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Research Article

Novel Delay-Decomposing Approaches to Absolute Stability Criteria for Neutral-Type Lur'e Systems

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The problem of absolute stability analysis for neutral-type Lur'e systems with time-varying delays is investigated. Novel delay-decomposing approaches are proposed to divide the variation interval of the delay into three unequal subintervals. Some new augment Lyapunov–Krasovskii functionals (LKFs) are defined on the obtained subintervals. The integral inequality method and the reciprocally convex technique are utilized to deal with the derivative of the LKFs. Several improved delay-dependent criteria are derived in terms of the linear matrix inequalities (LMIs). Compared with some previous criteria, the proposed ones give the results with less conservatism and lower numerical complexity. Two numerical examples are included to illustrate the effectiveness and the improvement of the proposed method.

1. Introduction

Over the last 30 years, time delay system has been one of the hottest research areas in control engineering for time delay often appears in many control systems either in the state, the control input, or the measurements [1, 2]. Since time delay frequently occurs in practical systems and is often the source of instability, there have been many results for stability of delayed systems [3–9]. Also stabilization [10–12], filtering [13], and adaptive control [14] of time-delay systems have received considerable attention. The neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar [15]. A considerable number of studies related to this topic have been reported (see, for example, [16–21], and the references therein).

Lur'e system is originally from a pilot robot [22], which is one of the important classes of nonlinear systems whose nonlinear element satisfies certain sector constraints. Many systems such as Chua's circuits, Goodwin models, Swarm models, n-scroll attractors, and hyperchaotic attractors can be represented as Lur'e-type systems [23, 24]. In recent years, stability analysis of the neutral-type Lur'e system with time delays has attracted the attention of many researchers [25–35]. The main purpose of stability analysis is to calculate the maximum allowable delay bounds (MADBs), which is a key index for judging the conservatism of stability criteria, such that the Lur'e system maintains absolute stability for any time delay less than the MADBs by using the LKF method. In [25], absolute stability of Lur'e systems with sector-bounded nonlinearities and constant delay was discussed by using a Lur'e-Postnikov function. To avoid involving a considerable number of free-weighting matrices and leading to a computationally expensive stability criterion in [26], the general free weighting matrix method was proposed and some improved delay-range-dependent stability criteria were obtained [27, 28]. By using integral inequality and the Wirtinger integral inequality method instead of free weighting matrix one, some improved delay-dependent robust stability criteria were derived in [29, 30], respectively. Also, by eliminating nonlinearity and reducing the number of free-weighting matrices, Lur'e systems with interval time-varying delays were discussed in [31]. By discretizing the delay interval into two segmentations with an unequal width, some delay-dependent sufficient conditions were given for the robust stability of neutral-type Lur'e systems in [32, 33]. By employing an augmented LKF method, reciprocally convex combination approach, and convex combination technique, several robust absolute

stability criteria for uncertain neutral-type Lur'e systems with time-varying delays were presented in [34]. Very recently, by constructing a LKF including both double-integral terms and triple-integral terms, using the piecewise analysis method, Wirtinger-based integral inequality, and the reciprocally convex combination technique, some new stability criteria were obtained in [35]. In fact, one of the term $\int_{(t-\tau_M)/2}^{t} \varrho_1^T(t)Q\varrho_1(t)\mathrm{d}s$ in LKFs in [35] is the special case of the delay-partitioning approach, where $\varrho_1(t)^T = [x^T(t), x^T((t-\tau_M)/2)]^T$. The delay-partitioning method is an effective one to reduce a criterion's conservativeness, which is widely used in the stability analysis for various systems (see details in [36–39]). However, when utilizing the delay-partitioning approach, the term $\int_{t-\tau_i}^{t} \varrho_2(t)^T Q\varrho_2(t)\mathrm{d}s$ is inevitably involved in LKFs, where $\varrho_2(t)^T = [x^T(t), x^T(t-\tau_1), \ldots, x^T(t-\tau_{N-1})]^T$, $\tau_i = (d/N)$ and N is the delay-partitioning number. It is clear that the derived conditions become more complicated and the computational burden grows bigger when N increases. In addition, to relate to the Wirtinger-based integral inequality and deal with the derivative of triple-integral terms introduced in LKFs, it is ineluctable that the extra terms $\int_{(t-\tau_M)/2}^{t} x^T(s)Q\mathrm{d}s$, $\int_{t-\tau_M}^{(t-\tau_M)/2} x^T(s)Q\mathrm{d}s$, $\int_{-\tau_M/2}^{0}\int_{t+\theta}^{t} x^T(s)Q\mathrm{d}s\mathrm{d}\theta$, and $\int_{-\tau_M}^{-\tau_M/2}\int_{t+\theta}^{t} x^T(s)Q\mathrm{d}s\mathrm{d}\theta$ had to be introduced into the derivation process in [35], which leads to a sharp increase in the dimensions of the LMIs involved.

Motivated by this mentioned above, the aim of this work is to revisit the stability analysis for the neutral-type Lur'e system. In this study, novel delay-decomposing approaches are proposed firstly. Different from the delay-partitioning approaches or the delay-decomposing ones in [32, 33, 36–40], the interval of the state time delay $[0, \tau_M]$ is divided into three unequal subintervals $[0, \tau_\alpha]$, $[\tau_\alpha, \tau_\beta]$, and $[\tau_\beta, \tau_M]$. In particular, to establish the relationship of the vectors such as $x(t), x(t-\tau_\alpha), x(t-\tau_\beta)$, and $x(t-\tau_M)$, the novel terms $\int_{t-\tau_\alpha}^{t} \varrho_3^T(t)Q\varrho_3(t)\mathrm{d}s$ and $\int_{t-\tau_\beta}^{t} \varrho_4^T(t)Q\varrho_4(t)\mathrm{d}s$ are introduced in LKFs in each of subintervals, which are more general than the ones in [28, 35], where $\varrho_3^T(t) = [x^T(t), x^T(t-\tau_\beta)]$ and $\varrho_4^T(t) = [x^T(t), x^T(t-\tau_\alpha)]$. It is worth mentioning that the merit of the proposed delay-decomposing method lies in that the dimensions of the LMIs involved are independent of the number of subinterval. In addition, to avoid introducing the extra vectors by Wirtinger-based integral inequality, the reciprocally convex combination method [41] and the integral inequality [42] are utilized to deal with the bounds of integral terms. Some novel LKFs related to the above inequalities are constructed on the obtained three subintervals. The presented stability criteria are given in terms of LMIs. Compared with the related literature, the conclusions of this paper have the advantages of less conservatism conservatism and the dimensions of the LMIs. Finally, two well-known numerical examples are given to demonstrate the effectiveness and less conservatism over the existing results.

Notation: in this paper, $\mathbb{R}^n$ denotes n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative). $\mathrm{diag}\{\cdots\}$ denotes the block diagonal matrix. The subscript "T" denotes the transpose of the matrix. I_n denotes the identity matrix.

2. Problem Statements and Preliminaries

Consider a class of Lur'e systems of neutral type with time-varying delays and sector-bound nonlinearities described as follows:

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-d(t)) = Ax(t) + Bx(t-\tau(t)) + Df(\sigma(t)), \\ \sigma(t) = H^T x(t), \quad t \geq 0, \\ x(s) = \phi(s), \\ \dot{x}(s) = \dot{\phi}(s), \\ s \in [-\max(\tau_M, d_M), 0], \end{cases} \tag{1}$$

where A, B, C, and D are constant matrices with appropriate dimensions. $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector, and $\sigma(t) = [\sigma_1(t), \sigma_2(t), \ldots, \sigma_m(t)]^T \in \mathbb{R}^m$ is the output vector. The time delay $\tau(t)$ is a time-varying continuous function satisfying

$$\begin{aligned} 0 \leq \tau(t) \leq \tau_M, \\ \dot{\tau}(t) \leq \mu_\tau, \\ 0 \leq d(t) \leq d_M, \\ 0 < \dot{d}(t) \leq \mu_d, \end{aligned} \tag{2}$$

where d_M, τ_M, μ_τ, and μ_d are the known constant scalars.

$f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_m(\cdot)]^T \in \mathbb{R}^m$ is the nonlinear function and $f_i(\sigma_i(t))\cdot(i = 1, 2, \ldots, m)$ is assumed to satisfy the finite sector restriction:

$$\begin{aligned} f_i(\sigma_i(t)) \in K_{[0,k_i]} = \{ & f_i(\sigma_i(t)) \mid f_i(0) = 0, 0 < \sigma(t) f_i(\sigma_i(t)) \\ & \leq k_i \sigma^2(t), \sigma_i(t) \neq 0\}, \end{aligned} \tag{3}$$

with known positive scalar k_i or the infinite sector restriction:

$$\begin{aligned} f_i(\sigma_i(t)) \in K_{[0,\infty]} = \{ & f_i(\sigma_i(t)) \mid f_i(0) \\ & = 0, \sigma(t) f_i(\sigma_i(t)) > 0, \sigma_i(t) \neq 0\}. \end{aligned} \tag{4}$$

The objective of this paper is to formulate the delay-dependent stability conditions of system (1). The following lemmas will play important roles in deriving the criteria.

Lemma 1 (see [41]). *Let $f_1, f_2, \ldots, f_N : \mathbb{R}^m \longmapsto \mathbb{R}$ have positive values in an open subset D of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\left\{\alpha_i \mid \alpha_i>0, \sum_i \alpha_i=1\right\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t), \tag{5}$$

subject to

$$\left\{ g_{i,j} : R^m \longrightarrow R, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}. \tag{6}$$

Lemma 2 (see [42]). *For any matrices $Q>0, M, N, X$ with compatible dimensions, any continuous time vector function $x(t)$ and $\eta(t)$ with compatible dimensions, and any scalar τ_M satisfying $0 \leq \tau(t) \leq \tau_M$, the following integral inequality holds:*

$$-\int_{t-\tau_M}^{t} x^{\mathrm{T}}(s) Q x(s) \mathrm{d}s \leq \tau_M \eta^{\mathrm{T}}(t) X \eta(t) + 2\eta^{\mathrm{T}}(t) \cdot \left\{ M \int_{t-\tau_M}^{t-\tau(t)} x(s) \mathrm{d}s + N \int_{t-\tau(t)}^{t} x(s) \mathrm{d}s \right\}, \tag{7}$$

if $\begin{bmatrix} Q & M^T \\ M & X \end{bmatrix} \geq 0$ *and* $\begin{bmatrix} Q & N^T \\ N & X \end{bmatrix} \geq 0$.

Remark 1. Let $M = N$ and $\eta^T(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-\tau_M)]$, the integral inequality reduces the one in [43].

3. Main Result

In this section, some new delay-dependent stability criteria are proposed for system (1).

Note $\tau_\alpha = \alpha\tau_M$ and $\tau_\beta = \beta\tau_M$ $(0 \leq \alpha \leq 0.5, \beta + \alpha = 1)$. It is easy to see that $0 \leq \tau_\alpha \leq \tau_\beta \leq \tau_M$ holds. Then, we divide the time interval $[0, \tau_M]$ into three subintervals $\Delta_1 =: [0, \tau_\alpha]$, $\Delta_2 =: [\tau_\alpha, \tau_\beta]$, and $\Delta_3 =: [\tau_\beta, \tau_M]$. In the following, we will propose some criteria for the three subintervals.

Now, we give the stability criteria for system (1) with conditions (2) and (4) when $\tau(t) \in \Delta_1$ as follows.

Theorem 1. *For given scalars $0 \leq \alpha \leq 0.5$, $\tau_M \geq 0$, $0 \leq \mu_d < 1$, $\mu_\tau \geq 0$, and $k_i > 0$, $(i = 1, 2, \ldots, m)$, system (1) with conditions (2) and (4) is absolutely stable if there exist $n \times n$ symmetric positive definite matrices $P, S_1, S_2, R_1, R_2, R_3$, $2n \times 2n$ symmetric positive definite matrices Q_1, Q_2, $2n \times 2n$ symmetric semi-positive definite matrices Y_{11}, Z_{11}, positive diagonal matrices $U = diag\{u_1, u_2, \ldots, u_m\}$, $L = diag\{l_1, l_2, \ldots, l_m\}$, any $n \times n$ matrix X_{12}, and $2n \times n$ matrices Y_{12}, Z_{12}, such that LMIs (5) and (6) hold when $\tau(t) \in \Delta_1$:*

$$\Xi_1 + \Xi_1^T + \Xi_2 + \Xi_3 + \Xi_4 + \Xi_4^T < 0, \tag{8}$$

$$\Phi_i \geq 0, \quad i = 1, 2, 3, \tag{9}$$

where

$$\begin{aligned}
\Phi_1 &= \begin{bmatrix} R_1 & X_{12} \\ X_{12}^T & R_1 \end{bmatrix}, \\
\Phi_2 &= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & R_2 \end{bmatrix}, \\
\Phi_3 &= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & R_3 \end{bmatrix}, \\
\Xi_1 &= \Xi_{10} + [e_3, e_4] Y_{12} \left(e_3^T - e_4^T\right) + \left([e_4, e_5] Z_{12} \left(e_4^T - e_5^T\right)\right)^T, \\
\Xi_{10} &= e_1 P e_8^T + e_7 L H^T e_8^T, \\
\Xi_{20} &= e_1 S_1 e_1^T - (1-\mu_\tau) e_2 S_1 e_2^T - (1-\mu_d) e_6 S_2 e_6^T \\
&\quad + [e_1, e_4] Q_1 [e_1, e_4]^T - [e_3, e_5] Q_1 [e_3, e_5]^T \\
&\quad + [e_1, e_3] Q_2 [e_1, e_3]^T - [e_4, e_5] Q_2 [e_4, e_5]^T, \\
\Xi_2 &= \Xi_{20} - 2 e_7 U e_7^T, \\
\Xi_3 &= e_8 \left[S_2 + \tau_\alpha^2 R_1 + \left(\tau_\beta - \tau_\alpha\right) R_2 + \left(\tau_M - \tau_\beta\right) R_3\right] e_8^T \\
&\quad - [e_2 - e_1, e_3 - e_2] \Phi_1 [e_2 - e_1, e_3 - e_2]^T \\
&\quad + \left(\tau_\beta - \tau_\alpha\right) [e_3, e_4] Y_{11} [e_3, e_4]^T + \left(\tau_M - \tau_\beta\right) \\
&\quad \cdot [e_4, e_5] Z_{11} [e_4, e_5]^T, \\
\Xi_4 &= e_1 \mathrm{HKU} e_7^T, \\
e_8 &= e_1 A^T + e_5 B^T + e_6 C^T + e_7 D^T, \\
K &= \mathrm{diag}\{k_1, k_2, \ldots, k_m\}, \\
e_1 &= [I_n, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times m}]^T, \\
e_2 &= [0_{n\times n}, I_n, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times m}]^T, \\
e_3 &= [0_{n\times n}, 0_{n\times n}, I_n, 0_{n\times n}, 0_{n\times n}, 0_{n\times m}]^T, \\
e_4 &= [0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, I_n, 0_{n\times m}]^T, \\
e_5 &= [0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, I_n, 0_{n\times m}]^T, \\
e_6 &= [0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times n}, 0_{n\times m}, I_n]^T, \\
e_7 &= [0_{m\times n}, 0_{m\times n}, 0_{m\times n}, 0_{m\times n}, 0_{m\times n}, I_m]^T.
\end{aligned} \tag{10}$$

Proof. For positive diagonal matrices $L = \mathrm{diag}\{l_1, l_2, \ldots, l_m\}$ and positive definite matrices P, R_j, S_i, and $Q_i \cdot (i = 1, 2, j = 1, 2, 3)$, let us consider the following LKF candidates for the case $0 \leq \tau(t) \leq \tau_\alpha$:

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)), \tag{11}$$

where

$$\begin{aligned}
V_1(x(t)) &= x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(s)S_1x(s)\mathrm{d}s \\
&\quad + \int_{t-d(t)}^{t} \dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s + 2\sum_{i=1}^{m} l_i \int_0^{\sigma_i} f_i(\sigma)\mathrm{d}\sigma, \\
V_2(x(t)) &= \int_{t-\tau_\alpha}^{t} \begin{bmatrix} x(s) \\ x(s-\tau_\beta) \end{bmatrix}^T Q_1 \begin{bmatrix} x(s) \\ x(s-\tau_\beta) \end{bmatrix} \mathrm{d}s \\
&\quad + \int_{t-\tau_\beta}^{t} \begin{bmatrix} x(s) \\ x(s-\tau_\alpha) \end{bmatrix}^T Q_2 \begin{bmatrix} x(s) \\ x(s-\tau_\alpha) \end{bmatrix} \mathrm{d}s, \\
V_3(x(t)) &= \tau_\alpha V_{31}(x(t)) + V_{32}(x(t)) + V_{33}(x(t)), \\
V_{31}(x(t)) &= \int_{-\tau_a}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta, \\
V_{32}(x(t)) &= \int_{-\tau_\beta}^{-\tau_\alpha} \int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta, \\
V_{33}(x(t)) &= \int_{-\tau_M}^{-\tau_\alpha} \int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta.
\end{aligned} \tag{12}$$

The time derivative of $V(x(t))$ along the trajectory of system (4) is given by

$$\begin{aligned}
\dot{V}_1(x(t)) &\le 2x^T(t)P\dot{x}(t) + 2f^T(x(t))\mathrm{LH}^T\dot{x}(t) + x^T(t)S_1x(t) \\
&\quad + \dot{x}^T(t)S_2\dot{x}(t) - (1-\mu_\tau)x(t-\tau(t))^T(t) \\
&\quad \cdot S_1x(t-\tau(t)) - (1-\mu_d)\dot{x}(t-d(t))^T(t)S_2\dot{x}(t-d(t)) \\
&= \xi^T(t)\left[e_1Pe_8^T + e_8Pe_1^T + e_1S_1e_1^T - (1-\mu_\tau)e_2S_1e_2^T\right. \\
&\quad \left. - (1-\mu_d)e_6S_2e_6^T + e_7LH^Te_8^T + e_8HL^Te_7^T + e_8S_2e_8^T\right]\xi(t),
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\xi^T(t) = \big[&x^T(t), x^T(t-\tau(t)), x^T(t-\tau_\alpha), x^T(t-\tau_\beta), \\
&x^T(t-\tau_M), \dot{x}^T(t-d(t)), f^T(\sigma(t))\big],
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{V}_2(x(t)) &= \begin{bmatrix} x(t) \\ x(t-\tau_\beta) \end{bmatrix}^T Q_1 \begin{bmatrix} x(t) \\ x(t-\tau_\beta) \end{bmatrix} \\
&\quad - \begin{bmatrix} x(t-\tau_\alpha) \\ x(t-\tau_M) \end{bmatrix}^T Q_1 \begin{bmatrix} x(t-\tau_\alpha) \\ x(s-\tau_M) \end{bmatrix} \\
&\quad + \begin{bmatrix} x(t) \\ x(t-\tau_\alpha) \end{bmatrix}^T Q_2 \begin{bmatrix} x(t) \\ x(t-\tau_\alpha) \end{bmatrix} \\
&\quad - \begin{bmatrix} x(t-\tau_\beta) \\ x(t-\tau_M) \end{bmatrix}^T Q_2 \begin{bmatrix} x(t-\tau_\beta) \\ x(s-\tau_M) \end{bmatrix} \\
&= \xi^T(t)\big\{[e_1,e_4]Q_1[e_1,e_4]^T - [e_3,e_5]Q_1[e_3,e_5]^T \\
&\quad + [e_1,e_3]Q_2[e_1,e_3]^T - [e_4,e_5]Q_2[e_4,e_5]^T\xi(t)\big\},
\end{aligned} \tag{15}$$

$$\begin{aligned}
\dot{V}_3(x(t)) &= \dot{x}^T(t)\left[\tau_\alpha^2R_1 + (\tau_\beta-\tau_\alpha)R_2 + (\tau_M-\tau_\beta)R_3\right]\dot{x}(t) \\
&\quad - \tau_\alpha\int_{t-\tau_\alpha}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s - \int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\
&\quad - \int_{t-\tau_M}^{t-\tau_\beta} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s.
\end{aligned} \tag{16}$$

If $0 \le \tau(t) \le \tau_\alpha$ holds, one can compute out the following according to Lemma 1:

$$\begin{aligned}
&-\tau_\alpha\int_{t-\tau_\alpha}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s \\
&= -\tau_\alpha\int_{t-\tau(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s - \tau_\alpha\int_{t-\tau_\alpha}^{t-\tau(t)} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s \\
&\le -\begin{bmatrix} x(t-\tau(t)) - x(t) \\ x(t-\tau_\alpha) - x(t-\tau(t)) \end{bmatrix}^T \Phi_1 \begin{bmatrix} x(t-\tau(t)) - x(t) \\ x(t-\tau_\alpha) - x(t-\tau(t)) \end{bmatrix} \\
&= -\xi^T(t)[e_2-e_1, e_3-e_2]\Phi_1[e_2-e_1, e_3-e_2]^T\xi(t),
\end{aligned} \tag{17}$$

where Φ_1 is given in (6).

Based on Lemma 2 (in which $M = N = Y_{12}$ and $\eta^T(t) = [x^T(t-\tau_\alpha), x^T(t-\tau_\beta)]$), when $\Phi_2 = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^t & R_2 \end{bmatrix} \ge 0$, the following inequality holds:

$$\begin{aligned}
&-\int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\
&\le (\tau_\beta-\tau_\alpha)\begin{bmatrix} x(t-\tau_\alpha) \\ x(t-\tau_\beta) \end{bmatrix}^T Y_{11} \begin{bmatrix} x(t-\tau_\alpha) \\ x(t-\tau_\beta) \end{bmatrix} \\
&\quad + 2\begin{bmatrix} x(t-\tau_\alpha) \\ x(t-\tau_\beta) \end{bmatrix}^T Y_{12}\left[x(t-\tau_\alpha) - x(t-\tau_\beta)\right] \\
&= \xi^T(t)\Big\{(\tau_\beta-\tau_\alpha)[e_3,e_4]Y_{11}[e_3,e_4]^T + [e_3,e_4] \\
&\quad \cdot Y_{12}(e_3^T - e_4^T) + \left([e_3,e_4]Y_{12}(e_3^T - e_4^T)\right)^T\Big\}\xi(t),
\end{aligned} \tag{18}$$

and using Lemma 2 again (in which $M = N = Z_{12}$ and $\eta^T(t) = [x^T(t-\tau_\beta), x^T(t-\tau_M)]$), when $\Phi_3 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^t & R_3 \end{bmatrix} \ge 0$, one can conclude

$$\begin{aligned}
&-\int_{t-\tau_M}^{t-\tau_\beta} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s \\
&\le (\tau_M-\tau_\beta)\begin{bmatrix} x(t-\tau_\beta) \\ x(t-\tau_M) \end{bmatrix}^T Z_{11} \begin{bmatrix} x(t-\tau_\beta) \\ x(t-\tau_M) \end{bmatrix} \\
&\quad + 2\begin{bmatrix} x(t-\tau_\beta) \\ x(t-\tau_M) \end{bmatrix}^T Z_{12}\left[x(t-\tau_\beta) - x(t-\tau_M)\right] \\
&= \xi^T(t)\Big\{(\tau_M-\tau_\beta)[e_4,e_5]Z_{11}[e_4,e_5]^T \\
&\quad + [e_4,e_5]Z_{12}(e_4^T - e_5^T) + \left([e_4,e_5]Z_{12}(e_4^T - e_5^T)\right)^T\Big\}\xi(t).
\end{aligned} \tag{19}$$

Under the assumption on nonlinear function (4), the following inequality holds

$$u_i f_i(\sigma(t))\left[k_i h_i^T x(t) - f_i(\sigma_i(t))\right] \geq 0, \quad i = 1, 2, \ldots, m. \tag{20}$$

It is equivalent to

$$2\left[x^T(t)\mathrm{HKU} f(\sigma(t)) - f^T(\sigma(t)) U f(\sigma(t))\right] \geq 0. \tag{21}$$

Rewrite the above as follows:

$$2\left[e_1 \mathrm{HKU} e_7^T - e_7 U e_7^T\right] \geq 0, \tag{22}$$

where

$$\begin{aligned} U &= \mathrm{diag}\{u_1, u_2, \ldots, u_m\}, \\ K &= \mathrm{diag}\{k_1, k_2, \ldots, k_m\}. \end{aligned} \tag{23}$$

Then, combining (3)–(15) leads to

$$\dot{V}(x(t)) \leq \xi^T(t)\left(\Xi_1 + \Xi_1^T + \Xi_2 + \Xi_3 + \Xi_4 + \Xi_4^T\right)\xi(t). \tag{24}$$

Therefore, if LMIs (5) and (6) hold, one has $\dot{V}(x(t)) < 0$, which shows the absolute stability of system (1) subject to (2) and (4), when $\tau(t)$ satisfies C_1. This completes our proof.

For the case $\tau(t) \in \Delta_2$, we construct the LKFs as follows:

$$\overline{V}(x(t)) = V_1(x(t)) + V_2(x(t)) + \overline{V}_3(x(t)), \tag{25}$$

where

$$\overline{V}_3(x(t)) = V_{31}(x(t)) + \left(\tau_\beta - \tau_\alpha\right) V_{32}(x(t)) + V_{33}(x(t)), \tag{26}$$

and $V_1(x(t)), V_2(x(t)), V_{31}(x(t))$, and $V_{33}(x(t))$ are defined in (7).

Then, the derivative of $\overline{V}_3(x(t))$ can be obtained as

$$\begin{aligned} \dot{\overline{V}_3}(x(t)) &= \dot{x}^T(t)\left[\tau_\alpha R_1 + \left(\tau_\beta - \tau_\alpha\right)^2 R_2 + \left(\tau_M - \tau_\beta\right) R_3\right]\dot{x}(t) \\ &\quad - \int_{t-\tau_\alpha}^{t} \dot{x}^T(s) R_1 \dot{x}(s)\mathrm{d}s - \left(\tau_\beta - \tau_\alpha\right)\int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s) R_2 \dot{x}(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_M}^{t-\tau_\beta} \dot{x}^T(s) R_3 \dot{x}(s)\mathrm{d}s. \end{aligned} \tag{27}$$

According to Lemma 1, if $\tau_\alpha \leq \tau(t) \leq \tau_\beta$, the following inequality holds:

$$\begin{aligned} &-\left(\tau_\beta - \tau_\alpha\right)\int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s) R_2 \dot{x}(s)\mathrm{d}s \\ &\quad = -\left(\tau_\beta - \tau_\alpha\right)\int_{t-\tau(t)}^{t-\tau_\alpha} \dot{x}^T(s) R_2 \dot{x}(s)\mathrm{d}s, \\ &\qquad -\left(\tau_\beta - \tau_\alpha\right)\int_{t-\tau_\beta}^{t-\tau(t)} \dot{x}^T(s) R_2 \dot{x}(s)\mathrm{d}s \\ &\quad \leq -\begin{bmatrix} x(t-\tau(t)) - x(t-\tau_\alpha) \\ x(t-\tau_\beta) - x(t-\tau(t)) \end{bmatrix}^T \\ &\qquad \cdot \overline{\Phi}_2 \begin{bmatrix} x(t-\tau(t)) - x(t-\tau_\alpha) \\ x(t-\tau_\beta) - x(t-\tau(t)) \end{bmatrix} \\ &\quad = -\xi^T(t)\left[e_2 - e_3, e_4 - e_2\right]\overline{\Phi}_2\left[e_2 - e_3, e_4 - e_2\right]^T \xi(t), \end{aligned} \tag{28}$$

where $\overline{\Phi}_2$ is given in (20).

In addition, by using Lemma 2 (in which $M = N = X_{12}$ and $\eta^T(t) = [x^T(t), x^T(t-\tau_\alpha)]$), if $\overline{\Phi}_1 = \begin{bmatrix} X_{11} & X_{12}^t \\ X_{12} & R_1 \end{bmatrix} \geq 0$, one can obtain

$$\begin{aligned} &-\int_{t-\tau_\alpha}^{t} \dot{x}^T(s) R_1 \dot{x}(s)\mathrm{d}s \\ &\quad \leq \tau_\alpha \begin{bmatrix} x(t) \\ x(t-\tau_\alpha) \end{bmatrix}^T X_{11} \begin{bmatrix} x(t) \\ x(t-\tau_\alpha) \end{bmatrix} \\ &\qquad + 2\begin{bmatrix} x(t) \\ x(t-\tau_\alpha) \end{bmatrix}^T X_{12}\left[x(t) - x(t-\tau_\alpha)\right] \\ &\quad = \xi^T(t)\left\{\tau_\alpha\left[e_1, e_3\right] X_{11}\left[e_1, e_3\right]^T + \left[e_1, e_3\right] X_{12}\left(e_1^T - e_3^T\right)\right. \\ &\qquad \left. + \left(\left[e_1, e_3\right] X_{12}\left(e_1^T - e_3^T\right)\right)^T\right\}\xi(t). \end{aligned} \tag{29}$$

The other procedure is straightforward from the proof of Theorem 1; we can cope with the situation of $\tau(t) \in \Delta_2$ and have the following. □

Theorem 2. *For given scalars $0 \leq \alpha \leq 0.5, \tau_M \geq 0, 0 \leq \mu_d < 1$, $\mu_\tau \geq 0$, and $k_i > 0$ ($i = 1, 2, \ldots, m$), system (1) with conditions (2) and (4) is absolutely stable if there exist $n \times n$ symmetric positive definite matrices $P, S_1, S_2, R_1, R_2, R_3$, $2n \times 2n$ symmetric positive definite matrices Q_1, Q_2, $2n \times 2n$ symmetric semi-positive definite matrices X_{11}, Z_{11}, positive diagonal matrices $U = diag\{u_1, u_2, \ldots, u_m\}$, $L = diag\{l_1, l_2, \ldots, l_m\}$, any $n \times n$ matrix Y_{12}, and $2n \times n$ matrices X_{12}, Z_{12}, such that LMIs (19) and (20) hold for the case $\tau(t) \in \Delta_2$:*

$$\overline{\Xi}_1 + \overline{\Xi}_1^T + \Xi_2 + \overline{\Xi}_3 + \Xi_4 + \Xi_4^T < 0, \tag{30}$$

$$\begin{aligned} \overline{\Phi}_1 &\geq 0, \\ \overline{\Phi}_2 &\geq 0, \\ \Phi_3 &\geq 0, \end{aligned} \tag{31}$$

where

$$\begin{aligned} \overline{\Phi}_1 &= \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^t & R_1 \end{bmatrix}, \\ \overline{\Phi}_2 &= \begin{bmatrix} R_2 & Y_{12} \\ Y_{12}^t & R_2 \end{bmatrix}, \\ \overline{\Xi}_1 &= \Xi_{10} + \left[e_1, e_3\right] X_{12}\left(e_1^T - e_3^T\right) \\ &\quad + \left[e_4, e_5\right] Z_{12}\left(e_4^T - e_5^T\right), \\ \overline{\Xi}_3 &= e_8\left[S_2 + \tau_\alpha R_1 + \left(\tau_\beta - \tau_\alpha\right)^2 R_2 + \left(\tau_M - \tau_\beta\right) R_3\right] e_8^T \\ &\quad - \left[e_2 - e_3, e_4 - e_2\right]\overline{\Phi}_2\left[e_2 - e_3, e_4 - e_2\right]^T \\ &\quad + \tau_\alpha\left[e_1, e_3\right] X_{11}\left[e_1, e_3\right]^T \\ &\quad + \left(\tau_M - \tau_\beta\right)\left[e_4, e_5\right] Z_{11}\left[e_4, e_5\right]^T \end{aligned} \tag{32}$$

and $\Phi_3, \Gamma_{10}, \Gamma_2$, and Γ_4 (e_i, $i = 1, 2, \ldots, 8$) and K are defined in Theorem 1.

For the case $\tau(t) \in \Delta_3$, we construct the following LKF:

$$\widehat{V}(x(t)) = V_1(x(t)) + V_2(x(t)) + \widehat{V}_3(x(t)), \tag{33}$$

where

$$\widehat{V}_3(x(t)) = V_{31}(x(t)) + V_{32}(x(t)) + \left(\tau_M - \tau_\beta\right)V_{33}(x(t)) \tag{34}$$

and $V_1(x(t)), V_2(t), V_{31}(t)$, and $V_{32}(t)$ are defined in (7).

Taking the derivative of $\widehat{V}_3(x(t))$ yields

$$\begin{aligned}\dot{\widehat{V}}_3(x(t)) &= \dot{x}^T(t)\left[\tau_\alpha R_1 + \left(\tau_\beta - \tau_\alpha\right)R_2 + \left(\tau_M - \tau_\beta\right)^2 R_3\right]\dot{x}(t) \\ &\quad - \int_{t-\tau_\alpha}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s - \left(\tau_M - \tau_\beta\right) \\ &\quad \cdot \int_{t-\tau_M}^{t-\tau_\beta} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s.\end{aligned} \tag{35}$$

By using Lemma 1, if $\tau_\beta \le \tau(t) \le \tau_M$, one can obtain

$$\begin{aligned}&-\left(\tau_M - \tau_\beta\right)\int_{t-\tau_M}^{t-\tau_\beta} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s \\ &\quad = -\left(\tau_M - \tau_\beta\right)\int_{t-\tau(t)}^{t-\tau_\beta} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s - \left(\tau_\beta - \tau_\alpha\right) \\ &\qquad \cdot \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s \\ &\quad \le -\begin{bmatrix} x(t-\tau(t)) - x\left(t-\tau_\beta\right) \\ x\left(t-\tau_M\right) - x(t-\tau(t))\end{bmatrix}^T \widehat{\Phi}_3 \\ &\qquad \cdot \begin{bmatrix} x(t-\tau(t)) - x\left(t-\tau_\beta\right) \\ x\left(t-\tau_M\right) - x(t-\tau(t))\end{bmatrix} \\ &\quad = -\xi^T(t)\left[e_2 - e_4, e_5 - e_2\right]\widehat{\Phi}_3\left[e_2 - e_4, e_5 - e_2\right]^T\xi(t),\end{aligned} \tag{36}$$

where $\widehat{\Phi}_3 = \begin{bmatrix} R_3 & Z_{12}^t \\ Z_{12} & R_3\end{bmatrix}$.

Handle the integral terms $-\int_{t-\tau_\alpha}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds$ and $-\int_{t-\tau_\beta}^{t-\tau_\alpha} \dot{x}^T(s)R_2\dot{x}(s)ds$ in the same way as in (13) and (17), respectively. Following the same procedure, we can cope with the situation of $\tau(t) \in \Delta_3$ and have the following.

Theorem 3. *For given scalars $0 \le \alpha \le 0.5, \tau_M \ge 0, 0 \le \mu_d < 1$, $\mu_\tau \ge 0$, and $k_i > 0$ ($i = 1, 2, \ldots, m$), system (1) with conditions (2) and (4) is absolutely stable if there exist $n \times n$ symmetric positive definite matrices $P, S_1, S_2, R_1, R_2, R_3$, $2n \times 2n$ symmetric positive definite matrices Q_1, Q_2, $2n \times 2n$ symmetric semi-positive definite matrices X_{11}, Y_{11}, positive diagonal matrices $U = diag\{u_1, u_2, \ldots, u_m\}$, $L = diag\{l_1, l_2, \ldots, l_m\}$, any $n \times n$ matrices Z_{12}, and $2n \times n$ matrices X_{12}, Y_{12}, such that LMIs (23) and (24) hold for the case $\tau(t) \in \Delta_3$:*

$$\widehat{\Xi}_1 + \widehat{\Xi}_1^T + \Xi_2 + \widehat{\Xi}_3 + \Xi_4 + \Xi_4^T < 0, \tag{37}$$

$$\begin{aligned}\overline{\Phi}_1 &\ge 0, \\ \Phi_2 &\ge 0, \\ \widehat{\Phi}_3 &\ge 0,\end{aligned} \tag{38}$$

where

$$\begin{aligned}\widehat{\Phi}_3 &= \begin{bmatrix} R_3 & Z_{12} \\ Z_{12}^t & R_3\end{bmatrix}, \\ \widehat{\Xi}_3 &= e_8\left[S_2 + \tau_\alpha R_1 + \left(\tau_\beta - \tau_\alpha\right)R_2 + \left(\tau_M - \tau_\beta\right)^2 R_3\right]e_8^T \\ &\quad + \tau_\alpha\left[e_1, e_3\right]X_{11}\left[e_1, e_3\right]^T \\ &\quad + \left(\tau_\beta - \tau_\alpha\right)\left[e_3, e_4\right]Y_{11}\left[e_3, e_4\right]^T, \\ \widehat{\Xi}_1 &= \Xi_{10} + \left[e_1, e_3\right]X_{12}\left(e_1^T - e_3^T\right) \\ &\quad + \left[e_3, e_4\right]Y_{12}\left(e_3^T - e_4^T\right),\end{aligned} \tag{39}$$

and $\Phi_2, \Gamma_{10}, \Gamma_2, \Gamma_4, e_i$ ($i = 1, 2, \ldots, 8$), K, and $\overline{\Phi}_1$ are defined in Theorems 1 and 2, respectively.

Remark 2. Theorems 1–3 present the absolute stability criteria for the Lur'e system. Unlike the delay-partitioning approach or delay-decomposing one used in [28, 32, 33, 35, 40], the interval of the state time delay $[0, \tau_M]$ has been divided into three subintervals $[0, \tau_\alpha]$, $[\tau_\alpha, \tau_\beta]$, and $[\tau_\beta, \tau_M]$. Obviously, the range of the subintervals $[0, \tau_\alpha]$, $[\tau_\alpha, \tau_\beta]$, and $[\tau_\beta, \tau_M]$ is $\alpha\tau_M$, $(1 - 2\alpha)\tau_M$, and $\alpha\tau_M$. Thus, to know more about the time-varying $\tau(t)$, one can select the value of the parameter α as close as possible to 0 for the case $\tau(t) \in \Delta_1$ or case $\tau(t) \in \Delta_3$ and as close as possible to 0.5 for the case $\tau(t) \in \Delta_2$, respectively. The following examples in the work will illustrate the point.

Remark 3. The terms $\int_{t-\tau_\alpha}^{t} \varrho_3^T(t)Q_1\varrho_3(t)\mathrm{d}s$ and $\int_{t-\tau_\beta}^{t} \varrho_4^T(t)Q_2\varrho_4(t)\mathrm{d}s$ have been introduced in LKFs in the work instead of the terms $\int_{t-\tau_M/2}^{t} \varrho_1^T(t)Q\varrho_1(t)\mathrm{d}s$ in [28, 35] and $\int_{t-\tau_i}^{t} \varrho_2(t)^T Q\varrho_2(t)\mathrm{d}s$ in [36–39], where $\varrho_1(t)^T = [x^T(t), x^T((t - \tau_M)/2)]^T$ and $\varrho_2(t)^T = [x^T(t), x^T(t-\tau_1), \ldots, x^T(t-\tau_{N-1})]^T$. And the relations between $x(t), x(t-\tau_\alpha), x(t-\tau_\beta)$, and $x(t-\tau_M)$ have been effectively expressed by the derivative of $V_2(x(t))$. One can easily see that $\int_{t-\tau_\alpha}^{t} \varrho_3^T(t)Q_1\varrho_3(t)\mathrm{d}s$ or $\int_{t-\tau_\beta}^{t} \varrho_4^T(t)Q_2\varrho_4(t)\mathrm{d}s$ deduces into $\int_{t-\tau_M/2}^{t} \varrho_1^T(t)Q\varrho_1(t)\mathrm{d}s$ when $\alpha = \beta = 0.5$. This is to say the established LKFs are more general than ones in [28, 35]. The idea is expected to reduce the conservatism of obtained criteria. On the other

hand, it can effectively overcome the weakness of the computational burden growing bigger when delay-partitioning number increases in [36–39].

Remark 4. In [35], the Wirtinger-based integral inequality and double-integral inequality were used to deal with bounds of the derivative of double-integral and triple-integral terms in LKFs. Thus, the extra terms $\int_{t-\tau_M/2}^{t} x^T(s)Q\,\mathrm{d}s$, $\int_{t-\tau_M}^{t-\tau_M/2} x^T(s)Q\,\mathrm{d}s$, $\int_{-\tau_M/2}^{0}\int_{t+\theta}^{t} x^T(s)Q\,\mathrm{d}s\,\mathrm{d}\theta$, and $\int_{-\tau_M}^{-\tau_M/2}\int_{t+\theta}^{t} x^T(s)Q\,\mathrm{d}s\,\mathrm{d}\theta$ were unavoidably introduced in the derivation process, which leads the dimensions of the LMIs involved to increase sharply. A detailed comparison is given in Example 1.

Remark 5. If function $f(\sigma(t))$ of system (1) satisfies sector condition (3), for any $u_i \geq 0$, $i = 1, 2, \ldots, m$, one has the following:

$$u_i f_i(\sigma_i(t)) h_i^T x(t) \geq 0, \quad i = 1, 2, \ldots, m, \tag{40}$$

which is equivalent to

$$\begin{gathered} 2x^T(t)\mathrm{HU}f(\sigma(t)) \geq 0 \\ \text{or } 2\xi^T(t)e_1\mathrm{HU}e_7^T\xi(t) \geq 0. \end{gathered} \tag{41}$$

Thus, if function $f(\sigma(t))$ of system (1) satisfies sector condition (3), we have the following corollary.

Corollary 1. *For given scalars $0 \leq \alpha \leq 0.5$, $\tau_M \geq 0$, $0 \leq \mu_d < 1$, $\mu_\tau \geq 0$, $k_i > 0$, $i = 1, 2, \ldots, m$, the system (1) with conditions (2) and (3) is absolutely stable if there exist symmetric positive definite matrices $P, S_1, S_2, R_1, R_2, R_3, Q_1, Q_2, X_{11}, Y_{11}$ with appropriate dimensions, positive diagonal matrices $U = diag\{u_1, u_2, \ldots, u_m\}$, $L = diag\{l_1, l_2, \ldots, l_m\}$, and matrices X_{11}, $Y_{11}, Z_{11}, X_{12}, Y_{12}, Z_{12}$ with appropriate dimensions and properties, such that LMIs (6) and (26) hold for the case $\tau(t) \in \Delta_1$, (3) and (28) hold for the case $\tau(t) \in \Delta_2$, and (20) and (28) hold for the case $\tau(t) \in \Delta_3$, respectively:*

$$\Xi_1 + \Xi_1^T + \Xi_{20} + \Xi_3 + \Xi_{40} + \Xi_{40}^t < 0, \tag{42}$$

$$\overline{\Xi}_1 + \overline{\Xi}_1^T + \Xi_{20} + \overline{\Xi}_3 + \Xi_{40} + \Xi_{40}^t < 0, \tag{43}$$

$$\widehat{\Xi}_1 + \widehat{\Xi}_1^T + \Xi_{20} + \widehat{\Xi}_3 + \Xi_{40} + \Xi_{40}^t < 0, \tag{44}$$

where $\Gamma_{40} = e_1 HUe_7^T$, $\Gamma_1, \overline{\Gamma}_1, \widehat{\Gamma}_1$, $\Gamma_{20}, \Gamma_3, \overline{\Gamma}_3, \widehat{\Gamma}_3$ are defined in Theorems 1–3.

4. Illustrative Example

In this section, we will use two well-known numerical examples to show the effectiveness and benefits of our results.

Example 1. Consider the following nominal neutral-type Lur'e system (1) subject to (2) and (4) with the parameters:

$$\begin{aligned} A &= \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \\ D &= \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}, \\ H &= \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}. \end{aligned} \tag{45}$$

The purpose of this example is to compute MADBs of τ_M such that the neutral-type Lur'e system (1) remains stable for different μ_τ and μ_d. For given $\mu_d = 0.9, \alpha = 0.45$, $\mu_d = 0.5$, $\alpha = 0.45$, and $\mu_d = 0.1, \alpha = 0.4$, the acceptable upper bounds of τ_M is 0.271, 2.277, and 2.652 when $\mu_\tau \geq 1$ by using Corollary 1 ($\tau(t) \in \Delta_1$) in our work, respectively. In order to make a comparison with some existing stability criteria, we calculate the MADBs and list them in Tables 1–3. Together with all derived MADBs listed in Tables 1–3, one can check that Corollary 1 can be superior over some present ones. The number of decision variables, maximal order of LMIs between our work, and the criteria in [29, 30, 33, 35] are listed in Table 4. It shows that our proposed method involves smaller decision variables or lower maximal order of LMIs than the relative ones. To confirm the obtained result, a simulation result is shown in Figure 1 when μ is unknown, $x(0) = [0, 0.4]^T$, $\tau_M = 5.886$, and $h(t) = 0.5 + 0.1\sin t$.

Example 2. Consider Chua's circuit example discussed in [35]:

$$\begin{cases} \dot{\theta}_1(t) = a(\theta_2 - g(\theta_1)), \\ \dot{\theta}_2(t) = \theta_1 - \theta_2 + \theta_3, \\ \dot{\theta}_3(t) = -b\theta_2. \end{cases} \tag{46}$$

The nonlinear function $g(\theta_1) = m_1\theta_1 + 0.5(m_0 - m_1)(|\theta_1 + c| - |\theta_1 - c|)$. Let $m_0 = -1/7, m_1 = 2/7, a = 9, b = 14.28$, and $c = 1$; then, Chua's circuit can be expressed as a Lur'e-type system with

Table 1: MADBs of τ_M for different μ_τ, when $\mu_d = 0.1$ for Example 1.

Method	$\mu_\tau = 0.2$	$\mu_\tau = 0.4$	$\mu_\tau = 0.6$	$\mu_\tau = 0.8$
[44]	2.6052	1.9538	1.6232	1.1659
[31]	2.6062	1.9730	1.6408	1.1854
[27]	2.6488	2.0056	1.6679	1.2049
[30]	2.8890	2.0678	1.7048	1.6059
[32]	2.9962	2.1316	1.7138	1.3204
[28]	2.9997	2.1012	1.7307	1.3359
[29]	3.0717	2.1171	1.7310	1.3367
[33]	3.1034	2.1579	1.7436	1.7088
[34]	3.1743	2.1789	1.7467	1.7153
[35]	3.4880	2.3787	1.8062	1.4625
Cor 1 ($\tau(t) \in \Delta_1, \alpha = 0.45$)	5.886	4.456	3.705	2.676
Cor 1 ($\tau(t) \in \Delta_2, \alpha = 0.3$)	3.784	2.865	2.385	2.176
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.4$)	3.060	2.157	2.000	1.999
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.04$)	3.729	3.729	3.729	3.729

Table 2: MADBs of τ_M for different μ_τ, when $\mu_d = 0.5$ for Example 1.

Method	$\mu_\tau = 0.2$	$\mu_\tau = 0.4$	$\mu_\tau = 0.6$	$\mu_\tau = 0.8$
[29]	2.5131	1.8146	1.4951	1.1168
[44]	2.1532	1.6986	1.3687	1.0066
[31]	2.1537	1.7041	1.3828	1.0095
[27]	2.1883	1.7322	1.4056	1.0262
[30]	2.3628	1.7752	1.5027	1.4364
[32]	2.4472	1.8289	1.4551	1.1058
[28]	2.4563	1.8009	1.4823	1.1157
[33]	2.5302	1.8461	1.5371	1.5208
[35]	2.8538	2.0236	1.5679	1.2425
Cor 1 ($\tau(t) \in \Delta_1, \alpha = 0.45$)	4.862	3.848	3.119	2.280
Cor 1 ($\tau(t) \in \Delta_2, \alpha = 0.3$)	3.126	2.474	2.101	1.922
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.04$)	2.877	2.877	2.877	2.877
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.4$)	2.504	1.847	1.738	1.737

Table 3: MADBs of τ_M for different μ_τ, when $\mu_d = 0.9$ for Example 1.

Method	$\mu_\tau = 0.2$	$\mu_\tau = 0.4$	$\mu_\tau = 0.6$	$\mu_\tau = 0.8$
[31]	0.1122	0.1088	0.1086	0.1086
[27]	0.1122	0.1089	0.1086	0.1086
[32]	0.1126	0.1102	0.1102	0.1102
[28]	0.1130	0.1105	0.1105	0.1105
[44]	0.1221	0.1087	0.1083	0.1083
[29]	0.1227	0.1197	0.1197	0.1197
[35]	0.1799	0.1697	0.1660	0.1658
[30]	0.2495	0.2366	0.2300	0.2285
[33]	0.2906	0.2745	0.2653	0.2621
Cor 1 ($\tau(t) \in \Delta_1, \alpha = 0.35$)	0.321	0.311	0.310	0.310
Cor 1 ($\tau(t) \in \Delta_1, \alpha = 0.4$)	0.281	0.272	0.271	0.271
Cor 1 ($\tau(t) \in \Delta_2, \alpha = 0.4$)	0.189	0.189	0.189	0.189
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.4$)	0.113	0.113	0.113	0.113
Cor 1 ($\tau(t) \in \Delta_3, \alpha = 0.04$)	0.119	0.119	0.119	0.119

Table 4: Number of decision variables (NDV) and maximal order of LMIs (MOL) of different methods.

Method	NDV	MOL
[29] Theorem 1	$12n^2 + 7n + 2m$	$7n + m$
[35] Theorem 10	$19.5n^2 + 6.5n + 2m$	$10n + m$
[30] Theorem 1	$11.5n^2 + 7n + 2m$	$10n + m$
[33] Theorem 1	$20n^2 + 6n + 2m$	$12n + m$
Our Cor 1	$16n^2 + 7n + 2m$	$6n + m$

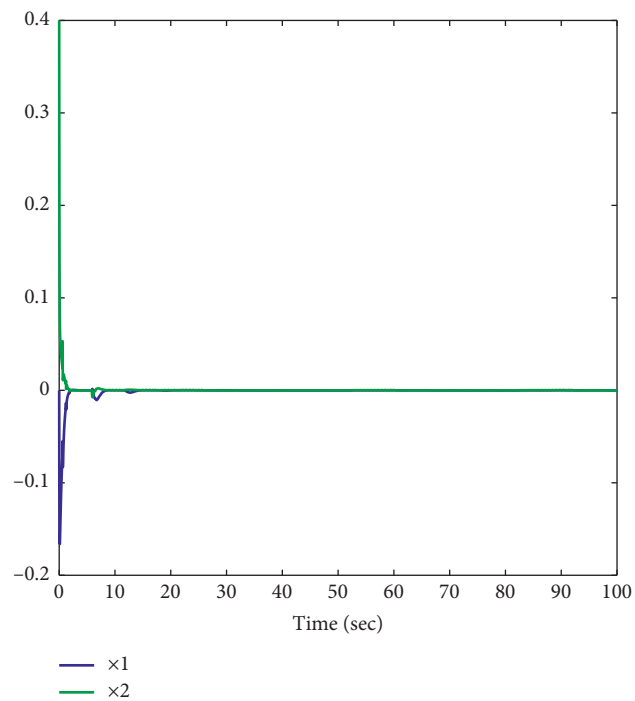

Figure 1: State responses of the system considered in Example 1.

$$A = \begin{bmatrix} -am_1 - 1 & a & 0 \\ 1 & -2 & 1 \\ 0 & -b & -1 \end{bmatrix},$$
$$B = \begin{bmatrix} -6.0029 & 0 & 0 \\ -1.3367 & 0 & 0 \\ 2.1264 & 0 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{47}$$
$$D = \begin{bmatrix} -a(m_0 - m_1) \\ 0 \\ 0 \end{bmatrix},$$
$$H = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The feedback nonlinear function belongs to $K_{[0,1]}$.

Now, we calculate MADBs of τ_M. For different μ_τ, the obtained results are given in Table 5. From this table, it is clear to see that Theorems 1 and 2 offer larger MADBs of τ_M than those methods in existing references.

5. Conclusions

This paper has investigated the absolute stability analysis for neutral-type Lur'e systems with time-varying delays. Based on the new delay-decomposition approaches in combination

Table 5: MADBs τ_M for different μ_τ, when $\mu_d = 0.1$ for Example 2.

Method	$\mu_\tau = 0$	$\mu_\tau = 0.3$	$\mu_\tau = 0.6$	$\mu_\tau = 0.9$	>1
[24]	0.1622	0.1591	0.1566	0.1541	0.1527
[27]	0.1745	0.1698	0.1698	0.1698	0.1698
[29]	0.1747	0.1710	0.1703	0.1703	0.1703
[32]	0.1771	0.1721	0.1715	0.1715	0.1715
[35]	0.2790	0.2691	0.2553	0.2463	0.2463
Theorem 1 ($\alpha = 0.45$)	0.361	0.359	0.359	0.359	0.359
Theorem 2 ($\alpha = 0.45$)	0.295	0.295	0.295	0.295	0.295
Theorem 3 ($\alpha = 0.01$)	0.163	0.163	0.163	0.163	0.163

with the integral inequality and reciprocally convex technique, several improved stability criteria have been derived by constructing some appropriate LKFs on the subintervals. The merit of the obtained stability criteria lies in the significant less conservativeness and lower computational complexity than some existing ones. Finally, two examples have been given to demonstrate the effectiveness and less conservatism of the proposed method. In the future works, we will be dedicated to study the stability analysis for systems with infinite delays and devote to the study of output feedback, tracking control and filtering of the neutral-type Lur'e systems with time-varying delays based on the method proposed in this paper.

Data Availability

All data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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