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Research Article

Reformulated Reciprocal Degree Distance and Reciprocal Degree Distance of the Complement of the Mycielskian Graph and Generalized Mycielskian

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The reformulated reciprocal degree distance is defined for a connected graph G as $\overline{R}_t(G) = (1/2)\sum_{u,v\in V(G)}((d_G(u)+d_G(v))/(d_G(u,v)+t)), t \geq 0$, which can be viewed as a weight version of the $t-$Harary index; that is, $\overline{H}_t(G) = (1/2)\sum_{u,v\in V(G)}(1/(d_G(u,v)+t)), t \geq 0$. In this paper, we present the reciprocal degree distance index of the complement of Mycielskian graph and generalize the corresponding results to the generalized Mycielskian graph.

1. Introduction

Throughout this paper we consider (nontrivial) simple graphs, which are finite and undirected graphs without loops or multiple edges. Let G be a connected graph. For vertices $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u,v)$, is the length of a shortest $(u,v)-$path in G and let $d_G(v)$ be the degree of a vertex $v \in V(G)$. A chemical graph is a graph whose vertices denote atoms and edges denote bonds between those atoms of the underlying chemical structure. A topological index for a (chemical) graph is a real number related to the graph; it does not depend on labeling or pictorial representation of a graph. In theoretical chemistry, molecular structure descriptors (also called topological indices) and graph invariants based on the distances between vertices of a graph or vertex degree are widely used for characterizing molecular graphs, establishing relations between structure and properties of molecules, predicting biological activity of chemical compounds, and making their chemical applications.

The Wiener index of G is defined as $W(G) = \sum_{\{u,v\}\in V(G)} d_G(u,v)$ with the summation going over all pairs of distinct vertices of G. Similarly, the Harary index of G is defined as $H(G) = (1/2)\sum_{u,v\in V(G)}(1/d_G(u,v))$.

Das et al. [1] consider the generalized version of Harary index, namely, the $t-$Harary index, which is defined as $\overline{H}_t(G) = (1/2)\sum_{u,v\in V(G)}(1/(d_G(u,v)+t)), t \geq 0$. Its applications and mathematical properties are well studied in [2–5].

Dobrynin and Kochetova [6] and Gutman [7] independently proposed a vertex-degree-weighted version of Wiener index called degree distance, which is defined for a connected graph G as $DD(G) = (1/2)\sum_{u,v\in V(G)}(d_G(u)+d_G(v))d_G(u,v)$. Note that the degree distance is a degree-weight version of the Wiener index. Hua and Zhang [8] introduced a new graph invariant named reciprocal degree distance, which can be as a degree-weight version of Harary index; that is, $RDD(G) = (1/2)\sum_{u,v\in V(G)}((d_G(u)+d_G(v))/d_G(u,v))$. The reciprocal degree distance of some graph operations is obtained in [9, 10].

Recently, Li et al. [11] introduced a vertex-degree-weighted version of $t-$Harary index called reformulated reciprocal degree distance, which is defined for a connected graph G as $\overline{R}_t(G) = (1/2)\sum_{u,v\in V(G)}((d_G(u)+d_G(v))/(d_G(u,v)+t)), t \geq 0$. In view of $\overline{H}_t(G)$, $\overline{R}_t(G)$ is just the additively weighted $t-$Harary index, while in view of $RDD(G)$ it is also the generalized version of reciprocal degree distance

of a connected graph G. The mathematical properties of the three edge-grafting transformations and some sharp upper bounds on the reformulated reciprocal degree distance of trees are presented in [11].

The first Zagreb index is defined as $M_1(G) = \sum_{u\in V(G)} d_G(u)^2 = \sum_{uv\in E(G)}(d_G(u) + d_G(v))$ and the second Zagreb index is defined as $M_2(G) = \sum_{u\in V(G)} d_G(u)^2 = \sum_{uv\in E(G)}(d_G(u)d_G(v))$. The Zagreb indices are found to have applications in QSPR and QSAR studies as well; see [12].

Mycielski [13] developed a graph transformation that transforms a graph G into a new graph $\mu(G)$, which is called the Mycielskian of G, in a search for triangle-free graphs with arbitrarily large chromatic number. The generalized Mycielskian is natural generalization of Mycielski graphs, which is also called by Tardif cones over graphs [14]. Let G be a graph with vertex set $V^0 = \{v_1^0, v_2^0, \ldots, v_n^0\}$ and edge set E^0. Given an integer $m \geqslant 1$, the m−Mycielskian of G, denoted by $\mu_m(G)$, is the graph with vertex set $V^0 \cup V^1 \cup V^2 \ldots \cup V^m \cup \{u\}$, where $V^i = \{v_j^i : v_j^0 \in V^0\}$ is the ith distinct copy of V^0 for $i = 1, 2, \ldots, m$ and edge set $E^0 \cup (\bigcup_{i=0}^{m-1}\{v_j^i v_{j'}^{i+1} : v_j^0 v_{j'}^0 \in E^0\}) \cup \{v_j^m u : v_j^m \in V^m\}$. Concerning on the other results for (generalized) Mycielskian graph, one can refer to papers [15–19].

2. Complement of the Mycielskian Graph

In this section, we obtain the reformulated reciprocal degree distance and reciprocal degree distance of the graph $\overline{\mu}(G)$.

The following lemmas follow from the structure of the complement of the Mycielskian graph.

Lemma 1. *Let G be a connected graph. Then the distances between the vertices of the Mycielskian graph $\overline{\mu}(G)$ of G are given as follows. For each $x, y \in V((\overline{\mu}(G)))$,*

$$(1)\ d_{\overline{\mu}(G)}(x,y) = \begin{cases} 1, & \text{if } x = u_i,\ y = u_j; \\ 1, & \text{if } x = v_i,\ y = v_j,\ d_G(v_i, v_j) > 1; \\ 2, & \text{if } x = v_i,\ y = v_j,\ d_G(v_i, v_j) = 1. \end{cases}$$

$$(2)\ d_{\overline{\mu}(G)}(x,y) = \begin{cases} 1, & \text{if } x = v_i,\ y = u_j,\ i = j; \\ 1, & \text{if } x = v_i,\ y = u_j,\ i \neq j,\ d_G(v_i, u_j) > 1; \\ 2, & \text{if } x = v_i,\ y = u_j,\ i \neq j,\ d_G(v_i, u_j) = 1. \end{cases} \tag{1}$$

$$(3)\ d_{\overline{\mu}(G)}(x,y) = \begin{cases} 2, & \text{if } x = u_i,\ y = \omega; \\ 1, & \text{if } x = v_i,\ y = \omega. \end{cases}$$

Lemma 2. *Let G be a graph on n vertices. Then the degree of any vertex*

$$x \in \overline{\mu}(G) \text{ is as following}: d_{\overline{\mu}(G)}(x) = \begin{cases} n, & \text{if } x = \omega; \\ 2n - 1 - d_G(v_i), & \text{if } x = u_i; \\ 2n - 2d_G(v_i), & \text{if } x = v_i. \end{cases} \tag{2}$$

Remark 3. Let G be a graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$. Then there are $n-1$ two element subsets in $V(G)$. Therefore $\sum_{\{v_i,v_j\}\subseteq V(G)}(d_G(v_i)+d_G(v_j)) = \sum_{i=1}^{n}(n-1)d_G(v_i) = 2(n-1)m_1$.

Theorem 4 (corresponding to Theorem 3.4 in [20]). *Let G be a graph on n vertices and m_1 edges with diameter 2. Then $\overline{R_t}(\overline{\mu}(G)) = (1/(1+t))(8n^3-3n^2+n-24nm_1+4m_1+5M_1(G)) + (1/(2+t))(3n^2-n+12nm_1-4m_1-5M_1(G))$.*

Proof. From the structure of the complement of Mycielskian graph, we consider the following cases of adjacent and nonadjacent pairs of vertices in $\overline{\mu}(G)$ to compute $\overline{R_t}(\overline{\mu}(G))$:

(i) If $\{x, y\} \subseteq U$, then

$$\sum_{\{u_i,u_j\}\subseteq U} \frac{d_{\overline{\mu}(G)}(u_i) + d_{\overline{\mu}(G)}(u_j)}{d_{\overline{\mu}(G)}(u_i, u_j) + t} = \sum_{\{u_i,u_j\}\subseteq U} \frac{2n - 1 - d_G(v_i) + 2n - 1 - d_G(v_j)}{1+t}; \tag{3}$$

by Lemmas 1 and 2

$$\begin{aligned} &= \frac{1}{1+t}\sum_{\{u_i,u_j\}\subseteq U}\left(4n - 2 - \left(d_G(v_i) + d_G(v_j)\right)\right) \\ &= \frac{1}{1+t}\left(\frac{n(n-1)}{2}(4n-2) - 2m_1(n-1)\right) \\ &= \frac{1}{1+t}\left(n(n-1)(2n-1) - 2m_1(n-1)\right). \end{aligned} \tag{4}$$

(ii) If $\{x, y\} \subseteq V(G)$, then $d_{\overline{\mu}(G)}(v_i, v_j) = 1$ for each $v_iv_j \notin E(G)$ and $d_{\overline{\mu}(G)}(v_i, v_j) = 2$ otherwise. Therefore

$$\begin{aligned} &\sum_{\{v_i,v_j\}\subseteq V(G)} \frac{d_{\overline{\mu}(G)}(v_i) + d_{\overline{\mu}(G)}(v_j)}{d_{\overline{\mu}(G)}(v_i, v_j) + t} \\ &= \frac{1}{1+t}\sum_{v_iv_j\notin E(G)}\left(2n - 2d_G(v_i) + 2n - 2d_G(v_j)\right) \\ &\quad + \frac{1}{2+t}\sum_{v_iv_j\in E(G)}\left(2n - 2d_G(v_i) + 2n - 2d_G(v_j)\right); \end{aligned} \tag{5}$$

by Lemmas 1 and 2

$$\begin{aligned}
&= \frac{1}{1+t}\left[\sum_{v_iv_j\notin E(G)}(4n)\right.\\
&\quad\left.-2\sum_{v_i,v_j\notin E(G)}\left(d_G(v_i)+d_G(v_j)\right)\right]\\
&\quad+\frac{1}{2+t}\left[\sum_{v_i,v_j\in E(G)}(4n)\right.\\
&\quad\left.-2\sum_{v_iv_j\in E(G)}\left(d_G(v_i)+d_G(v_j)\right)\right]\\
&= \frac{1}{t+1}\left[4n\left(\frac{n(n-1)}{2}-m_1\right)\right.\\
&\quad\left.-2\left(2(n-1)m_1-M_1(G)\right)\right]+\frac{1}{2+t}\left(4nm_1\right.\\
&\quad\left.-2M_1(G)\right)=\frac{1}{1+t}\left[2n^2(n-1)-8nm_1+4m_1\right.\\
&\quad\left.+2M_1(G)\right]+\frac{1}{2+t}\left(4nm_1-2M_1(G)\right).
\end{aligned}\tag{6}$$

(iii) If $x=v_i$ and $y=u_i$, $1\le i\le n$, then by Lemmas 1 and 2

$$\begin{aligned}
&\sum_{\{v_i,u_i\}\subseteq V(\overline{\mu}(G))}\frac{d_{\overline{\mu}(G)}(v_i)+d_{\overline{\mu}(G)}(u_i)}{d_{\overline{\mu}(G)}(v_i,u_i)+t}\\
&\quad=\sum_{i=1}^{n}\frac{d_{\overline{\mu}(G)}(v_i)+d_{\overline{\mu}(G)}(u_i)}{d_{\overline{\mu}(G)}(v_i,u_i)+t}\\
&\quad=\frac{1}{1+t}\sum_{i=1}^{n}\left(2n-2d_G(v_i)+2n-1-d_G(v_i)\right)\\
&\quad=\frac{1}{1+t}\left((4n-1)n-6m_1\right).
\end{aligned}\tag{7}$$

(iv) If $x=v_i$ and $y=u_j$, $i\neq j$, then by Lemmas 1 and 2

$$\begin{aligned}
&\sum_{\{v_i,u_j\}\subseteq V(\overline{\mu}(G)),i\neq j}\frac{d_{\overline{\mu}(G)}(v_i)+d_{\overline{\mu}(G)}(u_j)}{d_{\overline{\mu}(G)}(v_i,u_j)+t}=\frac{1}{1+t}\\
&\quad\cdot\sum_{(v_i,v_j),v_iv_j\notin E(G)}\left(2n-2d_G(v_i)+2n-1-d_G(v_j)\right)\\
&\quad+\frac{1}{2+t}\\
&\quad\cdot\sum_{(v_i,v_j),v_iv_j\in E(G)}\left(2n-2d_G(v_i)+2n-1-d_G(v_j)\right)\\
&=\frac{1}{1+t}\left[2(4n-1)\left(\frac{n(n-1)}{2}-m_1\right)\right.\\
&\quad-\sum_{(v_i,v_j),v_iv_j\notin E(G)}\left(d_G(v_i)+d_G(v_j)\right)\\
&\quad\left.-\sum_{(v_i,v_j),v_iv_j\notin E(G)}\left(d_G(v_i)\right)\right]+\frac{1}{2+t}\left[2(4n-1)m_1\right.\\
&\quad-\sum_{(v_i,v_j),v_iv_j\in E(G)}\left(d_G(v_i)+d_G(v_j)\right)\\
&\quad\left.-\sum_{(v_i,v_j),v_iv_j\in E(G)}\left(d_G(v_i)\right)\right]\\
&=\frac{1}{1+t}\left[(4n-1)\left(n(n-1)-2m_1\right)\right.\\
&\quad-2\left(2m_1(n-1)-M_1(G)\right)\\
&\quad\left.-\left(2m_1(n-1)-M_1(G)\right)\right]+\frac{1}{2+t}\left[(4n-1)2m_1\right.\\
&\quad\left.-2M_1(G)-M_1(G)\right]=\frac{1}{1+t}\left[n(n-1)(4n-1)\right.\\
&\quad\left.-2m_1(7n-4)+3M_1(G)\right]+\frac{1}{2+t}\left[(4n-1)2m_1\right.\\
&\quad\left.-3M_1(G)\right].
\end{aligned}\tag{8}$$

Each v_j can be paired with $n-1$ vertices v_i as (v_i,v_j), $i\neq j$, $\sum_{(v_i,v_j)}d_G(v_j)=(n-1)\sum_{j=1}d_G(v_j)=2m_1(n-1)$.

(v) If $x=\omega$ and $y\in U$, then by Lemmas 1 and 2

$$\begin{aligned}
&\sum_{\{\omega,u_i\}\subseteq V(\overline{\mu}(G))}\frac{d_{\overline{\mu}(G)}(\omega)+d_{\overline{\mu}(G)}(u_i)}{d_{\overline{\mu}(G)}(\omega,u_i)+t}\\
&\quad=\sum_{i=1}^{n}\frac{d_{\overline{\mu}(G)}(\omega)+d_{\overline{\mu}(G)}(u_i)}{d_{\overline{\mu}(G)}(\omega,u_i)+t}\\
&\quad=\frac{1}{2+t}\sum_{i=1}^{n}\left(n+2n-1-d_G(v_i)\right)\\
&\quad=\frac{(3n-1)n-2m_1}{2+t}.
\end{aligned}\tag{9}$$

(vi) If $x=\omega$ and $y\in V(G)$, then by Lemmas 1 and 2

$$\begin{aligned}
&\sum_{\{\omega,v_i\}\subseteq V(\overline{\mu}(G))}\frac{d_{\overline{\mu}(G)}(\omega)+d_{\overline{\mu}(G)}(v_i)}{d_{\overline{\mu}(G)}(\omega,v_i)+t}\\
&\quad=\sum_{i=1}^{n}\frac{d_{\overline{\mu}(G)}(\omega)+d_{\overline{\mu}(G)}(v_i)}{d_{\overline{\mu}(G)}(\omega,v_i)+t}\\
&\quad=\frac{1}{1+t}\sum_{i=1}^{n}\left(n+2n-2d_G(v_i)\right)=\frac{3n^2-4m_1}{1+t}.
\end{aligned}\tag{10}$$

Summarizing the total contributions of above cases of adjacent and nonadjacent pairs of vertices in $\mu(G)$, we can obtain the desired result. This completes the proof. □

Using $t = 0$ in Theorem 4, we obtain the reciprocal degree distance of the complement of the Mycielskian graph.

Corollary 5 (corresponding to Corollary 3.5 in [20]). *Let G be a graph on n vertices and m_1 edges with diameter 2. Then $RDD(\overline{\mu}(G)) = 8n^3 - (3/2)n^2 + (1/2)n - 18nm_1 + 2m_1 + (5/2)M_1(G)$.*

3. Complement of the Generalized Mycielskian Graph

In this section, we obtain the reformulated reciprocal degree distance and reciprocal degree distance of the graph $\overline{\mu}_m(G)$.

The following lemmas follow from the structure of the complement of the generalized Mycielskian graph.

Lemma 6. *Let G be a connected graph. Then the distances between the vertices of the generalized Mycielskian graph $\overline{\mu}_m(G)$ of G are given as follows. For each $u, v \in V(\overline{\mu}_m(G))$*

$$\begin{aligned}
&(1)\ d_{\overline{\mu}_m(G)}(u,v) = \begin{cases} 2, & if\ u = u_i^0,\ v = u_j^0,\ d_{\mu_m(G)}\left(u_i^0, u_j^0\right) = 1,\ i,j = 1,2,\ldots,n; \\ 1, & if\ u = u_i^0,\ v = u_j^0,\ d_{\mu_m(G)}\left(u_i^0, u_j^0\right) > 1,\ i,j = 1,2,\ldots,n \end{cases} \\
&(2)\ d_{\overline{\mu}_m(G)}(u,v) = 1, \quad u = u_i^l,\ v = u_j^l,\ l = 1,2,\ldots,m;\ i,j = 1,2,\ldots,n; \\
&(3)\ d_{\overline{\mu}_m(G)}(u,v) = 2, \quad u = u_i^m,\ v = \omega,\ i = 1,2,\ldots,n; \\
&(4)\ d_{\overline{\mu}_m(G)}(u,v) = 1, \quad u = u_i^l,\ v = \omega,\ l = 0,1,2,\ldots,m-1;\ i = 1,2,\ldots,n; \\
&(5)\ d_{\overline{\mu}_m(G)}(u,v) = 1, \quad u = u_i^l,\ v = u_i^h,\ l,h = 0,1,2,\ldots,m;\ i = 1,2,\ldots,n;\ l \neq h; \\
&(6)\ d_{\overline{\mu}_m(G)}(u,v) = \begin{cases} 2, & if\ u = u_i^l,\ v = u_j^h,\ d_{\mu_m(G)}\left(u_i^l, u_j^h\right) = 1,\ l \neq h,\ i \neq j,\ l,h = 0,1,2,\ldots,m;\ i,j = 1,2,\ldots,n; \\ 1, & if\ u = u_i^l,\ v = u_j^h,\ d_{\mu_m(G)}\left(u_i^l, u_j^h\right) > 1,\ l \neq h,\ i \neq j,\ l,h = 0,1,2,\ldots,m;\ i,j = 1,2,\ldots,n. \end{cases}
\end{aligned} \tag{11}$$

Lemma 7. *Let G be a graph with n vertices. Then the degree of any vertex $x \in \overline{\mu}_m(G)$ is as follows:*

$$d_{\overline{\mu}_m(G)}(x) = \begin{cases} nm, & if\ x = \omega; \\ n(m+1) - 2d_G\left(u_i^0\right), & if\ x = u_i^l,\ l = 0,1\ldots,m-1;\ i = 1,2\ldots,n; \\ n(m+1) - d_G\left(u_i^0\right) - 1, & if\ x = u_i^m,\ i = 1,2\ldots,n. \end{cases} \tag{12}$$

Theorem 8. *Let G be a graph on n vertices and m_1 edges with diameter 2. Then $\overline{R_t}(\overline{\mu}_m(G)) = (1/(1+t))(n^3m^3 + 3n^3m + 3n^3m^2 + n^2m^2 + n^3 + n - (m^2 - m - 5)M_1(G) - 2n^2m - 2m_1nm^2 - 18m_1nm + 6mm_1 - 6m_1m^2 + 4m_1 - 2n^2 - 4m_1n) + (1/(2+t))(6mm_1n + 4m_1nm^2 + 2m_1n + 2n^2m - 4m_1 + n^2 - n - (m^2 - m + 5)M_1(G))$.*

Proof. From the structure of the complement of Mycielskian graph, we consider the structure of the complement of generalized Mycielskian and then also consider the following cases of adjacent and nonadjacent pairs of vertices in $\overline{\mu}_m(G)$ to compute $\overline{R_t}(\overline{\mu}_m(G))$.

Case 1. If $\{u,v\} \subseteq V(\overline{\mu}_m(G)), u = u_i^0, v = u_j^0 \in V^0$, then

$$\frac{1}{2} \sum_{u=u_i^0, v=u_j^0 \in V^0} \frac{d_{\overline{\mu}_m(G)}\left(u_i^0\right) + d_{\overline{\mu}_m(G)}\left(u_j^0\right)}{d_{\overline{\mu}_m(G)}\left(u_i^0, u_j^0\right) + t} \tag{13}$$

and by Lemmas 6 and 7

$$\begin{aligned}
&= \frac{1}{2} \sum_{\substack{u=u_i^0, v=u_j^0 \in V^0 \\ u_i^0 u_j^0 \notin E(\mu_m(G))}} \frac{n(m+1) - 2d_G\left(u_i^0\right) + n(m+1) - 2d_G\left(u_j^0\right)}{1+t} \\
&\quad + \frac{1}{2} \\
&\quad \cdot \sum_{\substack{u=u_i^0, v=u_j^0 \in V^0 \\ u_i^0 u_j^0 \in E(\mu_m(G))}} \frac{n(m+1) - 2d_G\left(u_i^0\right) + n(m+1) - 2d_G\left(u_j^0\right)}{2+t} \\
&= \frac{1}{1+t} \sum_{\substack{u=u_i^0, v=u_j^0 \in V^0 \\ u_i^0 u_j^0 \notin E(\mu_m(G))}} \left(n(m+1) - \left(d_G\left(u_i^0\right) + d_G\left(u_j^0\right)\right)\right) \\
&\quad + \frac{1}{2+t} \sum_{\substack{u=u_i^0, v=u_j^0 \in V^0 \\ u_i^0 u_j^0 \in E(\mu_m(G))}} \left(n(m+1) - \left(d_G\left(u_i^0\right) + d_G\left(u_j^0\right)\right)\right)
\end{aligned}$$

$$= \frac{1}{1+t}\left\{\sum_{\substack{u=u_i^0,v=u_j^0\in V^0\\ u_i^0u_j^0\notin E(\mu_m(G))}}(n(m+1)) - \sum_{\substack{u=u_i^0,v=u_j^0\in V^0\\ u_i^0u_j^0\notin E(\mu_m(G))}}\left(\left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right)\right\}$$

$$+\frac{1}{2+t}\left\{\sum_{\substack{u=u_i^0,v=u_j^0\in V^0\\ u_i^0u_j^0\in E(\mu_m(G))}}(n(m+1)) - \sum_{\substack{u=u_i^0,v=u_j^0\in V^0\\ u_i^0u_j^0\in E(\mu_m(G))}}\left(\left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right)\right\}$$

$$= \frac{2}{1+t}\left\{\left(\frac{n(n-1)}{2}-m_1\right)(m+1)n - \left(2m_1(n-1)-M_1(G)\right)\right\} + \frac{2}{2+t}\left\{m_1n(m+1) - M_1(G)\right\}. \tag{14}$$

Case 2. If $\{u,v\}\subseteq V(\overline{\mu}_m(G))$, $u=u_i^l, v=u_j^l\in V^l, l=1,2,\ldots,m$, then

$$\frac{1}{2}\sum_{\substack{u=u_i^l,v=u_j^l\in V^l;\\ l=1,2,\ldots,m}}\frac{d_{\overline{\mu}_m(G)}\left(u_i^l\right)+d_{\overline{\mu}_m(G)}\left(u_j^l\right)}{d_{\overline{\mu}_m(G)}\left(u_i^l+u_j^l\right)+t} \tag{15}$$

and by Lemmas 6 and 7

$$= \frac{1}{2}\sum_{\substack{u=u_i^l,v=u_j^l\in V^l;\\ l=1,2,\ldots,m-1}}\frac{n(m+1)-2d_G\left(u_i^0\right)+n(m+1)-2d_G\left(u_j^0\right)}{1+t} + \frac{1}{2}\sum_{u=u_i^m,v=u_j^m\in V^m}\frac{n(m+1)-d_G\left(u_i^0\right)-1+n(m+1)-d_G\left(u_j^0\right)-1}{1+t}$$

$$= \frac{1}{2(1+t)}\sum_{\substack{u=u_i^l,v=u_j^l\in V^l;\\ l=1,2,\ldots,m-1}}\left(2n(m+1)-2\left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right) + \frac{1}{2(1+t)}\sum_{u=u_i^m,v=u_j^m\in V^m}\left(2n(m+1)-2 - \left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right) \tag{16}$$

$$= \frac{1}{1+t}\left\{\sum_{\substack{u=u_i^l,v=u_j^l\in V^l;\\ l=1,2,\ldots,m-1}}(n(m+1)) - \sum_{\substack{u=u_i^l,v=u_j^l\in V^l;\\ l=1,2,\ldots,m-1}}\left(\left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right)\right\}$$

$$+\frac{1}{1+t}\left\{\sum_{u=u_i^m,v=u_j^m\in V^m}(n(m+1)-1) - \frac{1}{2}\sum_{u=u_i^m,v=u_j^m\in V^m}\left(\left(d_G\left(u_i^0\right)+d_G\left(u_j^0\right)\right)\right)\right\}$$

$$= \frac{2}{1+t}\left\{\frac{n(n-1)}{2}(m-1)n(m+1)-2m_1(n-1)(m-1)\right\} + \frac{2}{1+t}\left\{\frac{n(n-1)}{2}(n(m+1)-1)-\frac{1}{2}\cdot 2m_1(n-1)\right\}.$$

Case 3. If $\{u,v\}\subseteq V(\overline{\mu}_m(G))$, $u=u_i^m\in V^m, v=\omega, i=1,2,\ldots,n$, then

$$\frac{1}{2}\sum_{\substack{u=u_i^m\in V^m,\\ v=\omega,i=1,2,\ldots,n}}\frac{d_{\overline{\mu}_m(G)}\left(u_i^m\right)+d_{\overline{\mu}_m(G)}(\omega)}{d_{\overline{\mu}_m(G)}\left(u_i^m,\omega\right)+t} \tag{17}$$

and by Lemmas 6 and 7

$$= \frac{1}{2}\sum_{\substack{u=u_i^m\in V^m,\\ v=\omega,i=1,2,\ldots,n}}\frac{n(m+1)-1-d_G\left(u_i^0\right)+nm}{2+t}$$

$$= \frac{1}{2(2+t)}\left\{\sum_{\substack{u=u_i^m\in V^m,\\ v=\omega,i=1,2,\ldots,n}}(nm+n(m+1)-1)\right.$$

$$
\begin{aligned}
&- \sum_{\substack{u=u_i^m\in V^m,\\ v=\omega, i=1,2,\ldots,n}} d_G\left(u_i^0\right)\Bigg\} \\
&= \frac{2}{2(2+t)}\left\{n(2nm+n-1)-2m_1\right\} \\
&= \frac{1}{2+t}\left\{n(2nm+n-1)-2m_1\right\}
\end{aligned} \tag{18}
$$

Case 4. If $\{u,v\} \subseteq V(\overline{\mu}_m(G)), u = u_i^l \in V^l, v = \omega, l = 0,1,\ldots,m-1; i = 1,2,\ldots,n$, then

$$
\frac{1}{2} \sum_{\substack{u=u_i^l\in V^l, v=\omega,\\ l=0,1,\ldots,m-1; i=1,2,\ldots,n}} \frac{d_{\overline{\mu}_m(G)}\left(u_i^l\right)+d_{\overline{\mu}_m(G)}(\omega)}{d_{\overline{\mu}_m(G)}\left(u_i^l,\omega\right)+t} \tag{19}
$$

and by Lemmas 6 and 7

$$
\begin{aligned}
&= \frac{1}{2} \sum_{\substack{u=u_i^l\in V^l, v=\omega,\\ l=0,1,\ldots,m-1; i=1,2,\ldots,n}} \frac{n(m+1)-2(d_G\left(u_i^0\right)+nm}{1+t} \\
&= \frac{1}{2(1+t)} \sum_{\substack{u=u_i^l\in V^l, v=\omega,\\ l=0,1,\ldots,m-1; i=1,2,\ldots,n}} (2nm+n \\
&\quad -2\left(d_G\left(u_i^0\right)\right) \\
&= \frac{1}{2(1+t)}\Bigg\{\sum_{\substack{u=u_i^l\in V^l, v=\omega,\\ l=0,1,\ldots,m-1; i=1,2,\ldots,n}} (2nm+n) \\
&\quad -2\sum_{\substack{u=u_i^l\in V^l, v=\omega,\\ l=0,1,\ldots,m-1; i=1,2,\ldots,n}} d_G\left(u_i^0\right)\Bigg\} \\
&= \frac{2}{2(1+t)}\left\{nm(2nm+n)-2\cdot 2m_1 m\right\} \\
&= \frac{1}{1+t}\left\{nm(2nm+n)-2\cdot 2m_1 m\right\}.
\end{aligned} \tag{20}
$$

Case 5. If $\{u,v\} \subseteq V(\overline{\mu}_m(G)), u = u_i^l, v = u_i^h, l \neq h; l,h = 0,1,\ldots,m; i = 1,2\ldots,n$, then

$$
\frac{1}{2} \sum_{\substack{u=u_i^l, v=u_i^h;\\ l\neq h; l,h=0,1,\ldots,m; i=1,2\ldots,n}} \frac{d_{\overline{\mu}_m(G)}\left(u_i^l\right)+d_{\overline{\mu}_m(G)}\left(u_i^h\right)}{d_{\overline{\mu}_m(G)}\left(u_i^l,u_i^h\right)+t} \tag{21}
$$

and by Lemmas 6 and 7

$$
\begin{aligned}
&= \frac{1}{2} \sum_{\substack{u=u_i^l, v=u_i^h; l\neq h;\\ l,h=0,1,\ldots,m-1; i=1,2\ldots,n}} \frac{n(m+1)-2d_G\left(u_i^0\right)+n(m+1)-2d_G\left(u_i^0\right)}{1+t} \\
&\quad + \frac{1}{2} \sum_{\substack{u=u_i^l, v=u_i^m;\\ l=0,1,\ldots,m-1; i=1,2\ldots,n}} \frac{n(m+1)-2d_G\left(u_i^0\right)+n(m+1)-d_G\left(u_i^0\right)-1}{1+t} \\
&= \frac{1}{2(1+t)} \sum_{\substack{u=u_i^l, v=u_i^h; l\neq h;\\ l,h=0,1,\ldots,m-1; i=1,2\ldots,n}} \left(2n(m+1)-4\left(d_G\left(u_i^0\right)\right)\right) + \frac{1}{2(1+t)} \sum_{\substack{u=u_i^l, v=u_i^m;\\ l=0,1,\ldots,m-1; i=1,2\ldots,n}} \left(2n(m+1)-1-3d_G\left(u_i^0\right)\right) \\
&= \frac{1}{2(1+t)}\Bigg\{\sum_{\substack{u=u_i^l, v=u_i^h; l\neq h;\\ l,h=0,1,\ldots,m-1; i=1,2\ldots,n}} 2n(m+1)-4\sum_{\substack{u=u_i^l, v=u_i^h; l\neq h;\\ l,h=0,1,\ldots,m-1; i=1,2\ldots,n}} d_G\left(u_i^0\right)\Bigg\} \\
&\quad + \frac{1}{2(1+t)}\Bigg\{\sum_{\substack{u=u_i^l, v=u_i^m;\\ l=0,1,\ldots,m-1; i=1,2\ldots,n}} (2n(m+1)-1))-3\sum_{\substack{u=u_i^l, v=u_i^m;\\ l=0,1,\ldots,m-1; i=1,2\ldots,n}} d_G\left(u_i^0\right)\Bigg\} \\
&= \frac{2}{2(1+t)}\left\{\frac{m(m-1)}{2}n\cdot 2n(m+1)-4\frac{m(m-1)}{2}\cdot 2m_1\right\} + \frac{2}{2(1+t)}\left\{nm(2n(m+1)-1)-3\cdot 2m_1 m\right\} \\
&= \frac{1}{1+t}\left\{\frac{m(m-1)}{2}n\cdot 2n(m+1)-4\frac{m(m-1)}{2}\cdot 2m_1\right\} + \frac{1}{1+t}\left\{nm(2n(m+1)-1)-3\cdot 2m_1 m\right\}.
\end{aligned} \tag{22}
$$

Case 6. If $\{u,v\} \subseteq V(\overline{\mu}_m(G)), u = u_i^l, v = u_j^h, l \neq h; i \neq j; l,h = 0,1,\ldots,m; i = 1,2,\ldots,n$, then and by Lemmas 6 and 7

$$\frac{1}{2} \sum_{\substack{u=u_i^l, v=u_j^h, l\neq h; i\neq j; \\ l,h=0,1,\ldots,m-1; i,j=1,2,\ldots,n}} \frac{d_{\overline{\mu}_m(G)}\left(u_i^l\right) + d_{\overline{\mu}_m(G)}\left(u_j^h\right)}{d_{\overline{\mu}_m(G)}\left(u_i^l, u_j^h\right) + t} \tag{23}$$

$$\begin{aligned}
&= \frac{1}{2} \sum_{\substack{u=u_i^l, v=u_j^h, l\neq h; i\neq j; \\ l,h=0,1,\ldots,m-1; i,j=1,2,\ldots,n}} \frac{n(m+1) - 2d_G\left(u_i^0\right) + n(m+1) - 2d_G\left(u_j^0\right)}{d_{\overline{\mu}_m(G)}\left(u_i^l, u_j^h\right) + t} + \frac{1}{2} \\
&\cdot \sum_{\substack{u=u_i^l, v=u_j^m; i\neq j; \\ l=0,1,\ldots,m-1; i,j=1,2,\ldots,n}} \frac{n(m+1) - 2d_G\left(u_i^0\right) + n(m+1) - d_G\left(u_j^0\right) - 1}{d_{\overline{\mu}_m(G)}\left(u_i^l, u_j^m\right) + t} = \frac{1}{2} \\
&\cdot \sum_{\substack{u=u_i^l, v=u_j^h, l\neq h; i\neq j; \\ l,h=0,1,\ldots,m-1; \\ i,j=1,2,\ldots,n; u_i^l u_j^h \notin E(\mu_m(G))}} \frac{2n(m+1) - 2\left(d_G\left(u_i^0\right) + d_G\left(u_j^0\right)\right)}{1+t} + \frac{1}{2} \\
&\cdot \sum_{\substack{u=u_i^l, v=u_j^h, l\neq h; i\neq j; \\ l,h=0,1,\ldots,m-1; \\ i,j=1,2,\ldots,n; u_i^l u_j^h \in E(\mu_m(G))}} \frac{2n(m+1) - 2\left(d_G\left(u_i^0\right) + d_G\left(u_j^0\right)\right)}{2+t} + \frac{1}{2} \\
&\cdot \sum_{\substack{u=u_i^l, v=u_j^m; i\neq j; \\ l=0,1,\ldots,m-1; \\ i,j=1,2,\ldots,n; u_i^l u_j^m \notin E(\mu_m(G))}} \frac{2n(m+1) - 1 - 2d_G\left(u_i^0\right) - d_G\left(u_j^0\right)}{1+t} + \frac{1}{2} \\
&\cdot \sum_{\substack{u=u_i^l, v=u_j^m; i\neq j; \\ l=0,1,\ldots,m-1; \\ i,j=1,2,\ldots,n; u_i^l u_j^m \in E(\mu_m(G))}} \frac{2n(m+1) - 1 - 2d_G\left(u_i^0\right) - d_G\left(u_j^0\right)}{2+t}
\end{aligned} \tag{24}$$

$$\begin{aligned}
&= \frac{2}{2(1+t)} \left\{\left(n(n-1)\frac{m(m-1)}{2} - (m-1)2m_1\right)2n(m+1) - 2\cdot\frac{m(m-1)}{2}\left(2m_1(n-1) - M_1(G)\right)\right\} \\
&+ \frac{2}{2(2+t)} \left\{(m-1)\cdot 2m_1 \cdot 2n(m+1) - 2\cdot\frac{m(m-1)}{2}\cdot M_1(G)\right\} \\
&+ \frac{2}{2(1+t)} \left\{\left(n(n-1) - 2m_1 + (m-1)n(n-1)\right)\left(2n(m+1) - 1\right) - 2\left(m(n-1)2m_1 - M_1(G)\right)\right. \\
&\left. - \left(m(n-1)2m_1 - M_1(G)\right)\right\} + \frac{2}{2(2+t)}\left\{2m_1\left(2n(m+1) - 1\right) - 2M_1(G) - M_1(G)\right\}; \\
&= \frac{1}{1+t} \left\{\left(n(n-1)\frac{m(m-1)}{2} - (m-1)2m_1\right)2n(m+1) - m(m-1)\left(2m_1(n-1) - M_1(G)\right)\right\} + \frac{1}{2+t}\left\{(m-1)\right. \\
&\left.\cdot 2m_1 \cdot 2n(m+1) - m(m-1)\cdot M_1(G)\right\} + \frac{1}{1+t}\left\{\left(n(n-1) - 2m_1 + (m-1)n(n-1)\right)\left(2n(m+1) - 1\right)\right. \\
&\left. - 2\left(m(n-1)2m_1 - M_1(G)\right) - \left(m(n-1)2m_1 - M_1(G)\right)\right\} + \frac{1}{2+t}\left\{2m_1\left(2n(m+1) - 1\right) - 2M_1(G) - M_1(G)\right\}.
\end{aligned}$$

Summarizing the total contributions of above cases of adjacent and nonadjacent pairs of vertices in $\overline{\mu}_m(G)$, we can obtain the desired result; this completes the proof. □

Using $t = 0$ in Theorem 8, we obtain the reciprocal degree distance of the complement of the generalized Mycielskian graph.

Corollary 9. *Let G be a graph on vertices and m edges with diameter 2. Then*

$$\begin{aligned} RDD\left(\overline{\mu}_m(G)\right) &= n^3m^3 + 3n^3m + 3n^3m^2 + n^2m + n^3 - 15mm_1n \\ &\quad - 3m_1n - n^2m \\ &\quad - \frac{3n^2 - n + \left(3m^2 - 3m - 5\right)M_1(G)}{2} + 6mm_1 \\ &\quad - 6m_1m^2 + 2m_1 \end{aligned} \tag{25}$$

Data Availability

In this paper, we correct the second part results in the paper [K. Pattabiraman, M. Vijayaragavan, Reformulated Reciprocal Degree Distance of Transformation Graph, Electronic Notes in Discrete Mathematics 53 (2016) 259-270] and generalize the corresponding results to the generalized Mycielskian graph.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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