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Research Article

An Outranking Multicriteria Method for Nominal Classification Problems with Minimum Performance Profiles

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In recent years, nominal classification problems have gained importance, especially in the context of strategic management of organizations. In this sense, this paper presents a novel multicriteria nominal classification method, derived from the concepts of PROMETHEE, applied to use in problems characterized by minimum performance profiles (MMP) for the classes. The main advantages of this proposal are criterion and alternative flexibility for classes; robustness, because it uses the concepts of a well-known method (PROMETHEE); and usefulness, because many real situations are characterized by MMP for the classes. Moreover, a real-world example is presented: a retailer's assignment in a bank, showing the applicability of the method. The proposal of a new multicriteria nominal classification method emerges from a need to devise a more flexible and realistic procedure for characterizing classes because the feature of criterion and alternative flexibility for classes has not been addressed in any extant multicriteria nominal classification procedure. The present proposal thereby endeavors to address this deficit in the multicriteria field.

1. Introduction

Multicriteria decision problems represent situations in which the decision maker (DM) confronts at least two alternatives, and the decision aims to achieve multiple objectives that are, most of the time, conflicting [1]. In these problems, the DM may pose the problem by choosing, ranking, classifying, or describing the alternatives. These modes of framing are referred to as problematics [2].

Classification allows a DM to assign alternatives to predefined classes, a process known as supervised assignment, or to nonpredefined classes, which is known as unsupervised classification and typically referred to as clustering. In both cases, according to [3], this problematic has compelling implications in numerous areas related to practical or scientific issues, such as the following fields: inventory classification [4–6]; supplier classification [7]; risk analysis in pipelines [8]; and cooperation classification [9]. For more detailed information on clustering approaches that permit allocation of alternatives into nonpredefined classes, see [10–15].

In the context of a supervised assignment, the predefined classes can either be ordered or not ordered. Sorting applies to cases involving ordered classes, and classification applies to problems involving nonordered classes, also known as nominal classification problems [3]. According to [17], in sorting problems, classes are either represented by the lower and upper bounds of a limiting profile (as in the case of ELECTRE TRI) or by a central profile as in [18].

The method proposed herein enables the allocation of alternatives into predefined classes. The proposal of a new multicriteria nominal classification method based on MPP (minimum performance profiles) emerges from a need to devise a more flexible and realistic procedure for characterizing classes, using concepts already associated with multicriteria methods. It is, however, worth noting that the method is easily adapted to apply to other types of problems with classes characterized by maximum performance profiles, central profiles, or alternatives representing the typical element of a class and pursuant to a proximity index. As such, this proposal's main advantages are as follows: criterion and alternative flexibility for classes; robustness, conceptualized in terms of a well-known method (i.e., PROMETHEE); and usefulness.

The paper is structured as follows. The next section—Theoretical Contributions—highlights the importance of methods devoted to nominal classification problems by outlining several potential applications and presents the gap of the literature which motivated the development of the method proposed herein. The section Materials and Methods comprises two subsections: the first subsection is devoted to describe nominal classification problems and the aim of the proposed method; the second subsection—Proposed Method: Features and Definitions—first presents this proposed method in detail, along with a summary of PROMETHEE concepts, assumptions, and notations, and goes on to describe the conditions and features used in the proposed nominal classification method. The section Application presents an illustrative example and a comparison among several nominal classification methods, and it is followed by a robustness analysis of the proposed method. The following section provides a discussion of the results. Finally, the section Conclusions presents some conclusions and final remarks.

2. Theoretical Contributions

Although [19] assert that, in recent years, nominal classification problems have grown more important, mainly in the context of managing of organizations and institutions, the same authors also acknowledge that this has not yielded a correspondingly vast literature on multiple criteria nominal classification. Indeed, due to its competitiveness, modern society is on a constant quest for patterns or homogeneity aimed at more effective implementation of its policies and strategies.

Five potential applications of multicriteria nominal classification problems are described in [19]. One such application is the problem of identifying or determining the most accurate disease class(es) for a given patient, based on his/her symptoms. Thus, patients assigned to the same class (es) of disease may be subject to identical medical procedure(s). Alternatively, the process of recruiting soldiers could also be handled as a nominal classification problem, as each candidate is assessed according to multiple individual features (i.e., physical fitness, intelligence, motivation, teamwork skills, and mental faculties) and subsequently assigned to one of several special core skill task units, where they will undertake special training courses. Another potential application relies on the fact that alerting people to information about public health events and risks, via social media, should be pursued differently, according to the specific type of user targeted. Whenever possible, users are characterized in terms of various features, such as age, health condition, frequency of travel, and degree of dependence on social networks. Users can then be assigned to one of several social groups, like "younger," "middle-aged," or "elderly." The fourth potential application concerns the problem of assigning responsibilities to risk owners (i.e., a person or entity responsible for managing an assigned risk). This is normally performed in risk management. Finally, the fifth potential application involves the task of determining the type of instrument(s) for issuing environmental policy best suited to manage each environmental issue in a way that achieves desired outcomes strategically, effectively, and efficiently. This is especially important, because policies play a key role in addressing complex environmental and health problems, and consequently, in improving the state of the environment.

In the multicriteria field, many approaches have been proposed to address the sorting problematic. Evidently, the ELECTRE TRI method [20, 21] is "the most popular", according to [16], and "the most used", according to [3], method of ordinal classification and based on limiting or boundary profiles. Adaptations of this method are exemplified by many works, including ELECTRE TRI-C, based on characteristic or central reference profile [22]; ELECTRE TRI-NC, where each class is defined by several central reference actions [23]; and ELECTRE-SORT [24], where classes are defined by central limiting profiles that can also be incomparable. It is possible to cite additional methods, along with the ELECTRE TRI, that deal with ordinal classification: PROMSORT [25]; AHP-Sort [26]; THESEUS [27]; TRICHOM [28]; N-TOMIC [29]; FlowSort [17]; a pairwise comparison-based method [18]; ORCLASS [30]; and a hybrid method based on AHP method, a veto system, and the K-means algorithm [31]. However, there are substantially fewer methods developed to address nominal classification problems than methods, proposed in the last few decades, intended to aid DMs in choosing, ranking, and even sorting problems.

Most current methods designed to handle nominal classifications problems are procedures based on reference actions, also called central profiles. Indeed, [32] argued that such problems usually require determination of whether an alternative a is close or similar to alternative b, or to an alternative representing a typical element of a class—also known as a prototype, and [33] explained preferences for criteria in terms of weights reflecting the importance of the criteria, relative to all classes. As such, the latter mode does not rely on a reference profile, as the weights define the classes. Moreover, [34] treated a problem defining nonordered classes by the least typical representative of each, referred to as the entrance threshold, and [35] defined each class by a given number of features, conditions, or constraints. Problems characterized by MPPs have drawn the attention of multiple researchers. For instance, [36] employed a nominal classification method aimed at enabling a construction company to select managers for different roles (i.e., the classes), according to different competencies and MPPs for classes; [37] applied the NeXClass nominal classification method to the project of assigning military students to one of multiple classes, characterized by MPPs consistent with predefined criteria; and [34, 38, 39] presented a real-world application of a classification method, using MPPs, to a problem in a banking environment.

Indeed, according to a literature review on classification methods, the feature of criterion and alternative flexibility for classes has not been addressed in any extant multicriteria nominal classification procedure, except in the method proposed by [33]. Although the proposal of [33] evaluates the alternatives, according to some criteria, the classification method relies on a binary linear programming approach, akin to a portfolio problem maximizing a valued objective

function. The present proposal thereby endeavors to address this deficit in the multicriteria field. The feature of criterion and alternative flexibility for classes will be fully discussed and described in the second subsection of the next section.

3. Materials and Methods

3.1. Nominal Classification Problems. Classification (nominal and ordinal) problems aim to assign alternatives to predefined classes, according to some evaluation criteria, thus, giving the following: a set of n alternatives $A = (a_1, a_2, \ldots, a_i)$, where $i = 1, \ldots, n$; a set of m criteria $G = (g_1, g_2, \ldots, g_j)$, where $j = 1, \ldots, m$; a set of c classes $C = (C_1, C_2, \ldots C_k)$, where $k = 1, \ldots, c$; and c sets of MPPs, for each class k and for all m criteria, $B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$-; the aim is to assign each alternative in a specific class by evaluating the alternative pursuant to the criteria relevant to the different classes and also according to the MPPs defined for each class. Mathematically, each class C_k is represented by a B_k of MPPs. Further, each class is defined by a unique MPP. These concepts are represented, schematically, in Figure 1.

According to Figure 1, an alternative a_i must meet the minimum performance profile b_{kj} for each criterion g_j, defined for a specific class k, to be able to belong to this class. The problem presented in Figure 1 is a sorting problem, as, when comparing two classes C_k and C_{k-1} for all criteria, the MPP required by C_k is always greater than that required by C_{k-1}.

The main feature distinguishing nominal classification from sorting problems is that, in the first, classes are nonordered regarding the criteria. Figure 2 illustrates this idea for a nominal classification problem characterized by MPPs.

As is observable in Figure 2, the MPP required by some criteria in some classes does not follow an order. To wit, the classes are nonordered. For example, the MPP required for one alternative, to be assigned to class C_k, is greater than the MPP required for the same alternative to be assigned to class C_{k-1} for criterion g_1. However, the MPP requirement in the case of criterion g_2 is greater for the C_{k-1} class than it is for the C_k class.

Regarding the methods applicable to multicriteria nominal classification problems, researchers have proposed some modes of assigning alternatives to classes, including the following: [41] proposed the fuzzy nominal classification method PROAFTN; [33] presented a multicriteria decision method with an additive linear function, based on SMART and with linear constraints; [32] developed a method based on the concepts of concordance and discordance; and [19] proposed a nominal classification method based on the concepts of similarity and dissimilarity. There are certainly more nominal classification proposals, such as those from the following researchers: [42], with TRINONFC; [43], with CLOSORT; [34], with NeXClass; [35], with a method based on selectability/rejectability measures; and [44].

As can be seen, there are numerous potential applications of multicriteria nominal classification problems. This is a clear motivation driving the development of the proposal presented in this paper. The problem stated here consists of assigning an alternative to a specific class, considering a set of alternatives, a set of predefined nonordered classes, and a set of evaluation criteria. Also, for each predefined class, the DM defines a MPP for each evaluation criterion, which represents the minimum requirements for the inclusion of an alternative in this class. In that way, the method proposed here differs from the methodological contributions described previously. Our proposal aims to assign each alternative to the most suitable class, or rather, the alternative that outranks the reference profile with a greater magnitude, thereby ensuring coherent classification is coherent.

The next subsection details the aspects of the proposed method, after presenting the general features of the outranking multicriteria approach—more specifically the PROMETHEE, in which the proposal is based.

3.2. Proposed Method: Features and Definitions. Following an outranking multicriteria approach, where two alternatives a_1 and $a_2 \in A$ are compared, the result must be expressed as a preference. Therefore, a preference function F / F: A × A $\longrightarrow$ (0, 1), representing the intensity of preference of alternative a_1 regarding alternative a_2, must be recognized, such that [2, 45–48].

(i) $F(a_1, a_2) = 0$ means indifference between a_1 and a_2, or no preference of a_1 over a_2;

(ii) $F(a_1, a_2) \sim 0$ means weak preference of a_1 over a_2;

(iii) $F(a_1, a_2) \sim 1$ means strong preference of a_1 over a_2;

(iv) $F(a_1, a_2) = 1$ means strict preference of a_1 over a_2.

It is worth stating that the symbol ~ stands for "close to" in the multicriteria literature [45–48].

Among methods for outranking multicriteria, the PROMETHEE, proposed by [48], is particularly simple and suitable method for achieving accuracy, where multiple evaluation criteria are involved [49]. The PROMETHEE methods use six types of preference functions associated with each criterion, as detailed by [48]. These were based on previous methods, such as ELECTRE III (linear criterion), or on preference modeling structures (usual, U-shape, and level criterion). In most practical applications, the six preference types provide the DM with a sufficient level of flexibility [40]. The six types of criteria and their respective descriptions are provided in Table 1.

As presented in Table 1, most types of preference functions used in PROMETHEE have a double threshold: p and q. Reference [50] has noted the importance of defining the structure of criteria in classification methods, by a double threshold (i.e., preference and indifference thresholds). According to this author, a double-threshold structure prevents improper classification. To wit, the absence of preference and indifference thresholds can lead to improper judgements between strict preference and indifference among alternatives and profiles of classes. In fact, several multicriteria classification methods, such as ELECTRE TRI or NeXClass, rely on the double-threshold structure.

Further, another justification for the double-threshold structure is that it facilitates avoidance of weak outranking

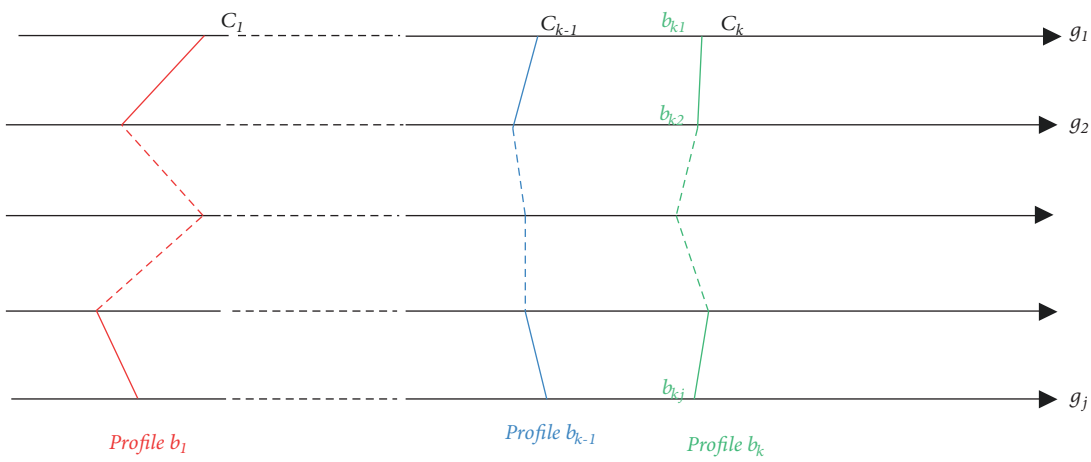

Figure 1: Profiles in ordinal classification problem [16].

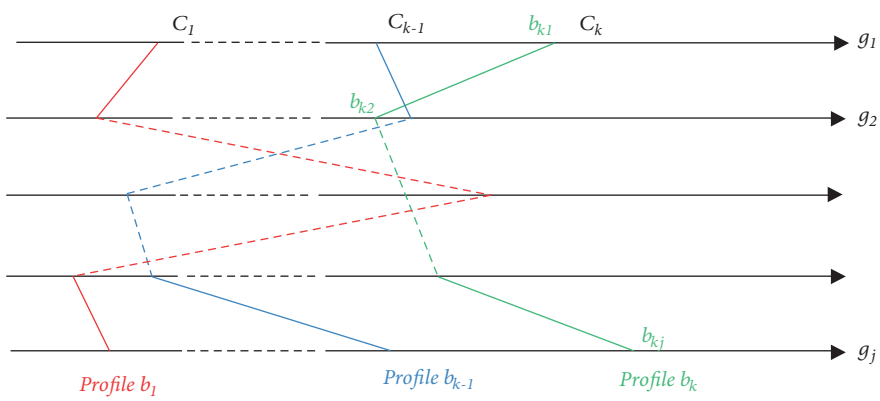

Figure 2: Profiles in a nominal classification problem characterized by MPPs.

relations between alternatives and profiles of classes that produce improper assignments to classes. Moreover, given that there must be imprecise and uncertain information about the MPPs, setting indifference and preference thresholds is recommended. Finally, in a case where the DM is absolutely sure about the values for the MPPs, the preference and indifference thresholds can equal zero. Therefore, our approach is flexible in the sense that it can use or not use a double-threshold structure.

For each pair of alternatives a_1 and $a_2 \epsilon A$, one first defines a preference index Π for a_1 regarding a_2 over all the m criteria. Suppose every criterion j $(j = 1, 2, \ldots m)$ has been identified as one of the six types considered (Table 1), so the preference functions F_j (a_1, a_2) have been defined for each j. The multicriteria preference index Π for a_1 with regard to a_2 over all the m criteria in the PROMETHEE method is therefore defined as the weighted average of preference functions F_j:

$$\Pi(a_1, a_2) = \frac{\sum_{j=1}^{m} \pi_j F_j(a_1, a_2)}{\sum \pi_j} \tag{1}$$

F_j (a_1, a_2) represents the preference function F of alternative a_1 regarding a_2 over the criterion j.

π_j represents the weight of criterion j.

Π (a_1, a_2) represents the intensity of preference of the DM of alternative a_1 over alternative a_2, given all the criteria simultaneously. It is a value between 0 and 1:

(i) $\Pi(a_1, a_2) \sim 0$ denotes a weak preference of a_1 over a_2 for all the criteria,

(ii) $\Pi(a_1, a_2) \sim 1$ denotes a strong preference of a_1 over a_2 for all the criteria.

In classification problems, nominal or ordinal, the outranking relationships are then generated by comparing alternatives to profiles. This comparison, in the approach proposed in this paper, is made through two indices that validate the claim $a_i S B_k$. These indices are defined in the following set of terms.

Definition 1 (intensity of membership). For any alternative a_i from A and any MPP B_k representing class C_k, Π (a_i, B_k) represents the intensity of the membership of a_i in B_k; to wit, the amount of evaluation criteria supports this membership.

$$\Pi(a_i, B_k) = \frac{\sum_{j=1}^{m} w_{kj} F_{kj}(a_i, B_k)}{\sum w_{kj}} \tag{2}$$

$F_{kj}(a_i, B_k)$ represents the preference function F of alternative a_i regarding the profile B_k over the criterion j for the class C_k.

Table 1: The six types of preference functions used in PROMETHEE [40].

Criterion	Type	Parameters	Description
Usual	I	None	It is used for qualitative criteria with few evaluation levels (up to 5-point-scale)
Quasi-criterion (U-shape)	II	q parameter (indifference threshold)	It is a special case of level one
Preference threshold (V-shape)	III	p parameter (preference threshold)	It is a special case of the linear criterion when there is no indifference threshold (q)
Pseudo-criterion (Level)	IV	p and q (preference and indifference thresholds)	It is used for qualitative criteria when one needs to differentiate smaller deviations from large ones
Indifference area (Linear)	V	p and q (preference and indifference thresholds)	It is used for quantitative criteria expressed on a continuous scale
Gaussian	VI	Standard deviation	It is more difficult to structure because its threshold value is somewhere between the q indifference threshold and the p preference threshold

Definition 2 (intensity of nonmembership). Having a_i from A and any MPP B_k representing class C_k, $\Pi(B_k, a_i)$ represents the amount of evaluation criteria opposed to the membership of a_i into B_k.

$$\Pi\left(B_k, a_i\right) = \frac{\sum_{j=1}^{m} w_{kj} F_{kj}\left(B_k, a_i\right)}{\sum w_{kj}} \tag{3}$$

$F_{kj}(B_k, a_i)$ represents the preference function F of the profile B_k regarding an alternative a_i over the criterion j for the class C_k.

The sets of parameters for application of the present nominal classification proposal are as follows:

$W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$: the set of all criteria weights for class k;

$B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$: the set of each MPP for each criterion j in class k;

$P_k = (p_{k1}, p_{k2}, \ldots, p_{kj})$: the set of each preference threshold for each criterion j in class k.

$Q_k = (q_{k1}, q_{k2}, \ldots, q_{kj})$: the set of each indifference threshold for each criterion j in class k.

Given that not all criteria are necessarily considered across all classes, and even when they are, they may vary in their preference functions, weights, or thresholds, depending on their relevance to and influence on each class, and the sets F_k, W_k, Q_k, and P_k may differ for each class.

Based on these two indices (intensity of membership and intensity of nonmembership), the assignment of an alternative a_i to a class C_k is determined by the intensity of the assignment $\Pi\ (a_i, C_k)$ described in the following.

Definition 3 (intensity of the assignment). For any alternative a_i from A and any MPP B_k, representing class C_k, $\Pi(a_i, C_k) = \Pi(a_i, B_k) - \Pi(B_k, a_i)$ represents the intensity of the assignment of a_i to C_k. Thus, in the proposed method, the objective is to max $\Pi(a_i, C_k)$.

The framework presented in Figure 3 summarizes the proposal through steps divided into three phases (Problem Definition, Evaluation, and Assignment).

The Problem Definition phase comprises the definition as follows:

(i) The set of n alternatives: $A = (a_1, a_2, \ldots, a_i)$, where $i = 1, \ldots, n$, that must be assigned to some nonordered classes;

(ii) The set of c nonordered classes: $C = (C_1, C_2, \ldots, C_k)$, where $k = 1, \ldots, c$;

(iii) The set of m criteria: $G = (g_1, g_2, \ldots, g_j)$, where $j = 1, \ldots, m$, which comprises all the criteria used to define the classes and specifies which alternatives will be evaluated.

For each class k, the following sets are also defined:

(i) MPPs: $B_k = (b_{k1}, b_{k2}, \ldots, b_{kj})$;

(ii) Criteria weights: $W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$;

(iii) Preference thresholds: $P_k = (p_{k1}, p_{k2}, \ldots, p_{kj})$;

(iv) The indifference thresholds: $Q_k = (q_{k1}, q_{k2}, \ldots, q_{kj})$.

In the Evaluation phase, each single alternative a_i is compared with each MPP B_k, and $\Pi\ (a_i,\ B_k)$ is calculated using (2). Further, each MPP B_k is compared with each single alternative a_i, and $\Pi\ (B_k, a_i)$ is calculated, using (3). Then, $\Pi\ (a_i, C_k)$ is defined for all alternatives as it bears on all classes.

The Assignment phase is performed via the allocation of each alternative a_i to a specific class C_k as a way of maximizing $\Pi\ (a_i, C_k)$.

In the proposed method, it is possible to apply different criteria subsets to different classes, given the possibility that some criteria may be applicable to characterizing some classes but unnecessary for other classes. It is worth stating that this is a specific characteristic of nominal classification problems and thus does not apply to sorting problems in which classes are ordered and are characterized by the same criteria. Therefore, a unique set of criteria, including all criteria considered for at least one class, is generated. Thus, the set of criteria weights of a class k, for example, is represented by the set $W_k = (w_{k1}, w_{k2}, \ldots, w_{kj})$. The value of a given criterion weight represents its relevance to each class. So, when a criterion g_2, for example, is neither relevant to nor

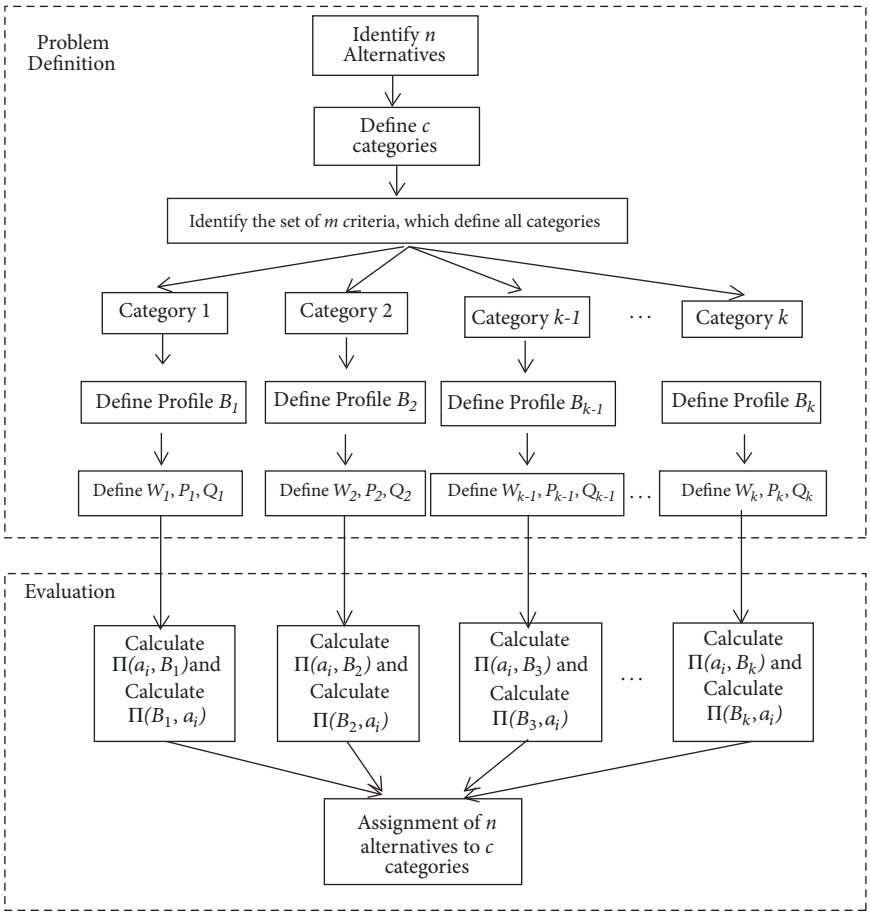

Figure 3: Outranking method for nominal classification.

even considered by a specific class k, w_{k2} assumes a null value. Indeed, other researchers have pointed out the property of criteria flexibility for classes. For instance, [41] claims that criteria weights should be defined in terms of the following two conditions: criterion g_j is not pertinent to the assignment of alternative a_i to class C_k and criterion g_j is the only criterion pertinent to the assignment of alternative a_i to class C_k.

In fact, there are several classification problems where some criteria characterize more than one class and some criteria are specific to one class. In medical diagnosis, for example, patients are assessed on the basis of different symptoms (e.g., fever, pain, headache, and cough) characterizing a very heterogeneous group of diseases (classes). According to the medical evaluation of the patient (alternative), given these various symptoms (criteria), the appropriate treatment is prescribed, to maximize the chances of success [19, 41].

In addition to the criteria flexibility for classes, given that it may not be appropriate to apply the same criteria set to different classes, another important feature in nominal classification problems, exemplified by the proposed method, is the alternative flexibility for classes, which means that some alternatives may be assigned to more than one class, and others may not be assigned to any class. As in the case of medical diagnoses, the patient could have symptoms that characterize different diseases and require different treatments. However, for the disease that represents the worst condition afflicting patient, the correspondent treatment takes priority. In this way, the minimum profile approach is used to identify the class (C_k) to which an alternative a_i gives the maximum contribution (the worse condition in the medical example), using the expression $\max \Pi(a_i, C_k)$, by means of assessing the alternative a_i for the criteria that characterize the class C_k.

Therefore, it is important to present formal and explicit definitions of criteria and alternative flexibility for classes.

Definition 4 (criteria flexibility for classes). For each class C_k, the criterion weight (w_{kj}) can assume the following values:

(a) 0, when the criterion g_j is not pertinent for the assignment of an alternative a_i to class C_k.

(b) $0 < g_j < 1$, when the criterion g_j is not the only pertinent criterion for the assignment of an alternative a_i to class C_k.

(c) 1, when the criterion g_j is the only pertinent criterion for the assignment of an alternative a_i to class C_k.

Definition 5 (alternative flexibility for classes). An alternative $a_i \in A$, will be as follows:

(a) assigned to only one class C_k, if $\Pi(a_i, C_k) \geq 0$ and this is the maximum value when comparing with other $\Pi(a_i, C_l)$, where $l \neq k, l = 1, \ldots, c$.

(b) assigned to $C_k \in S \subset C$, if $\Pi(a_i, C_k) \geq 0$ and it is the maximum value when comparing with other $\Pi(a_i, C_l)$ where $l \neq k, l = 1, \ldots, c$ and if $\Pi(a_i, C_t) = \Pi(a_i, C_k)$, $\forall C_t \in S, t = 1, \ldots, s$.

(c) not assigned to any class C_k, if $\Pi(a_i, C_k) < 0 \ \forall \ k$, k=1...c.

Despite the importance of flexibility, in relation to both criteria and alternatives, there are only a few works described in the literature, such as [33], which approach this flexibility in the context of proposing models for nominal classification. This flexibility is a particularly strong characteristic of the method proposed in the present work.

Other important properties, regarding nominal classification methods, are proposed by Costa et al. (2018) and regard the operations of merging, splitting, adding, and removing.

Definition 6 (merging operation). If two different classes, C_l and C_s, characterized by MPPs B_l and B_s, respectively, are merged to become a new one, C_t, characterized by the MPP B_t, then $B_t = \{B_{t1} = \min\{B_{l1,}, B_{s1}\}, B_{t2} = \min\{B_{l2,}, B_{s2}\}, \ldots, B_{tj} = \min\{B_{lj,}, B_{sj}\}\}$. As a result, all the alternatives previously assigned to classes C_l and C_s will be assigned to this new class C_t.

Definition 7 (splitting operation). If one class C_t, characterized by the MPP B_t, is separated into two different classes, C_l and C_s, characterized by two new MPPs B_l and B_s, respectively, then one of new classes is characterized by the MPP B_{tj}, that is, B_{lj} or $B_{sj} = B_{tj}$, for all criteria j. Consequently, all the alternatives previously assigned class C_t will be assigned to the new classes C_l and C_s.

Definition 8 (adding classes operation). If one class C_t is included in the problem, this operation leads to build a new MPP as well as the set of criteria weights for this class. Such a new class may receive alternatives previously assigned to other classes and alternatives which were previously not assigned to any class.

Definition 9 (removing classes operation). If one class C_t is removed from the problem, alternatives previously assigned to this class may be assigned to one, more than one, or none of the remaining classes.

The next section furnishes a better understanding of the proposed method through the application of the method to a real-world problem.

4. Application

To illustrate the proposal, this paper presents a real-world application that uses real data presented by [34], concerning the problem of assigning retailers to use bank services. The real-world problem involved a Greek bank aiming to reorganize its electronic payment network of retailers equipped with terminals for online payments. To improve service efficiency, the bank wants to assign retailers to four predefined nonordered classes that represent the potential and profitability characteristics, according to specific criteria. The bank uses a two-dimensional evaluation framework, which comprises the retailer's site potential and profitability dimensions to classify retailers.

Following the framework proposed in Figure 2, for this example, in the Problem Definition phase, the following were defined:

(i) 20 alternatives: 20 retailers, $A = (a_1, a_2, \ldots, a_{20})$ and

(ii) 4 nonordered classes, $C = C_1, C_2, \ldots, C_4$, described in Table 2.

The four classification classes (Table 2), defined on the basis of this segmentation, depict the importance of the retailer to the bank.

Further, the classes are also linked to a marketing strategy that the bank will follow as a result of the classification.

(i) 13 criteria, $G = (g_1, g_2, \ldots, g_{13})$, grouped into financial and nonfinancial dimensions, as shown in Table 3;

(ii) 4 profiles, $B_k = (b_{k1}, b_{k2}, \ldots, b_{k13})$; and

(iii) the set of criterion weights $W_k = (w_{k1}, w_{k2}, \ldots, w_{k13})$, which, in this problem, is the same for all classes.

Data regarding the evaluations of alternatives, criterion weights, and MPP of the classes are shown in Table 4.

Figure 4 illustrates the idea of MPPs, considering the minimum performance profiles (b_1 and b_2) required for 2 classes (C_1 and C_2) for 5 of the 13 criteria.

As can be seen in Figure 3, the MPPs for this application do not set boundaries between classes, as expected in nominal classification methods. It worth noting, further, that the profile of class 2 (blue line) is below the profile of class 1 (red line) for criteria g_1, g_2, and g_3, but the profile of class 2 is above the profile of class 1 for criteria g_4 and g_5.

(i) 4 sets of preference thresholds, $p_k = (p_{k1}, p_{k2}, \ldots, p_{k13})$ and

(ii) 4 sets of indifference thresholds, $q_k = (q_{k1}, q_{k2}, \ldots, q_{k13})$.

The data regarding preference and indifference thresholds for each class C_k according to each criterion g_j are shown in Table 5 and in Table 6. The values determined for this problem are exactly the same for all criteria and classes.

The results of the Evaluation phase, where $\Pi\ (a_i, b_k)$, $\Pi\ (b_k, a_i)$, and $\Pi\ (a_i, C_k)$ are calculated, can be seen in Appendix. Finally, the Assignment phase is performed through the allocation of each alternative a_i to a specific class C_k as a way to maximize $\Pi\ (a_i, C_k)$. Table 7 summarizes results of the comparison of this nominal classification proposal with three methods: the NeXClass by [37], the method presented by [35], and the one proposed by [33]—adapted for this example. The methods used in these three papers, as proposed in this paper, aim to help the DM address a nonordinal classification problem. Details about them were presented in the initial sections.

Table 2: Classes for retailer classification [34].

Class specification	C_1	C_2	C_3	C_4
Definition	Retailers with relative low potential and medium to high profitability.	Retailers with relative high potential and medium to high profitability.	Retailers with minimum to high potential and medium to low profitability.	Retailers with medium to low potential and low profitability.
Strategy	Bank will allocate substantial resources to strengthen retailer's potential.	Bank will allocate maximum resources to provide high added value innovative services.	Bank will minimize resource allocation and focus to top retailers of the class.	Bank will screen retailers for potential development, allocating a minimum level of resources.

Table 3: Criteria for the evaluation of retailers [37].

Criterion	Definition	Scale
g_1	Retailer size (average daily sales in 1000 Euros)	1-100
g_2	Intensity of EFT/PoS (percentage of daily sales through EFT/PoS	1-100
g_3	Average value per EFT/PoS transaction (in Euros)	1-100
g_4	Average cost per EFT/PoS terminal (in Euros)	1-100
g_5	EFT/PoS terminal profitability (average monthly revenue per terminal [in Euros]/average monthly cost per terminal [in Euros])	1-100
g_6	Average growth rate (indicator showing monthly increase in transaction ratio)	1-100
g_7	Merchant class (based on bank's merchant type definition, according to merchant activity)	1-100
g_8	Collaboration efficiency (index based on merchants calls to bank support center)	1-100
g_9	Exclusivity (index based on retailer's exclusive collaboration; normally a retailer has installed at the same place EFT/PoS terminals from several competing banks)	1-100
g_{10}	Location (Index based on retailer's distance factors from areas with high traffic)	1-100
g_{11}	Opening hours (index based on retailer's opening hours)	1-100
g_{12}	Training of employees (index expressing employees' expertise on EFT/PoS)	1-100
g_{13}	Alternative channels (index expressing usage degree of bank's alternative payment channels from retailer)	1-100

∗EFT/PoS: Electronic Fund Transfer at Point Sale.
Source: [37].

As can be seen, NeXClass [37] differs in three classifications, [35] in one classification, [33] in one classification, and this proposal in one classification, relative to the current procedure. It is important to note that [33, 35] did not apply thresholds to the problem; however, the structure with a double threshold (preference and indifference thresholds) used in this paper prevents improper classification, as stated before. Although our results are the same as those seen [35] and differ only in one classification from the results of [33], it is extremely important to analyze the results with different data.

5. A Scenario Analysis

Therefore, this paper addresses the robustness of the results obtained by the nominal classification method proposed herein, using this first illustrative example. According to [51], robustness is a key issue in the field of decision-aiding, as well as in operations research. As a result, numerous researchers have recently addressed this issue [51–62] and have proposed the use of performance measures for classification and clustering methods [63, 64]. The term robustness refers to a capacity for withstanding "vague approximations" and/or "zones of ignorance" to maintain certain properties [51].

In general, the values assigned to the parameters in multicriteria methods are not perfectly defined. Indeed, according to [57], a critical challenge faced by analysts utilizing a multicriteria decision aid (MCDA) framework is the elicitation of the criteria weights. In the proposed method, the aim is to provide recommendations concerning the classification of retailers that remain acceptable for a wide range of values of the parameters. Thus, robustness with respect to different scenarios was assessed by changing some preference parameters, such as criteria weights, profiles of classes, and preference and indifference thresholds. As a result, a total of 138 scenarios were tested: the combination of changing the values of 13 criteria weights, four profiles of classes according to each criterion, and preference and indifference thresholds in ±10%, following similar procedures to those presented in [19, 56].

The results of the analysis of 134 scenarios are shown in Table 8. As can be seen, results are unchanged for 19 out of 20 alternatives. The only alternative showing different classifications, according to different values, for the parameters is

Table 4: Evaluations of alternatives, criterion weights, and MPP of the classes [37].

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
a_1	29	22	28	25	69	25	61	52	25	39	58	61	68
a_2	80	78	88	69	59	30	50	45	48	42	22	15	27
a_3	77	90	88	61	63	28	35	33	51	33	22	28	33
a_4	16	39	26	25	55	25	50	51	43	65	37	38	73
a_5	28	56	51	21	34	8	37	61	30	37	55	66	98
a_6	79	75	80	65	60	25	30	34	22	19	22	18	21
a_7	50	625	54	25	38	21	47	41	40	57	65	65	88
a_8	44	19	31	55	49	29	80	70	73	55	48	29	45
a_9	49	43	28	29	61	22	67	42	25	39	51	62	55
a_{10}	30	25	30	51	55	44	82	84	90	74	32	15	32
a_{11}	30	29	32	87	86	80	77	46	28	49	25	29	33
a_{12}	49	17	54	25	37	21	47	39	42	54	65	55	98
a_{13}	42	14	27	51	43	22	74	67	69	53	40	25	92
a_{14}	25	19	26	90	81	79	70	44	32	45	28	24	30
a_{15}	42	14	27	51	56	46	81	78	82	52	40	25	33
a_{16}	80	77	79	69	65	22	31	37	28	22	19	21	29
a_{17}	21	15	22	86	79	83	68	40	30	41	20	19	25
a_{18}	18	12	25	82	81	79	64	38	29	39	19	15	27
a_{19}	22	18	26	49	51	41	80	80	86	69	24	11	26
a_{20}	41	35	44	29	34	21	47	61	50	57	62	61	98
W_k	10	12	4	13	13	8	10	4	4	8	4	8	2
b_1	75	70	75	60	55	20	25	35	20	15	15	10	20
b_2	15	10	20	75	70	75	60	30	25	35	15	10	20
b_3	15	10	20	45	45	40	75	70	75	60	15	10	20
b_4	55	10	20	15	10	20	35	30	40	70	75	60	55

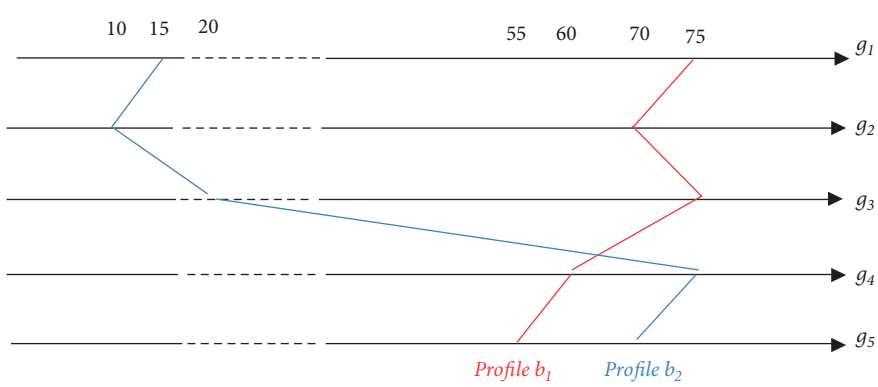

Figure 4: Minimum performance profiles (application of Rigopoulos et al., 2010a).

Table 5: Preference thresholds for each class, according to each criterion [34].

p_{kj}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
C_1	5	5	5	5	5	5	5	5	5	5	5	5	20
C_2	5	5	5	5	5	5	5	5	5	5	5	5	20
C_3	5	5	5	5	5	5	5	5	5	5	5	5	20
C_4	5	5	5	5	5	5	5	5	5	5	5	5	20

Table 6: Indifference thresholds for each class, according to each criterion [34].

q_{kj}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}
C_1	2	2	2	2	2	2	2	2	2	2	2	2	10
C_2	2	2	2	2	2	2	2	2	2	2	2	2	10
C_3	2	2	2	2	2	2	2	2	2	2	2	2	10
C_4	2	2	2	2	2	2	2	2	2	2	2	2	10

Table 7: Final classification comparison for the five procedures.

	NeXClass [37]	Method presented by [35]	Method presented by [33]	Nominal Classification with MPP – the method proposed	Existing Procedure (Benchmarking)
a_1	C_4	C_4	C_3	C_4	C_3
a_2	C_1	C_1	C_1	C_1	C_1
a_3	C_1	C_1	C_1	C_1	C_1
a_4	C_3	C_4	C_3	C_4	C_4
a_5	C_4	C_4	C_4	C_4	C_4
a_6	C_1	C_1	C_1	C_1	C_1
a_7	C_4	C_4	C_4	C_4	C_4
a_8	C_3	C_3	C_3	C_3	C_3
a_9	C_4	C_4	C_4	C_4	C_4
a_{10}	C_3	C_3	C_3	C_3	C_3
a_{11}	C_2	C_2	C_2	C_2	C_2
a_{12}	C_4	C_4	C_4	C_4	C_4
a_{13}	C_4	C_3	C_3	C_3	C_3
a_{14}	C_2	C_2	C_2	C_2	C_2
a_{15}	C_3	C_3	C_3	C_3	C_3
a_{16}	C_1	C_1	C_1	C_1	C_1
a_{17}	C_2	C_2	C_2	C_2	C_2
a_{18}	C_2	C_2	C_2	C_2	C_2
a_{19}	C_3	C_3	C_3	C_3	C_3
a_{20}	C_4	C_4	C_4	C_4	C_4

a_{13}. Depending on these values, a_{13} can be assigned mainly to classes C_2 (86.23%) and C_4 (86.96%). In addition, in 6.52% of scenarios, a_{13} is assigned to class C_3, which is the correct class, according to the existing procedure based on heuristics. It is worth noting that the method proposed in this paper assigns a_{13} to the same class—C_3 (using the initial values for parameters). Moreover, alternative a_{13} is assigned to more than one class in some scenarios analyzed (alternative flexibility for classes feature). In general, these results show that the proposed method leads to robust classification, according to the changes in the preference parameters.

6. Discussion

The illustrative example presented in the previous section demonstrates the applicability of the proposal for nominal classification problems using MPPs and it can be seen that the results achieved by this proposed method are similar to those determined by other nominal classification methods, including the existing procedure performed by the bank, which can be used as the benchmark. This example had twenty alternatives, to be assigned to four classes—each characterized by one profile—regarding thirteen criteria, accounting for thirteen preference thresholds and thirteen indifference thresholds. It is worth noting that, for this example, the criterion flexibility for classes was not verified, as all criteria influenced the assignment to the four classes. Thus, the profiles of each class were evaluated according to the same thirteen criteria.

From the example, it is possible to illustrate the importance of intensity of the nonmembership $\Pi\ (b_k, a_i)$ index for a correct assignment. The alternative a_9, for example, could be assigned to the class C_2 if only the intensity of the membership index $\Pi\ (a_i, b_k)$ had been taken into account. However, a_9 was assigned to C_4 after considering the intensity of the nonmembership index $\Pi\ (b_k, a_i)$. It demonstrates that, despite the fact that a_9 has good evaluation on some criteria to ensure it belongs to the C_2 class, this alternative did not meet other important (weighted) criteria for C_2, caused, in a balanced way, a_9 to become more pertinent in class C_4; thus, it was allocated to this class. Still, it is important to highlight that the proposed method aims to maximize the

Table 8: Results of the analysis of scenarios.

Alternatives	C_1	C_2	C_3	C_4	The proposed method (initial values)	Existing Procedure (Benchmarking)
a_1				138 (100%)	C_4	C_3
a_2	138 (100%)				C_1	C_1
a_3	138 (100%)				C_1	C_1
a_4				138 (100%)	C_4	C_4
a_5				138 (100%)	C_4	C_4
a_6	138 (100%)				C_1	C_1
a_7				138 (100%)	C_4	C_4
a_8			138 (100%)		C_3	C_3
a_9				138 (100%)	C_4	C_4
a_{10}			138 (100%)		C_3	C_3
a_{11}		138 (100%)			C_2	C_2
a_{12}				138 (100%)	C_4	C_4
a_{13}		119 (86.23%)	9 (6.52%)	120 (86.96%)	C_3	C_3
a_{14}		138 (100%)			C_2	C_2
a_{15}			138 (100%)		C_3	C_3
a_{16}	138 (100%)				C_1	C_1
a_{17}		138 (100%)			C_2	C_2
a_{18}		138 (100%)			C_2	C_2
a_{19}			138 (100%)		C_3	C_3
a_{20}				138 (100%)	C_4	C_4

overall allocation, i.e., to assign the alternative a_i to the class C_k that leads to $\max \Pi(a_i, C_k)$.

Another important characteristic of the present proposal concerns alternative flexibility for classes, which refers to an alternative—according to the intensity of assignment parameter $\Pi\ (a_i, C_k)$—that can be in zero, one, or more classes. The first possibility could occur when the $\Pi\ (a_i, C_k)$ for each class is below a DM's given minimum, such that the alternative does not belong to any class of the problem. A real-life example might involve some candidates, under consideration for employment by a company, one or more of whom cannot be assigned to any job vacancy, given the lack of required skills. The second possibility is the most common: an alternative is assignable to one, and only one, class. The last possibility refers to a situation where an alternative could be assigned to more than one class, due to a difference between two or more of the biggest $\Pi\ (a_i, C_k)$ that is too small or possibly even zero. This was the case for some scenarios considered in the robustness analysis, and it would be the case, in the context of the aforementioned real-life example previously presented, where one or some of the candidates have the skills required for more than one job.

To deal with the alternative flexibility, this work proposes assignment thresholds to be discussed and determined by the DM. These thresholds would be in accordance with a minimum-intensity assignment parameter and indifference between more than one of the biggest intensities of assignment parameters.

One can observe that the proposed method requires the definition of several parameters (criteria weights, preference and indifference thresholds, and MPP) which is a common requirement of most multicriteria methods. For this reason, in the last decades, there has been an increase in research dedicated to elicitation of parameters because the elicitation process is one of the most complex and critical tasks facing research and applications within the field of decision analysis [59]. Indeed, this is especially critical because such parameters can change the position of any alternative in a class [9]. Reference [16], for example, proposed a methodology for the ELECTRE TRI that encompasses this problem, by substituting assignment examples by direct elicitation of the parameters of the model. The values of the parameters are inferred via a certain form of regression on assignment examples, which can be extended to apply to our method.

Another important point is that more than one DM may participate in the nominal classification process and consequently a potential conflict can emerge regarding the numerical values of parameters. An interesting discussion regarding group decision process is provided by [60–62]. Finally, it is worth noting that it is possible to incorporate those methodologies related to the elicitation of parameters for group decision in our method.

7. Conclusions

As it can be seen, the type of classification problems which aims to assign alternatives in different classes according to particular characteristics is getting much attention from researchers and practitioners. The method proposed herein has three unique features, namely: flexibility, criterion and

Table 9

	$\Pi(a_i, b_k)$	$\Pi(b_k, a_i)$	$\Pi(a_{ni}, C_k)$
a_1 x b_1	0.8	0.2	0.6
a_1 x b_2	0.7	0	0.7
a_1 x b_3	0.99	0	0.99
a_1 x b_4	0.87	0	0.87
a_2 x b_1	0.8	0.2	0.6
a_2 x b_2	0.6	0.4	0.2
a_2 x b_3	0.31	0.68	-0.37
a_2 x b_4	0.45	0.39	0.06
a_3 x b_1	0.4	0.6	-0.2
a_3 x b_2	0.9	0.1	0.8
a_3 x b_3	0.62	0.37	0.25
a_3 x b_4	0.84	0.16	0.68
a_4 x b_1	0.923	0	0.923
a_4 x b_2	1	0.93	0.07
a_4 x b_3	0.62	0	0.62
a_4 x b_4	0.768	0	0.768
a_5 x b_1	0.8	0.2	0.6
a_5 x b_2	0.6	0.4	0.2
a_5 x b_3	0.68	0.31	0.37
a_5 x b_4	0.68	0.32	0.36
a_6 x b_1	0.4	0.6	-0.2
a_6 x b_2	0.9	0.1	0.8
a_6 x b_3	0.68	0.31	0.37
a_6 x b_4	0.74	0.26	0.48
a_7 x b_1	0.8	0.2	0.6
a_7 x b_2	0.7	0.3	0.4
a_7 x b_3	0.99	0	0.99
a_7 x b_4	0.87	0.13	0.74
a_8x b_1	0.6	0.2	0.4
a_8 x b_2	0.7	0	0.7
a_8 x b_3	0.302	0	0.302
a_8 x b_4	0.87	0	0.87
a_9 x b_1	0.8	0.2	0.6
a_9 x b_2	0.7	0	0.7
a_9 x b_3	0.68	0.31	0.37
a_9 x b_4	0.61	0.26	0.35
a_{10} x b_1	0.2	0.8	-0.6
a_{10} x b_2	0.6	0.292	0.308
a_{10} x b_3	0.68	0.31	0.37
a_{10} x b_4	0.61	0.343	0.267

alternative flexibility for classes; robustness, because it uses concepts of a well-known method (PROMETHEE); and usefulness, many real problems are characterized by MPP for the classes; thus, this novel approach demonstrably addresses this problem.

Moreover, because our method deals with nominal classification problems using the concept of MPP, the alternatives are designed to classes according to the concept of maximizing the overall performance of the assignment taking into account particular characteristics (criteria) of the classes. For instance, suppose that one is analyzing the health condition (class) of a patient (alternative) according to several symptoms (criteria). Using the proposed method, the patient would be assigned to a class in which the treatment would be efficient for all possible diseases.

For future work, given the relative ease of the proposed method and its practical utility, this research may be extended, by using interval operations to deal with imprecise data. Further investigations may account for the study of the proposed assignment thresholds. Yet, some problems may require classifying alternatives by similarity, to allow for comparisons to the profiles of the classes related to a proximity index. An important item to remember is that this proposal is easily modified to address all problems with maximum performance profiles. Finally, another subject for future research is the development of a decision support system (DSS) with the proposed multicriteria method to make it available in a convenient way.

Appendix

Evaluation Phase for the Illustrative Example

See Table 9.

Data Availability

Previously reported real data were used to support this study and are available at [10.3923/jas.2008.443.452]. This prior study is cited at relevant places within the text as reference [34].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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