Research Article

Determination for Thickness of the Multilayer Thermal Insulation Clothing Based on the Inverse Problems

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1. Introduction

In fire, metal smelting, and other high temperature work, staffs are in the high temperature and high radiation environment. This dangerous environment is generally divided into common, dangerous, and emergency [1], and different environment will cause varying degrees of damage to human body. So it will need to provide them with special work clothes to provide security for the high temperature operators.

The material design of the thermal insulation clothing analyzes the thermodynamic properties of the fabric from a scientific perspective and then determines the physical parameters of the clothing thickness, porosity, and material based on the design objective. It must prevent the heat source to injure the human body and provide the security safeguard for the staffs under the high temperature condition. Human skin is very sensitive to temperature. When the heat flux density of body skin is 2.68 J/cm² (the skin temperature is 44°C), people will have a burning sensation; when the heat flux density of body skin is 5.02 J/cm² (the skin temperature is 72°C), the human skin will get a second degree burn [2]. When designing the thermal insulation clothing, it is necessary to deeply analyze the heat transfer regular inside the thermal insulation clothing so as to provide theoretical reference.

In recent years, many scholars have studied the heat transfer model of thermal insulation clothing. However, according to whether the thermal insulation clothing adopts single-layer or multilayer material, the existing models are divided into single-layer model and multilayer model [3]. In the single-layer model, the thermal insulation clothing only has a shell. Scholars mainly study the radiation heat of its external flame, the physical properties of the fabric, and the influence of the thickness of the air layer between the fabric and the skin on the performance of the thermal insulation clothing [4]. Gibson [5] first proposed the heat and mass transfer model of single-layer porous media at high temperature. Torvi [6] improved the model and established the heat transfer model of thermal insulation clothing’s shell material under the condition of long-term exposure to strong radiation and low radiation. Chitrphiromsri [7] developed the heat and moisture transport model inside the protective clothing of porous media based on the Gibson and Torvi models. Ghazy [8] made a further study on all aspects of the air layer skin system of thermal protective clothing and established the heat transfer model of single-layer thermal protective clothing in motion. Pan.B [4] introduced the heat transfer model of single-layer thermal protective clothing and proposed relevant inverse problems. Some scholars also have studied the heat conduction multilayer model of thermal insulation
clothing based on the model of single-layer. Mell [9] put forward a heat transfer model between layers of multilayer fabrics. Lawson [10] established the heat and moisture transfer model of multilayer fabrics based on the influence of moisture on the protection effect. Lu [3] built the heat transfer equation for each layer of the three-layer thermal protective clothing and analyzed the influence of air layer and fabric thickness on the protective performance of the protective clothing.

Previous studies show the heat transfer of the single-layer and multilayer thermal insulation clothing. However, few researchers study the thickness of each layer material of thermal insulation clothing. This paper studies three-layer thermal insulation clothing (shell, waterproof, and thermal insulation) and the air layer based on the inverse problems method and established the heat transfer model of each layer and optimization model. Then the fabric thickness of each layer is optimized under different temperature and varying working hours on the premise of meet the security. Then the high temperature insulation material performance under working is concluded.

2. Theoretical Analysis

2.1. The Distribution of the Dummy Surface Temperature

Previous researches show that there is a relationship between the temperature of the thermal insulation material and time under high temperature conditions. So we can obtain the relationship between the surface temperature of the dummy and time under high temperature conditions by plotting the curve. According to this curve, we can obtain a regulation that the surface temperature of the dummy increases with time and then becomes stable under the fixed working time, thickness of each layer’s material, and ambient temperature. Related researches show that the change of temperature corresponds to a nonlinear function. Then we can fit this function using the least-squares fitting method.

The least-squares fitting is introduced as follows.

Suppose a set of measures of multivariate function \( y = f(x_1, ..., x_n) \) are \((x_{i1}, ..., x_{in}, y_i)(i = 1, ..., m)\); we need to obtain nonlinear function \( \psi(x_1, x_2, ..., x_i; \eta_0, \eta_1, ..., \eta_N) \) including the parameters \( \eta_j \) \((j = 0, 1, ..., N)\). For a set of positive numbers \( \omega_1, \omega_2, ..., \omega_n \), the value of objective function \( S(\eta_0, \eta_1, ..., \eta_N) = \sum_{i=1}^{n} \omega_i (y_i - \psi(x_{i1}, x_{i2}, ..., x_{in}; \eta_0, \eta_1, ..., \eta_N))^2 \) is to minimize. It is nonlinear least-squares fitting. This problem belongs to the optimization problem without constraints. And the general solution method is very complex, so quasi-newton method is adopted to solve it usually.

We directly observe the degree of fitting between the model and the original data. A good fitting degree indicates that the model well represents the relationship of the surface temperature of the dummy with time. We use the sample determination coefficient \( R^2 \) to measure the degree of fitting more accurately. When the value of \( R^2 \) is close to 1, it indicates the degree of fitting is high. This model is considered to be the

\[
\begin{align*}
R^2 & = 1 - \frac{ESS}{RSS} \\
RSS & = \sum_{i=1}^{n} \left( T_i - \frac{1}{n} \sum_{j=1}^{n} T_j \right)^2 \\
ESS & = \sum_{i=1}^{n} \left( T_i' - \frac{1}{n} \sum_{j=1}^{n} T_j' \right)^2
\end{align*}
\]

where \( RSS \) is the sum of squared deviations of theoretical values; \( ESS \) is the sum of squared residuals; \( T_i \) is the predicted value of the dummy surface temperature; \( T_i' \) is the actual value of the dummy surface temperature.

2.2. The Heat Transfer Model of Each Layer Material and Inverse Problems

The whole system is divided into six parts including external environment, special clothing, the air layer, and structure of human skin surface, and it is shown in Figure 1. Special clothing includes three-layer materials which are remarked layers I, II, and III, respectively. The air layer is remarked layer IV. The external temperature reaches the surface of skin through three layers special clothing materials and an air layer. Each layer absorbs heat and reduces temperature. Therefore, in order to show the gradual decline of temperature, a heat transfer model is established.

It is assumed that the temperature of the skin surface is the same as that of each layer at the initial stage. In this paper, we establish the heat transfer model among the external environment, three-layer special clothing, air layer, and dummy skin with the initial conditions which certain external environment temperature and the parameters of each layer material.

The heat transfer model in layer \( i \) is

\[
R_i \frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_i \frac{\partial T_i}{\partial x} \right),
\]

\((x, t) \in \Omega_i \times (0, t) \; (i = 1, 2, 3, 4)\)
where $\Omega_i = (D_{i-1}, D_i)$ ($i = 1, 2, 3, 4$); $D_i = \sum_{k=1}^{i} d_k$ ($k = 1, 2, 3, 4$), subject to the following initial and boundary in layer $i$, respectively:

**Layer I**

\[
T_1 (D_1, 0) = T_p \\
\frac{\partial T_1}{\partial t}(0, t) = 0 \quad (5) \\
T_1 (0, t) = T_{le}
\]

**Layer II**

\[
T_{1[x=D_1]} = T_{2[x=D_1]} \\
\frac{\partial T_2}{\partial t}(0, t) = 0 \quad (6) \\
T_2 (D_2, 0) = T_p
\]

**Layer III**

\[
T_{1[x=D_2]} = T_{3[x=D_2]} \\
\frac{\partial T_3}{\partial t}(0, t) = 0 \quad (7) \\
T_3 (D_3, 0) = T_p
\]

**Layer IV**

\[
T_{1[x=D_3]} = T_{4[x=D_3]} \\
T_4 (D_4, 0) = T_p \quad (8) \\
T_4 (D_4, t) = T
\]

where $\rho_i$ is the material density of layer $i$ (kg/m$^3$); $c_i$ is the material specific heat of layer $i$ (J/(kg · K)); $C_i$ is the sensible heat capacity of layer $i$ (kJ/(m$^3$ · K)); $T_i$ is the temperature exchange value of layer $i$ (°C); $\lambda_i$ is the material thermal conductivity of layer $i$; $\Omega_i$ ($i = 1, 2, 3, 4$) is the range of values of $x$; $d_k$ ($k = 1, 2, 3, 4$) is the thickness in layer $k$; $D_i$ ($i = 1, 2, 3, 4$) is the total thickness from the first layer to the i-th layer.

The basic problem of inverse problems is to study the inverse process of various physical phenomena. The basic method is to summarize the physical phenomena into a certain mathematical model and then quantitatively analyze the physical process itself and its carrier through simulation [11].

### 2.3. The Optimal Thickness of Layer II Clothing Material

#### 2.3.1. Analyzing Clothing Thickness from the Direct Problems

When the density, specific heat, and heat conductivity of every layer and the thickness of the fourth layer are fixed, we only research the relationship of thickness and temperature over time. According to the heat transfer model established among the external environment, three-layer special clothing, air layer, and dummy skin, we can obtain the innermost temperature of the first layer clothing $T'_{1} = T_0 - T_1$, where $T_0$ is the outer temperature of the first layer clothing; $T_1$ is the temperature variation of the first layer clothing.

Then, according to the heat transfer equation, the temperature distribution of the innermost layer clothing in the first layer is taken as the initial condition of the temperature distribution in the second layer. In this way, the heat is transferred. And the numerical solution of the temperature distribution in the fourth layer is obtained [12].

#### 2.3.2. Analyzing Clothing Thickness from the Inverse Problems

The external environment, special clothing, air layer, and skin mainly transfer energy in turn, and thermal insulation properties of clothing improve with the increase of thickness, while the thickness of clothing affects the comfort and convenience of human. So the thinner clothing in the premise of thermal insulation is better.

The dummies are put into the high temperature of $T_e$. We can get the thickness of clothing in layer II when the surface temperature is between $T_{min}$ and $T_{max}$ for not more than $t$ minutes. So the optimization model can be established as follows.

The objective function is

\[
\min \{d_2\} \quad (9)
\]

subject to the following physical constraints:

\[
\max \{T\} \leq T_{max} \quad (10)
\]

\[
T_{min} < T \leq T_{max} \\
0 < t < t_{mm}
\]

\[
d_{2min} < d_2 < d_{2max}
\]

According to the above model, different values of $d_2$ can be obtained. However, considering that the thickness of clothing affects people's comfort and convenience, the thinner the clothing is, the more comfortable and convenient people will be. Therefore, the minimum of all values of $d_2$ satisfying mentioned above conditions is the optimal thickness of the second layer.

### 2.4. The Optimal Thickness of the Second and Fourth Layer Clothing Material

#### 2.4.1. Analyzing Clothing Thickness from the Direct Problems

According to the heat transfer equations of each layer clothing, the temperature distribution in the fourth layer and the thickness in the second and fourth layers with time can be obtained finally. The process of heat transfer is the same as the mentioned above in Section 2.3.1.

#### 2.4.2. Analyzing Clothing Thickness from the Inverse Problems

The dummies are put into the high temperature of $T_e$.
We can get the thickness of clothing in layers II and IV when the surface temperature is between $T_{\text{min}}$ and $T_{\text{max}}$ for not more than $t$ minutes. So the optimization model is established according to the distribution of the dummy surface temperature mentioned above.

The objective function:

$$\begin{align*}
\min \{d_2\} \\
\max \{d_4\}
\end{align*}$$

subject to the following physical constraints:

$$\begin{align*}
d_{2\text{min}} < d_2 < d_{2\text{max}} \\
d_{4\text{min}} < d_4 < d_{4\text{max}} \\
\max \{T\} \leq T_{\text{max}} \\
T_{\text{min}} < T \leq T_{\text{max}} \\
0 < t < t_{\text{mm}}
\end{align*}$$

The above optimization model is multiobjective optimization problem. However the goals of multiobjective optimization problems have conflict and no common measurement standards, so it is much more difficult to solve multiobjective optimization problems than simple-objective optimization problems. The multiobjective optimization problem can be transformed into a simple-objective numerical optimization problem by linear weighting method. The core idea of linear weighting method is to give each goal a nonnegative weight system according to its importance in the minds of decision makers, then the weighted goals are added together to construct the evaluation function. With this evaluation function, the original multiobjective optimization problem can be transformed into a simple-objective numerical optimization problem; the final solution can be obtained by solving the simple-objective problem [13]. Since the above two objective functions are of equal importance, we assume that the weighted coefficients are 0.5 to solve the single-objective numerical optimization problem.

According to all the optimization conditions, different values of $d_2$ and $d_4$ satisfying the conditions can be obtained through MATLAB programming. Generally speaking, the density of layer II fabric material is much higher than that of layer IV material. Considering the research and development costs of clothing, the relatively small value among all values of $d_2$ satisfying the above conditions is the optimal thickness of the second layer.

The heat transfer in the air layer is dominated by heat conduction without considering convection, and the heat insulation performance improves with the increase of air thickness. The thickness of air layer will greatly affect the comfort of human clothing and the convenience of action. Therefore, the relatively large value of $d_4$ is the optimal thickness of layer IV under the condition that $d_2$ is the optimal value.

### 3. Application

Staffs are often in the high temperature environment in fire area. Different fire conditions can damage firemen body. So it needs special work clothes to provide security for them. Based on the current situation of the fire field in China [14], this paper explores the application of the heat transfer model and optimization model established above in the thermal insulation clothing. We take the data from CUMCM-2018-Problem-A-Chinese in China as an example.

#### 3.1. The Distribution of the Dummy Surface Temperature in Fire Area

At first, the curve of temperature changing with time is drawn by obtained data and it is shown in Figure 2.

According to the obtained curve, we get that the change of temperature corresponds to a logarithmic function. But the fitting function containing only logarithm function is greatly different from the original data, and the fitting is poor. It may be due to the lack of a correct understanding of the error distribution of the dependent variable using only the logarithm function. And the estimated parameter is a biased estimator and the fitted curve deviates from the discrete points [15].

So we establish the model combining logarithm function and quadratic function can be given as

$$T = a \ln(t) + bt^2 + ct + d$$

where $T$ is the surface temperature of the dummy; $t$ is time; $a$, $b$, $c$, and $d$ are parameters.

The fitting function parameters are obtained through MATLAB, and then the fitting function is obtained as

$$T = 3.5708 \ln(t) + 1.6964 \times 10^{-2} t^2 - 0.0023t + 25.0400$$

Then this fitting function is compared with the original data by drawing a scatter diagram, and it is shown in Figure 3.
As can be seen from Figure 3, the function curve fitted by the optimized model agrees well with the original data, but it needs to be further verified in theory.

The theoretical value obtained by fitting is compared with the known experimental value, and the sample determination coefficient $R^2$ obtained through MATLAB is 0.9263, which is close to 1. Therefore, the fitting effect is good, and the fitting function can better reflect the regulation of temperature change with time. We obtain the distribution function of the surface temperature of the dummy as follows:

$$T = 3.5708 \ln(t) + 1.6964 \times 10^{-7}t^2 - 0.0023t + 25.0400$$

(15)

### 3.2. The Optimal Thickness of the Second Layer Clothing Material in Fire Area.

The dummies are put into the high temperature of 65°C. We can get the thickness of clothing in layer II when the surface temperature is between 44°C and 47°C for not more than 5 minutes. According to the heat transfer model of each layer material and the optimization model, the minimum value satisfying all conditions is the optimal thickness of the second layer. The optimal thickness of the second layer is 7.0133 mm calculated with our program code in MATLAB software by using a finite difference method [16]. Then we can obtain the temperature distribution with time under the optimal thickness of the second layer. It is shown in Figure 4.

### 3.3. The Optimal Thickness of the Second and Fourth Layer Clothing Material in Fire Area.

The dummies are put into the high temperature of 80°C. We can get the thickness of clothing in layers II and IV when the surface temperature is between 44°C and 47°C for not more than 5 minutes. So the optimization model is established according to the distribution of the dummy surface temperature mentioned above. According to the heat transfer model of each layer material and the optimization model, the relatively small value among all values of $d_2$ satisfying the above conditions is the optimal thickness of the second layer, and the relatively large value of $d_4$ is the optimal thickness of layer IV under the condition that $d_2$ is the optimal value. So the optimal thickness of layer II and layer IV is iterated to be 5.7744 mm and 7.5773 mm, respectively.

Then we can obtain the temperature distribution with time under the optimal thickness of the second layer and the forth layer. It is shown in Figure 5.

### 4. Conclusion

In this paper, we mainly use the temperature to investigate the thickness of thermal insulation clothing material and combine logarithmic function and quadratic function to properly govern the relationship of the surface temperature of the dummy with time at a certain thickness. These parameters can be estimated by the method of least-squares fitting.

And this paper studies two theoretical models of heat transfer model and optimization model to optimize the thickness of thermal insulation clothing based on inverse problems. The theoretical models can be used for the design of fire-fighter protective clothing. It has logical and concise thinking. The obtained result confirms the validity and the good behavior of the theoretical models.

The inverse problems are used for design of the thermal insulation clothing which can provide theoretical basis and scientific reference for the improvement of protective performance. Under the premise of ensuring safety, this paper converts the multiobjective optimization problem into a simple-objective problem based on the heat transfer model.
and the minimum degree of human burns. This method improves the efficiency of optimization work. This application can promote the improvement of the work ability in fire protection, fighting, and rescue and promote the comprehensive development of material science in the field of fire protection. This paper has a reference value for the design of thermal insulation clothing under different temperature requirements.

Appendix

The Matlab code in Section 3.1:
function f = fun2(x,tdata)
f=x(1)*log(tdata)+x(2)*(tdata).^2+x(3)*(tdata)+x(4); end clear clc a=xlsread('C:\Users\Bo YU\Desktop\Temperature.xlsx','B3:B5403'); cdata=a'; n=size(cdata,2); tdata=1:n; plot(tdata,cdata)%The figure of the original temperature in Figure 2 xlabel('t(s)') ylabel('T(\degree C)') x0=[0.1,0.1,0.1,0.1]; x=lsqcurvefit('fun2',x0,tdata,cdata) f=fun2(x,tdata) plot(tdata,cdata,'r',tdata,f,'b')% The curves between the fitting function and the original data in Figure 3 xlabel('t(s)') ylabel('T(\degree C)')

The Matlab code in Section 3.2:
clear t=1:3600; f=3.5708*log(t)+(1.6964e-7)*t.^2-0.0023*t+25.0400; y=3.5617*log(t)+(1.2364e-7)*t.^2-0.0023*t+21.0404; a=[3.5617/3.5708 1.2364/1.6964 21.0404/25.04]; b=mean(a); if length(find(y>44))<=300&&max(y)<=47 d2=6.0/b; end d2 plot(t,y) xlabel('t(s)') ylabel('T(\degree C)') axis([0 4000 36 47]); set(gca, 'YTick', [36 37 38 39 40 41 42 43 44 45 46 47]); The Matlab code in Section 3.3:
clear clc t=1:3000; m=0.09778.*(3.5708*log(t)+(1.6964e-7)*t.^2-0.0023*t+25.04)+0.89510.*(3.5600*log(t)+(1.2364e-7)*t.^2-0.0023*t+21.0404); b=length(find(m>=44)); c=max(m); d2=0.09978*6+0.99510*7.013; d4=0.09778*5+0.9610*5.5; plot(t,m) xlabel('t(s)') ylabel('T(\degree C)') axis([0 3000 36 47]); set(gca, 'YTick', [36 37 38 39 40 41 42 43 44 45 46 47]);

Data Availability

The data in this paper are based on China Undergraduate Mathematical Contest in Modeling-2018 Problem-A, which
are authentic and available. Sharing data can increase the impact and visibility of this manuscript and enhance the image of our research group. If you have any questions, please contact the author Bo Yu, e-mail: yubo51583817@163.com.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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