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Research Article

Fuzzy State Observer-Based Adaptive Dynamic Surface Control of Nonlinear Systems with Time-Varying Output Constraints

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In this paper, a new fuzzy dynamic surface control approach based on a state observer is proposed for uncertain nonlinear systems with time-varying output constraints and external disturbances. An adaptive fuzzy state observer is used to estimate the states that cannot be measured in the systems. In our method, a time-varying Barrier Lyapunov Function (BLF) is used to ensure that the output does not violate time-varying constraints. In addition, dynamic surface control (DSC) technology is applied to overcome the problem of "explosion of complexity" in a backstepping control. Finally, the stability and signal boundedness of the system are confirmed by the Lyapunov method. The simulation results show the effectiveness and correctness of the proposed method.

1. Introduction

In practical engineering, there are many uncertain nonlinear electromechanical systems, such as robots, for which a mathematical model is difficult to determine. This leads to great difficulty in the design of their control systems [1–3]. Fuzzy logic systems (FLSs) have been widely used in adaptive control of uncertain nonlinear function modeling due to their universal approximation ability [4–8]. FLSs can be combined with backstepping design techniques to overcome the mismatched uncertainties problem. At the same time, backstepping control can provide a symmetric framework for controller design, so fuzzy backstepping control schemes have achieved great success in the control field [4, 9–13]. However, backstepping control needs to do repeated differentiations of the virtual control law. If there are nonlinear functions in the virtual control law, repeated differentiations will lead to the problem of "explosion of complexity" with increasing order of the system. This makes high order systems face great difficulties in controller implementation. Recently, Hedrick et al. proposed a dynamic surface control (DSC) method using first-order low-pass filters to avoid repetitive differential problems. It attracted great interest among researchers [14–18].

There is a lot of literature that focuses on fuzzy backstepping controls, but most of the exiting approaches are based on state feedback, for which all the states of the closed system should be directly measured. In practical engineering, it is impossible to measure all the states directly due to the limitation of sensors, installation positions, or measuring points. Therefore, the control scheme based on state feedback may not be applicable in practical engineering. References [19–24] describe the recent developments in adaptive output feedback control for uncertain nonlinear systems based on a state observer that identifies the unmeasurable states instead of directly measuring them. Reference [19] describes the problem of robust adaptive control for nontriangular stochastic nonlinear systems with unmeasurable states and unmodeled dynamics. Reference [20] describes a study of the problem of output feedback control for a class of SISO stochastic switched nonlinear systems with completely unknown functions, unmodeled dynamics, and arbitrary switching. In [21, 22], under the unified framework of adaptive backstepping control technology, an output feedback tracking control design method based on an adaptive fuzzy observer is proposed for uncertain nonlinear systems. Reference [23] contains a proposal for two adaptive fuzzy output feedback control methods for a class of uncertain stochastic nonlinear strict-feedback systems without state measurement. Reference [24] contains a proposal for a robust H∞ control of an observer-based repetitive-control system. The problems with the control methods in the literature mentioned above are that they are computationally complex and do not take engineering constraints into account.

Output constraints are important engineering constraints for many industrial systems. Without considering the problem of output constraints, equipment may be damaged and accidents can happen. Because a BLF grows to infinity when its related state is close to a certain limit, it has received extensive attention as a way to solve the output constraint problem. Therefore, as long as the BLF is bounded, the related states will not violate the constraints. References [25–27] describe how BLFs have been used to deal with the output constraints. References [25, 27] describe how a BLF can be used to solve the output constraint problem of a robot manipulator system. Reference [26] describes an adaptive neural network control that is designed for the control of a nonlinear affine system subject to external unknown disturbances for the conditions of an input dead zone and output constraints.

References [25–27] all focus on the static output constraints problem, but time-varying output constraints are more in line with practical engineering, leading some researchers to publish literature on this problem. Just as a conventional BLF can handle static output constraints, time-varying output constraints can be tackled by using a time-varying BLF [14, 28, 29]. Reference [14] describes the design of an adaptive state feedback control for uncertain strictly feedback nonlinear systems with asymmetric time-varying output constraints when input saturation occurs. Reference [28] describes how an asymmetric time-varying BLF can be used to prevent the output from exceeding the constraint bounds, and it shows that the output can start anywhere in the initial restricted output space. Reference [29] shows for the first time how time-varying output constraints can be extended to full-state time-varying constraints and describes an adaptive controller based on backstepping technology. However, the control methods in the research mentioned above are all based on state feedback control, for which all the states in closed-loop systems must be measurable.

Because few references consider the output feedback control based on DSC of uncertain nonlinear systems with time-varying constraints, we have tried in this paper to deal with this more difficult and practical problem for the design of an adaptive control based on a state observer for uncertain nonlinear systems with asymmetric time-varying output constraints and unknown external disturbances. Our main contributions lie in two points that contrast with existing works. (1) It is the first time that an adaptive DSC based on a fuzzy state observer has been addressed for uncertain nonlinear systems with time-varying output constraints and external disturbances. The system in this paper is more general and practical, and the control method is simple, which avoids the traditional computational complexity. (2) The control method described in this paper does not require n-order differentiable and bounded conditions for input signals, and it reduces the requirement of hypothetical conditions.

2. System Description and Basic Knowledge

The goal of the study described in this paper was to develop a nonlinear system with a strict-feedback structure that fits the following equations:

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1) + d_1(t) \\ \dot{x}_2 &= x_3 + f_2(x_1, x_2) + d_2(t) \\ &\vdots \\ \dot{x}_i &= x_{i+1} + f_i(x_1, x_2, \ldots, x_i) + d_i(t), \quad i = 1, 2, \ldots, n-1 \\ &\vdots \\ \dot{x}_n &= u(t) + f_n(x_1, x_2, \ldots, x_n) + d_n(t) \\ y &= x_1, \end{aligned} \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are the state variables and only x_1 can be measured. $u \in R$ and $y \in R$ are the input and output of the system, respectively. $f_i(X_i)$ $(X_i = (x_1, x_2, \ldots, x_i)^T$, $i = 1, 2, \ldots, n)$ represents unknown smooth functions. $d_i(t)$ $(i = 1, 2, \ldots, n)$ represents the external disturbances with unknown boundaries. The output $y(t)$ requirements meet the boundary constraints:

$$\underline{k}_{c1}(t) \le y(t) \le \overline{k}_{c1}(t), \quad \forall t > 0 \tag{2}$$

where $\overline{k}_{c1}(t) : R_+ \longrightarrow R$ and $\underline{k}_{c1}(t) : R_+ \longrightarrow R$, such that $\overline{k}_{c1}(t) > \underline{k}_{c1}(t)$, $\forall t \ge 0$.

System (1) can be rewritten as

$$\begin{aligned} \dot{X} &= AX + Ky + \sum_{i=1}^{n} B_i \left[f_i(X_i) + d_i \right] + Bu \\ y &= CX \end{aligned} \tag{3}$$

where $X = [x_1, x_2, \ldots, x_n]^T$, $A = \begin{bmatrix} -k_1 & & \\ \vdots & I & \\ -k_n & 0 & 0 \end{bmatrix}$, $K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$, $B_i = [0 \ \ldots \ 1 \ \ldots \ 0]^T$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $C = [1 \ \ldots \ 0 \ \ldots \ 0]$, and K is chosen such that A is a Hurwitz matrix. Thus, given a positive definite diagonal matrix $Q > 0$, there exists a positive definite symmetric matrix $P > 0$ satisfying

$$A^T P + PA = -2Q. \tag{4}$$

Control Objective. A state observer is designed to estimate the unmeasurable state. An adaptive controller is designed to use this estimate to create the output $y(t)$ tracking the desired trajectory $y_d(t)$ and ensure that the output $y(t)$ satisfies time-varying constraints. All signals involved in the closed-loop system are bounded, and the tracking error remains in the sufficiently small range.

Assumption 1 (see [30]). External disturbance $d_i(t)$ is bounded by the positive unknown constant d_{iM}; that is, $|d_i(t)| \leq d_{iM}$.

Assumption 2 (see [14]). There are constants $\overline{K}_{ci}$ and $\underline{K}_{ci}(i = 0, 1, 2, \ldots, n)$ such that $\overline{k}_{c1}(t) < \overline{K}_{c0}$, $\underline{k}_{c1}(t) < \underline{K}_{c0}$, and $|\overline{k}_{c1}^{(i)}(t)| \leq \overline{K}_{ci}$, $|\underline{k}_{c1}^{(i)}(t)| < \underline{K}_{ci}(i = 1, 2, \ldots, n)\forall t \geq 0$.

Assumption 3 (see [31]). There are functions $\overline{Y}_0 : R_+ \longrightarrow R_+$ and $\underline{Y}_0 : R_+ \longrightarrow R_+$ that satisfy $\overline{Y}_0 < \overline{k}_{c1}(t)$ and $\underline{Y}_0 > \underline{k}_{c1}(t)$, $\forall t > 0$, and there is a positive constant Y_1 such that the desired trajectory $y_d(t)$ and its time derivative satisfy $\underline{Y}_0(t) \leq y_d(t) \leq \overline{Y}_0(t)$ and $|\dot{y}(t)| \leq Y_1$, $\forall t > 0$.

3. Fuzzy System and Its Approximation

A Fuzzy system is a universal approximator that is used to approximate unknown nonlinear functions. By defining the fuzzy basis function vector as $\xi(x)$ and the adjustable weight parameter vector as $\theta \in R^N$, the general output form of the fuzzy system can be written as follows.

$$\hat{f}(x \mid \theta) = \theta^T \xi(x) \tag{5}$$

According to the universal approximation theorem of fuzzy systems, if $f(x)$ is a continuous function defined based on the compact set Ω and if a fuzzy system $\hat{f}(x \mid \theta)$ is used to approximate $f(x)$, there exists a parameter vector θ such that $\sup_{x\in\Omega}|f(x) - \theta^{\mathrm{T}}\xi(x)| \leq \varepsilon$ for any given small constant, ε, such that $0 < \varepsilon < \varepsilon_M$ [32].

4. Adaptive Control and Observer Design

In this paper, the states $x_2, \ldots, x_n$ of system (1) are not available for feedback, so a state observer needs to be established to estimate the states. Therefore, we defined the estimate of X_i as $\hat{X}_i$, $i = 1, 2, \ldots, n$. According to the universal approximation of fuzzy systems, the uncertain nonlinear function $f_i(X_i)$ $(i = 1, 2, \ldots, n)$ can be expressed as

$$f_i(X_i) = \theta_i^T \xi_i(\hat{X}_i) + \varepsilon_i \tag{6}$$

$$f_i(X_i) = \theta_i^{*T} \xi_i(\hat{X}_i) + \varepsilon_i^* \tag{7}$$

where ε_i is the approximation error, θ_i^* is the optimal parameter vector, and ε_i^* is the minimal approximation error.

We designed the fuzzy state observer as follows.

$$\begin{aligned} \dot{\hat{X}} &= A\hat{X} + Ky + \sum_{i=1}^{n} B_i \hat{f}_i(\hat{X}_i \mid \theta_i) + Bu \\ \hat{y} &= C\hat{X} \end{aligned} \tag{8}$$

By defining the observer error vector as $\tilde{X} = X - \hat{X}$, from (3) and (8), the observer errors equation becomes

$$\begin{aligned} \dot{\tilde{X}} &= A\tilde{X} + \sum_{i=1}^{n} B_i \left[f_i(X_i) - \hat{f}_i(\hat{X}_i \mid \theta_i) + d_i\right] \\ &= A\tilde{X} + \sum_{i=1}^{n} B_i [\varepsilon_i + d_i] = A\tilde{X} + \sum_{i=1}^{n} B_i\delta_i = A\tilde{X} + \delta \end{aligned} \tag{9}$$

where $\delta = [\delta_1, \delta_2, \ldots, \delta_n]^T$ and $\delta_i = \varepsilon_i + d_i$.

Step 1. Define $z_1 = y - \omega_0$ $(\omega_0 = y_d)$ as the tracking error and $z_2 = \hat{x}_2 - \omega_1$ as the virtual error for the second step. Define the first virtual control law as α_1. Let α_1 pass through a first-order filter that has the time constant υ_1. We can then obtain ω_1:

$$\begin{aligned} \upsilon_1 \dot{\omega}_1 + \omega_1 &= \alpha_1; \\ \omega_1(0) &= \alpha_1(0). \end{aligned} \tag{10}$$

Defining the output error of this filter as e_1 leads to $e_1 = \omega_1 - \alpha_1$ and $\dot{\omega}_1 = -e_1/\upsilon_1$, so the time derivative of z_1 is

$$\begin{aligned} \dot{z}_1 &= x_2 + f_1 + d_1 - \dot{\omega}_0 \\ &= z_2 + e_1 + \alpha_1 + \tilde{x}_2 + \theta_1^{*T} \xi_1(\hat{x}_1) + \varepsilon_1^* + d_1 - \dot{y}_d. \end{aligned} \tag{11}$$

Let $D_1 = \varepsilon_1^* + d_1$. Because $|\varepsilon_1^*| \leq \varepsilon_{1M}$ and $|d_1| \leq d_{1M}$, there exists an unknown constant $D_{1M} > 0$ such that $|D_1| \leq D_{1M}$. Define $\tilde{\theta}_1 = \theta_1^* - \theta_1$. Now the time-varying asymmetric BLF can be chosen as

$$\begin{aligned} V_0 = {} & \frac{1}{2}\tilde{X}^T P \tilde{X} + \frac{q(z_1)}{2} \log \frac{k_b^2(t)}{k_b^2(t) - z_1^2} \\ & + \frac{1 - q(z_1)}{2} \log \frac{k_a^2(t)}{k_a^2(t) - z_1^2} + \frac{1}{2\gamma_1} \tilde{\theta}_1^T \tilde{\theta}_1 \end{aligned} \tag{12}$$

where $\gamma_1 > 0$ is the positive design parameter. The time-varying barriers are defined as

$$k_a(t) = y_d(t) - \underline{k}_{c1}(t) \tag{13}$$

$$k_b(t) = \overline{k}_{c1}(t) - y_d(t) \tag{14}$$

$$q(z_1) = \begin{cases} 1 & z_1 > 0 \\ 0 & z_1 \leq 0. \end{cases} \tag{15}$$

Define $\varsigma_a = z_1(t)/k_a(t)$, $\varsigma_b = z_1(t)/k_b(t)$, and $\varsigma = q\varsigma_b + (1 - q)\varsigma_a$; then (12) can be rewritten as

$$V_0 = \frac{1}{2}\tilde{X}^T P \tilde{X} + \frac{1}{2} \log \frac{1}{1 - \varsigma^2} + \frac{1}{2\gamma_1} \tilde{\theta}_1^T \tilde{\theta}_1. \tag{16}$$

The time derivative of V_0 is described as

$$\dot{V}_0 = \frac{1}{2}\dot{\widetilde{X}}^T P\widetilde{X} + \frac{1}{2}\widetilde{X}^T P\dot{\widetilde{X}} + \frac{\varsigma\dot{\varsigma}}{1-\varsigma^2} - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1 = \frac{1}{2}\widetilde{X}^T\left[PA^T + AP\right]\widetilde{X} + \widetilde{X}^T P\delta + \frac{\varsigma\dot{\varsigma}}{1-\varsigma^2} - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1. \quad (17)$$

Because

$$\frac{\varsigma\dot{\varsigma}}{1-\varsigma^2} = \frac{q\varsigma_b + (1-q)\varsigma_a}{1-\varsigma^2}\left(q\dot{\varsigma}_b + (1-q)\dot{\varsigma}_a\right) \quad (18)$$

$$\dot{\varsigma}_b = \frac{\dot{z}_1 k_b(t) - z_1\dot{k}_b(t)}{k_b{}^2(t)} \quad (19)$$

$$\dot{\varsigma}_a = \frac{\dot{z}_1 k_a(t) - z_1\dot{k}_a(t)}{k_a{}^2(t)} \quad (20)$$

we can obtain

$$\begin{aligned}\dot{V}_0 &= -\widetilde{X}^T Q\widetilde{X} + \widetilde{X}^T P\delta + \frac{q\varsigma_b + (1-q)\varsigma_a}{1-\varsigma^2}\left(q\dot{\varsigma}_b + (1-q)\dot{\varsigma}_a\right) - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1 \\ &= -\widetilde{X}^T Q\widetilde{X} + \widetilde{X}^T P\delta + \frac{q\varsigma_b}{1-\varsigma^2}\dot{\varsigma}_b + \frac{(1-q)\varsigma_a}{1-\varsigma^2}\dot{\varsigma}_a - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1 \\ &= -\widetilde{X}^T Q\widetilde{X} + \widetilde{X}^T P\delta + \frac{q\varsigma_b}{k_b(1-\varsigma_b{}^2)}\left(\dot{z}_1 - \frac{z_1\dot{k}_b}{k_b}\right) + \frac{(1-q)\varsigma_a}{k_a(1-\varsigma_a{}^2)}\left(\dot{z}_1 - \frac{z_1\dot{k}_a}{k_a}\right) - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1.\end{aligned} \quad (21)$$

Assuming that $\mu = q/(k_b{}^2 - z_1^2) + (1-q)/(k_a{}^2 - z_1^2)$, we get

$$\begin{aligned}\dot{V}_0 &= -\widetilde{X}^T Q\widetilde{X} + \widetilde{X}^T P\delta + \mu z_1\left(\dot{z}_1 - q\frac{z_1\dot{k}_b}{k_b} - (1-q)\frac{z_1\dot{k}_a}{k_a}\right) - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1 = -\widetilde{X}^T Q\widetilde{X} + \widetilde{X}^T P\delta \\ &+ \mu z_1(\widetilde{x}_2 + D_1) + \mu z_1\left(z_2 + \alpha_1 + e_1 + \theta_1^{*T}\xi_1(\widehat{x}_1) - \dot{y}_d - q\frac{z_1\dot{k}_b}{k_b} - (1-q)\frac{z_1\dot{k}_a}{k_a}\right) - \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\dot{\theta}_1.\end{aligned} \quad (22)$$

By using the inequality $2ab \le a^2 + b^2$, we get

$$\begin{aligned}&\widetilde{X}^T P\delta + \mu z_1(\widetilde{x}_2 + D_1) \\ &\le \frac{1}{2}\left\|\widetilde{X}\right\|^2 + \frac{1}{2}\|P\delta\|^2 + \frac{1}{2}D_1{}^2 + \frac{1}{2}\left|\widetilde{x}_2\right|^2 + (\mu z_1)^2 \\ &\le \left\|\widetilde{X}\right\|^2 + \frac{1}{2}\|P\delta\|^2 + \frac{1}{2}D_1{}^2 + (\mu z_1)^2.\end{aligned} \quad (23)$$

Substituting (23) into (22) results in

$$\begin{aligned}\dot{V}_0 &\le -(\lambda_{\min}(Q) - 1)\left\|\widetilde{X}\right\|^2 + \frac{1}{2}\|P\delta\|^2 + \frac{1}{2}D_1{}^2 + \mu z_1\left[z_2 + \mu z_1 + \alpha_1 + \theta_1{}^T\xi_1(\widehat{x}_1) - \dot{y}_d + q\frac{z_1\dot{k}_b}{k_b} + (1-q)\frac{z_1\dot{k}_a}{k_a}\right] \\ &+ \frac{1}{\gamma_1}\widetilde{\theta}_1{}^T\left(\gamma_1\mu z_1\xi_1(\widehat{x}_1) - \dot{\theta}_1\right) + \mu z_1 e_1.\end{aligned} \quad (24)$$

We choose the first virtual control law α_1 and the parameter adaptive law θ_1 to be

$$\alpha_1 = -\lambda_1 z_1 - \lambda_1' z_1 - \mu z_1 - \theta_1{}^T\xi_1(\widehat{x}_1) + \dot{y}_d \quad (25)$$

$$\dot{\theta}_1 = \gamma_1\mu z_1\xi_1(\widehat{x}_1) - 2\sigma_1\theta_1 \quad (26)$$

where $\lambda_1 > 0$ and $\sigma_1 > 0$ are positive design parameters, and $\lambda_1' = \sqrt{(\dot{k}_b/k_b)^2 + (\dot{k}_a/k_a)^2 + \beta}$ with β as a positive design constant.

Substituting (25) and (26) into (24) results in

$$\dot{V}_0 < -(\lambda_{\min}(Q) - 1)\left\|\widetilde{X}\right\|^2 - \lambda_1\mu z_1^2 + \mu z_1 z_2 + \mu z_1 e_1 + \frac{1}{2}\|P\delta\|^2 + \frac{1}{2}D_1{}^2 + \frac{2\sigma_1}{\gamma_1}\widetilde{\theta}_1{}^T\theta_1. \quad (27)$$

There are Young's inequalities in (28) and (29)

$$\frac{2\sigma_1}{\gamma_1}\widetilde{\theta}_1{}^T\theta_1 \le -\frac{\sigma_1}{\gamma_1}\theta_1{}^T\theta_1 + \frac{\sigma_1}{\gamma_1}\theta_1^{*T}\theta_1^* \quad (28)$$

$$z_1 e_1 \le z_1{}^2 + \frac{1}{4}e_1{}^2. \quad (29)$$

Substituting (28) and (29) into (27) leads to

$$\begin{aligned}\dot{V}_0 &< -(\lambda_{\min}(Q) - 1)\left\|\widetilde{X}\right\|^2 - (\lambda_1 - 1)\mu z_1^2 + \mu z_1 z_2 - \frac{\sigma_1}{\gamma_1}\theta_1{}^T\theta_1 + \frac{\sigma_1}{\gamma_1}\theta_1^{*T}\theta_1^* + \frac{1}{4}\mu e_1{}^2 + \frac{1}{2}\|P\delta\|^2 \\ &+ \frac{1}{2}D_1{}^2.\end{aligned} \quad (30)$$

By using the following inequality:

$$-\frac{1}{2}\widetilde{\theta}_1{}^T\widetilde{\theta}_1 \ge -\theta_1^{*T}\theta_1^* - \theta_1{}^T\theta_1, \quad (31)$$

we obtain

$$\begin{aligned}\dot{V}_0 <& -(\lambda_{\min}(Q)-1)\left\|\widetilde{X}\right\|^2-(\lambda_1-1)\mu z_1^2+\mu z_1 z_2\\&-\frac{\sigma_1}{2\gamma_1}\widetilde{\theta}_1{}^T\widetilde{\theta}_1+\frac{2\sigma_1}{\gamma_1}\theta_1^{*T}\theta_1^*+\frac{1}{4}\mu e_1{}^2+\frac{1}{2}\left\|P\delta\right\|^2\\&+\frac{1}{2}D_1{}^2.\end{aligned}\tag{32}$$

Now, consider the following Lyapunov function candidate:

$$V_1=V_0+\frac{1}{2}e_1^2.\tag{33}$$

Then we can have

$$\begin{aligned}\dot{V}_1 &= \dot{V}_0+e_1\left(-\frac{e_1}{\upsilon_1}-\dot{\alpha}_1\right)\le\dot{V}_0-\frac{e_1{}^2}{\upsilon_1}+e_1{}^2+\frac{1}{4}\psi_1{}^2\\&<-(\lambda_{\min}(Q)-1)\left\|\widetilde{X}\right\|^2-(\lambda_1-1)\mu z_1^2+\mu z_1z_2\\&\quad-\frac{\sigma_1}{2\gamma_1}\widetilde{\theta}_1{}^T\widetilde{\theta}_1+\frac{2\sigma_1}{\gamma_1}\theta_1^{*T}\theta_1^*-\left(\frac{1}{\upsilon_1}-1-\frac{1}{4}\mu\right)e_1{}^2\\&\quad+\frac{1}{2}\left\|P\delta\right\|^2+\frac{1}{2}D_1{}^2+\frac{1}{4}\psi_1{}^2.\end{aligned}\tag{34}$$

Here ψ_1 is the maximum absolute value of $\dot{\alpha}_1$.

Step 2. Define $z_3=\widehat{x}_3-\omega_2$. Define the second virtual control law as α_2. Let α_2 pass through a first-order filter that has the time constant υ_2. We can then obtain ω_2:

$$\begin{aligned}\upsilon_2\dot{\omega}_2+\omega_2&=\alpha_2;\\\omega_2(0)&=\alpha_2(0).\end{aligned}\tag{35}$$

By defining the output error of this filter as e_2, we get $e_2=\omega_2-\alpha_2$ and $\dot{\omega}_2=-e_2/\upsilon_2$.

So the time derivative of z_2 is as follows:

$$\begin{aligned}\dot{z}_2&=\widehat{x}_3+k_2\widetilde{x}_1+\widehat{f}_2-\dot{\omega}_1\\&=z_3+e_2+\alpha_2+k_2\widetilde{x}_1+\theta_2{}^T\xi_2\left(\widehat{X}_2\right)-\dot{\omega}_1.\end{aligned}\tag{36}$$

Define $\widetilde{\theta}_2=\theta_2{}^*-\theta_2$, and the Lyapunov Function can be chosen as

$$V_2=V_1+\frac{1}{2}z_2^2+\frac{1}{2}e_2^2+\frac{1}{2\gamma_2}\widetilde{\theta}_2{}^T\widetilde{\theta}_2\tag{37}$$

where $\gamma_2>0$ is the positive design parameter.

The time derivative of V_2 is described as follows.

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+z_2\left(z_3+e_2+\alpha_2+k_2\widetilde{x}_1+\theta_2{}^{*T}\xi_2\left(\widehat{X}_2\right)\right.\\&\quad\left.+\varepsilon_2{}^*-\varepsilon_2-\dot{\omega}_1\right)+e_2\dot{e}_2-\frac{1}{\gamma_2}\widetilde{\theta}_2{}^T\dot{\theta}_2=\dot{V}_1+z_2\left(z_3\right.\\&\quad\left.+e_2+\alpha_2+k_2\widetilde{x}_1+\theta_2{}^{*T}\xi_2\left(\widehat{X}_2\right)+D_2-\dot{\omega}_1\right)+e_2\dot{e}_2\\&\quad-\frac{1}{\gamma_2}\widetilde{\theta}_2{}^T\dot{\theta}_2.\end{aligned}\tag{38}$$

Here $D_2=\varepsilon_2^*-\varepsilon_2$. $|D_2|\le D_{2M}$, and D_{2M} is an unknown positive constant.

Then we can obtain

$$\begin{aligned}\dot{V}_2&<\dot{V}_1+z_2\left(z_3+e_2+\alpha_2+k_2\widetilde{x}_1+D_2\right.\\&\quad\left.+\theta_2{}^T\xi_2\left(\widehat{X}_2\right)-\dot{\omega}_1\right)+e_2\dot{e}_2+\frac{1}{\gamma_2}\widetilde{\theta}_2{}^T\left(\gamma_2z_2\xi_2\left(\widehat{x}_2\right)\right.\\&\quad\left.-\dot{\theta}_2\right)<\dot{V}_1+z_2\left(z_3+e_2+\frac{1}{2}z_2+\alpha_2+k_2\widetilde{x}_1\right.\\&\quad\left.+\theta_2{}^T\xi_2\left(\widehat{X}_2\right)-\dot{\omega}_1\right)+e_2\dot{e}_2+\frac{1}{\gamma_2}\widetilde{\theta}_2{}^T\left(\gamma_2z_2\xi_2\left(\widehat{x}_2\right)\right.\\&\quad\left.-\dot{\theta}_2\right)+\frac{1}{2}D_2{}^2.\end{aligned}\tag{39}$$

The virtual control law α_2 and the parameter adaptive law θ_2 can be described as

$$\begin{aligned}\alpha_2&=-\lambda_2z_2-\mu z_1-\frac{1}{2}z_2-k_2\widetilde{x}_1-\theta_2{}^T\xi_2\left(\widehat{X}_2\right)\\&\quad-\frac{\omega_1-\alpha_1}{\upsilon_1}\end{aligned}\tag{40}$$

$$\dot{\theta}_2=\gamma_2z_2\xi_2\left(\widehat{x}_2\right)-2\sigma_2\theta_2\tag{41}$$

where $\lambda_2>0$ and $\sigma_2>0$ are positive design parameters.

Substituting (40) and (41) into (39) results in

$$\begin{aligned}\dot{V}_2&<\dot{V}_1-\lambda_2z_2{}^2-\mu z_1z_2+z_2z_3+z_2e_2+e_2\dot{e}_2\\&\quad+\frac{2\sigma_2}{\gamma_2}\widetilde{\theta}_2{}^T\theta_2+\frac{1}{2}D_2{}^2\\&<\dot{V}_1-(\lambda_2-1)z_2{}^2-\mu z_1z_2+z_2z_3+\frac{1}{4}e_2{}^2\\&\quad+e_2\left(-\frac{e_2}{\upsilon_2}-\dot{\alpha}_2\right)+\frac{2\sigma_2}{\gamma_2}\widetilde{\theta}_2{}^T\theta_2+\frac{1}{2}D_2{}^2\\&<-(\lambda_{\min}(Q)-1)\left\|\widetilde{X}\right\|^2-(\lambda_1-1)\mu z_1^2\\&\quad-(\lambda_2-1)z_2{}^2+z_2z_3-\sum_{i=1}^{2}\frac{\sigma_i}{2\gamma_i}\widetilde{\theta}_i{}^T\widetilde{\theta}_i\\&\quad-\left(\frac{1}{\upsilon_1}-1-\frac{1}{4}\mu\right)e_1{}^2-\left(\frac{1}{\upsilon_2}-1-\frac{1}{4}\right)e_2{}^2\\&\quad+\sum_{i=1}^{2}\frac{2\sigma_i}{\gamma_i}\theta_i^{*T}\theta_i^*+\frac{1}{2}\left\|P\delta\right\|^2+\frac{1}{2}\sum_{i=1}^{2}D_i{}^2+\frac{1}{4}\sum_{i=1}^{2}\psi_i{}^2\end{aligned}\tag{42}$$

where ψ_2 is the maximum absolute value of $\dot{\alpha}_2$.

Next, we step i $(i=3,4,\ldots,n-1)$. Define $z_i=\widehat{x}_i-\omega_{i-1}$ as the virtual error of the i^{th} step and $z_{i+1}=\widehat{x}_{i+1}-\omega_i$ as the virtual error of the $(i+1)^{th}$ step. Define the i^{th} virtual control law as α_i. Let α_i pass through a first-order filter that has the time constant υ_i. We can then obtain ω_i:

$$\begin{aligned}\upsilon_i\dot{\omega}_i+\omega_i&=\alpha_i;\\\omega_i(0)&=\alpha_i(0).\end{aligned}\tag{43}$$

Defining the output error of this filter as e_i yields $e_i = \omega_i - \alpha_i$ and $\dot{\omega}_i = -e_i/\upsilon_i$.

Therefore, the time derivative of z_i is as follows:

$$\begin{aligned}\dot{z}_i &= \widehat{x}_{i+1} + k_i\widetilde{x}_1 + \widehat{f}_i - \dot{\omega}_{i-1} \\ &= z_{i+1} + e_i + \alpha_i + k_i\widetilde{x}_1 + \theta_i^{T}\xi_i\left(\widehat{X}_i\right) - \dot{\omega}_{i-1}.\end{aligned} \tag{44}$$

Define $\widetilde{\theta}_i = \theta_i^{*} - \theta_i$, and choose the Lyapunov Function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}e_i^2 + \frac{1}{2\gamma_i}\widetilde{\theta}_i^{T}\widetilde{\theta}_i \tag{45}$$

where $\gamma_i > 0$ is the positive design parameter.

The time derivative of V_i is described as follows.

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i\left(z_{i+1} + e_i + \alpha_i + k_i\widetilde{x}_1 + \theta_i^{T}\xi_i\left(\widehat{X}_i\right)\right. \\ &\quad \left.- \dot{\omega}_{i-1}\right) + e_i\dot{e}_i - \frac{1}{\gamma_i}\widetilde{\theta}_i^{T}\dot{\theta}_i = \dot{V}_{i-1} + z_i\left(z_{i+1} + e_i + \alpha_i\right. \\ &\quad \left.+ k_i\widetilde{x}_1 + \theta_i^{*T}\xi_i\left(\widehat{X}_i\right) + D_i - \dot{\omega}_{i-1}\right) + e_i\dot{e}_i - \frac{1}{\gamma_i}\widetilde{\theta}_i^{T}\dot{\theta}_i.\end{aligned} \tag{46}$$

Here $D_i = \varepsilon_i^{*} - \varepsilon_i$, $|D_i| \le D_{iM}$, and D_{iM} is a unknown positive constant.

Then we can obtain

$$\begin{aligned}\dot{V}_i &< \dot{V}_{i-1} + z_i\left(z_{i+1} + e_i + \alpha_i + k_i\widetilde{x}_1 + \theta_i^{T}\xi_i\left(\widehat{X}_i\right) + D_i\right. \\ &\quad \left.- \dot{\omega}_{i-1}\right) + e_i\dot{e}_i + \frac{1}{\gamma_i}\widetilde{\theta}_i^{T}\left(\gamma_i z_i\xi_i\left(\widehat{x}_i\right) - \dot{\theta}_i\right) < \dot{V}_{i-1} \\ &\quad + z_i\left(z_{i+1} + e_i + \alpha_i + k_i\widetilde{x}_1 + \frac{1}{2}z_i + \theta_i^{T}\xi_i\left(\widehat{X}_i\right)\right. \\ &\quad \left.- \dot{\omega}_{i-1}\right) + e_i\dot{e}_i + \frac{1}{\gamma_i}\widetilde{\theta}_i^{T}\left(\gamma_i z_i\xi_i\left(\widehat{x}_i\right) - \dot{\theta}_i\right) + \frac{1}{2}D_i^{2}.\end{aligned} \tag{47}$$

The virtual control law α_i and the parameter adaptive law θ_i can be described as

$$\begin{aligned}\alpha_i &= -\lambda_i z_i - z_{i-1} - \frac{1}{2}z_i - k_i\widetilde{x}_1 - \theta_i^{T}\xi_i\left(\widehat{X}_i\right) \\ &\quad - \frac{\omega_{i-1} - \alpha_{i-1}}{\upsilon_{i-1}}\end{aligned} \tag{48}$$

$$\dot{\theta}_i = \gamma_i z_i\xi_i\left(\widehat{x}_i\right) - 2\sigma_i\theta_i \tag{49}$$

where $\lambda_i > 0$ and $\sigma_i > 0$ are positive design parameters.

Substituting (48) and (49) into (47) results in

$$\begin{aligned}\dot{V}_i &< \dot{V}_{i-1} - \lambda_i z_i^{2} - z_i z_{i-1} + z_i z_{i+1} + z_i e_i + e_i\dot{e}_i \\ &\quad + \frac{2\sigma_i}{\gamma_i}\widetilde{\theta}_i^{T}\theta_i + \frac{1}{2}D_i^{2} \\ &< \dot{V}_{i-1} - (\lambda_i - 1) z_i^{2} - z_i z_{i-1} + z_i z_{i+1} \\ &\quad - \left(\frac{1}{\upsilon_i} - 1 - \frac{1}{4}\right)e_i^{2} - \frac{\sigma_i}{2\gamma_i}\widetilde{\theta}_i^{T}\widetilde{\theta}_i + \frac{2\sigma_i}{\gamma_i}\theta_i^{*T}\theta_i^{*} \\ &\quad + \frac{1}{4}\psi_i^{2} + \frac{1}{2}D_i^{2} \\ &< -(\lambda_{\min}(Q) - 1)\left\|\widetilde{X}\right\|^2 - (\lambda_1 - 1)\mu z_1^2 \\ &\quad - \sum_{k=2}^{i}(\lambda_k - 1) z_k^{2} + z_k z_{k+1} - \sum_{k=1}^{i}\frac{\sigma_k}{2\gamma_k}\widetilde{\theta}_k^{T}\widetilde{\theta}_k \\ &\quad - \left(\frac{1}{\upsilon_1} - 1 - \frac{1}{4}\mu\right)e_1^{2} - \sum_{k=2}^{i}\left(\frac{1}{\upsilon_k} - 1 - \frac{1}{4}\right)e_k^{2} \\ &\quad + \sum_{k=1}^{i}\frac{2\sigma_k}{\gamma_k}\theta_k^{*T}\theta_k^{*} + \frac{1}{2}\|P\delta\|^2 + \frac{1}{2}\sum_{k=1}^{i}D_k^{2} \\ &\quad + \frac{1}{4}\sum_{k=1}^{i}\psi_k^{2}.\end{aligned} \tag{50}$$

Here ψ_i is the maximum absolute value of $\dot{\alpha}_i$.

Step n. Because $z_n = \widehat{x}_n - \omega_{n-1}$, the time derivative of z_n is

$$\begin{aligned}\dot{z}_n &= u + k_n\widetilde{x}_1 + \widehat{f}_n - \dot{\omega}_{n-1} \\ &= u + k_n\widetilde{x}_1 + \theta_n^{T}\xi_n\left(\widehat{X}_n\right) - \dot{\omega}_{n-1}.\end{aligned} \tag{51}$$

Define $\widetilde{\theta}_n = \theta_n^{*} - \theta_n$, and the Lyapunov function can be chosen as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\gamma_n}\widetilde{\theta}_n^{T}\widetilde{\theta}_n \tag{52}$$

where $\gamma_n > 0$ is the positive design parameter.

The time derivative of V_n is equal to

$$\begin{aligned}&\dot{V}_n \\ &\quad = \dot{V}_{n-1} \\ &\qquad + z_n\left(u + k_n\widetilde{x}_1 + \theta_n^{*T}\xi_n\left(\widehat{X}_n\right) + \varepsilon_n^{*} - \varepsilon_n - \dot{\omega}_{n-1}\right) \\ &\qquad - \frac{1}{\gamma_n}\widetilde{\theta}_n^{T}\dot{\theta}_n \\ &\quad = \dot{V}_{n-1} + z_n\left(u + k_n\widetilde{x}_1 + \theta_n^{*T}\xi_n\left(\widehat{X}_n\right) + D_n - \dot{\omega}_{n-1}\right) \\ &\qquad - \frac{1}{\gamma_n}\widetilde{\theta}_n^{T}\dot{\theta}_n.\end{aligned} \tag{53}$$

Here $D_n = \varepsilon_n^* - \varepsilon_n$, $|D_n| \le D_{nM}$, and D_{nM} is an unknown positive constant.

Then we can obtain

$$\begin{aligned}\dot{V}_n &< \dot{V}_{n-1} \\ &\quad + z_n\left(u + k_n\tilde{x}_1 + \theta_n{}^T\xi_n\left(\widehat{X}_n\right) + \frac{1}{2}z_n - \dot{\omega}_{n-1}\right) \\ &\quad + \frac{1}{\gamma_n}\tilde{\theta}_n{}^T\left(\gamma_n z_n\xi_n\left(\hat{x}_n\right) - \dot{\theta}_n\right) + \frac{1}{2}D_n.\end{aligned} \tag{54}$$

Choose the control law u and the parameter adaptive law θ_n as follows:

$$\begin{aligned}u &= -\lambda_n z_n - z_{n-1} - \frac{1}{2}z_n - k_n\tilde{x}_1 - \theta_n{}^T\xi_n\left(\widehat{X}_n\right) \\ &\quad - \frac{\omega_{n-1} - \alpha_{n-1}}{\upsilon_{n-1}}\end{aligned} \tag{55}$$

$$\dot{\theta}_n = \gamma_n z_n\xi_n\left(\hat{x}_n\right) - 2\sigma_n\theta_n \tag{56}$$

where $\lambda_n > 0$ and $\sigma_n > 0$ are positive design parameters.

Substituting (55) and (56) into (54) results in

$$\begin{aligned}\dot{V}_n &< \dot{V}_{n-1} - \lambda_n z_n{}^2 - z_n z_{n-1} + \frac{2\sigma_n}{\gamma_n}\tilde{\theta}_n{}^T\theta_n + \frac{1}{2}D_n \\ &< \dot{V}_{n-1} - \lambda_n z_n{}^2 - z_n z_{n-1} - \frac{\sigma_n}{2\gamma_n}\tilde{\theta}_n{}^T\tilde{\theta}_n \\ &\quad + \frac{2\sigma_n}{\gamma_n}\theta_n^{*T}\theta_n^* + \frac{1}{2}D_n \\ &< -\left(\lambda_{\min}(Q) - 1\right)\left\|\widetilde{X}\right\|^2 - \left(\lambda_1 - 1\right)\mu z_1^2 \\ &\quad - \sum_{i=2}^{n-1}\left(\lambda_i - 1\right)z_i{}^2 - \lambda_n z_n{}^2 - \sum_{i=1}^{n}\frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i{}^T\tilde{\theta}_i \\ &\quad - \left(\frac{1}{\upsilon_1} - 1 - \frac{1}{4}\mu\right)e_1{}^2 - \sum_{i=2}^{n-1}\left(\frac{1}{\upsilon_i} - 1.25\right)e_i{}^2 \\ &\quad + M\end{aligned} \tag{57}$$

where $M = \sum_{i=1}^{n}(2\sigma_i/\gamma_i)\theta_i^{*T}\theta_i^* + (1/2)\|P\delta_M\|^2 + (1/2)\sum_{i=1}^{n}D_{iM}{}^2 + (1/4)\sum_{i=1}^{n-1}\psi_i{}^2$.

It can be seen from (25), (40), (48), and (55) that the proposed control method not only has overcome the difficulty in backstepping control design due to the "explosion of complexity", but also has removed the restrictive assumption that is widely used in [33, 34] that the input signal should be n-order differentiable and bounded. Moreover, the proposed control method can easily obtain adaptive control of nonlinear systems with various output constraints and unmeasurable states.

To further illustrate the advantages of our method, we will make some comparisons with previous results that considered adaptive control of nonlinear systems with multiple constraints, but for which all the states of the control system need to be measured [14–16, 29, 33, 34]. This paper describes the design of a fuzzy state observer, such that only the output of the system needs to be measured. Previous papers [19–24, 31] described the development of adaptive control of nonlinear systems based on a fuzzy sate observer, but these adaptive control methods cannot deal with the problem of output constraints. Because of the "explosion of complexity", these methods have a heavy computation burden. In addition, these control methods all assume that the input signal should be n-order differentiable with bounded derivatives.

5. Stability Analysis

Define $V = V_n$ as the Lyapunov function of the closed-loop system, so the derivation of V is (57).

Select the positive matrix Q and the positive coefficients λ_i, λ_n, υ_1, and υ_i as

$$\lambda_{\min}(Q) - 1 > 0 \tag{58}$$

$$\lambda_i - 1 > 0, \quad i = 1, 2, \ldots, n-1 \tag{59}$$

$$\lambda_n > 0 \tag{60}$$

$$\frac{1}{\upsilon_1} - 1 - \frac{1}{4}\mu > 0 \tag{61}$$

$$\frac{1}{\upsilon_i} - 1.25 > 0. \tag{62}$$

Based on lemma 2 in [14], we get

$$\begin{aligned}\dot{V} &< -2\frac{\lambda_{\min}(Q) - 1}{\lambda_{\max}(P)}\left(\frac{1}{2}\widetilde{X}^T P\widetilde{X}\right) \\ &\quad - 2\left(\lambda_1 - 1\right)\frac{1}{2}\log\frac{z_1^2}{k_b{}^2 - z_1^2} - \sum_{i=2}^{n-1}2\left(\lambda_i - 1\right)\frac{1}{2}z_i{}^2 \\ &\quad - 2\lambda_n\frac{1}{2}z_n{}^2 - \sum_{i=1}^{n}\frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i{}^T\tilde{\theta}_i \\ &\quad - 2\left(\frac{1}{\upsilon_1} - 1 - \frac{1}{4}\mu\right)\frac{1}{2}e_1{}^2 \\ &\quad - \sum_{i=2}^{n-1}2\left(\frac{1}{\upsilon_i} - 1.25\right)\frac{1}{2}e_i{}^2 + M.\end{aligned} \tag{63}$$

Define the positive parameter as

$$\begin{aligned}C = \min\Bigg\{&2\frac{\lambda_{\min}(Q) - 1}{\lambda_{\max}(P)}, 2\left(\lambda_i - 1\right), 2\lambda_n, \\ &2\left(\frac{1}{\upsilon_1} - 1 - \frac{1}{4}\mu\right), 2\left(\frac{1}{\upsilon_i} - 1.25\right), \sigma_i;\ i = 1, 2, \cdots, n\Bigg\}.\end{aligned} \tag{64}$$

Then (63) can be rewritten as

$$\dot{V} \le -CV + M. \tag{65}$$

The initial condition requirement $\underline{k}_{c1}(0) \le y(0) \le \overline{k}_{c1}(0)$ implies that $-k_a(0) < z_1(0) < k_b(0)$ and $|\varsigma(0)| < 1$. Then, based on lemma 1 in [35], we can have $|\varsigma(t)| < 1$, $\forall t > 0$, where V is bounded in the set of $[0, \infty)$. Because $-k_a(t) < z_1(t) < k_b(t)$ and $y(t) = z_1(t) + y_d(t)$, we can assume that for all $t > 0$, $-k_a(t) + y_d(t) < y(t) < k_b(t) + y_d(t)$, and $\underline{k}_{c1}(t) < y(t) < \overline{k}_{c1}(t)$, $\forall t > 0$, can be deduced.

Multiply both sides of (65) by e^{Ct} to obtain

$$e^{Ct}\dot{V} \leq (-CV + M) e^{Ct} \tag{66}$$

$$\frac{d}{dt}\left(Ve^{Ct}\right) \leq Me^{Ct} \tag{67}$$

$$Ve^{Ct} - V(0) \leq \frac{M}{C}\left(e^{Ct} - 1\right) \tag{68}$$

$$\begin{aligned} 0 \leq V(t) &\leq V(0) e^{-Ct} + \frac{M}{C}\left(1 - e^{-Ct}\right) \\ &\leq V(0) + \frac{M}{C}. \end{aligned} \tag{69}$$

From (69), we can see that if $V(0) \leq \nu$ and $V(t) \leq \nu + M/C$, $\forall t > 0$, the boundedness of ς and V guarantees that all signals of the closed-loop system, such as $x_i(t)$, $\hat{x}_i(t)$, $z_i(t)$, $a_i(t)$, and $u(t)$, are semiglobally uniformly ultimately bounded (SGUUB) [36, 37]. Based on (69) and the definitions of C and M, it can be seen that $z_1(t)$ can be made arbitrarily small by appropriate design parameters.

6. Simulations

Consider a system governed by the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 e^{-0.5x_1} + 0.1\sin(2t) \\ \dot{x}_2 &= u(t) + x_1 \sin\left(x_2^2\right) + 0.01\cos(10t) \\ y &= x_1 \end{aligned} \tag{70}$$

where $f_1(x_1) = x_1 e^{-0.5x_1}$ and $f_2(x_1, x_2) = x_1 \sin(x_2^2)$ are unknown functions. $d_1(t) = 0.1\sin(2t)$, $d_2(t) = 0.01\cos(10t)$. The input tracking signal is $y_d = 0.5\sin(t)$. $\underline{k}_{c1} = -0.5 + 0.4\sin(t)$, $\overline{k}_{c1} = 0.6 + 0.1\cos(t)$.

By choosing the fuzzy membership function as

$$\begin{aligned} \mu_{F_1^l}(\hat{x}_1) &= \exp\left[-\frac{(\hat{x}_1 + 2.5 - l/2)^2}{2}\right], \quad l = 1, 2, \ldots, 9 \\ \mu_{F_2^l}(\hat{x}_1, \hat{x}_2) &= \exp\left[-\frac{(\hat{x}_1 + 2.5 - l/2)^2}{2}\right] \\ &\quad \times \exp\left[-\frac{(\hat{x}_2 + 2.5 - l/2)^2}{2}\right], \quad l = 1, 2, \ldots, 9, \end{aligned} \tag{71}$$

defining the fuzzy basis functions as

$$\begin{aligned} \xi_{1l}(\hat{x}_1) &= \frac{\mu_{F_1^l}(\hat{x}_1)}{\sum_{j=1}^{9} \mu_{F_1^j}(\hat{x}_1)}, \quad l = 1, 2, \ldots, 9 \\ \xi_{2l}(\hat{x}_1, \hat{x}_2) &= \frac{\mu_{F_1^l}(\hat{x}_1) \times \mu_{F_2^l}(\hat{x}_2)}{\sum_{j=1}^{9} \mu_{F_1^j}(\hat{x}_1) \times \mu_{F_2^j}(\hat{x}_2)}, \quad l = 1, 2, \ldots, 9, \end{aligned} \tag{72}$$

and choosing the parameters in the controller and in the adaptive laws as

$$\begin{aligned} \lambda_1 &= 20, \\ \lambda_2 &= 20, \\ k_1 &= 20, \\ k_2 &= 10, \\ \gamma_1 &= \gamma_2 = 0.1, \\ \sigma_1 &= \sigma_2 = 0.1, \\ \beta &= 0.01 \end{aligned} \tag{73}$$

we can obtain

$$K = [20, 10]^T \quad \text{and } A = \begin{bmatrix} -20 & 1 \\ -10 & 0 \end{bmatrix}. \tag{74}$$

$Q = \mathrm{diag}[5, 5]$ is the given symmetric positive matrix. By solving the Lyapunov equation (4), we can get the symmetric positive matrix P:

$$P = \begin{bmatrix} 2.75 & -5 \\ -5 & 10.275 \end{bmatrix}. \tag{75}$$

The initial conditions of the system and the observer are chosen as

$$\mathbf{x}(0) = [0, -0.2]^T \quad \text{and } \hat{\mathbf{x}}(0) = [0, 0.3]^T. \tag{76}$$

Initial values of adaptive parameters are chosen as

$$\begin{aligned} &\theta_1(0) \\ &= [0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15]^T \\ &\theta_2(0) \\ &= [0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15, 0.15]^T. \end{aligned} \tag{77}$$

The simulation results are shown in Figures 1–5. Figure 1 shows the output $y(t)$ and the asymmetric constraints, $\underline{k}_{c1}(t) \leq y(t) \leq \overline{k}_{c1}(t)$, $\forall t \geq 0$. We can see that the

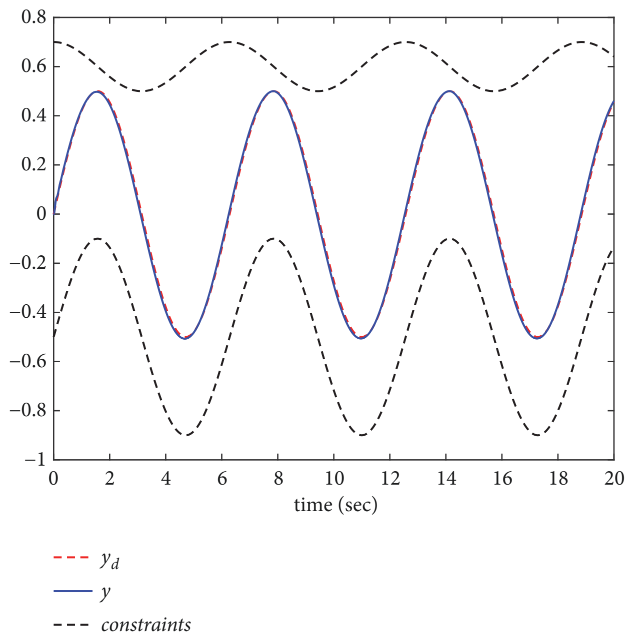

Figure 1: Trajectories of y, y_d and the output constraints.

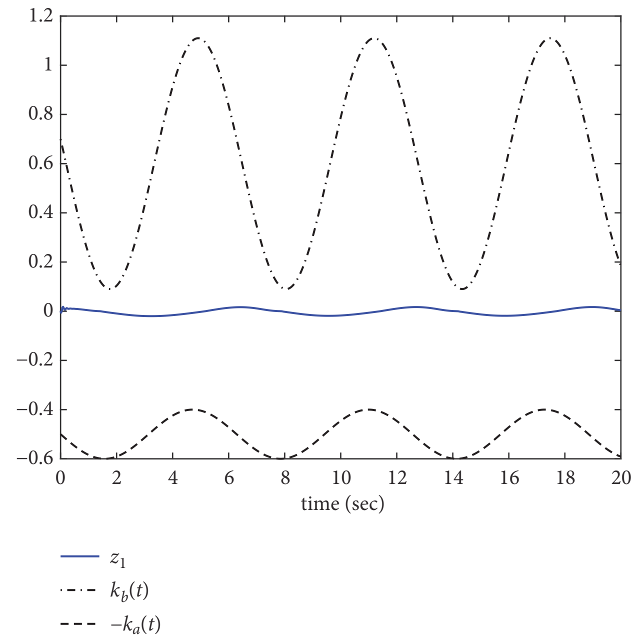

Figure 2: The tracking error z_1 and the error bounds.

output $y(t)$ can track the desired trajectory $y_d(t)$ very well. Figure 2 shows the trajectories of tracking error $z_1(t)$ and the error boundaries. It shows that $z_1(t)$ always satisfies $-k_a(t) < z_1(t) < k_b(t)$, $\forall t \geq 0$. Figure 3 shows the trajectories of state x_1 and its estimate $\hat{x}_1$. Figure 4 shows the trajectories of state x_2 and its estimate $\hat{x}_2$. Figure 5 shows the control input signal $u(t)$. From Figures 1, 2, and 5, we can see that, when $y(t)$ and $z_1(t)$ get close to their constraints, the amplitude of $u(t)$ increases rapidly. This is predictable, and when y and z_1 come close to the limit boundaries, the controller will provide a large control effort to keep the output and error away from the constraints. The simulation results show that, in the presence of external disturbances, the proposed output control scheme is capable of guaranteeing the boundedness of all the signals in the closed-loop system, such as, x_1, $\hat{x}_1$, x_2, $\hat{x}_2$, and u, without violating the asymmetric time-varying output constraints.

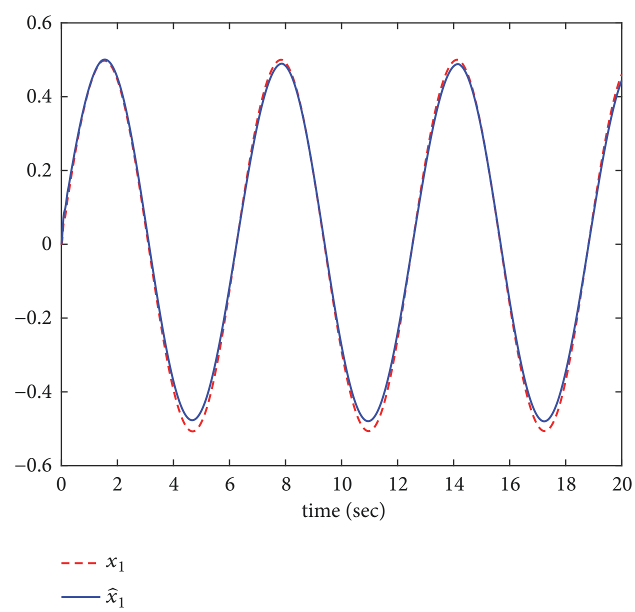

Figure 3: The trajectories of x_1 and $\hat{x}_1$.

7. Conclusion

This paper has proposed an adaptive DSC scheme based on a fuzzy state observer for uncertain nonlinear systems with asymmetric time-varying output constraints in the presence of external disturbances. As part of this scheme, a fuzzy adaptive state observer has been designed to estimate the unmeasured states, and an asymmetric time-varying BLF is employed to prevent the output from violating the asymmetric time-varying constraints. The problem of "explosion of complexity" is avoided by employing the DSC design. Finally, the stability of the closed-loop system has been confirmed by using Lyapunov method. The semiglobal uniform ultimate boundedness of all the signals can be guaranteed, and the tracking error remains within a sufficiently small boundary. Our future research will include extending of the results described here to nonstrict-feedback MIMO nonlinear systems and stochastic nonlinear systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

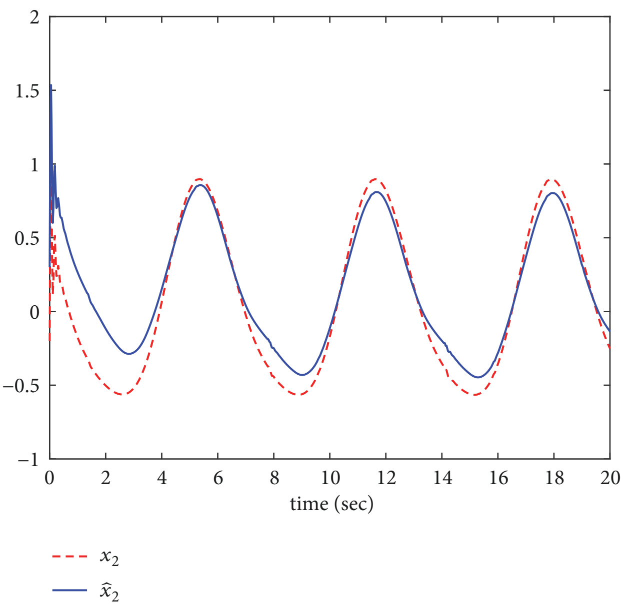

FIGURE 4: The trajectories of x_2 and $\hat{x}_2$.

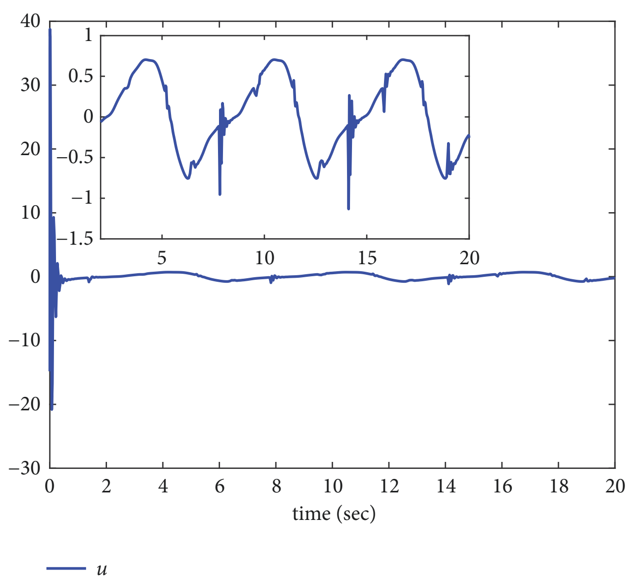

FIGURE 5: The control input $u(t)$.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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