\varepsilon_1 < \Delta q < \varepsilon_2) \\ &= P\ (\Delta q < \varepsilon_2) - P\ (\Delta q < \varepsilon_2) = \Phi\left(\frac{\varepsilon_2 - \mu}{\sigma}\right) \\ &\quad - \Phi\left(\frac{\varepsilon_1 - \mu}{\sigma}\right) \end{aligned} \tag{18}$$

Because the output error of the mechanism is a series of random values that conform to normal distribution based on the stress-strength interference theory and reliability theory, the allowable range about the mechanism does not need to be determined. The allowable range is not a fixed value but variable. For the sake of uniformity and convenience of calculation, the allowable error can be regarded as having normal distribution.

The mean value of the allowable error is μ_ε, and the standard deviation is σ_ε; then the allowable error ε is subject to the normal distribution $N \sim (\mu_\varepsilon, \sigma_\varepsilon{}^2)$.

When the allowable error ε obeys the normal distribution, let the function be $G(Z)$.

$G(Z) = \varepsilon - \Delta q > 0$, where ε is the maximum value of allowable error.

The above equation indicates that the output error is smaller than the allowable error. Since both the output error Δq and the allowable error ε obey the normal distribution, it is easy to know that their probability density functions are as follows:

$$f_1(x_1) = \Delta q = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left[-\frac{1}{2}\left(\frac{x - \mu_s}{\sigma_s}\right)^2\right] \tag{19}$$

$$f_2(x_2) = \varepsilon = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp\left[-\frac{1}{2}\left(\frac{y - \mu_\varepsilon}{\sigma_\varepsilon}\right)^2\right]. \tag{20}$$

In the above equation, $\mu_s, \sigma_s, \mu_\varepsilon$, and σ_ε are the mean value and standard deviation of Δq and ε, respectively. According to the stress-strength interference model, the reliability can be expressed as follows:

$$\begin{aligned} R &= P\ (\varepsilon > \Delta q) \\ &= \int_{-\infty}^{\infty} f_2(x_2) \left[\int_{-\infty}^{x_2} f_1(x_1)\, \mathrm{d}x_1\right] \mathrm{d}x_2 \\ &= \int_0^{\infty} f(z)\, \mathrm{d}z \\ &= \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{1}{2}\left(\frac{z - \mu_z}{\sigma_z}\right)^2\right] \mathrm{d}z. \end{aligned} \tag{21}$$

Turning it into a standard normal distribution, let $\mu = (Z - \mu_z)/\sigma_z$.

Then the following equation can be obtained:

$$\begin{aligned} R &= P\ (\varepsilon > \Delta q) = \int_0^{\infty} f(z)\, \mathrm{d}z = \\ &= \int_{-\beta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}u^2\right] \mathrm{d}u = \Phi(\beta). \end{aligned} \tag{22}$$

where $f(z) = (1/\sqrt{2\pi}\sigma_z)\exp[-(1/2)((z - \mu_z)/\sigma_z)^2]$, $\beta = \mu_z/\sigma_z = (\mu_\varepsilon - \mu_s)/\sqrt{\sigma_\varepsilon{}^2 + \sigma_s{}^2}$, $Z = \sigma - \Delta q$.

The reliability coefficient can be obtained from the definition of reliability and the coupling equation of the reliability calculation:

$$z_R = \frac{\mu_\varepsilon - \mu_s}{\sqrt{\sigma_\varepsilon{}^2 + \sigma_s{}^2}}. \tag{23}$$

Then the motion reliability of parallel mechanism in the$x$, y, and z directions is obtained:

$$R_q = \Phi(z_R), \quad q = x, y, z, \tag{24}$$

where $\Phi(\bullet)$is the standard normal distribution function.

5. Numerical Example

This paper takes the robot of Chen Xing (Tianjin) automation equipment company in our laboratory as an example. The model of this robot is D3PM–1000. It is mainly composed of a fixed platform, moving platform, driving arm, and driven arm. The robot's workspace covers a diameter of 500–1400

Table 2: Structural parameters of the robot.

Structural parameters	Numerical value
Radius of the fixed platform (r_1/mm)	125
Radius of the moving platform (r_2/mm)	50
The length of the driving arm (a/mm)	400
The length of the driven arm (c/mm)	950

Figure 5: The diagram of D3PM–1000 robot.

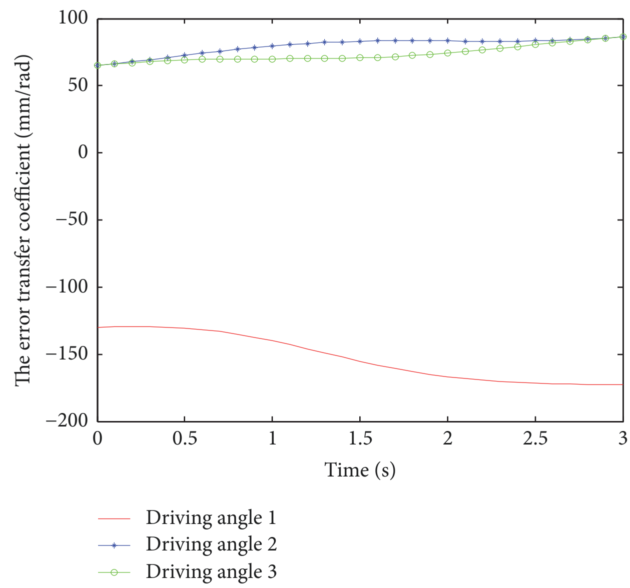

Figure 6: The error transfer coefficient of the driving angle $\Delta\theta_{i1}$ in the x direction.

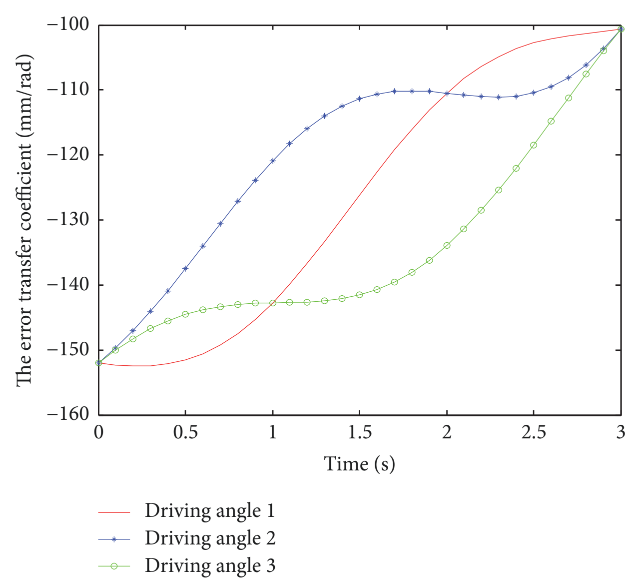

Figure 7: The error transfer coefficient of the driving angle $\Delta\theta_{i1}$ in the y direction.

mm, and its grasping speed is 75–150 times per minute. Its maximum acceleration and maximum velocity are 100 m/s^2 and 8 m/s, respectively. It has the characteristics of moving in the x, y, and z directions and is widely used in the field of high-speed sorting and packaging. The robot is shown in Figure 5.

Through the product instruction manual and actual measurement, the structural parameters of this robot are shown in Table 2.

The distribution angles of the driving arm are $\alpha_1 = 0^o$, $\alpha_2 = 120^o$, and $\alpha_3 = 240^o$, respectively. The trajectory of the center point of the mechanism end actuator is as follows: $x = 30\sin(120^o t)$ (mm), $y = -500 - 20t$ (mm), and $z = -30\cos(120^o) + 30$ (mm), and the exercise time is 3 s.

(1) The expression of each error source shows that the error transfer coefficient of the seven error sources $(\Delta r_{i1}, \Delta r_{i2}, \Delta a_i, \Delta c_i, \Delta\alpha_i, R_{ei}, \rho)$ is small, and the errors are compensatory on the three kinematic chains; they are not the main factors affecting the motion accuracy of the mechanism. The error transfer coefficient of the driving angle $\Delta\theta_{i1}$ is obviously greater than the other error transfer coefficients, which has a greater impact on the position error of the mechanism, so it should be strictly controlled.

The error transfer coefficient of the driving angle $\Delta\theta_{i1}$ is shown in Figures 6–8.

Similarly, the error transfer coefficient graphs of other error sources can be obtained but are omitted here.

(2) When t=1s, the motion reliability analysis of the end effector of the mechanism is made (the permissible precision of the output error is ε, $\varepsilon \sim N(1, 0.1^2)$).

The original errors are independent and consistent with normal distribution. From the real discrete distribution data of each original error, the distribution law of each original error can be determined according to Monte Carlo

TABLE 3: The mean value and standard deviation of the input errors.

Original input errors (i=1,2,3)	Mean value	Standard deviation
Dimension error of the fixed platform Δr_{i1}/mm	0	0.01
Dimension error of the moving platform Δr_{i2}/mm	0	0.01
Length error of the driving arm Δa_i/mm	0	0.01
Length error of the driven arm Δc_i/mm	0	0.01
Installation error $\Delta \alpha_i$/rad	0	0.01
Driving error $\Delta \theta_{i1}$/rad	0.01	0.01
The error of revolute joint clearance R_{ei}/mm	0.01	0.01
The error of spherical joint clearance ρ/mm	0.01	0.01

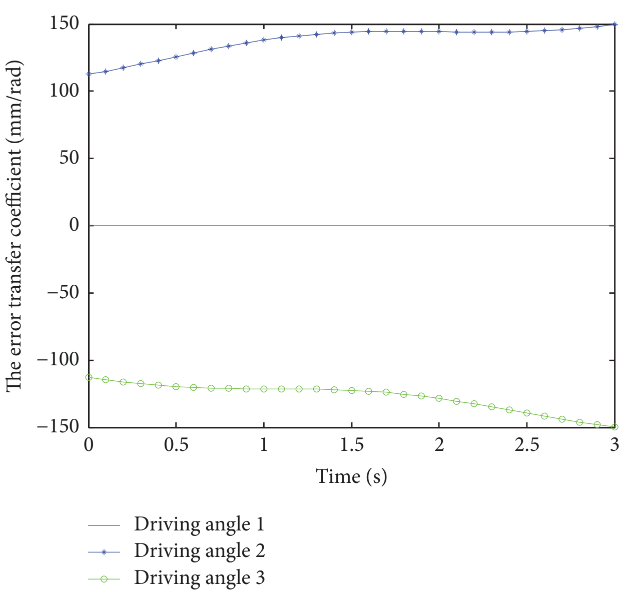

FIGURE 8: The error transfer coefficient of the driving angle $\Delta\theta_{i1}$ in the z direction.

theory [19], and then the mean and standard deviation can be calculated. Finally, the reliability can be obtained according to the distribution parameters and their mean value and standard deviation are shown in Table 3.

According to the mean value and standard deviation of each original error source in Table 3 and the error transfer coefficient of each error, the distribution law of the error caused by each error can be determined. When the mean value and standard deviation of the total error are determined, the motion reliability of the robot can be calculated.

Taking t=1s, the error transfer coefficient of the error sources on each chain is shown in Table 4.

The error transfer coefficients of the error sources in Table 4 show that the error transfer coefficients of Δr_{i1}, Δr_{i2}, Δa_i, Δc_i, $\Delta \alpha_i$, R_{ei}, and ρ are relatively small and have certain compensations in the x, y, and z directions, while the error source of $\Delta\theta_{i1}$ has a relatively large influence on the output error.

The error transfer coefficients in Table 4 provide the data source for determining the mean value and the variance of the total error by the superposition principle.

According to $\mu_s = \sum_{i=1}^{n} Kq_{si}\mu_{si}$ and $\sigma_s = \sqrt{D_s} = \sqrt{\sum_{i=1}^{n} Kq^2_{si}\sigma^2_{si}}$, the mean value and variance of output error can be deduced from the original error. The mean value and the variance of output error in the x, y, and z directions are shown in Table 5.

(3) From the above data, the calculation of motion reliability is as follows.

From Table 5, the following is obtained.

The mean of output error in the x direction: μ_x= 0.10.

The mean of output error in the y direction: μ_y= −3.96.

The mean of output error in the z direction: μ_z= 0.16.

The variance of output error in the x direction: σ_x^2= 1.07, then σ_x= 1.03.

The variance of output error in the y direction: σ_y^2= 5.13, then σ_y= 2.26.

The variance of output error in the z direction: σ_z^2= 0.68, then σ_z= 0.82.

When t = 1 s, the reliability of the mechanism in the x direction is

$$R_x = \Phi\left(\frac{\mu_\varepsilon - \mu_x}{\sqrt{\sigma_\varepsilon^2 + \sigma_x^2}}\right) = \Phi\left(\frac{1 - 0.10}{\sqrt{0.1^2 + 1.03^2}}\right) = \Phi(0.87) = 0.8079 \tag{25}$$

The reliability of the mechanism in the y direction is

$$R_y = \Phi\left(\frac{\mu_\varepsilon - \mu_y}{\sqrt{\sigma_\varepsilon^2 + \sigma_y^2}}\right) = \Phi\left(\frac{1 - (-3.96)}{\sqrt{0.1^2 + 2.26^2}}\right) = \Phi(2.19) = 0.9857 \tag{26}$$

Table 4: Error transfer coefficient of each error source on each kinematic chain.

Kq_{si}		Δr_{i1}	Δr_{i2}	Δa_i	Δc_i
x direction	$i = 1$	1	−1	0.6889	−0.2080
	$i = 2$	−0.5	0.5	−0.2883	0.1760
	$i = 3$	−0.5	0.5	−0.3444	0.2434
y direction	$i = 1$	0	0	−0.7249	−0.9716
	$i = 2$	0	0	−0.8171	−0.9360
	$i = 3$	0	0	−0.7249	−0.9716
z direction	$i = 1$	0	0	0	0.1125
	$i = 2$	−0.866	0.866	−0.4993	0.3048
	$i = 3$	0.866	−0.866	0.5966	−0.1239
Kq_{si}		$\Delta \alpha_i$	$\Delta \theta_{i1}$	R_{ei}	ρ
x direction	$i = 1$	45.0000	−144.9759	0.6889	−0.2080
	$i = 2$	22.0829	81.7097	−0.2883	0.1760
	$i = 3$	69.7518	72.4879	−0.3444	0.2434
y direction	$i = 1$	0	−137.7752	−0.7249	−0.9716
	$i = 2$	0	115.3001	−0.8171	−0.9360
	$i = 3$	0	−137.7752	−0.7249	−0.9716
z direction	$i = 1$	−54.5617	0	0	0.1125
	$i = 2$	−12.7496	141.5253	−0.4993	0.3048
	$i = 3$	−11.6903	−125.5528	0.5966	−0.1239

Table 5: The mean value and the variance of output error.

The direction	The mean value	The variance
x direction	0.10	1.07
y direction	−3.96	5.13
z direction	0.16	0.68

The reliability of the mechanism in the z direction is

$$R_z = \Phi\left(\frac{\mu_\varepsilon - \mu_z}{\sqrt{{\sigma_\varepsilon}^2 + {\sigma_z}^2}}\right) = \Phi\left(\frac{1 - 0.16}{\sqrt{0.1^2 + 0.82^2}}\right) = \Phi(1.01) = 0.8438 \quad (27)$$

Therefore, the reliability of the mechanism is

$$R_S = \frac{1}{n}\sum_{i=1}^{n} R_i = \frac{1}{3}\left(R_x + R_y + R_z\right) = 0.8791 \quad (28)$$

When the mechanism errors are not considered, we can obtain that the mean value and the standard deviation of output error are $\mu' = 0$, $\sigma' = 0$.

Therefore, the reliability of the mechanism is

$$R_S' = \Phi\left(\frac{\mu_\varepsilon - \mu'}{\sqrt{{\sigma_\varepsilon}^2 + {\sigma'}^2}}\right) = \Phi\left(\frac{1 - 0}{\sqrt{0.1^2 + 0^2}}\right) = \Phi(10) = 0.9999. \quad (29)$$

Setting ΔR_S as the difference between the calculated values of reliability in two cases, we can obtain $\Delta R_S = R_S' - R_S = 0.9999 - 0.8791 = 0.1208 = 12.08\%$.

That is to say, when there are mechanism errors, the motion reliability of the mechanism will decrease by about 12%. The error transfer coefficient of the driving arm angle $\Delta\theta_{i1}$ is obviously greater than the other error transfer coefficients, which has a greater impact on the position error of the mechanism. Therefore, the mechanism errors, especially the errors of driving arm angle, have a significant effect on the motion accuracy and reliability of the robot.

When considering the mechanism errors, the reliability calculation result obtained in the y direction is basically the same as the result without considering the mechanism error, and both are around 0.99. Setting ΔR_y as the difference between the calculated values of reliability, we can obtain $\Delta R_y = R_y' - R_y = R_S' - R_y = 0.9999 - 0.9857 = 0.0142 = 1.42\%$. So, the motion accuracy of the mechanism in the y direction is relatively easy to control.

The results of this paper can be applied to other high-speed and high-precision mechanisms, such as high-speed parallel machine tools. In order to improve the manufacturing precision, the influence of the mechanism errors should be considered. In addition, the reliability analysis of the mechanism can provide a reference for setting reliability standards for the parallel mechanism.

6. Conclusions

The paper establishes a kinematic model and the error model for the Delta robot and analyzes the motion reliability of the robot considering mechanism errors. According to the simulation results and computational analysis, driving error is the main factor affecting motion accuracy of the mechanism. The influence of driving error on the motion reliability of the mechanism is decisive, so it should be strictly controlled. The effect of the mechanism errors on the motion reliability varies with the direction. The mechanism errors have little influence on the reliability of the mechanism in the y direction, while the mechanism errors have greater influence on the reliability in the x and z directions.

Additionally, the influence of each error source on the motion error and reliability of the mechanism is considered comprehensively. Under the comprehensive influence of mechanism errors, the reliability of the mechanism is reduced by about 12%. Therefore, the mechanism errors should be minimized; the driving angle, especially, should be controlled to improve the motion accuracy of the robot. The numerical example in this paper shows that the calculation methods and results of motion reliability provide a reference for reliability analysis of other parallel mechanisms.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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