Research Article

Mixed Simultaneous Iterative Algorithms for the Extended Multiple-Set Split Equality Common Fixed-Point Problem with Lipschitz Quasi-Pseudocontractive Operators

Meixia Li¹,^{ID} Xueling Zhou,² and Haitao Che¹^{ID}

¹School of Mathematics and Information Science, Weifang University, Weifang, Shandong 261061, China

²College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China

Correspondence should be addressed to Meixia Li; limeixia001@163.com

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In this paper, we study a kind of extended multiple-set split equality common fixed-point problem with Lipschitz quasi-pseudocontractive operators, which is an extension of multiple-set split equality common fixed-point problem with quasi-nonexpansive operator. We propose two mixed simultaneous iterative algorithms, in which the selecting of the stepsize does not need any priori information about the operator norms. Furthermore, we prove that the sequences generated by the mixed simultaneous iterative algorithms converge weakly to the solution of this problem. Some numerical results are shown to illustrate the feasibility and efficiency of the proposed algorithms.

1. Introduction

Throughout this paper, we always assume that H_1, H_2 , and H_3 are real Hilbert spaces with inner product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$. Let C and Q be nonempty closed and convex subsets of H_1 and H_2 , respectively. $A : H_1 \rightarrow H_2$ is a bounded linear operators. Let I denote the identity operator. The split feasibility problem (SFP) originally introduced by Censor and Elfving [1] which is to find

$$x \in C \quad \text{such that} \quad Ax \in Q. \quad (1)$$

It can be used in various disciplines such as image restoration, radiation therapy treatment planning [2, 3]. These applications are in finite-dimensional Hilbert spaces [4–7]. It also can be found in an infinite-dimensional real Hilbert space [8–10] and references therein.

In [11], Censor et al. introduce the multiple-set split feasibility problem (MSFP) which finds application in intensity-modulated radiation therapy. It is formulated as finding a point x satisfying the following property:

$$x \in \bigcap_{i=1}^p C_i \quad \text{such that} \quad Ax \in \bigcap_{j=1}^r Q_j, \quad (2)$$

where $p, r \geq 1$ are integers and $\{C_i\}_{i=1}^p$ and $\{Q_j\}_{j=1}^r$ are nonempty closed convex subsets of H_1 and H_2 , respectively. When $p = r = 1$, then MSFP (2) is known as SFP (1). About the corresponding results in finite-dimensional spaces, we can see [12–14] and references therein.

Since every closed convex subset of a Hilbert space is the fixed-point set of its associating projection, problems (1) and (2) are all special cases of the so-called multiple-set split common fixed-point problem (MSCFP) which is to find

$$x \in \bigcap_{i=1}^p \text{Fix}(S_i) \quad \text{such that} \quad Ax \in \bigcap_{j=1}^r \text{Fix}(T_j), \quad (3)$$

where $p, r \geq 1$ are integers, $\{S_i\}_{i=1}^p : H_1 \rightarrow H_1$ and $\{T_j\}_{j=1}^r : H_2 \rightarrow H_2$ are nonlinear operators. In particular, if $p = r = 1$, then MSCFP (3) reduces to the split common fixed-point problem (SCFP) [15–17] which is to find

$$x \in \text{Fix}(S) \quad \text{such that} \quad Ax \in \text{Fix}(T), \quad (4)$$

where $S : H_1 \rightarrow H_1$ and $T : H_2 \rightarrow H_2$ are nonlinear operators.

Recently, Moudafi [18] introduced the split equality common fixed-point problem (SECFP) which is to find

$$\begin{aligned} x &\in \text{Fix}(S), \\ y &\in \text{Fix}(T) \end{aligned} \quad (5)$$

such that $Ax = By$,

where $A : H_1 \rightarrow H_3$ and $B : H_2 \rightarrow H_3$ are two bounded linear operators. It allows asymmetric and partial relations between the variables x and y . The interest is to cover many situations, for instance, applications in decomposition methods for PDEs, in game theory, and in intensity-modulated radiation therapy (IMRT). At the same time, Moudafi [18] introduced the following iterative method

$$\begin{aligned} x_{n+1} &= S(x_n - \gamma_n A^*(Ax_n - By_n)), \\ y_{n+1} &= T(y_n + \gamma_n B^*(Ax_n - By_n)) \end{aligned} \quad (6)$$

for firmly quasi-nonexpansive mappings S and T . The weak convergence property was proved where γ_n was dependent in the spectral radii of A^*A and B^*B . Furthermore, Che and Li [19] proposed the following iterative algorithm:

$$\begin{aligned} u_n &= x_n - \gamma_n A^*(Ax_n - By_n), \\ x_{n+1} &= \alpha_n x_n + (1 - \alpha_n) Su_n, \\ v_n &= y_n + \gamma_n B^*(Ax_n - By_n), \\ y_{n+1} &= \alpha_n y_n + (1 - \alpha_n) Tv_n \end{aligned} \quad (7)$$

for quasi-nonexpansive mappings S and T and the weak convergence of scheme (7) can also be established. Chang, Wang, and Qin [20] modified the iterative scheme (7) and provided the following framework:

$$\begin{aligned} u_n &= x_n - \gamma_n A^*(Ax_n - By_n), \\ x_{n+1} &= \alpha_n x_n \\ &\quad + (1 - \alpha_n) [(1 - \xi)I + \xi S((1 - \eta)I + \eta S)] u_n, \\ v_n &= y_n + \gamma_n B^*(Ax_n - By_n), \\ y_{n+1} &= \alpha_n y_n \\ &\quad + (1 - \alpha_n) [(1 - \xi)I + \xi T((1 - \eta)I + \eta T)] v_n \end{aligned} \quad (8)$$

for quasi-pseudocontractive mappings S and T with weak convergence property.

In [21], Zhao et al. study the multiple-set split equality common fixed-point problem (MSECFP) which is to find

$$\begin{aligned} x &\in \bigcap_{i=1}^p \text{Fix}(S_i), \\ y &\in \bigcap_{j=1}^r \text{Fix}(T_j) \end{aligned} \quad (9)$$

such that $Ax = By$,

where $p, r \geq 1$ are integers and $\{S_i\}_{i=1}^p : H_1 \rightarrow H_1$ and $\{T_j\}_{j=1}^r : H_2 \rightarrow H_2$ are nonlinear operators. In particular,

if $p = r = 1$, then MSECFP (9) reduces to the split equality common fixed-point problem (SECFP) (5). They proposed the following iterative algorithm:

$$\begin{aligned} u_n &= x_n - \gamma_n A^*(Ax_n - By_n), \\ x_{n+1} &= \alpha_n^0 x_n + \alpha_n^1 S_1(x_n) + \dots + \alpha_n^p S_p(x_n) \\ v_n &= y_n + \gamma_n B^*(Ax_n - By_n), \\ y_{n+1} &= \beta_n v_n + (1 - \beta_n) T_{j(n)}(v_n) \end{aligned} \quad (10)$$

for quasi-nonexpansive mappings, where $\{\alpha_n^i\} \subset [0, 1]$, $i = 0, 1, 2, \dots, p$; $n = 1, 2, \dots$ such that $\sum_{i=0}^p \alpha_n^i = 1$ for every $n = 1, 2, \dots$ and $\{\beta_n\} \subset [0, 1]$. $j(n) = n(\text{mod } r) + 1$. Weak convergence results were obtained under certain assumption conditions.

Recently, Li et al. [22] studied the extended split equality common fixed-point problem (ESECFP) which is to find

$$\begin{aligned} x &\in \text{Fix}(S), \\ y &\in \text{Fix}(T) \end{aligned} \quad (11)$$

such that $Ax = By \in K$,

where $K \subset H_3$ is a nonempty set. For L -Lipschitz and quasi-pseudocontractive mappings S and T , as well as a kind of special $K = \{x \in H_3 : h(x) \leq 0\}$, they proposed the following iterative algorithm:

$$\begin{aligned} \omega_n &= P_{K_n} \left(\frac{Ax_n + By_n}{2} \right), \\ K_n &= \left\{ z \in H_3 : h \left(\frac{Ax_n + By_n}{2} \right) \right. \\ &\quad \left. + \left\langle \varsigma_n, z - \frac{Ax_n + By_n}{2} \right\rangle \leq 0 \right\}, \\ u_n &= x_n - \xi_n A^*(Ax_n - \omega_n), \\ x_{n+1} &= \alpha_n x_n + (1 - \alpha_n) Uu_n, \\ v_n &= y_n - \xi_n B^*(By_n - \omega_n), \\ y_{n+1} &= \alpha_n y_n + (1 - \alpha_n) Vv_n, \end{aligned} \quad (12)$$

where $U := (1 - \xi)I + \xi S((1 - \eta)I + \eta S)$, $V := (1 - \xi)I + \xi T((1 - \eta)I + \eta T)$, $0 < \xi < \eta < 1/(1 + \sqrt{1 + L^2})$, and $\varsigma_n \in \partial h((Ax_n + By_n)/2)$. They also proved the weak convergence results under mild conditions.

Inspired by the above results, in this paper, we study the following extended multiple-set split equality common fixed-point problem (EMSECFP) which is to find

$$\begin{aligned} x &\in \bigcap_{i=1}^p \text{Fix}(S_i), \\ y &\in \bigcap_{j=1}^r \text{Fix}(T_j) \end{aligned} \quad (13)$$

such that $Ax = By \in K$,

where $p, r \geq 1$ are integers and $S_i : H_1 \rightarrow H_1 (i = 1, 2, \dots, p)$ and $T_j : H_2 \rightarrow H_2 (j = 1, 2, \dots, r)$ are L -Lipschitzian mappings with $L \geq 1$ and quasi-pseudocontractive mappings with nonempty fixed-point sets which is more general than the class of quasi-nonexpansive mappings. When $K = H_3$, the EMSECFP (13) is MSECFP (9). When $p = r = 1$, the EMSECFP (13) is ESECFP (11). Therefore, EMSECFP (13) is the extension of MSECFP (9) and ESECFP (11). In this paper, we propose two mixed simultaneous iterative algorithms for solving EMSECFP (13), which avoids using the projection and the selecting of stepsize which do not depend on the operator norms. Furthermore, we prove that the sequences generated by the algorithms weakly converge to a solution of the EMSECFP (13). Numerical examples show the feasibility and efficiency of these algorithms.

2. Preliminaries

In this paper, we use $x_n \rightarrow x$ and $x_n \rightharpoonup x$ to indicate that the sequence $\{x_n\}$ converges weakly to x and $\{x_n\}$ converges strongly to x , respectively. We write $\omega_w(x_k) = \{x : \exists x_{k_j} \rightharpoonup x\}$ to stand for the weak ω -limit set of $\{x_k\}$ and use Ω to stand for the solution set of the EMSECFP (13).

Definition 1. A mapping $T : H \rightarrow H$ is called

- (i) Nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in H$.
- (ii) Quasi-nonexpansive, if $\text{Fix}(T) \neq \emptyset$ and $\|Tx - x^*\| \leq \|x - x^*\|$ for all $x \in H$ and $x^* \in \text{Fix}(T)$.
- (iii) Quasi-pseudocontractive, if $\text{Fix}(T) \neq \emptyset$ and $\|Tx - x^*\|^2 \leq \|x - x^*\|^2 + \|Tx - x\|^2$ for all $x \in H$ and $x^* \in \text{Fix}(T)$.

Definition 2. An operator $T : H \rightarrow H$ is called demiclosed at the origin, for any sequence $\{x_n\}$ which weakly converges to x , and if the sequence $\{Tx_n\}$ strongly converges to 0, then $Tx = 0$.

In the proof of our results, we need the following lemmas.

Lemma 3 (see [23]). *Let H be a real Hilbert space, then the following conclusions hold:*

$$\|x - y\|^2 = \|x\|^2 + \|y\|^2 - 2\langle x, y \rangle, \quad (14)$$

for all $x, y \in H$,

$$\begin{aligned} \|\alpha x + (1 - \alpha)y\|^2 &= \alpha\|x\|^2 + (1 - \alpha)\|y\|^2 \\ &\quad - \alpha(1 - \alpha)\|x - y\|, \end{aligned} \quad (15)$$

for all $x, y \in H, \alpha \in [0, 1]$.

Lemma 4 (see [20]). *Let H be a real Hilbert space and $T : H \rightarrow H$ be a L -Lipschitzian mapping with $L \geq 1$. Denote*

$$U := (1 - \xi)I + \xi T((1 - \eta)I + \eta T). \quad (16)$$

If $0 < \xi < \eta < 1/(1 + \sqrt{1 + L^2})$, then the following conclusions hold:

- (i) $\text{Fix}(T) = \text{Fix}((1 - \eta)I + \eta T) = \text{Fix}(U)$.
- (ii) *If T is demiclosed at 0, then U is also demiclosed at 0.*

(iii) *In addition, if $T : H \rightarrow H$ is quasi-pseudocontractive, then the mapping U is quasi-nonexpansive; that is,*

$$\begin{aligned} \|Ux - u^*\| &\leq \|x - u^*\|, \\ \forall x \in H, u^* \in \text{Fix}(T) &= \text{Fix}(U). \end{aligned} \quad (17)$$

Lemma 5 (see [21]). *Let H be a real Hilbert space. Then*

$$\begin{aligned} &\|\alpha_0 x_0 + \alpha_1 x_1 + \dots + \alpha_r x_r\|^2 \\ &\leq \sum_{i=0}^r \alpha_i \|x_i\|^2 - \alpha_s \alpha_t \|x_s - x_t\|^2, \end{aligned} \quad (18)$$

for any $s, t \in \{0, 1, 2, \dots, r\}$ and for $x_i \in H, i = 0, 1, 2, \dots, r$ with $\alpha_0 + \alpha_1 + \dots + \alpha_r = 1$ and $0 < \alpha_i < 1$.

Lemma 6 (see [16]). *Let H be a real Hilbert space and $T : H \rightarrow H$ be a quasi-nonexpansive mapping. Set $T_\alpha = \alpha I + (1 - \alpha)T$ for $\alpha \in [0, 1]$. Then the following properties are reached for all $(x, q) \in H \times F(T)$:*

- (i) $\langle x - Tx, x - q \rangle \geq (1/2)\|x - Tx\|^2$ and $\langle x - Tx, q - Tx \rangle \leq (1/2)\|x - Tx\|^2$;
- (ii) $\|T_\alpha x - q\|^2 \leq \|x - q\|^2 - \alpha(1 - \alpha)\|Tx - x\|^2$;
- (iii) $\langle x - T_\alpha x, x - q \rangle \geq ((1 - \alpha)/2)\|x - Tx\|^2$.

3. Main Results

In this section, we give the following assumption.

Assumption 7. Assume that

(i) H_1, H_2 , and H_3 are real Hilbert spaces and R^n is n -dimensional Euclidean space. $A : H_1 \rightarrow H_3$ and $B : H_2 \rightarrow H_3$ are two bounded linear operators; A^* and B^* are their adjoint operators, respectively.

(ii) $K = \{x \in H_3 : h(x) \leq 0\}$, where $h : H_3 \rightarrow R$ is convex function which is subdifferentiable and its subdifferentials are bounded on bounded sets.

(iii) $S_i : H_1 \rightarrow H_1$ and $T_j : H_2 \rightarrow H_2$ are all L -Lipschitz, quasi-pseudocontractive with $L \geq 1, \text{Fix}(S_i) \neq \emptyset$ and $\text{Fix}(T_j) \neq \emptyset$ and $S_i - I$ and $T_j - I$ are demiclosed at origin, where $i = 1, 2, \dots, p; j = 1, 2, \dots, r$.

(iv) $U_i = (1 - \xi)I + \xi S_i((1 - \eta)I + \eta S_i), i = 1, 2, \dots, p$, and $V_j = (1 - \xi)I + \xi T_j((1 - \eta)I + \eta T_j), j = 1, 2, \dots, r$, where $0 < \xi < \eta < 1/(1 + \sqrt{1 + L^2})$.

Theorem 8. *Let Assumption 7 be true and $\delta > 0$ be small enough. Assume that $\{\alpha_n^i\} \subset [0, 1], i = 0, 1, 2, \dots, p; n = 1, 2, \dots$, such that $\sum_{i=0}^p \alpha_n^i = 1$ for every $n = 1, 2, \dots$ and $\{\beta_n\} \subset [0, 1], j(n) = n(\text{mod } r) + 1$. Suppose that $x_0 \in H_1$,*

$y_0 \in H_2$ are arbitrary and $\{x_n\}$, $\{y_n\}$, $\{\omega_n\}$, $\{u_n\}$, and $\{v_n\}$ are generated by the following iteration scheme:

$$\begin{aligned} \omega_n &= P_{K_n} \left(\frac{Ax_n + By_n}{2} \right), \\ K_n &= \left\{ z \in H_3 : h \left(\frac{Ax_n + By_n}{2} \right) \right. \\ &\quad \left. + \left\langle \zeta_n, z - \frac{Ax_n + By_n}{2} \right\rangle \leq 0 \right\}, \\ u_n &= x_n - \gamma_n A^* (Ax_n - \omega_n), \\ x_{n+1} &= \alpha_n^0 u_n + \alpha_n^1 U_1 u_n + \cdots + \alpha_n^p U_p u_n, \\ v_n &= y_n - \gamma_n B^* (By_n - \omega_n), \\ y_{n+1} &= \beta_n v_n + (1 - \beta_n) V_{j(n)} v_n, \end{aligned} \quad (19)$$

where

$$\zeta_n \in \partial h \left(\frac{Ax_n + By_n}{2} \right) \quad (20)$$

and

$$\begin{aligned} \gamma_n &\in \left(\delta, (1 - \delta) \right. \\ &\quad \left. \cdot \frac{\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2}{\|A^* (Ax_n - \omega_n)\|^2 + \|B^* (By_n - \omega_n)\|^2} \right). \end{aligned} \quad (21)$$

Assume $\Omega \neq \emptyset$, $\liminf_{n \rightarrow \infty} \alpha_n^i > 0$, $i = 1, 2, \dots, p$, and $\beta_n \subset (\sigma, 1 - \sigma)$ for small enough $\sigma > 0$. Then $(x_n, y_n) \rightarrow (\tilde{x}, \tilde{y})$, $(u_n, v_n) \rightarrow (\tilde{x}, \tilde{y})$, and $\omega_n \rightarrow \tilde{\omega} := A\tilde{x} = B\tilde{y}$, where $(\tilde{x}, \tilde{y}) \in \Omega$.

Proof. It is obvious that $K \subset K_n$ for any $n \in N$. Let $(x^*, y^*) \in \Omega$, that is, $x^* \in \bigcap_{i=1}^p \text{Fix}(S_i)$, $y^* \in \bigcap_{j=1}^r \text{Fix}(T_j)$ and $\omega^* = Ax^* = By^* \in K$. By Lemma 3, we have

$$\begin{aligned} \|\omega_n - \omega^*\| &= \left\| P_{K_n} \left(\frac{Ax_n + By_n}{2} \right) - P_{K_n} (\omega^*) \right\|^2 \\ &\leq \left\| \frac{1}{2} (Ax_n - \omega^*) + \frac{1}{2} (By_n - \omega^*) \right\|^2 \\ &= \frac{1}{2} \|Ax_n - \omega^*\|^2 + \frac{1}{2} \|By_n - \omega^*\|^2 \\ &\quad - \frac{1}{4} \|Ax_n - By_n\|^2. \end{aligned} \quad (22)$$

$$\begin{aligned} \|u_n - x^*\|^2 &= \|x_n - x^* - \gamma_n A^* (Ax_n - \omega_n)\|^2 \\ &= \|x_n - x^*\|^2 + \gamma_n^2 \|A^* (Ax_n - \omega_n)\|^2 \\ &\quad - 2 \langle x_n - x^*, \gamma_n A^* (Ax_n - \omega_n) \rangle \\ &= \|x_n - x^*\|^2 + \gamma_n^2 \|A^* (Ax_n - \omega_n)\|^2 \\ &\quad - \gamma_n \|Ax_n - \omega^*\|^2 - \gamma_n \|Ax_n - \omega_n\|^2 \\ &\quad + \gamma_n \|\omega_n - \omega^*\|^2. \end{aligned} \quad (23)$$

Similarly,

$$\begin{aligned} \|v_n - y^*\|^2 &= \|y_n - y^*\|^2 + \gamma_n^2 \|B^* (By_n - \omega_n)\|^2 \\ &\quad - \gamma_n \|By_n - \omega^*\|^2 - \gamma_n \|By_n - \omega_n\|^2 \\ &\quad + \gamma_n \|\omega_n - \omega^*\|^2. \end{aligned} \quad (24)$$

By (23) and (24), we have

$$\begin{aligned} \|u_n - x^*\|^2 + \|v_n - y^*\|^2 &= \|x_n - x^*\|^2 + \|y_n - y^*\|^2 + \gamma_n^2 \|A^* (Ax_n - \omega_n)\|^2 \\ &\quad - \gamma_n \|Ax_n - \omega^*\|^2 - \gamma_n \|Ax_n - \omega_n\|^2 \\ &\quad + \gamma_n^2 \|B^* (By_n - \omega_n)\|^2 - \gamma_n \|By_n - \omega^*\|^2 \\ &\quad - \gamma_n \|By_n - \omega_n\|^2 + 2\gamma_n \|\omega_n - \omega^*\|^2. \end{aligned} \quad (25)$$

Combining (22) and (25), we have

$$\begin{aligned} \|u_n - x^*\|^2 + \|v_n - y^*\|^2 &\leq \|x_n - x^*\|^2 + \|y_n - y^*\|^2 + \gamma_n^2 \|A^* (Ax_n - \omega_n)\|^2 \\ &\quad - \gamma_n \|Ax_n - \omega_n\|^2 + \gamma_n^2 \|B^* (By_n - \omega_n)\|^2 \\ &\quad - \gamma_n \|By_n - \omega_n\|^2 - \frac{1}{2} \gamma_n \|Ax_n - By_n\|^2. \end{aligned} \quad (26)$$

Since S_i , $i = 1, 2, \dots, p$, are quasi-pseudocontractive, from Lemma 4, U_i , $i = 1, 2, \dots, p$, are quasi-nonexpansive. Furthermore, from Lemma 5, we have

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\leq \alpha_n^0 \|u_n - x^*\|^2 + \alpha_n^1 \|U_1(u_n) - x^*\|^2 \\ &\quad + \cdots + \alpha_n^p \|U_p(u_n) - x^*\|^2 \\ &\quad - \alpha_n^0 \alpha_n^1 \|U_1(u_n) - u_n\|^2 \\ &\leq \alpha_n^0 \|u_n - x^*\|^2 + \alpha_n^1 \|u_n - x^*\|^2 + \cdots \\ &\quad + \alpha_n^p \|u_n - x^*\|^2 \\ &\quad - \alpha_n^0 \alpha_n^1 \|U_1(u_n) - u_n\|^2 \\ &= \|u_n - x^*\|^2 - \alpha_n^0 \alpha_n^1 \|U_1(u_n) - u_n\|^2. \end{aligned} \quad (27)$$

Similarly, since $T_j, j = 1, 2, \dots, r$ are quasi-pseudo-contractive, from Lemma 4, $V_j, j = 1, 2, \dots, r$, are quasi-nonexpansive. By Lemma 6 (ii), we have

$$\begin{aligned} \|y_{n+1} - y^*\|^2 &\leq \|v_n - y^*\|^2 \\ &\quad - \beta_n (1 - \beta_n) \|V_{j(n)}(v_n) - v_n\|^2. \end{aligned} \quad (28)$$

Combining (27) and (28) with (26), we have

$$\begin{aligned} &\|x_{n+1} - x^*\|^2 + \|y_{n+1} - y^*\|^2 \\ &\leq \|u_n - x^*\|^2 + \|v_n - y^*\|^2 - \alpha_n^0 \alpha_n^1 \|U_1(u_n) - u_n\|^2 \\ &\quad - \beta_n (1 - \beta_n) \|v_n - V_{j(n)}(v_n)\|^2 \\ &\leq \|x_n - x^*\|^2 + \|y_n - y^*\|^2 + \gamma_n^2 \|A^*(Ax_n - \omega_n)\|^2 \\ &\quad + \gamma_n^2 \|B^*(By_n - \omega_n)\|^2 - \gamma_n \|Ax_n - \omega_n\|^2 \\ &\quad - \gamma_n \|By_n - \omega_n\|^2 - \frac{1}{2} \gamma_n \|Ax_n - By_n\|^2 \\ &\quad - \alpha_n^0 \alpha_n^1 \|U_1(u_n) - u_n\|^2 \\ &\quad - \beta_n (1 - \beta_n) \|V_{j(n)}(v_n) - v_n\|^2. \end{aligned} \quad (29)$$

Since $\gamma_n \in (\delta, (1-\delta)((\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2)/(\|A^*(Ax_n - \omega_n)\|^2 + \|B^*(By_n - \omega_n)\|^2)))$, one has

$$\begin{aligned} &\gamma_n^2 (\|A^*(Ax_n - \omega_n)\|^2 + \|B^*(By_n - \omega_n)\|^2) \\ &\quad - \gamma_n (\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2) \\ &= \gamma_n [\gamma_n (\|A^*(Ax_n - \omega_n)\|^2 + \|B^*(By_n - \omega_n)\|^2) \\ &\quad - (\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2)] \\ &\leq -\delta \gamma_n (\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2) \leq 0. \end{aligned} \quad (30)$$

As a result, we have from (29) and (30) that

$$\begin{aligned} &\|x_{n+1} - x^*\|^2 + \|y_{n+1} - y^*\|^2 \\ &\leq \|x_n - x^*\|^2 + \|y_n - y^*\|^2. \end{aligned} \quad (31)$$

We see that the sequence $\{\|x_n - x^*\|^2 + \|y_n - y^*\|^2\}$ is decreasing and lower bounded by 0. Consequently it converges to some finite limit. Hence the sequences $\{x_n\}$ and $\{y_n\}$ are bounded.

According to the assumption of $\{\gamma_n\}$ and (29), we have

$$\lim_{n \rightarrow \infty} \|Ax_n - By_n\| = 0. \quad (32)$$

Furthermore, by the conditions on $\{\alpha_n^i\} (0 \leq i \leq p)$ and $\{\beta_n\}$, we obtain

$$\lim_{n \rightarrow \infty} \|U_1(u_n) - u_n\| = \lim_{n \rightarrow \infty} \|V_{j(n)}(v_n) - v_n\| = 0. \quad (33)$$

From Lemma 5, (19) and the conference of (27) for $2 \leq i \leq p$, we have

$$\lim_{n \rightarrow \infty} \|U_i(u_n) - u_n\| = 0 \quad (34)$$

and

$$\begin{aligned} \lim_{n \rightarrow \infty} \|Ax_n - \omega_n\| &= 0, \\ \lim_{n \rightarrow \infty} \|By_n - \omega_n\| &= 0, \end{aligned} \quad (35)$$

which combining (19) implies that

$$\lim_{n \rightarrow \infty} \|u_n - x_n\| = \lim_{n \rightarrow \infty} \|\gamma_n A^*(Ax_n - \omega_n)\| = 0 \quad (36)$$

and

$$\lim_{n \rightarrow \infty} \|v_n - y_n\| = \lim_{n \rightarrow \infty} \|\gamma_n B^*(By_n - \omega_n)\| = 0. \quad (37)$$

From (33), (34), and (36), we can conclude that

$$\lim_{n \rightarrow \infty} \|U_i(u_n) - x_n\| = 0, \quad \text{for } 1 \leq i \leq p. \quad (38)$$

According to (36), (38), and

$$\begin{aligned} \|x_{n+1} - x_n\| &\leq \alpha_n^0 \|u_n - x_n\| + \alpha_n^1 \|U_1(u_n) - x_n\| + \dots \\ &\quad + \alpha_n^p \|U_p(u_n) - x_n\|, \end{aligned} \quad (39)$$

we have

$$\lim_{n \rightarrow \infty} \|x_{n+1} - x_n\| = 0 \quad (40)$$

which infers that $\{x_n\}$ is asymptotically regular. From (33) and (37), we have

$$\lim_{n \rightarrow \infty} \|V_{j(n)}(v_n) - y_n\| = 0. \quad (41)$$

As a result,

$$\begin{aligned} &\lim_{n \rightarrow \infty} \|y_{n+1} - y_n\| \\ &= \lim_{n \rightarrow \infty} \|\beta_n (v_n - y_n) + (1 - \beta_n) (V_{j(n)}(v_n) - y_n)\| \\ &= 0, \end{aligned} \quad (42)$$

which implies that $\{y_n\}$ is asymptotically regular.

It follows from

$$\|v_{n+1} - v_n\| \leq \|v_{n+1} - y_{n+1}\| + \|y_{n+1} - y_n\| + \|y_n - v_n\| \quad (43)$$

that

$$\lim_{n \rightarrow \infty} \|v_{n+1} - v_n\| = 0. \quad (44)$$

Furthermore,

$$\lim_{n \rightarrow \infty} \|v_{n+j} - v_n\| = 0, \quad 1 \leq j \leq r. \quad (45)$$

Taking $(\tilde{x}, \tilde{y}) \in \omega_\omega(x_n, y_n)$, from $\lim_{n \rightarrow \infty} \|u_n - x_n\| = 0$ and $\lim_{n \rightarrow \infty} \|v_n - y_n\| = 0$, we have $(\tilde{x}, \tilde{y}) \in \omega_\omega(u_n, v_n)$. Let an

index $j \in \{1, 2, \dots, r\}$ be fixed. Noting that the pool of indexes is finite, from (45) and the boundedness of $\{v_n\}$, there exists a subsequence $\{v_{n_m}\} \subset \{v_n\}$ such that $v_{n_m} \rightharpoonup \bar{y}$ as $n \rightarrow \infty$ and $j(n_m) = j$ for all m . It is easy to see from (33) that

$$\lim_{m \rightarrow \infty} \|V_j(v_{n_m}) - v_{n_m}\| = \lim_{m \rightarrow \infty} \|V_{j(n_m)}(v_{n_m}) - v_{n_m}\| = 0. \quad (46)$$

According to the demiclosednesses of $U_i - I$ and $V_j - I$ at 0, it yields from (33), (34) and (46) that $U_i(\bar{x}) = \bar{x}$ and $V_j(\bar{y}) = \bar{y}$ for $1 \leq i \leq p$ and $1 \leq j \leq r$. Hence, $\bar{x} \in \bigcap_{i=1}^p F(U_i)$ and $\bar{y} \in \bigcap_{j=1}^r F(V_j)$. From Lemma 4, we have $\bar{x} \in \bigcap_{i=1}^p F(S_i)$ and $\bar{y} \in \bigcap_{j=1}^r F(T_j)$.

Furthermore, $A\bar{x} - B\bar{y} \in \omega_\omega(Ax_n - By_n)$ and the weakly lower semicontinuity of the norm imply that

$$\|A\bar{x} - B\bar{y}\| \leq \liminf_{n \rightarrow \infty} \|Ax_n - By_n\| = 0, \quad (47)$$

that is, $A\bar{x} = B\bar{y}$.

From $\lim_{n \rightarrow \infty} \|Ax_n - \omega_n\| = 0$ and $\lim_{n \rightarrow \infty} \|By_n - \omega_n\| = 0$, we have

$$\omega_n \rightharpoonup \bar{\omega} = A\bar{x} = B\bar{y}. \quad (48)$$

As follows, we prove $\bar{\omega} \in K$.

According to the boundedness of $\{\varsigma_n\}$, there exists a constant $\bar{M} > 0$, such that $\|\varsigma_n\| \leq \bar{M}$.

Noting that $\omega_n = P_{K_n}((Ax_n + By_n)/2) \in K_n$, we obtain

$$h\left(\frac{Ax_n + By_n}{2}\right) + \left\langle \varsigma_n, \omega_n - \frac{Ax_n + By_n}{2} \right\rangle \leq 0. \quad (49)$$

As a result,

$$\begin{aligned} h\left(\frac{Ax_n + By_n}{2}\right) &\leq -\left\langle \varsigma_n, \omega_n - \frac{Ax_n + By_n}{2} \right\rangle \\ &\leq \bar{M} \left\| \omega_n - \frac{Ax_n + By_n}{2} \right\|. \end{aligned} \quad (50)$$

From the lower semicontinuity of h , (49), and (50), we have

$$\begin{aligned} h(\bar{\omega}) &\leq \liminf_{n \rightarrow \infty} h\left(\frac{Ax_n + By_n}{2}\right) \\ &\leq \bar{M} \liminf_{n \rightarrow \infty} \left\| \omega_n - \frac{Ax_n + By_n}{2} \right\| = 0. \end{aligned} \quad (51)$$

Thus $\bar{\omega} \in K$.

Next, we prove the uniqueness of the weak cluster point of $\{(x_n, y_n)\}$. Assume that $(\bar{x}, \bar{y})$ be other weak cluster point of $\{(x_n, y_n)\}$, then $(\bar{x}, \bar{y}) \in \Gamma$. Define $\rho_n(x, y) = \|x_n - x\|^2 + \|y_n - y\|^2$, then

$$\begin{aligned} \rho_n(\bar{x}, \bar{y}) &= \|x_n - \bar{x}\|^2 + \|\bar{x} - \bar{x}\|^2 + 2\langle x_n - \bar{x}, \bar{x} - \bar{x} \rangle \\ &\quad + \|y_n - \bar{y}\|^2 + \|\bar{y} - \bar{y}\|^2 \\ &\quad + 2\langle y_n - \bar{y}, \bar{y} - \bar{y} \rangle \\ &= \rho_n(\bar{x}, \bar{y}) + \|\bar{x} - \bar{x}\|^2 + 2\langle x_n - \bar{x}, \bar{x} - \bar{x} \rangle \\ &\quad + \|\bar{y} - \bar{y}\|^2 + 2\langle y_n - \bar{y}, \bar{y} - \bar{y} \rangle. \end{aligned} \quad (52)$$

Without loss of generality, we suppose that $x_n \rightharpoonup \bar{x}$, $y_n \rightharpoonup \bar{y}$. By passing to the limit in relation (52), we have

$$\rho(\bar{x}, \bar{y}) = \rho(\bar{x}, \bar{y}) + \|\bar{x} - \bar{x}\|^2 + \|\bar{y} - \bar{y}\|^2. \quad (53)$$

Reserving the role of $(\bar{x}, \bar{y})$ and $(\bar{x}, \bar{y})$, we also have

$$\rho(\bar{x}, \bar{y}) = \rho(\bar{x}, \bar{y}) + \|\bar{x} - \bar{x}\|^2 + \|\bar{y} - \bar{y}\|^2. \quad (54)$$

By adding the two last equalities, we obtain $\bar{x} = \bar{x}$ and $\bar{y} = \bar{y}$, which shows that whole sequence $\{(x_n, y_n)\}$ weakly converges to the unique solution of (13). The proof is completed. $\square$

Similar to the proof of Theorem 8, we can easily prove the following theorem.

Theorem 9. Let Assumption 7 be true and $\delta > 0$ be small enough. Assume that $\{\alpha_n\} \subset [0, 1]$, $i(n) = n(\bmod p) + 1$ and $\{\beta_n^i\} \subset [0, 1]$, $i = 0, 1, 2, \dots, r$; $n = 1, 2, \dots$ such that $\sum_{i=0}^r \beta_n^i = 1$ for every $n = 1, 2, \dots$. Suppose that $x_0 \in H_1$, $y_0 \in H_2$ are arbitrary and $\{x_n\}$, $\{y_n\}$, $\{\omega_n\}$, $\{u_n\}$, and $\{v_n\}$ are generated by the following iteration scheme:

$$\begin{aligned} \omega_n &= P_{K_n}\left(\frac{Ax_n + By_n}{2}\right), \\ K_n &= \left\{z \in H_3 : h\left(\frac{Ax_n + By_n}{2}\right) + \left\langle \varsigma_n, z - \frac{Ax_n + By_n}{2} \right\rangle \leq 0\right\}, \\ u_n &= x_n - \gamma_n A^*(Ax_n - \omega_n), \\ x_{n+1} &= \alpha_n u_n + (1 - \alpha_n) U_{i(n)} u_n, \\ v_n &= y_n - \gamma_n B^*(By_n - \omega_n), \\ y_{n+1} &= \beta_n^0 v_n + \beta_n^1 V_1 v_n + \dots + \beta_n^r V_r v_n, \end{aligned} \quad (55)$$

where

$$\varsigma_n \in \partial h\left(\frac{Ax_n + By_n}{2}\right) \quad (56)$$

and

$$\begin{aligned} \gamma_n &\in \left(\delta, (1 - \delta) \cdot \frac{\|Ax_n - \omega_n\|^2 + \|By_n - \omega_n\|^2}{\|A^*(Ax_n - \omega_n)\|^2 + \|B^*(By_n - \omega_n)\|^2}\right). \end{aligned} \quad (57)$$

Assume $\Omega \neq \emptyset$, $\alpha_n \in (\sigma, 1 - \sigma)$ for small enough $\sigma > 0$ and $\liminf_{n \rightarrow \infty} \beta_n^0 \beta_n^i > 0$, $i = 1, 2, \dots, r$. Then $(x_n, y_n) \rightharpoonup (\bar{x}, \bar{y})$, $(u_n, v_n) \rightharpoonup (\bar{x}, \bar{y})$, and $\omega_n \rightharpoonup \bar{\omega} := A\bar{x} = B\bar{y}$, where $(\bar{x}, \bar{y}) \in \Omega$.

4. Numerical Examples

In this section, we give an example to show some insight into the behavior of the algorithms presented in this paper. The algorithms presented in Theorems 8 and 9 are denoted as

TABLE 1: Number of iterations and the cpu time for different ϵ in Algorithm 1.

ϵ	Iter.	Sec.	Fina.	
10^{-2}	12	0.002	$x = (0.0028, 0.0014)^T$,	$y = (0.0114, -0.0064, -0.0001)^T$
10^{-4}	55	0.004	$x = (-0.2230, 0.2539)^T \times 10^{-4}$,	$y = (-0.0189, -0.1302, 0.1469)^T \times 10^{-3}$
10^{-6}	104	0.006	$x = (-0.1723, 0.4902)^T \times 10^{-6}$,	$y = (-0.0264, -0.1526, 0.1556)^T \times 10^{-5}$

TABLE 2: Number of iterations and the cpu time for different ϵ in Algorithm 2.

ϵ	Iter.	Sec.	Fina.	
10^{-2}	12	0.002	$x = (0.0044, 0.0008)^T$,	$y = (0.0106, -0.0080, 0.0011)^T$
10^{-4}	50	0.005	$x = (-0.1526, 0.5347)^T \times 10^{-4}$,	$y = (-0.0096, -0.1518, 0.1633)^T \times 10^{-3}$
10^{-6}	112	0.014	$x = (-0.0627, 0.4938)^T \times 10^{-6}$,	$y = (-0.0229, -0.1262, 0.1157)^T \times 10^{-5}$

Algorithms 1 and 2 separately. The whole codes are written in Matlab 7.0. All the numerical results are carried out on a personal Lenovo Thinkpad computer with Intel(R) Core(TM) i7-6500U CPU 2.50GHz and RAM 8.00GB.

In this section, we need the following lemmas.

Lemma 10 (see [24, 25]). *For every $k \geq 0$, let $x^k \in R^n$, $C_k = \{x \in R^n | c(x^k) + \langle \xi^k, x - x^k \rangle \leq 0\}$. Then for any $x \in R^n$, we have*

$$P_{C_k}(x) = \begin{cases} x - \frac{c(x^k) + \langle \xi^k, x - x^k \rangle}{\|\xi^k\|^2} \xi^k, & \text{if } c(x^k) + \langle \xi^k, x - x^k \rangle > 0 \\ x, & \text{otherwise.} \end{cases} \quad (58)$$

Lemma 11 (see [26]). *Suppose $h : R^n \rightarrow R$ is a finite convex function, then it is subdifferentiable everywhere and its subdifferentials are uniformly bounded on any bounded subset of R^n .*

Example 12. Let $H_1 = R^2$, $H_2 = R^3$, and $H_3 = R^4$. Assume that $S_1 x = -3x$, $x \in H_1$, $S_2 x = -4x$, $x \in H_1$, $T_1 x = -6x$, $x \in H_2$, $T_2 x = -7x$, $x \in H_2$, $K = \{x \in H_3 | h(x) = x_1^2 + 2x_2^2 + 4x_3^2 + x_4^2 - 2 \leq 0\}$, and

$$A = \begin{pmatrix} 0.5836 & 0.9962 \\ 0.5118 & 0.3545 \\ 0.0826 & 0.9713 \\ 0.7196 & 0.3464 \end{pmatrix}, \quad (59)$$

$$B = \begin{pmatrix} 0.8865 & 0.1257 & 0.6938 \\ 0.4547 & 0.3089 & 0.0098 \\ 0.4134 & 0.7261 & 0.8432 \\ 0.2177 & 0.7829 & 0.9223 \end{pmatrix}.$$

Find $x \in \bigcap_{i=1}^2 \text{Fix}(S_i)$, $y \in \bigcap_{j=1}^2 \text{Fix}(T_j)$ such that $Ax = By \in K$.

It is easy to see that $S_i : H_1 \rightarrow H_1$, $i = 1, 2$ and $T_j : H_2 \rightarrow H_2$, $j = 1, 2$ are all Lipschitz quasi-pseudocontractive

operators, but not quasi-nonexpansive operators. Therefore, the Example 12 cannot be calculated by the algorithms in [21].

In the experiments, we take $\delta = 0.6$, $L = 7$, $\xi = 1/3L$, $\eta = 1/2L$, $\alpha_n^0 = \alpha_n^1 = 0.25$, $\alpha_n^2 = 0.5$, and $\beta_n = 0.3$. And the stopping criterion is $\|Ax_n - w_n\| + \|By_n - w_n\| \leq \epsilon$.

Tables 1 and 2 show the number of iterations, the cpu time, and the final iteration points for different ϵ when the initial point is $x_0 = (0.1, 0.1)^T$, $y_0 = (0.1, 0.1, 0.1)^T$ in Algorithms 1 and 2. We denote by Iter., Sec. and Fina. the number of iterations, the cpu time in seconds, and the final iteration points, respectively.

Figures 1 and 2 present the behaviors of $\|Ax_n - By_n\|$ for $\epsilon = 10^{-4}$ and $\epsilon = 10^{-6}$ when the initial point is $x_0 = (0.1, 0.1)^T$, $y_0 = (0.1, 0.1, 0.1)^T$ in Algorithms 1 and 2.

It is easy to see that the presentation reveals that $Ax = By$.

Furthermore, for testing the stationary property of iterative numbers, we carry out 500 experiments for different initial points which are presented randomly, such as

$$\begin{aligned} x &= 0.1 * \text{rand}(2, 1), \\ y &= 0.1 * \text{rand}(3, 1). \end{aligned} \quad (60)$$

Figures 3 and 4 illustrate the behaviors of iterative numbers for $\epsilon = 10^{-4}$ and $\epsilon = 10^{-6}$, which reveals the stationary property of iterative numbers of the Algorithms 1 and 2.

Especially, Figure 5 illustrates the behaviors of $\|x_{n+1} - x_n\|$ and $\|y_{n+1} - y_n\|$ for $\epsilon = 10^{-4}$ in Algorithm 1. Figure 6 shows the behaviors of iterative numbers for different ϵ in Algorithm 1.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

FIGURE 1: The behaviors of $\|Ax_n - By_n\|$ for different ϵ in Algorithm 1. FIGURE 2: The behaviors of $\|Ax_n - By_n\|$ for different ϵ in Algorithm 2. FIGURE 3: The behaviors of iterative numbers for different ϵ in Algorithm 1.

FIGURE 4: The behaviors of iterative numbers for different ϵ in Algorithm 2.

FIGURE 5: The behaviors of $\|x_{n+1} - x_n\|$ and $\|y_{n+1} - y_n\|$ in Algorithm 1.

FIGURE 6: The behaviors of iterative numbers for different ϵ in Algorithm 1.

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