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Research Article

An Enhanced Reliability Index Method and Its Application in Reliability-Based Collaborative Design and Optimization

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When designing complex mechanical equipment, uncertainties should be considered to enhance the reliability of performance. The Reliability Index Method (RIM) is a powerful tool which has been widely utilized in engineering design under uncertainties. To reduce computational cost in RIM, first or second order Taylor approximation is introduced to convert nonlinear probability constraint to the equivalent linear constraint during optimization process. Generally, this approximation process is performed at Most Probable Point (MPP) to reduce the loss of reliability analysis accuracy. However, it is difficult for the original RIM to be utilized in the situation that MPP is collinear and RIM has the same direction with the gradient of performance function at MPP. To tackle the above challenges, an Enhanced RIM (ERIM) is proposed in this study. The Collaborative Optimization (CO) strategy is combined with ERIM. The formula of CO using ERIM is given to solve reliability-based multidisciplinary design and optimization problems. A design problem of the speed reducer is utilized in this study to show the effectiveness of the proposed method.

1. Introduction

The Reliability-Based Multidisciplinary Design Optimization (RBMDO) has obtained more and more attention for the high reliability and safety of complex engineering systems [1–10]. Generally, the RBMDO process involves a three-level optimization loop [11–16]. The inner loop deals with the interactions between coupled variables and the outer loop explores design space to obtain optimal design solutions. Between the inner and outer loop is uncertainty analysis loop. If RBMDO is performed directly, the heavy computational cost will affect the whole optimization process significantly [17–20]. To deal with the three-level optimization loop structure, many sophisticated optimization strategies have been developed [21–25]. According to the integration strategies of optimization and uncertainty analysis, these methods can be roughly categorized into single-loop, decouple-loop, and double-loop approaches [26–33].

The single-loop approaches are suitable for design problems with moderate nonlinear performance function. The Karush-Kuhn-Tucker conditions are adopted by single-loop approaches to replace the uncertainty analysis loop in an optimization process, while the decouple-loop approaches perform deterministic design optimization and uncertainty analysis sequentially. When new design solutions are obtained by deterministic optimization, uncertainty analysis will be conducted to find Most Probable Point (MPP). The acquisition of MPP is important for constructing shifting vectors. The shifting vectors are utilized to move limit state constraints into the safer feasible region. Compared with the single-loop and the decouple-loop approaches, the strategy of double-loop approaches is simple and robust. Many strategies have been introduced into double-loop approaches to reduce the computational cost. In general, these strategies include modifying the formulation of probability constraint [34–36] and enhancing efficiencies of optimization algorithms in reliability analysis [31, 37–39].

The Reliability Index Method (RIM) is an effective tool which can modify the formulation of probability constraint in RBMDO [40, 41]. In RIM, the first or second order Taylor

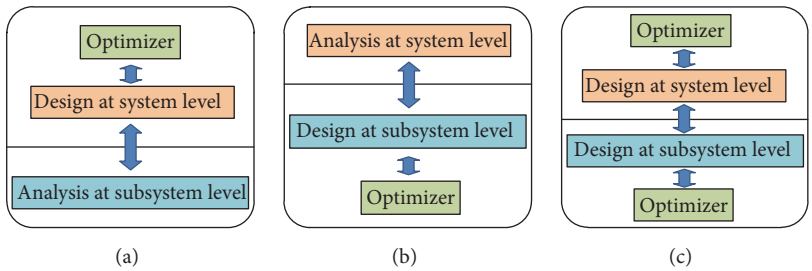

Figure 1: The single-level methods and multilevel methods for MDO.

approximation expansion is introduced to convert nonlinear probability constraint to the equivalent linear constraint. MPP is the expansion point, at which the accuracy loss of reliability analysis due to approximation can be minimized. However, in some practical optimization processes, MPP may be collinear and RIM has the same direction with the gradient of performance function at MPP. It will result in the low efficiency of the original RIM. To tackle the above challenges, an Enhanced RIM (ERIM) is proposed here. The formulation of CO using ERIM (CO-ERIM) is also proposed to solve RBMDO problems.

The rest of this study will be given as follows. In Section 2, the general formulation of RBMDO is given. Also, the uncertainty information in practical engineering is discussed and the strategy of RBMDO is briefly reviewed. The RIM and the Performance Measure Approach (PMA) are introduced in detail in Section 3. The proposed ERIM is also discussed in this part. The CO-ERIM is proposed in Section 4, including the formulation and procedure. An RBMDO problem of the speed reducer is given in Section 5 to show the efficiency and accuracy of CO-ERIM. The conclusions are given in Section 6.

2. Brief Review of RBMDO

When dealing with the design problems of complex engineering, there are two aspects of challenges which should be taken into consideration [43–46]. One is the complexity of multidisciplinary system analysis. The other is the information exchange of coupling disciplines involved. The multidisciplinary system analysis process is based on iterative calculations between coupled disciplines. This process requires the high computational cost. How to reduce the computational cost and improve the efficiency of system analysis is important for the application of Multidisciplinary Design and Optimization (MDO) in practical engineering. Furthermore, the interaction effect between coupling disciplines complicates the exchange of information in complex engineering systems. How to organize and manage the interaction information transmission between coupling disciplines effectively is another important problem which should be taken into consideration in practical engineering.

MDO is a methodology which can deal with the design problems of complex coupling engineering systems effectively [42, 47–50]. The main motivation of MDO is to drive the performance of an engineering system not only by each discipline but also by their coupling interactions.

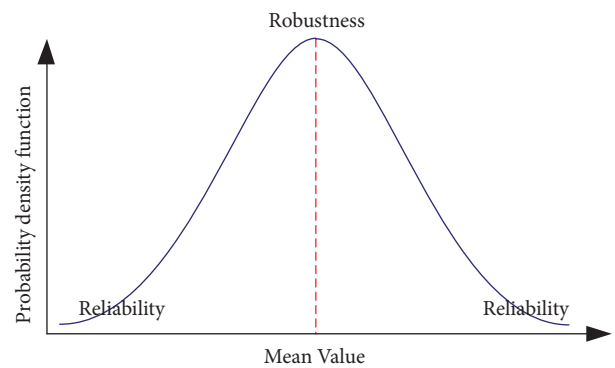

Figure 2: Two categories of uncertainty-based design [42].

By introducing MDO strategies into the early stage of engineering design, designers can enhance the performance of design solutions effectively. Also, the design cost can be reduced simultaneously. Considering these coupling interactions in an MDO problem requires sound mathematical strategies and their corresponding formulations. In general, the mathematical strategies of MDO can be classified into single-level methods and multilevel methods [51, 52]. As shown in Figures 1(a) and 1(b), single-level methods have a single optimizer. They utilize the nonhierarchical structure directly. Compared with single-level methods, multilevel methods introduce a hierarchical structure instead of a nonhierarchical structure, which is shown in Figure 1(c). There are optimizers at each level of the hierarchical structure. Because a specific MDO method cannot be suitable to all of the practical problems universally, appropriate MDO method should be chosen to satisfy the industrial requirements.

Furthermore, to achieve the high reliability and safety of complex industry systems, uncertainties in practical engineering should be considered. Uncertainties have different taxonomies in practical engineering. Correspondingly, there are two types of uncertainty-based design, reliability-based and robust-based, respectively [42]. The connection and difference between them are illustrated in Figure 2. The reliability-based design deals with extreme events which happen at tails of a probability density function, such as failure of performance, catastrophe and so on. The robust-based design mainly considers fluctuations of performance

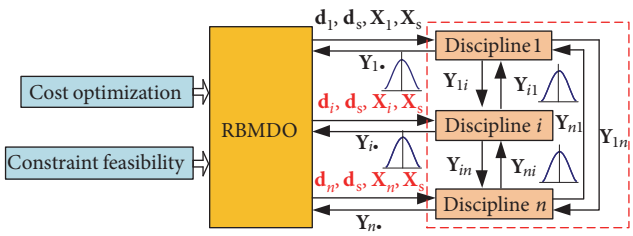

Figure 3: The design for a complex system based on RBMDO strategy.

around the mean value, such as degradation, deterioration, quality loss.

In this study, reliability-based design problems are primarily considered. The general formulation of RBMDO can be given as

$$\begin{aligned}
\min_{(\mathbf{d}_i,\mathbf{d}_s,\boldsymbol{\mu}_{\mathbf{X}_i},\boldsymbol{\mu}_{\mathbf{X}_s})} \quad & f\left(\mathbf{d}_i, \mathbf{d}_s, \boldsymbol{\mu}_{\mathbf{X}_i}, \boldsymbol{\mu}_{\mathbf{X}_s}\right) \\
\text{s.t.} \quad & \Pr_i\left[g_i\left(\mathbf{d}_i, \mathbf{d}_s, \mathbf{X}_i, \mathbf{X}_s, \mathbf{P}\right) \le 0\right] \le \Phi\left(-\beta_t\right) \\
& \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}} = \mathbf{Y}_{i\bullet}\left(\mathbf{d}_i, \mathbf{d}_s, \boldsymbol{\mu}_{\mathbf{X}_i}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right) \\
& \mathbf{d}_i^L \le \mathbf{d}_i \le \mathbf{d}_i^U, \\
& \mathbf{d}_s^L \le \mathbf{d}_s \le \mathbf{d}_s^U, \\
& \mathbf{X}_i^L \le \boldsymbol{\mu}_{\mathbf{X}_i} \le \mathbf{X}_i^U, \\
& \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s} \le \mathbf{X}_s^U, \\
& \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}} \le \mathbf{Y}^U, \\
& i = 1, 2, \ldots, n
\end{aligned} \tag{1}$$

where $f(\bullet)$ is a cost-type objective function; $g(\bullet) \le 0$ and $\Pr[\bullet] \le \Phi(-\beta_t)$ are the inequality constraint and its corresponding probability constraint, respectively; β_t is the safety reliability index; $\Phi(-\beta_t)$ is the allowable failure probability for $g(\bullet) \le 0$; $\Phi(\bullet)$ is the Cumulative Distribution Function (CDF) of the standard normal random variable; the subscript "s" denotes the sharing design information to all disciplines; the subscript "i" denotes the ith discipline in a complex system; $\mathbf{d}$ is the input design information which is not accompanied with with uncertainties; $\mathbf{X}$ and $\mathbf{P}$ are the input design information and parameters which are accompanied with uncertainties; $\boldsymbol{\mu}$ is the mean value of uncertainty information; $\mathbf{Y}$ is the coupling information; $\mathbf{Y}_{\bullet i}$ is the input coupling information for the ith discipline from other disciplines while $\mathbf{Y}_{i\bullet}$ is the output coupling information for the ith discipline to other disciplines; the superscripts "L" and "U" denote the lower and upper bounds of input design information, respectively; n is the number of disciplines.

As shown in Figure 3, uncertainties will be propagated among coupled disciplines in multidisciplinary systems. If the RBMDO problem in (1) is solved directly, a triple-loop strategy will be utilized, which is shown in Figure 4. The outer loop performs the optimization for objective function to obtain design point; the intermediate loop performs the reliability analysis on the design point; the inner loop performs the multidisciplinary analysis (MDA) between the subdisciplines.

Using the triple-loop strategy, the multidisciplinary system optimization problem requires reliability analysis in each iterative operation. Meanwhile, each reliability analysis operation involves MDA. Both of them result in a high computational burden. To solve this problem, ERIM and PMA are introduced in this study. The corresponding formulation of CO-ERIM is also proposed to improve the efficiency of RBMDO.

3. The Performance Measure Approach (PMA) Using RIM in Sequential Optimization and Reliability Assessment

Because of the existences of probability constraints, the PMA using RIM (PMA-RIM) strategy has been utilized widely in RBMDO to reduce computational cost. Researches on this strategy mainly include two aspects: the modifying formulation of probability constraint, and the enhanced efficiencies of reliability analysis and optimization algorithms [34].

In this study, the limitations of the original PMA-RIM are discussed. Then, a PMA based on Enhanced RI (PMA-ERIM) is proposed here, which is on the condition of accepting the approximate accuracy of the First Order Reliability Method (FORM).

3.1. The Strategy of RIM. In (1), the probability constraint can be reexpressed using Φ^{-1}:

$$\beta_{s_i} = -\Phi^{-1}\left(F_{g_i}(0)\right) \ge \beta_t \tag{2}$$

where β_{s_i} is the safety reliability index of the ith probability constraint. The input design information with uncertainties in (1) is treated as random variables in this study. Then, in RIM, the first order safety reliability index $\beta_{s,\mathrm{FORM}}$ can be obtained using FORM. This process mainly includes two steps.

First, all random variables x of the set of $\mathbf{X}$ ($x \in \mathbf{X}$) in the X-space are transformed into the standard normal distribution variable u in the U-space using the Rosenblatt transformation. $\mathbf{U}$ is a set of the standard normal distribution variable u, $u \in \mathbf{U}$. The standard normal variable can be denoted as

$$u = \Phi^{-1}\left(F_x(x)\right) \tag{3}$$

where $F_x(\bullet)$ is the CDF of a random variable x. Then the performance function $g_i(\mathbf{X})$ is transformed into the U-space as $g_i(\mathbf{U})$.

Second, an optimization problem formulated as follows is solved:

$$\begin{aligned}
\min \quad & \|\mathbf{U}\|_2 \\
\text{s.t.} \quad & g(\mathbf{U}) = 0
\end{aligned} \tag{4}$$

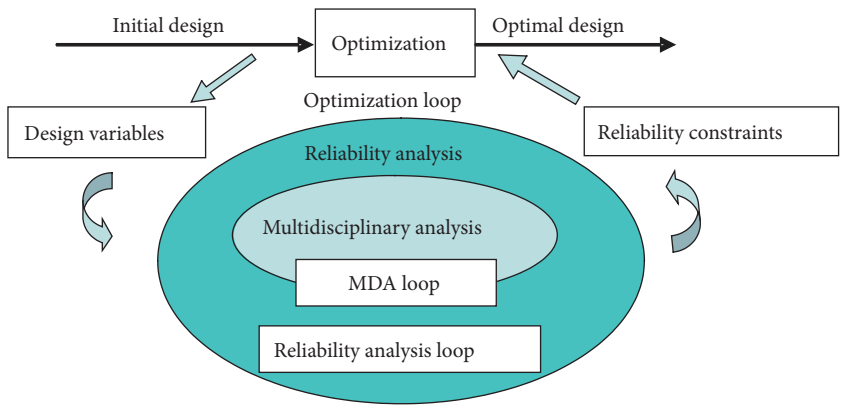

Figure 4: The triple-loop strategy of RBMDO [11].

where $\| \bullet \|_2$ is the magnitude of a vector. The optimum solution of (4) on the failure surface ($g(\mathbf{U}) = 0$) is called MPP $\mathbf{u}^*_{g(\mathbf{U})=0}$. Also, $\beta_{s,\mathrm{FORM}} = \|\mathbf{u}^*_{g(\mathbf{U})=0}\|_2$ [34, 35].

3.2. The Strategy of PMA-RIM. Here, the probability constraint in (1) is converted into its equivalent form by $F^{-1}_{g_i}$:

$$G_{p_i} = F^{-1}_{g_i}\left[\Phi\left(-\beta_t\right)\right] \geq 0 \tag{5}$$

where G_{p_i} is the probabilistic performance measure of the ith probability constraint.

If the value $g_p \geq 0$, then $\Pr(g(\mathbf{X}) \leq 0) \leq \Phi(-\beta_t)$; if $g_p < 0$, $\Pr(g(\mathbf{X}) \leq 0) > \Phi(-\beta_t)$. g_p corresponds to the $\Phi(-\beta_t)$ percentile of the CDF of performance function.

At first, all uncertainty inputs are converted into the standard normal random inputs in the U-space. The first order probabilistic performance measure $G_{p,\mathrm{FORM}}$ can be obtained by solving

$$\begin{aligned} &\min \quad G(\mathbf{U}) \\ &\text{s.t.} \quad \|\mathbf{U}\|_2 = \beta_t \end{aligned} \tag{6}$$

where the optimal point on the surface $\|\mathbf{U}\|_2 = \beta_t$ is identified as MPP $\mathbf{u}^*_{\beta=\beta_t}$. Furthermore, $G_{p,\mathrm{FORM}} = G(\mathbf{u}^*_{\beta=\beta_t})$.

3.3. The Strategy of PMA-ERIM. To improve the efficiency and accuracy of the original RIM, the ERIM is discussed in this section. Recall the statistic description of the failure of a performance function g:

$$\Pr\left(g(\mathbf{X}) \leq 0\right) = F_g(0) = \int_{g(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}. \tag{7}$$

Step 1. Using the Rosenblatt transformation, (7) is equivalent to

$$\begin{aligned} \Pr\left(g(\mathbf{U}) \leq 0\right) = &\int \cdots \int_{g(u_1,u_2,\cdots,u_n)\leq 0} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \\ &\cdot \exp\left(-\frac{1}{2}u_i^2\right) du_1 du_2 \cdots du_n. \end{aligned} \tag{8}$$

Step 2. To evaluate the integration more easily, the integrand boundary $g(\mathbf{U}) = 0$ is approximated. FORM utilizes the first order Taylor expansion as

$$g(\mathbf{U}) \approx L_g(\mathbf{U}) = g(\mathbf{u}^*) + \left[\nabla g(\mathbf{u}^*)\right]^T (\mathbf{U} - \mathbf{u}^*) \tag{9}$$

where $L_g(\mathbf{U})$ is the linearized performance function; $\mathbf{u}^*$ is the expansion point; T denotes transpose; $\nabla g(\mathbf{u}^*)$ is the gradient of g at $\mathbf{u}^*$:

$$\nabla g(\mathbf{u}^*) = \left\{\left(\frac{\partial g}{\partial u_1}, \frac{\partial g}{\partial u_2}, \ldots, \frac{\partial g}{\partial u_n}\right)\bigg|_{\mathbf{U}=\mathbf{u}^*}\right\}^T. \tag{10}$$

To minimize the accuracy loss, the performance function should be expanded at MPP. MPP can be obtained by (4).

Because at MPP $g(\mathbf{u}^*) = 0$, the performance function is linearized as

$$\begin{aligned} g(\mathbf{U}) \approx L_g(\mathbf{U}) &= \left[\nabla g(\mathbf{u}^*)\right]^T (\mathbf{U} - \mathbf{u}^*) \\ &= -\left[\nabla g(\mathbf{u}^*)\right]^T \mathbf{u}^* + \left[\nabla g(\mathbf{u}^*)\right]^T \mathbf{U}. \end{aligned} \tag{11}$$

If the gradient $\nabla g(\mathbf{u}^*)$ at $\mathbf{u}^*$ is equal to zero, then the performance function is linearized as $g(\mathbf{U}) \approx L_g(\mathbf{U}) = 0$. In this case, the linear approximation of the integrand boundary will cause a large error about the integration because of the highly nonlinear character of performance function. Therefore, if the above case appears, the FORM is not suitable to deal with the problem. In the following, the case of the gradient of performance function at MPP unequal to zero is discussed.

Since $L_g(\mathbf{U})$ is a linear function of standard normal variables, $L_g(\mathbf{U})$ is normally distributed. Thus, based on (11), the mean value and standard deviation of $L_g(\mathbf{U})$ are $\mu_{L_g} = -[\nabla g(\mathbf{u}^*)]^T \mathbf{u}^*$ and $\sigma_{L_g} = \|\nabla g(\mathbf{u}^*)\|_2$, which can be utilized in the derivation process in (12).

Therefore, the probability of failure is calculated as

$$\begin{aligned} \Pr\left(g(\mathbf{U}) \leq 0\right) \approx \Pr\left(L_g(\mathbf{U}) \leq 0\right) &= \Phi\left(\frac{0 - \mu_{L_g}}{\sigma_{L_g}}\right) \\ &= \Phi\left\{\left(\frac{\nabla g(\mathbf{u}^*)}{\|\nabla g(\mathbf{u}^*)\|_2}\right)^T \mathbf{u}^*\right\}. \end{aligned} \tag{12}$$

At the optimal point, $\mathbf{u}^* = \nabla g(\mathbf{u}^*)/\|\nabla g(\mathbf{u}^*)\|_2 \cdot \|\mathbf{u}^*\|_2$ or $\mathbf{u}^* = -\nabla g(\mathbf{u}^*)/\|\nabla g(\mathbf{u}^*)\|_2 \cdot \|\mathbf{u}^*\|_2$. Hence the probability of failure is

$$\Pr\left(g\left(\mathbf{U}\right) \le 0\right) \approx \begin{cases} \Phi\left(\left\|\mathbf{u}^*\right\|_2\right) & \text{if } \mathbf{u}^* = \dfrac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2} \cdot \left\|\mathbf{u}^*\right\|_2 \\ \Phi\left(-\left\|\mathbf{u}^*\right\|_2\right) & \text{if } \mathbf{u}^* = -\dfrac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2} \cdot \left\|\mathbf{u}^*\right\|_2 . \end{cases} \tag{13}$$

From (12) and (13), the difference of judgment on the satisfaction of probability constraint between RIM and the practical situation is caused when $\mathbf{u}^* = \nabla g(\mathbf{u}^*)/\|\nabla g(\mathbf{u}^*)\|_2 \cdot \|\mathbf{u}^*\|_2$. Utilizing (11), the probabilities of failure of the performance functions in (1) can be recalculated.

To satisfy the probability constraint, from (12), the probability of failure should satisfy

$$\Phi\left\{\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^*\right\} \le \Phi\left(-\beta_t\right). \tag{14}$$

Equation (14) can be written as

$$\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^* \le -\beta_t \tag{15}$$

or

$$-\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^* \ge \beta_t. \tag{16}$$

The reliability R of the performance function is

$$\begin{aligned} R &= 1 - \Pr\left(g\left(\mathbf{X}\right) \le 0\right) \\ &= 1 - \Phi\left\{\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^*\right\} \\ &= \Phi\left\{-\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^*\right\} \end{aligned} \tag{17}$$

and the reliability index is defined as

$$\beta = -\left(\frac{\nabla g\left(\mathbf{u}^*\right)}{\left\|\nabla g\left(\mathbf{u}^*\right)\right\|_2}\right)^T \mathbf{u}^*. \tag{18}$$

4. The CO and the Formulation of CO-ERIM

As a well-known multilevel method for MDO, the strategy of CO is suitable for large-scale distributed engineering systems. The CO algorithm decomposes, coordinates, and optimizes complex engineering problems. Every discipline in a system can enjoy good autonomy, regardless of the influence of other disciplines. The consistency between disciplines is guaranteed by compatibility constraints attached to the system-level optimization problem. While the value of compatibility constraint is obtained through subject-level optimization problem. The objectives of subject-level optimization problems are to minimize the inconsistency between disciplines while satisfying the constraints of discipline design.

Using CO, the RBMDO problem in (1) is converted into the system-level and subject-level optimization problems. The formulation of the optimization problem in system level is

$$\begin{aligned} \min_{\left(\mathbf{d}_i', \mathbf{d}_s', \boldsymbol{\mu}_{\mathbf{X}_i}', \boldsymbol{\mu}_{\mathbf{X}_s}', \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}'\right)} \quad & f\left(\mathbf{d}_i', \mathbf{d}_s', \boldsymbol{\mu}_{\mathbf{X}_i}', \boldsymbol{\mu}_{\mathbf{X}_s}', \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}'\right) \\ \text{s.t.} \quad & J_i \\ & = \left(\mathbf{d}_i' - \mathbf{d}_i\right)^2 + \left(\mathbf{d}_s' - \mathbf{d}_s\right)^2 \\ & \quad + \left(\boldsymbol{\mu}_{\mathbf{X}_i}' - \boldsymbol{\mu}_{\mathbf{X}_i}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_s}' - \boldsymbol{\mu}_{\mathbf{X}_s}\right)^2 \\ & \quad + \left(\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}' - \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right)^2 \le \varepsilon \\ & i = 1, 2, \ldots, n \end{aligned} \tag{19}$$

where J_i is the compatibility constraints. $\mathbf{d}_i'$, $\mathbf{d}_s'$,$\boldsymbol{\mu}_{\mathbf{X}_i}'$,$\boldsymbol{\mu}_{\mathbf{X}_s}'$, and $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}'$ are the design variables at the system level. The formulation of the optimization problem in subject-level optimization problems is

$$\begin{aligned} \min_{\left(\mathbf{d}_i, \mathbf{d}_s, \boldsymbol{\mu}_{\mathbf{X}_i}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right)} \quad & J_i \\ & = \left(\mathbf{d}_i' - \mathbf{d}_i\right)^2 + \left(\mathbf{d}_s' - \mathbf{d}_s\right)^2 \\ & \quad + \left(\boldsymbol{\mu}_{\mathbf{X}_i}' - \boldsymbol{\mu}_{\mathbf{X}_i}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_s}' - \boldsymbol{\mu}_{\mathbf{X}_s}\right)^2 \\ & \quad + \left(\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}' - \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right)^2 \\ \text{s.t.} \quad & \Pr_i\left[g_i\left(\mathbf{d}_i, \mathbf{d}_s, \mathbf{X}_i, \mathbf{X}_s, \mathbf{P}\right) \le 0\right] \\ & \le \Phi\left(-\beta_t\right) \\ & \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}} = \mathbf{Y}_{i\bullet}\left(\mathbf{d}_i, \mathbf{d}_s, \boldsymbol{\mu}_{\mathbf{X}_i}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right) \\ & \mathbf{d}_i^L \le \mathbf{d}_i \le \mathbf{d}_i^U, \\ & \mathbf{d}_s^L \le \mathbf{d}_s \le \mathbf{d}_s^U, \\ & \mathbf{X}_i^L \le \boldsymbol{\mu}_{\mathbf{X}_i} \le \mathbf{X}_i^U, \\ & \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s} \le \mathbf{X}_s^U, \\ & \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}} \le \mathbf{Y}^U, \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{20}$$

The corresponding bilevel strategy of CO is also shown in Figure 5.

Considering uncertainties in practical engineering, all discipline probability reliability constraints in (20) are converted into the corresponding RIM-based reliability constraints. These RIM-based reliability constraints only consider the performance reliability at MPP, which can reduce the cost of reliability analysis and improve the computational

Table 1: The design information of speed reducer.

Variables	Lower and upper bound	Distribution	Mean	Standard deviation
gear face width, x_1 (cm)	[2.6, 3.6]	-	-	-
teeth module, x_2 (cm)	[0.3, 1.0]	-	-	-
number of teeth of pinion, x_3	[17, 28]	-	-	-
distance between bearings 1, x_4 (cm)	[7.3, 8.3]	Normal	μ_{x_4}	$0.01\mu_{x_4}$
distance between bearings 2, x_5 (cm)	[7.3, 8.3]	Normal	μ_{x_5}	$0.01\mu_{x_5}$
diameter of shaft 1, x_6 (cm)	[2.9, 3.9]	Normal	μ_{x_6}	$0.01\mu_{x_6}$
diameter of shaft 2, x_7 (cm)	[5, 5.5]	Normal	μ_{x_7}	$0.01\mu_{x_7}$

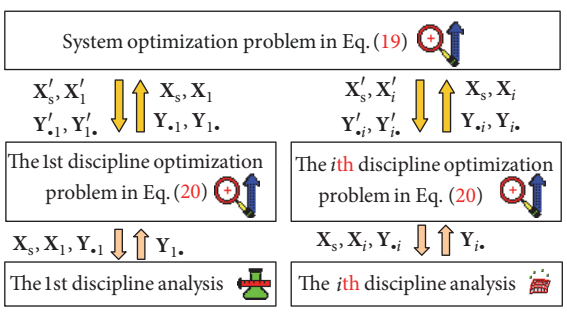

Figure 5: The bilevel strategy of CO.

efficiency of RBMDO. The formulation of the subject-level optimization problem in the U-space using RIM-based reliability constraints can be denoted as

$$\begin{aligned}
\min_{(\mathbf{d}_i,\mathbf{d}_s,\boldsymbol{\mu}_{\mathbf{X}_i},\boldsymbol{\mu}_{\mathbf{X}_s},\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}})} \quad & J_i \\
&= \left(\mathbf{d}'_i - \mathbf{d}_i\right)^2 + \left(\mathbf{d}'_s - \mathbf{d}_s\right)^2 \\
&\quad + \left(\boldsymbol{\mu}'_{\mathbf{X}_i} - \boldsymbol{\mu}_{\mathbf{X}_i}\right)^2 + \left(\boldsymbol{\mu}'_{\mathbf{X}_s} - \boldsymbol{\mu}_{\mathbf{X}_s}\right)^2 \\
&\quad + \left(\boldsymbol{\mu}'_{\mathbf{Y}_{\bullet i}} - \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right)^2 \\
\text{s.t.} \quad & \Phi\left\{-\left(\frac{\nabla g_i(\mathbf{u}^*)}{\left\|\nabla g_i(\mathbf{u}^*)\right\|_2}\right)^T \mathbf{u}^*\right\} \\
& \le \Phi(-\beta_t) \\
& \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}} = \mathbf{Y}_{i\bullet}\left(\mathbf{d}_i, \mathbf{d}_s, \boldsymbol{\mu}_{\mathbf{X}_i}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}\right) \\
& \mathbf{u} = \{u \mid \text{The Rosenblatt} \\
& \quad \text{transformation of } \mathbf{X}\} \\
& \mathbf{d}_i^L \le \mathbf{d}_i \le \mathbf{d}_i^U, \\
& \mathbf{d}_s^L \le \mathbf{d}_s \le \mathbf{d}_s^U, \\
& \mathbf{X}_i^L \le \boldsymbol{\mu}_{\mathbf{X}_i} \le \mathbf{X}_i^U, \\
& \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s} \le \mathbf{X}_s^U, \\
& \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}} \le \mathbf{Y}^U, \\
& i = 1, 2, \ldots, n.
\end{aligned} \tag{21}$$

The detail information of CO-ERIM is as follows.

Step 1. Input the original design information; the cycle number $k = 0$.

Step 2. Solve the system-level optimization problem in (19). During this process, $\mathbf{d}'_i$, $\mathbf{d}'_s$, $\boldsymbol{\mu}'_{\mathbf{X}_i}$, $\boldsymbol{\mu}'_{\mathbf{X}_s}$, and $\boldsymbol{\mu}'_{\mathbf{Y}_{\bullet i}}$ are treated as the design parameters.

Step 3. Transform random variables in X-space into random variables in U-space using the Rosenblatt transformation.

Step 4. Solve the subject-level optimization problems in (21). Then send the design solutions to the system level.

Step 5. Obtain the value of J_i, $i = 1, 2, \ldots, n$. If $J_i \le \varepsilon$ and the difference between the objective function values of two consecutive iterations is not more than a small number in the optimization iteration process, carry out Step 6; Otherwise, $k = k + 1$ and carry out Step 2.

Step 6. Stop and output the design solutions.

The flowchart of CO-ERIM is shown in Figure 6.

5. Example

Speed reducers are generally used in low-speed, high-torque transmission equipment. In this study, a speed reducer RBMDO problem is introduced to illustrate the utilization of the proposed method. There are seven design variables in this example, which is listed in Table 1. Twenty-five constraints are introduced to ensure that the design solutions can satisfy the strength, stiffness, and space requirements. The optimization object is to minimize the overall weight. Further information can be obtained in [14, 43].

There are three disciplines in this RBMDO problem, Bearing-Shaft 1, Bearing-Shaft 2, and Gears, which is shown in Figure 7. The CO-ERIM strategy for this problem is shown in Figure 8, where $\beta_t = 2.07$, $\Phi(-\beta_t) = 0.02$, and $\varepsilon = 0.001$.

To illustrate the accuracy of design solutions, the Monte Carlo Simulation (MCS) method is also introduced here as

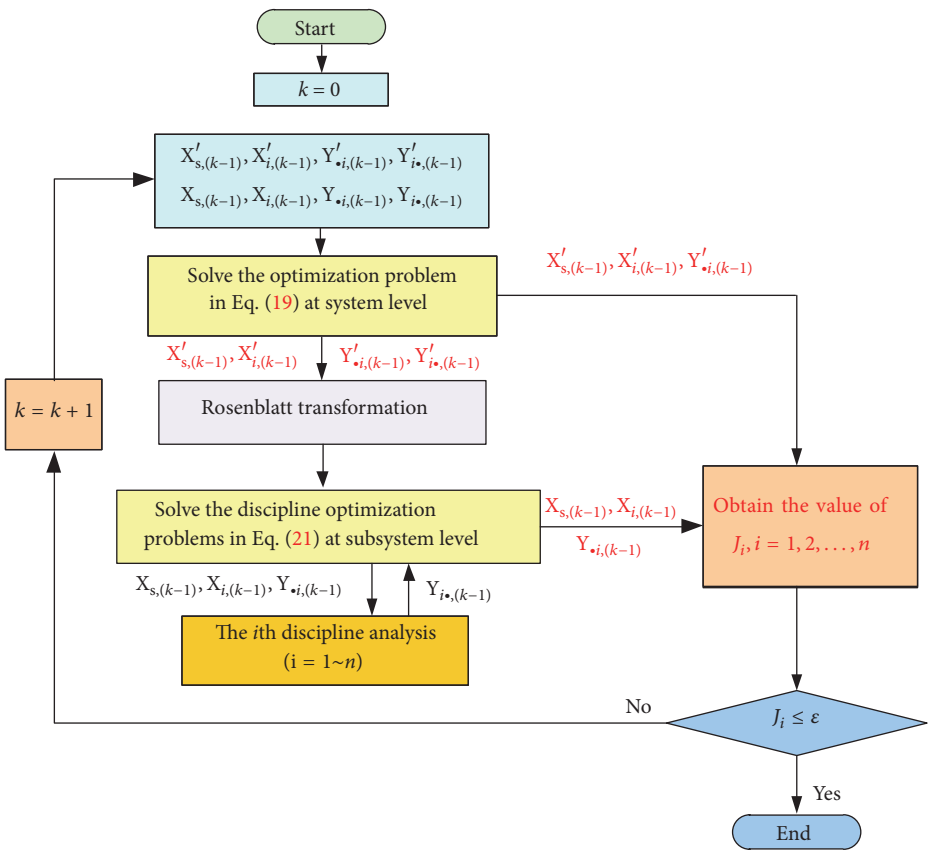

Figure 6: The flowchart of CO-ERIM.

Table 2: Optimization results of the reducer design.

	x_1	x_2	x_3	μ_{x_4}	μ_{x_5}	μ_{x_6}	μ_{x_7}	f
ERIM	3.4238	0.6493	18	7.3001	7.6902	3.3201	5.2646	2987.8558
RIM	3.4254	0.6502	18	7.3004	7.6865	3.3251	5.2637	2966.7482
MCS	3.4237	0.6487	18	7.3000	7.6893	3.3214	5.2657	2993.4750

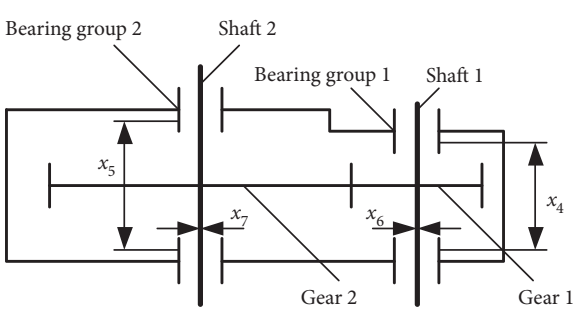

Figure 7: The speed reducer design [14, 43].

the reference. The software Isight is utilized in the computations of optimization. The solutions from CO-ERIM are compared with the ones from original PMA-RIM based CO and MCS based CO, which is listed in Table 2. From the comparison of solutions, the design results from ERIM are closer to the design results from MCS. Furthermore, the calculation time of ERIM is 17min23s, and the calculation time of RIM is 25min17s, which means the proposed method enjoys higher computational efficiency.

6. Conclusions

In this study, the efficiency problem of RBMDO is studied. The RIM strategy is reviewed and the corresponding algorithm of ERIM is discussed in detail. Furthermore, the CO-ERIM strategy is proposed, including its formulation and procedure. In CO-ERIM, the concurrent design idea is adapting to the development of modern engineering systems. Compatibility constraints are introduced into subsystem-level and system-level optimization problems, respectively. The consistency between different disciplines can be guaranteed when the RBMDO solutions are obtained. The introduction of ERIM reduces the computational burden of reliability analysis during optimization iteration process. Under the condition that the first order method is acceptable, the reliability analysis accuracy of the proposed method is similar

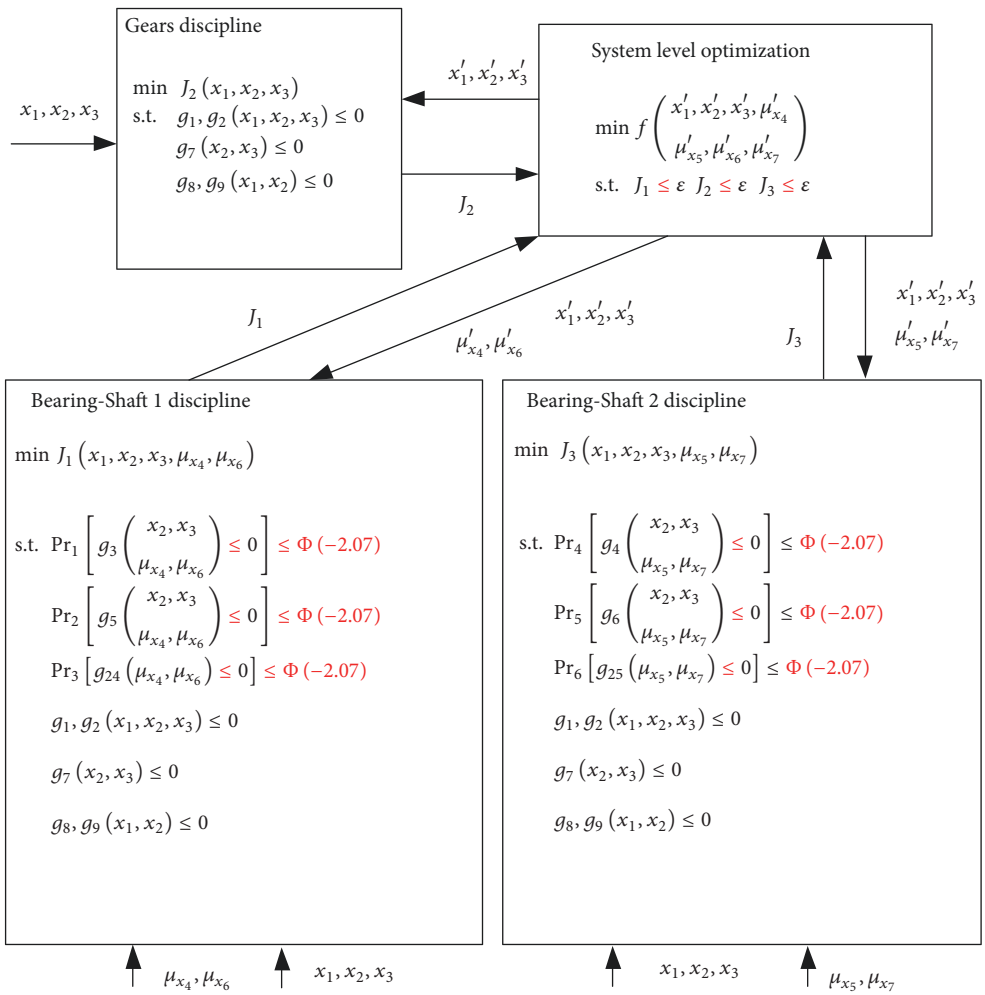

Figure 8: The MDO strategy for the speed reducer problem.

to the accuracy of MCS. The speed reducer example illustrates the effectiveness of the proposed method.

Data Availability

All data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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