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Research Article

Stability and Stabilization for Polytopic LPV Systems with Parameter-Varying Time Delays

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In this paper, we deal with the problem of stability and stabilization for linear parameter-varying (LPV) systems with time-varying time delays. The uncertain parameters are assumed to reside in a polytope with bounded variation rates. Being main difference from the existing achievements, the representation of the time derivative of the time-varying parameter is under a polytopic structure. Based on the new representation, delay-dependent sufficient conditions of stability and stabilization are, respectively, formulated in terms of linear matrix inequalities (LMI). Simulation examples are then provided to confirm the effectiveness of the given approach.

1. Introduction

Linear time-varying (LPV) systems, which depend on unknown but measurable time-varying parameters, have received much attention. This is mainly due to the context of gain scheduled control of nonlinear systems. In the framework of gain scheduled control, the design can be regarded as applicable for the underlying LPV system, which is a weighted combination of linearized systems. This type of system was first presented in [1], which was extensively studied by many researchers in [2–4] and the references therein. Many approaches used the parameter-independent Lyapunov approach and parameter-dependent Lyapunov approach; see [5–12] and the references therein. It is well known that much less conservative results can be obtained by using the parameter-dependent Lyapunov approach than using the parameter-independent one. However, the difficulty of using a parameter-dependent Lyapunov function is that if the parameter is time-varying, the rate of variation needs to be taken into account. Thus, for the parameter-dependent Lyapunov function, how to seek an appropriate method to represent the derivative of the time-varying parameter is particularly important.

On the other hand, time delay is frequently encountered in control systems [13–16]. It is well recognized that the time delay is a source of instability and degrades the system performance. Therefore, it is important to investigate the time-delay effects on the stability of LPV systems. Recently, there has been increasing interest in the stability analysis of LPV systems with parameter-varying time delays. For example, in [17], the analysis and state-feedback control synthesis problem for LPV systems with parameter-dependent state delays are considered. The corresponding analysis and synthesis conditions for stabilization and induced ℓ_2 norm performance are obtained in terms of LMIs that can be solved by interior-point algorithms. In [18], by using parameter-dependent Lyapunov functionals and interior-point algorithms, the stability, H_∞ gain performance, L_2 gain performance, and L_2-to-L_∞ gain performance are explored for LPV systems with parameter-varying time delays. Moreover, polytopic LPV systems whose parameter is contained in an a priori given set also have aroused the general scholars' attention; see [19] and the references therein. However, to the best of our knowledge, there have not been any available results on the stability and stabilization on polytopic LPV systems with parameter-varying time delays.

In the light of the above, this paper investigates the stability and stabilization of polytopic LPV systems with parameter-varying time delays. Firstly, an innovative representation for the rate of variation of the parameter is stated. Secondly, based on this representation and parameter-dependent Lyapunov functionals, delay-dependent sufficient conditions for the stability and stabilization are derived in terms of LMIs. Finally, two examples are given to illustrate the effectiveness of the methods presented in this paper.

Notation. $\mathbb{R}$ stands for the set of real numbers and $\mathbb{R}^+$ stands for the nonnegative real numbers. $\mathbb{R}^{m\times n}$ is the set of $m \times n$ matrices. For a symmetric block matrix, we use $*$ as an ellipsis for the terms that are introduced by symmetry. For a real matrix X, X^T denotes its transpose. And, finally, we use the symbol $He(X) = X + X^T$, and $X > 0 (X \geq 0)$ means that X is symmetric and positive-definite (positive semidefinite). $\otimes$ denotes the Kronecker products. The space of continuous functions will be denoted by C and the corresponding norm is $\|\phi(t)\| = \sup_t \|\phi(t)\|$.

2. Problem Statement

Consider a polytopic system with time parameter-varying time delays described by state-space equations.

$$\dot{x}(t) = A(\varrho(t))\,x(t) + B(\varrho(t))\,x(t - \tau(\varrho(t))) + C(\varrho(t))\,u(t), \tag{1a}$$

$$x(\theta) = \phi(\theta), \quad \theta \in [-\tau(\varrho(0)), 0], \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, the real-valued initial function $\phi(\theta)$ in (1b) is a given function in $C([-\varrho(0), 0], \mathbb{R}^n)$, and $\tau(t)$ is a differentiable scalar function representing the parameter-varying delay. The system matrices $A(\varrho(t)), B(\varrho(t)), C_1(\varrho(t)), C_2(\varrho(t))$ and the delay $\tau(\varrho(t))$ are dependent on the parameters $\varrho_i(t)$; that is,

$$\begin{aligned} A(\varrho(t)) &= \sum_{i=1}^{N} \varrho_i(t)\,A_i, \\ B(\varrho(t)) &= \sum_{i=1}^{N} \varrho_i(t)\,B_i, \\ C(\varrho(t)) &= \sum_{i=1}^{N} \varrho_i(t)\,C_i, \\ \tau(\varrho(t)) &= \sum_{i=1}^{N} \varrho_i(t)\,\tau_i, \end{aligned} \tag{2}$$

where $\varrho(t) = [\varrho_1(t)\ \varrho_2(t)\ \ldots\ \varrho_N(t)]^T \in \mathbb{R}^N$ is the time-varying parameter. We make the following assumptions:

(A1) The time-varying parameter $\varrho(t)$ varies in a polytope given by

$\varrho(t) \in \Lambda^N$, where $\Lambda^N := \{\varrho(t) \in \mathbb{R}^N : \sum_{i=1}^{N} \varrho_i(t) = 1,\ \varrho_i(t) \geq 0\}$.

(A2) The time derivative $\dot{\varrho}_i(t)$ of the parameter is such that $|\dot{\varrho}_i(t)| \leq r$, where, $r \geq 0$.

(A3) The delay is bounded and the function $t - \tau(t)$ is monotonically increasing; that is, $\tau(t)$ lies in the set $\Omega = \{\tau(t) \in C(\mathbb{R}, \mathbb{R}) : 0 \leq \tau(t) \leq \overline{\tau} < \infty,\ \dot{\tau}(t) < \omega \leq 1,\ \forall t \in \mathbb{R}^+\}$.

In this paper, we will establish stability and stabilization conditions for system (1a) with (1b) by using Lyapunov-Krasovskii functional and a novel representation of the time derivative of the parameter (t).

3. New Representation of the Time Derivative of the Parameter

Lemma 1. *If parameter $\varrho(t)$ satisfies assumptions (A1) and (A2), then its time derivative can be written as*

$$\dot{\varrho}_i(t) = \frac{N.r}{2}\left(\zeta_i(t) - \eta_i(t)\right), \tag{3}$$

where

$$\begin{aligned} \zeta(t) &= [\zeta_1(t)\ \ \zeta_2(t)\ \ \ldots\ \ \zeta_N(t)]^T \in \Lambda^N, \\ \eta(t) &= [\eta_1(t)\ \ \eta_2(t)\ \ \ldots\ \ \eta_N(t)]^T \in \Lambda^N. \end{aligned} \tag{4}$$

Proof. To complete the proof of Lemma 1, we need to distinguish two cases: $r = 0$ and $r > 0$.

Case 1 ($r = 0$). It is known that $\dot{\varrho}_i(t) = 0,\ i = 1, 2, \ldots, N$. In this situation, Lemma 1 is obviously right.

Case 2 ($r > 0$). From assumptions (A1) and (A2), it is known that $\dot{\varrho}_i(t),\ i = 1, 2, \ldots, N$ satisfies the following inequality:

$$\begin{gathered} -r \leq \dot{\varrho}_i(t) \leq r, \\ \sum_{i=1}^{N} \dot{\varrho}_i(t) = 0. \end{gathered} \tag{5}$$

$\dot{\varrho}_i(t),\ i = 1, 2, \ldots, N$ can be written as follows:

$$\dot{\varrho}_i(t) = \frac{N.r}{2}\left[\left(\frac{1}{N} + \frac{\dot{\varrho}_i(t)}{Nr}\right) - \left(\frac{1}{N} - \frac{\dot{\varrho}_i(t)}{Nr}\right)\right], \quad i = 1, 2, \ldots, N. \tag{6}$$

Let

$$\begin{aligned} \zeta_i(t) &= \frac{1}{N} + \frac{\dot{\varrho}_i(t)}{Nr}, \\ \eta_i(t) &= \frac{1}{N} - \frac{\dot{\varrho}_i(t)}{Nr}, \end{aligned} \qquad i = 1, 2, \ldots, N, \tag{7}$$

and then

$$\dot{\varrho}_i(t) = \frac{N.r}{2}\left(\zeta_i(t) - \eta_i(t)\right). \tag{8}$$

It follows from (5) that

$$
\begin{aligned}
&0 \leq \zeta_i(t) \leq 1, \\
&\sum_{i=1}^{N} \zeta_i(t) = 1, \\
&0 \leq \eta_i(t) \leq 1, \\
&\sum_{i=1}^{N} \eta_i(t) = 1.
\end{aligned} \tag{9}
$$

It follows that $\zeta(t) = [\zeta_1(t)\ \zeta_2(t)\ \ldots\ \zeta_N(t)]^T \in \Lambda^N$ and $\eta(t) = [\eta_1(t)\ \eta_2(t)\ \ldots\ \eta_N(t)]^T \in \Lambda^N$.

The proof is completed. □

Remark 2. The main difference for studying the stability problems of LPV time-dependent systems is how to represent the derivative of the time-varying parameter.

We note, for instance, the results presented in [20], where the rate of variation $\dot{\theta}_i$ is well defined at all times and satisfies

$$\dot{\theta}_i \leq \upsilon_i \theta_i, \quad \upsilon_i \text{ is constant}, \quad i = 1, 2, \ldots, N. \tag{10}$$

The weakness of this representation is that, in numerous practical cases, it cannot be physically justified.

We mention in a second case the results given in [21], where the time derivative is defined in a polytope such that

$$\dot{\theta}_i(t) = \sum_{j=1}^{M} \mu_j(t) h_j, \quad i = 1, 2, \ldots, N \tag{11}$$

where $\mu(t) \in \Lambda_M = \{\mu \in \mathbb{R}^M : \sum_{j=1}^{M} \mu_j = 1,\ \mu_j \geq 0\}, h_j \in \mathbb{R}^N$ for all $i = 1, 2, \ldots, M$ are given vectors. And $|\dot{\theta}_i(t)| \leq r$, $i = 1, 2, \ldots, N$. Contrary to (10), expression (11) does not impose any particular condition on the derivative of the uncertain parameter, although it does not give any idea about the dimension of the polyhedral convex set, where it evolves.

The main advantage of the new representation $\dot{\varrho}_i(t) = (N.r/2)(\zeta_i(t) - \eta_i(t))$ is that it does not directly depend on the parameter itself and that it is simply defined as a difference between two parameters that evolve in two known and well-defined polytopes.

4. Stability Analysis

Now, we use Lyapunov-Krasovskii functional and Lemma 1 to develop robust stability condition for the unforced system (1a) with the initial condition (1b). The following technical lemma will be employed to establish our robust stability condition.

Lemma 3 (see [22]). *Let X be a full-rank matrix; then, for an arbitrary negative definite matrix Y, the following inequality is satisfied:*

$$X^T Y X < 0. \tag{12}$$

Theorem 4. *Consider the unforced system (1a) with the initial condition (1b). If there exist matrices $0 < Q \in \mathbb{R}^{n \times n}, 0 < P_i \in \mathbb{R}^{n \times n}, E_j \in \mathbb{R}^{n \times n}, G_j \in \mathbb{R}^{n \times n}, F_j \in \mathbb{R}^{n \times n}$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$ such that*

$$
\begin{aligned}
&\begin{bmatrix} \dfrac{N.r}{2}\left(P_j - P_k\right) + Q & 0 & P_i \\ 0 & -\left[1 - \dfrac{N.r}{2}\left(\tau_j - \tau_k\right)\right] Q & 0 \\ P_i & 0 & 0 \end{bmatrix} \\
&\quad + He\left(\begin{bmatrix} E_j \\ G_j \\ F_j \end{bmatrix} \begin{pmatrix} A_i & B_i & -I \end{pmatrix} \right) < 0
\end{aligned} \tag{13}
$$

is satisfied for $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, $k = 1, 2, 3, \ldots, N$, then the unforced system (1a) with the initial condition (1b) is asymptotically stable.

Proof. Consider the following Lyapunov-Krasovskii type functional:

$$V(x_t, \varrho) = x^T P(\varrho(t)) x + \int_{t-\tau(t)}^{t} x^T(\xi) Q x(\xi)\, d\xi, \tag{14}$$

where $P(\varrho(t)) = \sum_{i=1}^{N} \varrho_i(t) P_i$.

Let

$$
\begin{aligned}
\overline{\lambda}_P &= \max_{i \in [1,N]} \left(\lambda_{max}(P_i)\right), \\
\underline{\lambda}_P &= \min_{i \in [1,N]} \left(\lambda_{min}(P_i)\right),
\end{aligned} \tag{15}
$$

and then we have that, for $\forall x \in \mathbb{R}^n$,

$$\underline{\lambda}_P \|x\|^2 \leq V(x_t, \varrho) \leq \left(\overline{\lambda}_P + \overline{\tau}\lambda_{max}(Q)\right) \|x\|^2. \tag{16}$$

The time derivative of $V(x_t, \varrho(t))$ along the trajectories of unforced system (1a) is given by

$$
\begin{aligned}
\frac{dV}{dt} &= x^T(t) \frac{dP(\varrho(t))}{dt} x(t) + 2x^T(t) P(\varrho(t)) \dot{x}(t) + x^T(t) Q x(t) - \left(1 - \frac{d\tau(\varrho(t))}{dt}\right) x^T(t - \tau(t)) Q x(t - \tau(t)) \\
&= \begin{bmatrix} x^T(t) & x^T(t - \tau(t)) \end{bmatrix} \begin{bmatrix} \Xi & * \\ B^T(\varrho(t)) P(\varrho(t)) & -\left(1 - \dfrac{d\tau(\varrho(t))}{dt}\right) Q \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix},
\end{aligned} \tag{17}
$$

where

$$\Xi = \frac{dP(\varrho(t))}{dt} + P(\varrho(t))A(\varrho(t)) + A^T(\varrho(t))P(\varrho(t)) + Q. \tag{18}$$

Notice that the time derivatives of $P(\varrho(t))$ and $\tau(\varrho(t))$ are, respectively, given by

$$\frac{dP(\varrho(t))}{dt} = \sum_{i=1}^{N} \dot{\varrho}_i(t) P_i, \qquad \frac{d\tau(\varrho(t))}{dt} = \sum_{i=1}^{N} \dot{\varrho}_i(t) \tau_i. \tag{19}$$

It follows from Lemma 1 that there exist $\zeta_i(t)$ and $\eta_i(t)$ such that

$$\dot{\varrho}_i(t) = \frac{N.r}{2}(\zeta_i(t) - \eta_i(t)), \quad i = 1, 2, \ldots, N, \tag{20}$$

where

$$\zeta(t) = [\zeta_1(t) \ \ \zeta_2(t) \ \ \ldots \ \ \zeta_N(t)]^T \in \Lambda^N, \qquad \eta(t) = [\eta_1(t) \ \ \eta_2(t) \ \ \ldots \ \ \eta_N(t)]^T \in \Lambda^N. \tag{21}$$

Multiplying condition (20), respectively, by P_i and τ_i, we get

$$\frac{dP(\varrho(t))}{dt} = \frac{N.r}{2}(P(\zeta(t)) - P(\eta(t))), \qquad \frac{d\tau(\varrho(t))}{dt} = \frac{N.r}{2}(\tau(\zeta(t)) - \tau(\eta(t))). \tag{22}$$

Multiplying condition (13), respectively, by $\varrho_i(t), \zeta_j(t)$ and $\eta_k(t)$ and summing up for $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, $k = 1, 2, \ldots, N$, we have

$$\Psi_{11} + \Psi_{22} < 0, \tag{23}$$

where

$$\Psi_{11} = \begin{bmatrix} \frac{N.r}{2}(P(\zeta(t)) - P(\eta(t))) + Q & 0 & P(\varrho(t)) \\ 0 & -\left[1 - \frac{N.r}{2}(\tau(\zeta(t)) - \tau(\eta(t)))\right]Q & 0 \\ P(\varrho(t)) & 0 & 0 \end{bmatrix},$$
$$\Psi_{22} = He\left(\begin{bmatrix} E(\zeta(t)) \\ G(\zeta(t)) \\ F(\zeta(t)) \end{bmatrix} [A(\varrho(t)) \ \ B(\varrho(t)) \ \ -I]\right). \tag{24}$$

Pre- and postmultiplying condition (23), respectively, by a full-row matrix

$$\begin{bmatrix} I & 0 & A^T(\varrho(t)) \\ 0 & I & B^T(\varrho(t)) \end{bmatrix}, \tag{25}$$

as well as its transpose, which combined with Lemma 3 yields

$$\begin{bmatrix} \frac{N.r}{2}(P(\zeta(t)) - P(\eta(t))) + P(\varrho(t))A(\varrho(t)) + A^T(\varrho(t))P(\varrho(t)) + Q & P(\varrho(t))B(\varrho(t)) \\ * & -\left[1 - \frac{N.r}{2}(\tau(\zeta(t)) - \tau(\eta(t)))\right]Q \end{bmatrix} < 0 \tag{26}$$

Substituting condition (22) into the above inequality, we can easily obtain that condition (17) is negative. Thus, $V(x_t, \varrho)$ is a Lyapunov functional and the unforced system (1a) with initial condition (1b) is asymptotically stable.

The proof is completed. □

5. Stabilization

In this paper, we consider the following control law:

$$u(t) = K(\varrho(t))x(t) = \sum_{i=1}^{N} \varrho_i(t) K_i x(t), \tag{27}$$

where matrices K_i, $i = 1, 2, \ldots N$ are to be designed. Then, the closed-loop system from (1a) and (27) reads

$$\dot{x}(t) = (A(\varrho(t)) + C(\varrho(t))K(\varrho(t)))x(t) + B(\varrho(t))x(t - \tau(\varrho(t))). \tag{28}$$

Before giving a solution to the robust stabilization problem of system (1a) with the initial condition (1b), the following technical lemmas will be employed to establish our main results.

Lemma 5 (see [10]). *Let Φ be a symmetric matrix and N, M are matrices of appropriate dimensions. The following statements are equivalent:*

(1)

$$\begin{aligned} &\Phi < 0, \\ &\Phi + NM^T + MN^T < 0. \end{aligned} \tag{29}$$

(2) *There exists a matrix F such that*

$$\begin{bmatrix} \Phi & M + NF \\ M^T + F^T N^T & -F - F^T \end{bmatrix} < 0. \tag{30}$$

Lemma 6. *Consider the system matrices $A(\varrho(t))$ and $B(\varrho(t))$; the following two conditions are equivalent:*

(1) *there exist matrices $P(\varrho(t)) > 0$ and $Q > 0$ such that*

$$He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt + Q}{2} + P(\varrho(t))A(\varrho(t)) & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1 - d\tau(t)/dt)Q}{2} \end{bmatrix}\right) < 0; \tag{31}$$

(2) *for the above matrices $P(\varrho(t)) > 0$ and $Q > 0$, there exist a sufficient large scalar $\beta > 0$ and matrices $E(\sigma(t)), F(\sigma(t)), G(\sigma(t)), H(\sigma(t))$ such that*

$$He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt + Q}{2} - \beta E(\sigma(t)) & 0 & 0 & 0 \\ -\beta F(\sigma(t)) & -\dfrac{(1 - d\tau(t)/dt)Q}{2} & 0 & 0 \\ \begin{pmatrix} -E^T(\sigma(t)) + P(\varrho(t)) \\ -\beta G(\sigma(t)) \end{pmatrix} & -F^T(\sigma(t)) & -G(\sigma(t)) & 0 \\ \begin{pmatrix} E^T(\sigma(t)) + H^T(\sigma(t))A(\varrho(t)) \\ +\beta H^T(\sigma(t)) \end{pmatrix} & \begin{pmatrix} F^T(\sigma(t)) \\ +H^T(\sigma(t))B(\varrho(t)) \end{pmatrix} & G^T(\sigma(t)) & -H^T(\sigma(t)) \end{bmatrix}\right) < 0, \tag{32}$$

where

$$\begin{aligned} \sigma(t) &= \begin{bmatrix} \sigma_1(t) & \sigma_2(t) & \dots & \sigma_N(t) \end{bmatrix}^T \in \Lambda^N, \\ E(\sigma(t)) &= \sum_{i=1}^{N} \sigma_2(t) E_i, \\ F(\sigma(t)) &= \sum_{i=1}^{N} \sigma_2(t) F_i, \\ G(\sigma(t)) &= \sum_{i=1}^{N} \sigma_2(t) G_i, \\ H(\sigma(t)) &= \sum_{i=1}^{N} \sigma_2(t) H_i. \end{aligned} \tag{33}$$

Proof. For necessity, we perform multiplication on the left by

$$\begin{bmatrix} I & 0 & 0 & \beta I \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix}, \tag{34}$$

and on the right we do it by its transpose; we have

$$\Pi(E(\sigma(t)), F(\sigma(t)), G(\sigma(t)), H(\sigma(t))) < 0, \tag{35}$$

where

$$\Pi\left(E(\sigma(t)),F(\sigma(t)),G(\sigma(t)),H(\sigma(t))\right)$$
$$= He\left(\begin{bmatrix} \begin{pmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2} \\ +\beta H^T(\sigma(t))A(\varrho(t)) \end{pmatrix} & 0 & 0 & 0 \\ \beta B^T(\varrho(t))H(\sigma(t)) & -\dfrac{(1-d\tau(t)/dt)Q}{2} & 0 & 0 \\ \begin{pmatrix} E^T(\sigma(t))+H^T(\sigma(t))A(\varrho(t)) \\ -\beta H(\sigma(t)) \end{pmatrix} & \begin{pmatrix} F^T(\sigma(t)) \\ +H^T(\sigma(t))B(\varrho(t)) \end{pmatrix} & -H(\sigma(t)) & 0 \\ -E^T(\sigma(t))+P(\varrho(t)) & -F(\sigma(t)) & G(\sigma(t)) & -G(\sigma(t)) \end{bmatrix}\right). \tag{36}$$

If we let

$$E(\sigma(t)) = P(\varrho(t)),$$
$$F(\sigma(t)) = 0,$$
$$H(\sigma(t)) = \frac{P(\varrho(t))}{\beta}, \tag{37}$$

then we obtain

$$\Pi\left(P(\varrho(t)),0,G(\sigma(t)),\frac{P(\varrho(t))}{\beta}\right)$$
$$= He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+P(\varrho(t))A(\varrho(t)) & 0 & 0 & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1-d\tau(\varrho(t))/dt)Q}{2} & 0 & 0 \\ \dfrac{P(\varrho(t))}{\beta}A(\varrho(t)) & \dfrac{P(\varrho(t))}{\beta}B(\varrho(t)) & -\dfrac{P(\varrho(t))}{\beta} & 0 \\ 0 & 0 & G(\sigma(t)) & -G(\sigma(t)) \end{bmatrix}\right) < 0. \tag{38}$$

Defining matrices

$$\Gamma = He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+P(\varrho(t))A(\varrho(t)) & 0 & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1-d\tau(\varrho(t))/dt)Q}{2} & 0 \\ \dfrac{P(\varrho(t))}{\beta}A(\varrho(t)) & \dfrac{P(\varrho(t))}{\beta}B(\varrho(t)) & -\dfrac{P(\varrho(t))}{\beta} \end{bmatrix}\right),$$
$$\Lambda_1 = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T,$$
$$\Lambda_2 = \begin{bmatrix}0 & 0 & I\end{bmatrix}^T, \tag{39}$$

condition (38) then can be rewritten as

$$\begin{bmatrix} \Gamma & \Lambda_1+\Lambda_2 G^T \\ \Lambda_1^T+G\Lambda_2^T & -G^T-G \end{bmatrix} < 0. \tag{40}$$

From conditions (40) and Lemma 5, we can easily verify that condition (38) is satisfied if and only if

$$\Gamma < 0,$$
$$\Gamma + \Lambda_2\Lambda_1^T + \Lambda_1\Lambda_2^T < 0; \tag{41}$$

that is

$$\Gamma < 0. \tag{42}$$

Applying Schur complement to the above inequality, we have

$$He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+P(\varrho(t))A(\varrho(t)) & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1-d\tau(\varrho(t))/dt)Q}{2} \end{bmatrix}\right) < -\frac{1}{2\beta}\begin{bmatrix} A^T(\varrho(t)) \\ B^T(\varrho(t)) \end{bmatrix} P(\varrho(t))\left[A(\varrho(t)) \;\; B(\varrho(t))\right], \tag{43}$$

which implies that condition (32) is satisfied for a sufficiently large scalar $\beta > 0$ of condition (31).

For Sufficiency, defining

$$\begin{aligned} \Xi &= He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}-\beta E(\varrho(t)) & 0 & 0 \\ -\beta F(\sigma(t)) & -\dfrac{(1-d\tau(\varrho(t))/dt)Q}{2} & 0 \\ -E^T(\sigma(t))+P(\varrho(t))-\beta G(\sigma(t)) & -F^T(\sigma(t)) & -G(\sigma(t)) \end{bmatrix}\right), \\ \Psi_1 &= \left[E^T(\sigma(t)) \;\; F^T(\sigma(t)) \;\; G^T(\sigma(t))\right]^T, \\ \Psi_2 &= \left[A(\varrho(t))+\beta I \;\; B(\varrho(t)) \;\; 0\right]^T, \end{aligned} \tag{44}$$

condition (32) can be written as

$$\begin{bmatrix} \Xi & \Psi_1+\Psi_2 H(\sigma(t)) \\ \Psi_1^T+H^T(\sigma(t))\Psi_2^T & -H(\sigma(t))-H^H(\sigma(t)) \end{bmatrix} < 0. \tag{45}$$

By Lemma 5, condition (45) is equivalent to

$$\begin{aligned} &\Xi+\Psi_2\Psi_1^T+\Psi_1\Psi_2^T \\ &= He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+E(\varrho(t))A(\varrho(t)) & 0 & 0 \\ B^T(\varrho(t))E^T(\varrho(t))+F(\varrho(t))A(\varrho(t)) & -\dfrac{(1-d\tau(\varrho(t))/dt)Q}{2}+F(\varrho(t))B(\varrho(t)) & 0 \\ -E^T(\varrho(t))+P(\varrho(t))+G(\varrho(t))A(\varrho(t)) & G(\varrho(t))B(\varrho(t))-F^T(\varrho(t)) & -G(\varrho(t)) \end{bmatrix}\right) \\ &< 0. \end{aligned} \tag{46}$$

We perform multiplication on the left by

$$\begin{bmatrix} I & 0 & A^T(\varrho(t)) \\ 0 & I & B^T(\varrho(t)) \end{bmatrix}, \tag{47}$$

and on the right we do it by its transpose; we obtain

$$He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+P(\varrho(t))A(\varrho(t)) & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1-d\tau(t)/dt)Q}{2} \end{bmatrix}\right) < 0. \tag{48}$$

The proof is completed. □

Theorem 7. *If there exist a sufficient large scalar $\beta > 0$ and matrices $0 < X_i \in \mathbb{R}^{n\times n}, 0 < Y \in \mathbb{R}^{n\times n}, L_i \in \mathbb{R}^{m\times n}, U_j \in \mathbb{R}^{n\times n}, V_j \in \mathbb{R}^{n\times n}, W_j \in \mathbb{R}^{n\times n}$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, and $M \in \mathbb{R}^{n\times n}$ such that the inequalities*

$$He\left(\begin{bmatrix} \dfrac{(N.r/2)\left(X_j - X_k\right) + Y}{2} - \beta U_j & 0 & 0 & 0 \\ -V_j & -\dfrac{\left(1 - (N.r/2)\left(\tau_j - \tau_k\right)\right)}{2} Y & 0 & 0 \\ -U_j^T + X_i - \beta W_j & -V_j^T & -W_j & 0 \\ \mathbf{I}\left(U_j + \beta M\right)^T + \mathbf{A}M + \mathbf{C}L_i & \mathbf{I}V_j^T + \mathbf{B}M & \mathbf{I}W_j^T & -\left(I_N \otimes M\right) \end{bmatrix}\right) < 0, \quad i = 1, 2, \ldots, N, \; j = 1, 2, \ldots, N, \; k = 1, 2, \ldots, N, \tag{49}$$

hold, where

$$\mathbf{A} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} I_n \\ I_n \\ \vdots \\ I_n \end{bmatrix} \in R^{Nn\times n}, \tag{50}$$

then the matrix gains $K_i = L_i M^{-1}$, $i = 1, 2, \ldots, N$, stabilize the closed-loop system (28) with the initial condition (1b).

Proof. Let

$$\begin{aligned} X_j &= M^T P_j M, \\ U_j &= M^T E_j M, \\ V_j &= M^T F_j M, \\ W_j &= M^T G_j M,, \\ L_i &= K_i M, \\ Y &= M^T Q M. \end{aligned} \tag{51}$$

Multiplying condition (49) at the left by full-row matrix

$$\begin{pmatrix} M^{-T} & 0 & 0 & 0 \\ 0 & M^{-T} & 0 & 0 \\ 0 & 0 & M^{-T} & 0 \\ 0 & 0 & 0 & \left(I_N \otimes M^{-T}\right) \end{pmatrix} \tag{52}$$

and at the right by its transpose, then, using Lemma 3, we have

$$He\left(\begin{bmatrix} \dfrac{(N.r/2)\left(P_j - P_k\right) + Q}{2} - \beta E_j & 0 & 0 & 0 \\ -F_j & -\dfrac{\left(1 - (N.r/2)\left(\tau_j - \tau_k\right)\right)}{2} Q & 0 & 0 \\ -E_j^T + P_i - \beta G_j & -F_j^T & -G_j & 0 \\ \mathbf{I}\left(E_j + \beta H\right)^T + \left(I_N \otimes H^T\right)\left(\mathbf{A} + \mathbf{C}K_i\right) & \mathbf{I}F_j^T + \left(I_N \otimes H^T\right)\mathbf{B} & \mathbf{I}G_j^T & -\left(I_N \otimes H\right) \end{bmatrix}\right) < 0, \tag{53}$$

and similar to the proof of Theorem 4, there exist

$$\zeta(t) = \left[\zeta_1(t) \;\; \zeta_2(t) \;\; \ldots \;\; \zeta_N(t)\right]^T \in \Lambda^N, \quad \eta(t) = \left[\eta_1(t) \;\; \eta_2(t) \;\; \ldots \;\; \eta_N(t)\right]^T \in \Lambda^N. \tag{54}$$

Therefore

$$\frac{dP(\varrho(t)}{dt} = \frac{N.r}{2}\left(P(\zeta(t)) - P(\eta(t))\right),$$
$$\frac{d\tau(\varrho(t)}{dt} = \frac{N.r}{2}\left(\tau(\zeta(t)) - \tau(\eta(t))\right). \tag{55}$$

Multiplying (53), respectively, by ϱ_i, ζ_j and η_k and summing up for $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, $k = 1, 2, \ldots, N$, we have

$$He\left(\begin{bmatrix} \frac{(N.r/2)(P(\zeta(t)) - P(\eta(t)))) + Q}{2} - \beta E(\zeta(t)) & 0 & 0 & 0 \\ -F(\zeta(t))) & -\frac{(1-(N.r/2)(\tau(\zeta(t)) - \tau(\eta(t)))}{2}Q & 0 & 0 \\ -E(\zeta(t)))^T + P(\varrho(t)) - \beta G(\zeta(t))) & -F(\zeta(t)))^T & -G(\zeta(t))) & 0 \\ \mathbf{I}(E(\zeta(t))) + \beta H)^T + (I_N \otimes H^T)(\mathbf{A} + \mathbf{C}K(\varrho(t))) & \mathbf{I}F(\zeta(t)))^T + (I_N \otimes H^T)\mathbf{B} & \mathbf{I}G(\zeta(t)))^T & -(I_N \otimes H) \end{bmatrix} < 0\right). \tag{56}$$

Substituting (55) into (56) gives

$$He\left(\begin{bmatrix} \frac{dP(\varrho(t))/dt + Q}{2} - \beta E(\zeta(t)) & 0 & 0 & 0 \\ -F(\zeta(t))) & -\frac{(1-d\tau(\varrho(t))/dt}{2}Q & 0 & 0 \\ -E(\zeta(t)))^T + P(\varrho(t)) - \beta G(\zeta(t))) & -F(\zeta(t)))^T & -G(\zeta(t))) & 0 \\ \mathbf{I}(E(\zeta(t))) + \beta H)^T + (I_N \otimes H^T)(\mathbf{A} + \mathbf{C}K(\varrho(t))) & \mathbf{I}F(\zeta(t)))^T + (I_N \otimes H^T)\mathbf{B} & \mathbf{I}G(\zeta(t)))^T & -(I_N \otimes H) \end{bmatrix} < 0\right). \tag{57}$$

Multiplying condition (57) at the left by full-row matrix $\left[I_{3n} \quad (\varrho(t) \otimes I)^T\right]$ and at the right by its transpose, then, using Lemma 3, we obtain

$$He\left(\begin{bmatrix} \frac{dP(\varrho(t))/dt + Q}{2} - \beta E(\zeta(t)) & 0 & 0 & 0 \\ -\beta F(\zeta(t)) & -\frac{(1-d\tau(t)/dt)Q}{2} & 0 & 0 \\ \begin{pmatrix} -E^T(\zeta(t)) + P(\varrho(t)) \\ -\beta G(\zeta(t)) \end{pmatrix} & -F^T(\zeta(t)) & -G(\zeta(t)) & 0 \\ \begin{pmatrix} E^T(\zeta(t)) + H^T A(\varrho(t)) \\ +\beta H^T \end{pmatrix} & \begin{pmatrix} F^T(\zeta(t)) \\ +H^T B(\varrho(t)) \end{pmatrix} & G^T(\zeta(t)) & -H^T \end{bmatrix}\right) < 0, \tag{58}$$

and applying Lemma 6 to (58), we have

$$He\left(\begin{bmatrix} \dfrac{dP(\varrho(t))/dt+Q}{2}+P(\varrho(t))A(\varrho(t)) & 0 \\ B^T(\varrho(t))P(\varrho(t)) & -\dfrac{(1-d\tau(t)/dt)Q}{2}\end{bmatrix}\right)<0; \tag{59}$$

consider the Lyapunov functional

$$V(x_t,\varrho)=x^TP(\varrho(t))x+\int_{t-\tau(t)}^{t}x^T(\xi)Qx(\xi)\,d\xi, \tag{60}$$

and its time derivative along the trajectories of system (28) is given by

$$\begin{aligned}\frac{dV}{dt}&=x^T(t)\frac{dP(\varrho(t))}{dt}+2x^T(t)P(\varrho(t))\dot{x}(t)\\&\quad+x^T(t)Qx(t)\\&\quad-\left(1-\frac{d\tau(\varrho(t))}{dt}\right)x^T(t-\tau(t))Qx(t-\tau(t))\\&=\left[x^T(t)\ \ x^T(t-\tau(t))\right]He\left(\begin{bmatrix}\Xi' & * \\ B^T(\varrho(t))P(\varrho(t)) & -\left(1-\dfrac{d\tau(\varrho(t))}{dt}\right)Q\end{bmatrix}\right)\\&\quad\cdot\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix},\end{aligned} \tag{61}$$

where

$$\begin{aligned}\Xi'&=\frac{dP(\varrho(t))}{dt}\\&\quad+P(\varrho(t))(A(\varrho(t))+C(\varrho(t))K(\varrho(t)))\\&\quad+(A(\varrho(t))+C(\varrho(t))K(\varrho(t)))^TP(\varrho(t))+Q.\end{aligned} \tag{62}$$

It follows from (59) that, for $\forall x\in\mathbb{R}^n$,

$$\frac{dV}{dt}<0, \tag{63}$$

which combined with (16) yielding that $V(x_t,\varrho)$ is a Lyapunov functional. Thus, system (28) with initial condition (1b) is asymptotically stable.

The proof is completed. □

6. Simulation Results

In this section, we give two examples to demonstrate the effectiveness of the proposed methods.

Example 1. Let us consider a polytopic system in the form of (1a) with $u(t)=0$. Assume that the system data are given by

$$\begin{aligned}A_1&=\begin{bmatrix}-4&-2\\-3&-7\end{bmatrix},\\A_2&=\begin{bmatrix}-13&-15\\-1&-10\end{bmatrix},\\B_1&=\begin{bmatrix}2&-2\\1&1.5\end{bmatrix},\\B_2&=\begin{bmatrix}1&-1\\2&0.5\end{bmatrix},\\\varrho_1(t)&=\frac{1}{2}+\frac{1}{2}\sin(2t),\\\varrho_1(t)&=\frac{1}{2}-\frac{1}{2}\sin(2t),\\\tau_1&=0.5,\\\tau_2&=1.\end{aligned} \tag{64}$$

One can verify that assumptions (A1)–(A3) are satisfied with parameters

$$\begin{aligned}N&=2,\\r&=1,\\\overline{\tau}&=1,\\\omega&=0.5.\end{aligned} \tag{65}$$

Using Theorem 4 and MATLAB LMI Control Toolbox, we can obtain a set of feasible solutions as

$$\begin{aligned}P_1&=\begin{bmatrix}0.1226&0.0354\\0.0354&0.3726\end{bmatrix},\\P_2&=\begin{bmatrix}0.0698&0.1188\\0.1188&0.4658\end{bmatrix},\\Q&=\begin{bmatrix}0.2150&0.0727\\0.0727&1.4302\end{bmatrix},\\E_1&=\begin{bmatrix}0.0786&-0.0281\\-0.0539&0.1857\end{bmatrix},\end{aligned}$$

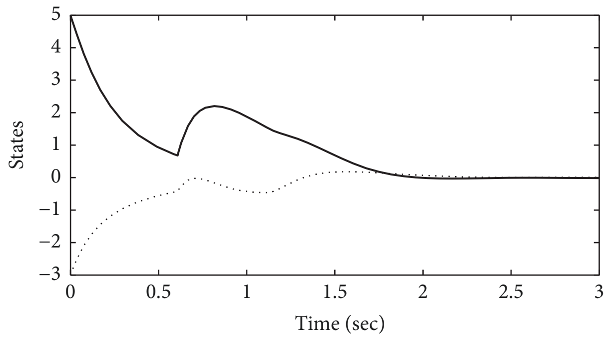

Figure 1: System response: x_1 (solid) and x_2 (dotted).

$$E_2 = \begin{bmatrix} 0.0616 & -0.0114 \\ -0.0502 & 0.1711 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.0085 & 0.0175 \\ -0.0657 & 0.0953 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.0108 & 0.0056 \\ -0.0185 & 0.0398 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.0124 & 0.0002 \\ -0.0033 & 0.0314 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0.0095 & 0.0053 \\ -0.0079 & 0.0454 \end{bmatrix}. \tag{66}$$

For an initial condition $(x_1(0), x_2(0)) = (5, -3)$, we simulate the open-loop behavior of the system. The states are shown in Figure 1. Note that both states x_1 and x_2 converge to zero.

Example 2. Let us consider a polytopic system in the form of (1a). Assume that the system data are given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 1 \\ 0.5 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.3 \\ -0.4 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.2 \\ 0.5 \end{bmatrix},$$

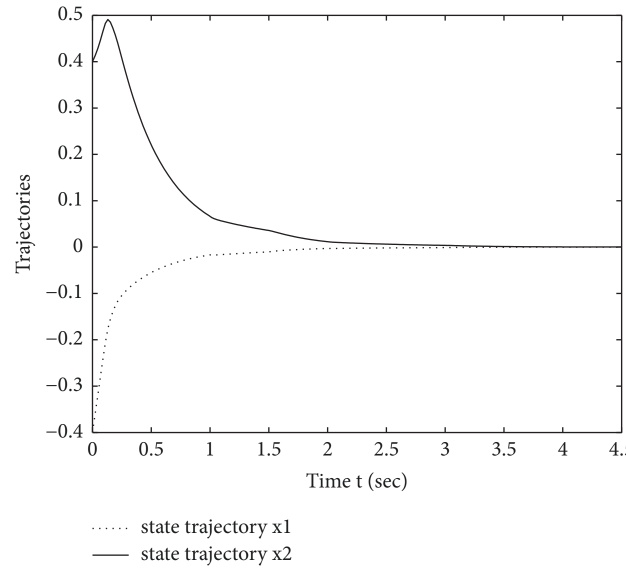

Figure 2: System response: x_1 (dotted) and x_2 (solid).

$$\varrho_1(t) = \frac{1}{2} + \frac{1}{2}\sin(2t),$$

$$\varrho_1(t) = \frac{1}{2} - \frac{1}{2}\sin(2t),$$

$$\tau_1 = 0.5,$$

$$\tau_2 = 1. \tag{67}$$

One can verify that assumptions (A1)–(A3) are satisfied with parameters

$$N = 2,\quad r = 1,\quad \overline{\tau} = 1,\quad \omega = 0.5. \tag{68}$$

Now, we choose $\beta = 3$ and then, using Theorem 7 and MATLAB LMI Control Toolbox, we can obtain the control gains as

$$K_1 = \begin{bmatrix} -0.0932 & -0.0341 \end{bmatrix},\quad K_2 = \begin{bmatrix} -0.0861 & -0.0969 \end{bmatrix}. \tag{69}$$

For an initial condition $(x_1(0), x_2(0)) = (-0.4, 0.4)$, we simulate the closed-loop behavior of the system. The states are shown in Figure 2. Note that both states x_1 and x_2 converge to zero.

7. Conclusions

This paper develops the stability and stabilization of polytopic LPV systems with parameter-varying time delays. For this purpose, an innovative representation for the rate of variation of the parameter is addressed. By using this representation and parameter-dependent Lyapunov functionals, delay-dependent sufficient conditions for the stability and stabilization are then derived in terms of LMIs. Two examples are given to illustrate the effectiveness of the methods presented in this paper.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Fu Chen carried out the molecular genetic studies, participated in the sequence alignment, and drafted the manuscript. Shugui Kang conceived of the study and participated in its design and coordination. Fangyuan Li gave some important insights and revised the first draft.

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