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Research Article

Multiobjective Optimization of 6-DOF Parallel Manipulator for Desired Total Orientation Workspace

Hongbin Qiang [1], Lihang Wang [1], Jisong Ding [1], and Lijie Zhang [1,2]

[1] *Hebei Key Laboratory of Heavy Machinery Fluid Power Transmission and Control, Yanshan University, Qinhuangdao 0660004, China*
[2] *Key Laboratory of Advanced Forging & Stamping Technology and Science, Yanshan University, Qinhuangdao 066004, China*

Correspondence should be addressed to Lijie Zhang; ljzhang@ysu.edu.cn

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A desired total orientation workspace for a parallel manipulator is usually an essential requirement in a practical application. At present, for the multiobjective optimization method of 6-DOF parallel manipulator for desired total orientation workspace, it is needed to predefine maximal and minimal lengths of the legs to serve as the constraint, and then the numerical method is used to solve the length of the legs and judge whether the solved maximal and minimal leg lengths meet the constraint. Predefining maximal and minimal length of the legs limits of the optimal range, the numerical method has heavy calculation burden and low calculating accuracy. In this paper, a hybrid method for solving the maximal and minimal lengths of the legs of 6-DOF parallel manipulator with desired total orientation workspace is proposed, and the actuator stroke length is calculated according to the maximal and minimal leg lengths. By judging whether the actuator stroke length can be solved to serve as the constraint, without the predefined maximal and minimal leg lengths to serve as the constraint, the optimal range is enlarged. Aiming at the physical size of the parallel manipulator and the proposed desired workspace condition index (DWCI), the optimization of the geometric parameters of the parallel manipulator is conducted based on the multiobjective optimization algorithm (NSGA-II), which is subject to the actuator stroke length. Stewart platform is set as the example; the geometric parameters of the platform whose workspace contains the desired total orientation workspace are optimized and the hybrid method is proved to be more accurate and efficient compared to the traditional numerical method. This method provides the optimization guidance for engineering designers to design the parallel manipulator for desired total orientation workspace.

1. Introduction

Parallel manipulator has the advantages of compact structure, strong bearing capacity, high precision of motion, and low inertia and therefore is widely used in the flight simulator, ship motion simulation, ship stabilized platform, space docking manipulator, parallel machine tools, and robot wrist [1, 2]. However, the limited workspace and dexterity are the major defects of the parallel manipulator [3–6]. As a result, it is of great importance to design a set of geometric parameters of the parallel manipulator with ideal workspace and good performance.

At present, in the design of the parallel manipulator for the workspace requirements, there are two types [6]. One is to get the geometry parameters of the parallel manipulator that maximizes the workspace. Lou [6] defined the effective regular workspace and aimed at the maximal side length of the effective regular workspace for the constant orientation, and the optimization of the geometric parameters is conducted by the controlled random search (CRS) method, which is subject to the dexterity and the leg lengths. Toz [7] defined the dexterous workspace by the condition index and minimum singular value (MSV) and used the particle swarm optimization (PSO) to optimize the dexterous workspace of asymmetric parallel manipulator, which is subject to kinematics and geometric constraints. Aiming at the condition number, stiffness, and the reachable workspace solved by geometric method, Shirazi [8] optimized the geometric parameters of the 6-DOF parallel manipulator.

However, maximizing the workspace blindly does not meet the engineering requirements, since a desired regular shape workspace is usually an essential requirement in

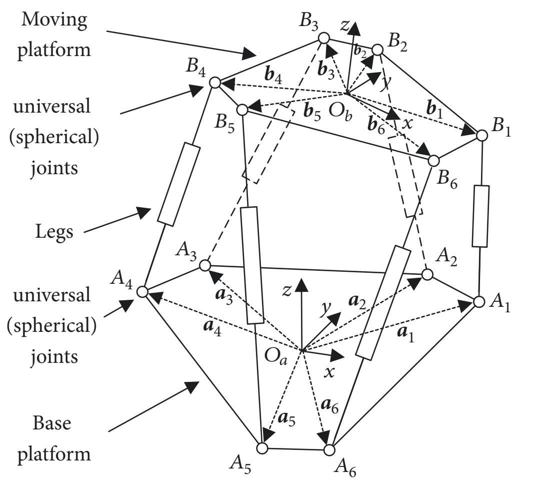


Figure 1: The structure sketch of 6-DOF parallel manipulator.

practice. The other type is to obtain the geometric parameters of a parallel manipulator whose workspace contains a desired workspace. Liu [9] used analytic method to design geometric parameters of a DELTA manipulator to get a desired total orientation workspace. In [10], a genetic algorithm was applied to optimize geometric parameters of a DELTA manipulator whose workspace contains a desired three-dimensional workspace. As for a 6-DOF parallel manipulator whose workspace is six- dimensional, it is necessary to optimize geometric parameters of parallel manipulator for six- dimensional workspace. At present, some scholars have optimized the geometric parameters of the 6-DOF parallel manipulator for a desired lower-mobility workspace. Liu [11] optimized the geometric parameters of the 6-DOF parallel manipulator for a desired single DOF workspace. Cirillo [12] used genetic algorithm to optimize the geometric parameters of the 6-DOF parallel manipulator in aeronautics applications for the known trajectory. In addition, some scholars have designed geometric parameters of parallel manipulator for a desired total orientation workspace [13]. Zhao [14] exploited an algebraic algorithm to optimize the length of the legs of a parallel manipulator to get a desired total orientation workspace. Fu [15] proposed a novel 3-leg 6-DOF parallel manipulator whose three translational and three rotational motions are decoupled, and an algorithm of optimal designing is proposed to find the smallest dimensional parameters of the proposed robot for the desired workspace based on analytical solutions of the forward kinematics. Yang [16] gave the size and position of the desired total orientation workspace, and the leg lengths were obtained by the six-dimensional discretization of the workspace based on the numerical method, and then aiming at the linear combination of the physical size and dexterity index, the geometric parameters of the 6-DOF platform were optimized by the genetic algorithm, which is subject to the given leg lengths. Xie [17] solved the global transmission indexes for the desired total orientation workspace with different sets of geometric parameters; the optimal geometric parameters were obtained based on the changing characteristics of indexes with structural parameters.

At present, there are few studies on multiobjective optimization of 6-DOF parallel manipulator for desired total orientation workspace. In general, numerical method is used to discretize the six- dimensional desired total orientation workspace to solve the length of the legs; however, it consumes a lot of time. Furthermore, the solved length of the legs should be subject to the predefined maximal and minimal length, which limits the optimization design of the geometric parameters. Therefore, a hybrid method for solving the maximal and minimal length of the legs of 6-DOF parallel manipulator with desired total orientation workspace is proposed in this paper, and it is more accurate and efficient compared to the numerical method. And the actuator stroke length is calculated according to the maximal and minimal leg lengths, by judging whether the actuator stroke length can be solved to serve as the constraint; without the predefined maximal and minimal leg lengths to serve as the constraint, the optimal range is enlarged. And then aiming at the physical size of the parallel manipulator and the proposed DWCI, the optimization of the geometric parameters of the parallel manipulator is conducted based on the multiobjective optimization algorithm (NSGA-II), which is subject to the actuator stroke length. Stewart platform is set as the example; the geometric parameters of the platform whose workspace contains the desired total orientation workspace are optimized and the hybrid method is proved to be more accurate and efficient compared to the traditional numerical method. This method provides the optimization guidance for engineering designers to design the parallel manipulator for desired total orientation workspace.

2. 6-DOF Parallel Manipulator

The structural sketch of 6-DOF parallel manipulator is shown in Figure 1. The manipulator is composed of the base platform, the moving platform, the legs, and the universal (spherical) joints. The basic platform is fixed and the moving platform is at the end of the manipulator. Two platforms are connected by the legs and the universal (spherical) joints.

Letting A_i ($i = 1, \ldots, 6$) denote the center points of the universal (spherical) joints which connect the legs and base platform and B_i ($i = 1, \ldots, 6$) denote the center points of the universal (spherical) joints which connect the legs and moving platform, A_i and B_i are at both ends of the same leg. A symmetrical hexagon is constructed by the point A_i, and O_a is the center of the symmetrical hexagon, the point O_a is the origin of the reference coordinate frame $\{O_a\}$, and the direction of z axes perpendicular to the base platform points upwards, the direction of x axes parallel to the base platform points to the midpoint of A_1 and A_6, and the direction of y axes is determined by the right-hand rule. A symmetrical hexagon is constructed by the point B_i, and O_b is the center of the symmetrical hexagon, the point O_b is the origin of the reference coordinate frame $\{O_b\}$, and the direction of z axes perpendicular to the moving platform points upwards, the direction of x axes parallel to the moving platform points to the midpoint of B_1 and B_6, and the direction of y axes is determined by the right-hand rule. The reference coordinate frame $\{O_a\}$ and the moving coordinate frame $\{O_b\}$

are assigned to the two platforms, respectively. The point O_a is the origin of $\{O_a\}$ located at the base platform and the point O_b is the origin of $\{O_b\}$ located at the moving platform. The vectors $\boldsymbol{a}_i = [x_{ai}, y_{ai}, 0]$ and $\boldsymbol{b}_i = [x_{bi}, y_{bi}, 0]$ represent the vectors of the points A_i and B_i in $\{O_a\}$ and $\{O_b\}$, respectively. In the initial state, the z axis of $\{O_a\}$ is recombined with the z axis of $\{O_b\}$, the x axis and y axis of $\{O_a\}$ are parallel to the x axis and y axis of $\{O_b\}$, and the vector of the point O_b in $\{O_a\}$ is $\boldsymbol{h} = [0, 0, H]$. The geometric parameters of the parallel manipulator can be determined by the vectors $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, and $\boldsymbol{h}$.

2.1. Inverse Kinematics. The total orientation workspace of the 6-DOF parallel manipulator can be denoted as $\Omega = \{(x, y, z, \alpha, \beta, \gamma)\}$, where the position in the workspace of the manipulator translation is defined as $\mathrm{P} = \{(x, y, z)\}$ and the posture in the workspace of the manipulator orientation is defined as $\mathrm{Q} = \{(\alpha, \beta, \gamma)\}$. In this paper, the Roll-Pitch-Yaw (RPY) angles are used to describe the orientation transformation. First rotate coordinate frame $\{O_b\}$ around the x axis of an angle γ (Yaw), then rotate the resulting coordinate frame around the y axis of an angle β (Pitch), and finally rotate coordinate frame around the z axis of an angle α (Roll). The transformation matrix from moving coordinate frame to the reference coordinate frame can be expressed as follows:

$$
\begin{aligned}
{}_B^A\boldsymbol{R} &= \boldsymbol{R}(z,\alpha)\,\boldsymbol{R}(y,\beta)\,\boldsymbol{R}(x,\gamma) \\
&= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}
\end{aligned} \tag{1}
$$

where $c\alpha \equiv \cos\alpha$, $s\alpha \equiv \sin\alpha$, $c\beta \equiv \cos\beta$, $s\beta \equiv \sin\beta$, $c\gamma \equiv \cos\gamma$, and $s\gamma \equiv \sin\gamma$.

According to the position P and posture Q in the workspace and the transformation matrix ${}_B^AR$, the leg length of parallel manipulator can be solved. The ith leg vector can be obtained by the difference between the vectors $\boldsymbol{b}_i$ and $\boldsymbol{a}_i$ in $\{O_a\}$. The ith actuator can be expressed by (2). The vector diagram for the ith actuator is given in Figure 2. $\{O_b\}$ represented by the dotted line is the initial state, and represented by the solid line is the arbitrary position state in the total orientation workspace.

$$
\begin{aligned}
\boldsymbol{l}_i &= \left({}_B^A\boldsymbol{R}\cdot\boldsymbol{b}_i + {}^A\boldsymbol{p}_B + \boldsymbol{h}\right) - \boldsymbol{a}_i = \begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^{\mathrm{T}} \\
&= \begin{bmatrix} c\alpha c\beta x_{bi} + (c\alpha s\beta s\gamma - s\alpha c\gamma)\,y_{bi} + x - x_{ai} \\ s\alpha c\beta x_{bi} + (s\alpha s\beta s\gamma + c\alpha c\gamma)\,y_{bi} + y - y_{ai} \\ -s\beta x_{bi} + c\beta s\gamma y_{bi} + z + H \end{bmatrix}
\end{aligned} \tag{2}
$$

where ${}^AP_B$ denotes the position P.

Then the length of the ith leg can be obtained by (3) and the square of the length of the ith leg can be got by (4).

$$e_i(x, y, z, \alpha, \beta, \gamma) = \sqrt{l_{ix}^{\,2} + l_{iy}^{\,2} + l_{iz}^{\,2}} \tag{3}$$

$$f_i(x, y, z, \alpha, \beta, \gamma) = l_{ix}^{\,2} + l_{iy}^{\,2} + l_{iz}^{\,2} \tag{4}$$

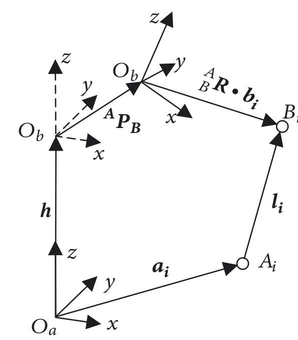


Figure 2: The vector diagram for the ith actuator.

3. The Maximal and Minimal Leg Lengths and the Actuator Stroke Length

The most usual types of workspace of 6-DOF parallel manipulator are reachable workspace, constant orientation workspace, orientation workspace, total orientation workspace, etc. Total orientation workspace is all the locations that may be reached with all the orientations among a set defined by ranges on the orientation angles [13]. The desired total orientation workspace is set by engineering designers according to engineering requirements. The desired total orientation workspace is a subset of the total orientation workspace, and it is usually geometry in a regular shape, such as a cube, a ball, and a cylinder. In this paper, a cube is set as the desired total orientation workspace Ω_{t}, as follows:

$$
\Omega_{\mathrm{t}} = \left\{ (x, y, z, \alpha, \beta, \gamma) \mid \begin{array}{c} x_{\min} \le x \le x_{\max}, y_{\min} \le y \le y_{\max} \\ z_{\min} \le z \le z_{\max}, \alpha_{\min} \le \alpha \le \alpha_{\max} \\ \beta_{\min} \le \beta \le \beta_{\max}, \gamma_{\min} \le \gamma \le \gamma_{\max} \end{array} \right\} \tag{5}
$$

3.1. Numerical Method for Calculating the Maximal and Minimal Leg Lengths. Numerical method [18] involves discretizing the six-dimensional desired total orientation workspace, calculating the leg lengths for each pose (discretized positions and postures), and getting the maximal and minimal leg lengths. Specific steps are as follows.

(1) Set a set of geometric parameters of the 6-DOF parallel manipulator, which includes $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, and $\boldsymbol{h}$.

(2) Divide the desired total orientation workspace Ω_{t} into the position workspace Ω_{p} and the posture workspace Ω_{q}. The position workspace and the posture workspace can be expressed by (5) and (6), respectively, as follows:

$$
\Omega_{\mathrm{p}} = \left\{ (x, y, z) \mid \begin{array}{c} x_{\min} \le x \le x_{\max}, y_{\min} \le y \le y_{\max} \\ z_{\min} \le z \le z_{\max} \end{array} \right\} \tag{6}
$$

$$\Omega_q = \left\{ (\alpha,\beta,\gamma) \mid \begin{array}{c} \alpha_{\min} \le \alpha \le \alpha_{\max}, \beta_{\min} \le \beta \le \beta_{\max} \\ \gamma_{\min} \le \gamma \le \gamma_{\max} \end{array} \right\} \quad (7)$$

(3) Discretize the position and posture workspace by the grid method. The translation step lengths along x axis, y axis, and z axis are S_x, S_y, and S_z. The orientation step lengths around x axis, y axis, and z axis are S_α, S_β, and S_γ. The number of discrete points in each dimension can be obtained by (8) as follows:

$$N_v = \text{floor}\left(\frac{v_{\max} - v_{\min}}{S_v}\right) + 1 \quad (8)$$

where floor(▪) denotes a function for rounding down and the subscript v denotes each dimension ($v = x, y, z, \alpha, \beta, \gamma$). Then, the total number of discrete points in all dimensions can be obtained by (9). The discretized position and posture can be expressed by (10) and (11).

$$N_T = N_x N_y N_z N_\alpha N_\beta N_\gamma \quad (9)$$

$$\left[x_{m_1}, y_{m_2}, z_{m_3}\right] = \left[x_{\min} + m_1 S_x, y_{\min} + m_2 S_y, z_{\min} + m_3 S_z\right] \quad (10)$$

$$\left[\alpha_{n_1}, \beta_{n_2}, \gamma_{n_3}\right] = \left[\alpha_{\min} + n_1 S_\alpha, \beta_{\min} + n_2 S_\beta, \gamma_{\min} + n_3 S_\gamma\right] \quad (11)$$

where $m_1 = 0, 1, \ldots, N_x$; $m_2 = 0, 1, \ldots, N_y$; $m_3 = 0, 1, \ldots, N_z$; $n_1 = 0, 1, \ldots, N_\alpha$; $n_2 = 0, 1, \ldots, N_\beta$; $n_3 = 0, 1, \ldots, N_\gamma$.

(4) According to (3), calculate the six leg lengths for each pose.

(5) Get the six maximal and minimal leg lengths by comparing the leg lengths in each position.

3.2. Algebraic Method for Calculating the Maximal and Minimal Leg Lengths. Equation (3) used to calculate the leg lengths is a six-dimensional function that involves the variables of $x, y, z, \alpha, \beta, \gamma$. Therefore, the maximal and minimal leg lengths are the maximum and minimum of a six-dimensional function. The functional maximum and minimum exist in the functional extremum or the boundary value. The extremal points of (3) are identical to (4) [19]. Therefore, for the convenience of solving the extremal points, we can obtain the extremal points of (3) by solving the extremal points of (4). Equation (12) is the partial derived function of (4), and the extremal points of (4) are obtained by (12) [20].

$$\begin{aligned}
\frac{\partial f}{\partial x} &= 2\left(c\alpha c\beta x_{bi} + \left(c\alpha s\beta s\gamma - s\beta c\gamma\right) y_{bi} + x - x_{ai}\right) \\
\frac{\partial f}{\partial y} &= 2\left(s\alpha c\beta x_{bi} + \left(s\alpha s\beta s\gamma + c\alpha c\gamma\right) y_{bi} + y - y_{ai}\right) \\
\frac{\partial f}{\partial z} &= 2\left(-s\beta x_{bi} + c\beta s\gamma y_{bi} + (z + H)\right) \\
\frac{\partial f}{\partial \alpha} &= 2\left(\left(x_{ai} - x\right)\left(s\alpha c\beta x_{bi} + \left(s\alpha s\beta s\gamma y_{bi} + c\alpha c\gamma\right) y_{bi}\right)\right. \\
&\quad \left. + \left(y - y_{ai}\right)\left(c\alpha c\beta x_{bi} + \left(c\alpha s\beta s\gamma y_{bi} - s\alpha c\gamma\right) y_{bi}\right)\right. \\
\frac{\partial f}{\partial \beta} &= 2\left(\left(\left(x_{ai} - x\right) c\alpha + \left(y - y_{ai}\right) s\alpha\right)\right. \\
&\quad \left. \cdot \left(c\beta s\gamma y_{bi} - s\beta x_{bi}\right) - \left(c\beta x_{bi} + s\beta s\gamma y_{bi}\right)(z + H)\right) \\
\frac{\partial f}{\partial \gamma} &= 2\left(\left(x - x_{ai}\right)\left(c\alpha s\beta c\gamma + s\alpha s\gamma\right) + \left(y - y_{bi}\right)\right. \\
&\quad \left. \cdot \left(s\alpha s\beta c\gamma - c\alpha s\gamma\right) + (z + H)\, c\beta c\gamma\right) y_{bi}
\end{aligned} \quad (12)$$

The maximum and minimum of six-dimensional function exist in the extremum of six-dimensional function or the maximum and minimum of five-dimensional function (one of the six variables is the boundary point); the maximum and minimum of a five-dimensional function exist in the extremum of five-dimensional function or the maximum and minimum of four-dimensional function (two of the six variables are the boundary points). By this analogy, the rest can be done as shown in Figure 3. Particularly, the maximum and minimum of a one-dimensional function exist in the extremum of one-dimensional function or the maximum and minimum of a zero-dimensional function (all of the six variables equal to the boundary points). Since each variable has two boundary values and arbitrary combinations with other variables, the number of the functions of k variables can be obtained by

$$N_m = 2^{6-m} \times C_6^{6-m} \quad (m = 1, 2, \cdots, 6) \quad (13)$$

where m is the number of variables of the function.

The solving process of function of k variables is shown in Figure 3. The algebraic method contains solving the function of $m(m = 1, \ldots, 6)$ variables, getting the extremal points of $m(m = 1, \ldots, 6)$ variables and 64 boundary points, excluding the extremal points that are not in the desired total orientation workspace, and calculating the extremum of (3) by the obtained extremal points and boundary points. Specific steps are shown as follows.

(1) Set a set of geometric parameters of the 6-DOF parallel manipulator, which includes $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, and $\boldsymbol{h}$.

(2) According to (12) and Figure 3, the function of $m(m = 1, \ldots, 6)$ variables is solved.

(3) Get the extremal points of $m(m = 1, \ldots, 6)$ variables and 64 boundary points.

(4) Exclude the extremal points that are not in the desired total orientation workspace.

(5) Get the extremum and the boundary value of (3).

(6) Get the six maximal and minimal leg lengths by comparing the leg lengths obtained in step (5).

3.3. Hybrid Method for Calculating the Maximal and Minimal Leg Lengths. The numerical method takes much time to calculate the maximal and minimal leg lengths, due to the large calculation quantity on the leg length for each discretized

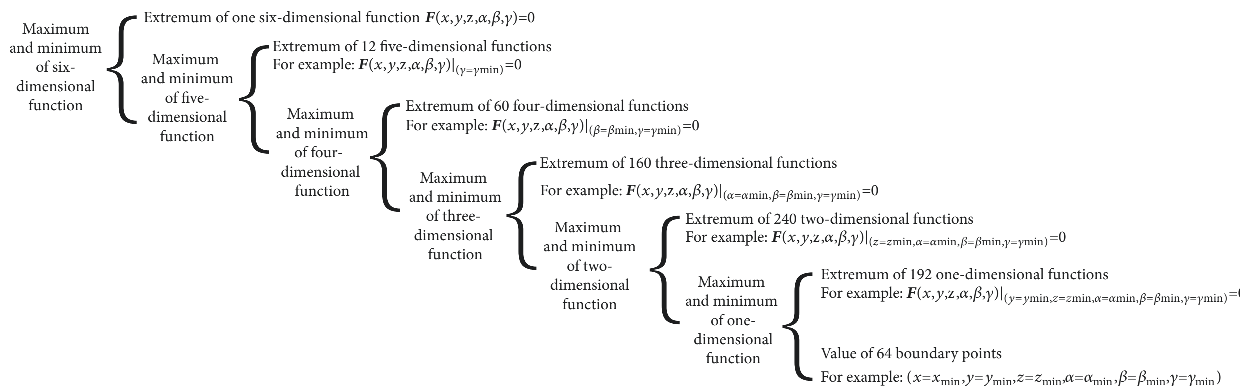


FIGURE 3: Solving process of function of m(m = 1, . . . , 6) variables.

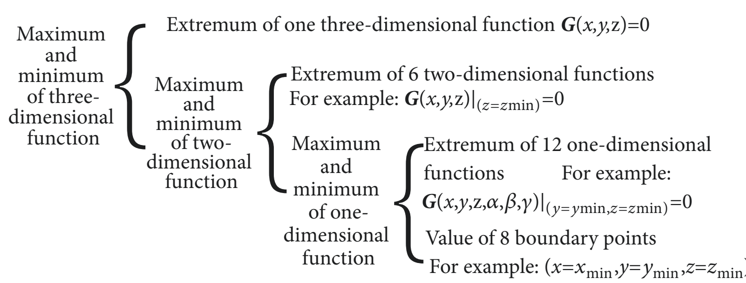


FIGURE 4: Solving process of function of n(n = 1, 2, 3) variables.

pose. And the precision is affected by the step lengths; usually a smaller step length leads to the higher precision and more time. The algebraic method takes less time to calculate the explicit maximal and minimal leg lengths. However, it is difficult to obtain the extremal points of the six-element function, and the extremal points of the function can be only obtained with the specified orientation variables (α, β, γ) which are equal to the boundary points.

Therefore, a hybrid method including the numerical method and algebraic method is proposed, where the numerical method is used for orientation variables and the algebraic method is used for translation variables. Firstly, the three-dimensional posture workspace is discretized and the discretized postures are obtained. Then the functions of 3 variables (x, y, z) in every discretized posture are solved. Finally, the maximal and minimal leg lengths are obtained. The extremal points can be obtained by the hybrid method based on the three-element function without solving six-element function. And compared with the numerical method, it is more accurate and efficient.

Equation (14) is the partial derived function of the functions of 3 variables (x, y, z) and (15) is the extremal points.

$$\begin{aligned}\frac{\partial f}{\partial x} &= c\alpha c\beta x_{bi} + \left(c\alpha s\beta s\gamma - s\beta c\gamma\right) y_{bi} + x - x_{ai}\\ \frac{\partial f}{\partial y} &= s\alpha c\beta x_{bi} + \left(s\alpha s\beta s\gamma + c\alpha c\gamma\right) y_{bi} + y - y_{ai}\\ \frac{\partial f}{\partial z} &= -s\beta x_{bi} + c\beta s\gamma y_{bi} + z + H\end{aligned} \tag{14}$$

$$\begin{aligned}x &= x_{ai} - c\alpha c\beta x_{bi} - \left(c\alpha s\beta s\gamma - s\beta c\gamma\right) y_{bi}\\ y &= y_{ai} - s\alpha c\beta x_{bi} - \left(s\alpha s\beta s\gamma + c\alpha c\gamma\right) y_{bi}\\ z &= s\beta x_{bi} - c\beta s\gamma y_{bi} - H\end{aligned} \tag{15}$$

The solving process of functions of $n(n = 1, 2, 3)$ variables is shown in Figure 4. Specific steps of the hybrid method for solving the maximal and minimal leg lengths of 6-DOF parallel manipulator for desired total orientation workspace are shown as follows.

(1) Set a set of geometric parameters of the 6-DOF parallel manipulator, which includes $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, and $\boldsymbol{h}$.

(2) Discretize the posture workspace by the grid method. The orientation step lengths around x axis, y axis, and z axis are $S_\alpha, S_\beta, S_\gamma$. The discretized postures can be got by (11). The number of discrete points in each dimension can be got by (8). The total number of discretized postures can be got by (16).

$$N_{TP} = N_\alpha N_\beta N_\gamma \tag{16}$$

(3) Select one posture in N_{TP} discretized postures.

(4) According to (14) and Figure 4, solve the function of $n(n = 1, 2, 3)$ variables.

(5) Get the extremal points of $n(n = 1, 2, 3)$ variables and 8 boundary points.

(6) Exclude the extremal points that are not in the desired total orientation workspace.

(7) Get the extremum and the boundary value of (3).

(8) Get the maximal and minimal leg lengths of each discretized posture.

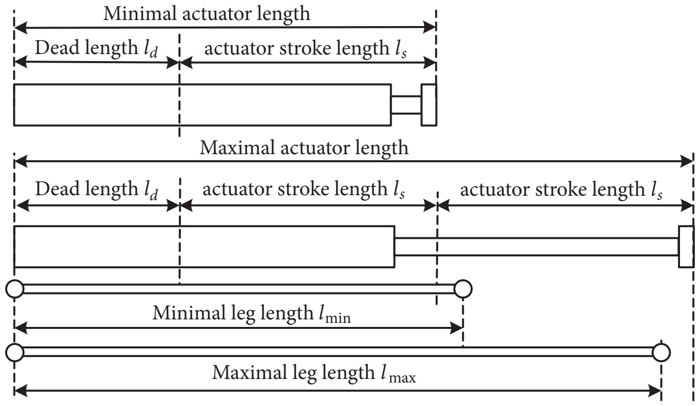


Figure 5: The schematic diagram of actuator.

(9) Select the next discretized posture in step (3) until all of the N_{TP} discretized postures have been calculated.

(10) Get the six maximal and minimal leg lengths by comparing the leg lengths in the N_{TP} discretized postures.

3.4. Actuator Stroke Length. The maximal and minimal leg length solved for the desired total orientation workspace should be between the maximum and minimum of the actuator length. The maximal and minimal values of the actuator can be determined by dead length and stroke length of the actuator. The dead length is the part which is not in the extension part of the cylinder, and the stroke length is the part in the extension part of the cylinder [21], as shown in Figure 5. According to Figure 5, the actuator stroke length can be obtained, as shown in (17).

$$\begin{aligned} l_{s\min} &\geq l_{\max} - l_{\min} \\ l_{s\min} &\geq \frac{(l_{\max} - l_d)}{2} \\ l_{s\max} &\leq l_{\min} - l_d \end{aligned} \tag{17}$$

where $l_{\min}$ denotes the minimum of leg length, $l_{\max}$ denotes the maximum of leg length, l_d denotes the dead length of actuator, and l_s denotes the actuator stroke length. Hydraulic cylinders, pneumatic cylinders, or electric cylinders are generally used as actuators, and the dead length l_d usually can be determined according to the actual condition.

According to (17), the minimum $l_{s\min}$ and the maximum $l_{s\max}$ of the actuator stroke length can be got by (18) and (19), respectively. Equation (20) can be defined based on (18) and (19), which is expressed as follows:

$$l_{s\min} = \max\left((l_{\max} - l_{\min}), \left(\frac{(l_{\max} - l_d)}{2} \right) \right) \tag{18}$$

$$l_{s\max} = l_{\min} - l_d \tag{19}$$

$$g = l_{s\max} - l_{s\min} \tag{20}$$

If the value of g in (20) is nonnegative, it means there is the solution to the actuator stroke length of the 6-DOF parallel manipulator under this set of geometric parameters; thus an actuator can be selected to achieve the maximum and minimum leg lengths for the desired total orientation workspace. Otherwise, if the value of g is negative, there is no solution to the actuator stroke length.

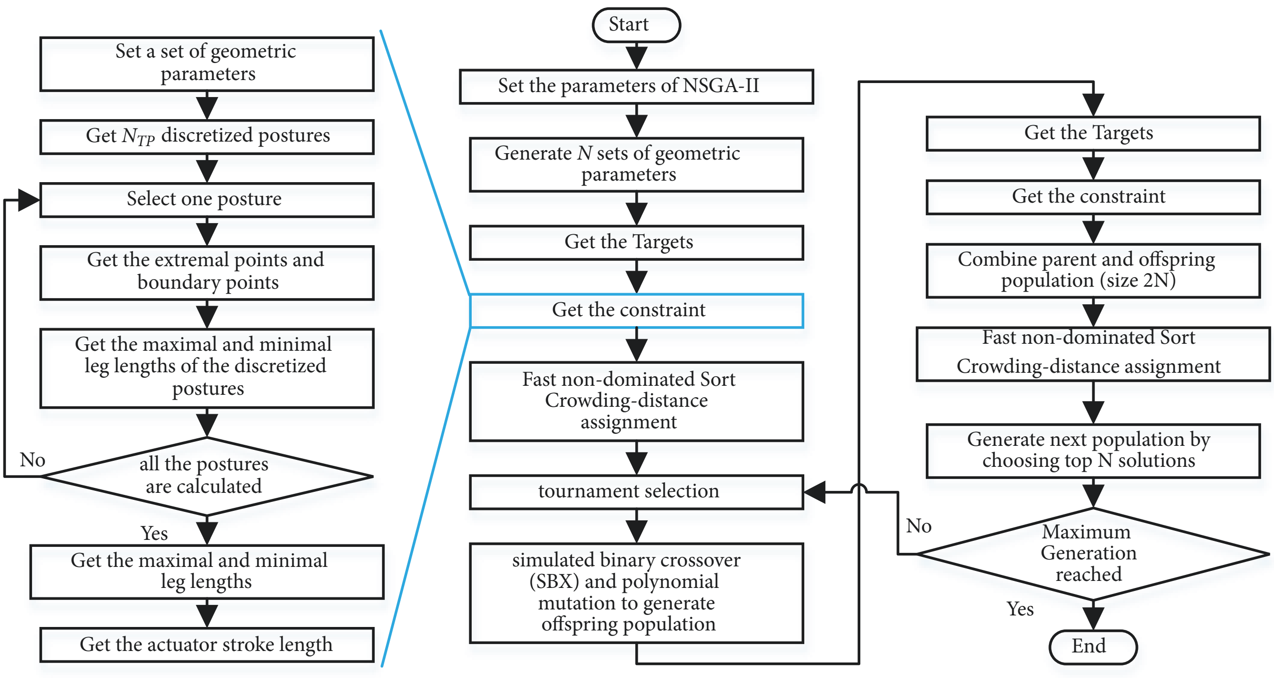
4. Multiobjective Optimization Algorithm

The Elitist Nondominated Sorting Genetic Algorithm version II (NSGA-II) is one of the most classical and popular multiobjective evolutionary algorithms, which is proposed by Deb [22] et al. In this paper, NSGA-II is employed to solve the multiobjective optimal design problems of 6-DOF parallel manipulator for desired total orientation workspace; the specific process is shown in Figure 6, and the simulated binary crossover (SBX) operator and polynomial mutation operator are applied to the algorithm [11].

4.1. Targets. By the determined volume of the desired total orientation workspace, the larger physical size of the parallel manipulator leads will meet the workspace requirements better. So the physical size of the parallel manipulator needs to be optimized to achieve higher economy and practicability. In general, the maximal leg length [14] or the volume of the manipulator [16] is the target of the physical size of the parallel manipulator.

One of the kinematics and dynamics indexes should be selected as the target to ensure the good performance of the manipulator. The global performance indexes such as the global condition index (GCI) [23], global manipulability index, and global transmission index [17] are calculated over the whole workspace. However, manipulator always works in the desired workspace rather than the whole workspace. Therefore, a performance index is proposed in this paper, which is used for the parallel manipulator within the desired workspace. For example, the desired workspace condition index (DWCI) can be got by

$$k = \frac{\int_W \eta dW}{\int_W dW} \tag{21}$$



(a) The solving process of the actuator stroke length

b) The process of NSGA II

FIGURE 6: The detailed process of NSGA II.

where W denotes the desired workspace and η denotes the local conditional index (LCI).

When the Monte Carlo method is used to randomly generate m poses for the desired workspace, the DWCI can be obtained by (22) [24],

$$k = \sum_{i=1}^{m} \frac{\eta_i}{m} \tag{22}$$

where m denotes the number of random poses and η_i denotes the local conditional index of the ith ($i = 1, \ldots, m$) random poses.

The range of k is $(0, 1]$, where 0 means the poor dexterity and 1 means the best dexterity. The larger the k is, the more flexible the mechanism is [23].

4.2. Constraint. The actuator stroke length of parallel manipulator for desired total orientation workspace is taken as the constraint. If (20) is nonnegative, the constraint is satisfied. If (20) is negative, the constraint is not satisfied.

Calculate the maximal and minimal leg lengths by the hybrid method proposed in this paper, and then calculate the stroke length. The specific process is shown in Figure 6.

5. The Example Application

5.1. The Desired Total Orientation Workspace and Mechanism Configuration. The proposed optimization method is used to optimize the ship motion simulation platform. The ship moves in six DOF, which includes heave (along z axis), sway (along y axis), surge (along x axis), roll (around x axis), pitch (around y axis), and yaw (around z axis). The amplitudes of roll, pitch, and heave are larger compared to yaw, sway, and surge and have great influence on ship navigation. Therefore, the desired total orientation workspace of ship movement is set as shown in Table 1.

TABLE 1: The desired total orientation workspace.

	x axis	y axis	z axis
Translation Amplitude(mm)	±35	±35	±50
Rotation Amplitude(°)	±10	±8	±5

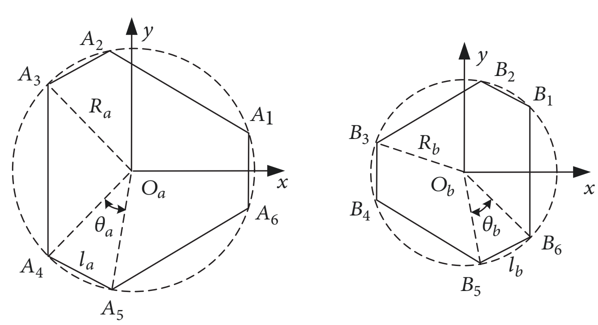


FIGURE 7: Positions of the joints on the base and moving platform.

The Stewart parallel manipulator is chosen as the ship motion simulation platform, whose joint position is shown in Figure 7,.where a symmetrical hexagon is constructed by the point A_i, which is inscribed in a circle with the radius of

Ra around the point O_a. l_a denotes the length of the shorter edge of the symmetrical hexagon and θ_a denotes the central angle corresponding to the short side l_a. Also, a symmetrical hexagon is constructed by the point B_i, which is inscribed in a circle with the radius of R_b around point O_b. lb denotes the length of the shorter edge of the symmetrical hexagon and θ_b denotes the central angle corresponding to the short sides l_b.

The central angles θ_a and θ_b can be obtained by (23) and (24) as follows:

$$\theta_a = 2\arcsin\left(\frac{0.5l_a}{R_a}\right) \tag{23}$$

$$\theta_b = 2\arcsin\left(\frac{0.5l_b}{R_b}\right) \tag{24}$$

The vectors $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ can be obtained by (25) and (26) as follows:

$$\boldsymbol{a}_i = \left[R_a\cos(\eta_i) \quad R_a\sin(\eta_i) \quad 0\right] \tag{25}$$

$$\boldsymbol{b}_i = \left[R_b\cos(\varphi_i) \quad R_b\sin(\varphi_i) \quad 0\right] \tag{26}$$

where

$$\begin{aligned} \varphi_i &= \frac{i\pi}{3} - \frac{\theta_b}{2}, \quad (i = 1,3,5) \\ \varphi_i &= \frac{(i-1)\pi}{3} + \frac{\theta_b}{2}, \quad (i = 2,4,6) \\ \eta_i &= \frac{(i-1)\pi}{3} + \frac{\theta_a}{2}, \quad (i = 1,3,5) \\ \eta_i &= \frac{i\pi}{3} - \frac{\theta_a}{2}, \quad (i = 2,4,6) \end{aligned} \tag{27}$$

Therefore, the geometric parameters of Stewart parallel manipulator can be determined by R_a, R_b, and H.

5.2. The Cost Function. R_a, R_b, and H are chosen as the design variables of the cost function.

The volume of the Stewart parallel manipulator and the DWCI are chosen as targets of the optimization algorithm. The volume of the Stewart parallel manipulator can be got by (28) as follows:

$$V = Sh = \pi R_a^2 h \tag{28}$$

The small length of short side leads to the good dexterity of the Stewart parallel manipulator [25]. To ensure the manipulator performs better without the interference of universal (spherical) joints installation, the short side lengths are set as 150mm. Hydraulic cylinders are chosen as the actuator, and the dead length ld of the hydraulic cylinder is 500mm. The orientation step length is set as $S_\alpha = S_\beta = S_\gamma = 1^\circ$.

Table 2: The parameters of NSGA-II.

parameters	values
Population size	50
Number of iterations	300
Crossover probability	0.9
Mutation probability	1/3
Distribution index for SBX	20
Distribution index for polynomial mutation	100

The cost function can be expressed as

$$\begin{aligned} \text{Minimize} \quad & \left[V\left(R_a R_b H\right), -k\left(R_a R_b H\right)\right] \\ \text{Subject to} \quad & 300 \le R_a \le 700\text{mm}; \\ & 250 \le R_b \le 700\text{mm}; \\ & 600 \le H \le 900\text{mm}; \\ & R_a \ge R_b; \\ & l_a = l_b = 170\text{mm}; \\ & S_\alpha = S_\beta = S_\gamma = 1^\circ; \\ & l_d = 500\text{mm}; \\ & \Omega_t = \left\{ (x, y, z, \alpha, \beta, \gamma) \;\middle|\; \begin{matrix} x_{\min} \le x \le x_{\max}, y_{\min} \le y \le y_{\max} \\ z_{\min} \le z \le z_{\max}, \alpha_{\min} \le \alpha \le \alpha_{\max} \\ \beta_{\min} \le \beta \le \beta_{\max}, \gamma_{\min} \le \gamma \le \gamma_{\max} \end{matrix} \right\}; \\ & g = l_{s\max} - l_{s\min} \ge 0. \end{aligned} \tag{29}$$

5.3. Optimization Results. All the parameters of NSGA-II are presented in Table 2.

The Pareto front, which is the optimization solutions including 50 sets of geometric parameters, is shown in Figure 8. V denotes the volume of the Stewart parallel manipulator, and k denotes the DWCI. The total orientation workspace of the Stewart parallel manipulator of each set of geometric parameters contains the desired total orientation workspace represented by Table 1. Known from Figure 8, the large volume of manipulator V leads to the high value of the DWCI k. The optimal set of geometric parameters can be selected among the 50 sets of geometric parameters according to the requirements of manipulability, stiffness, or power.

A set of geometric parameters is selected to verify the hybrid method proposed in this paper, as shown in Table 3.

The numerical method and the hybrid method are, respectively, used to obtain the maximal and minimal lengths of 3 legs, and the results are shown in Table 4. It can be known that the small step length leads to the high precision and long time. The computer runs Windows 7 operating system

Table 3: The set of geometric parameters for verifying the hybrid method.

Design variables			Targets	
R_a	R_b	H	V	$-k$
522.71mm	322.91mm	853.95mm	7.33×108mm^3	-0.25917

Table 4: The maximal and minimal length of 3 legs.

Step length	Numerical method		Hybrid method
	5mm, 1°	2mm, 1°	1°
Maximum(mm)	1041.0121	1041.0121	1041.0121
pose	(35,-35,50,5,8,10)	(35,-35,50,5,8,10)	(35,-35,50,5,8,10)
Minimum(mm)	796.3186	796.3167	796.3160
pose	(-20,-35,-50,-5,-8,-10)	(-23,-35,-50,-5,-8,-10)	(-22.01,-35,-50,-5,-8,-10)
time/s	457.47	8425.23	0.33

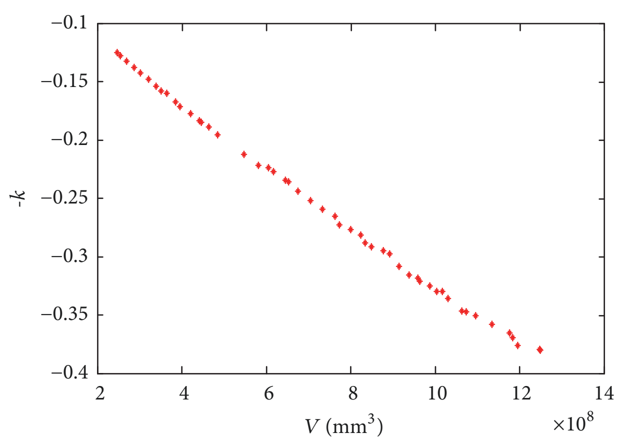


Figure 8: The optimization solutions.

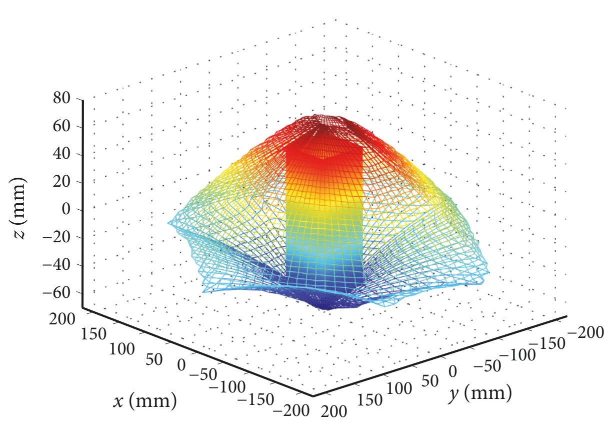


Figure 9: The desired total orientation workspace and total orientation workspace.

and has the Intel(R)Core(TM) i7-4770CPU@3.4GHz processor and a 8.00GB memory. Compared with the numerical method, the hybrid method is more accurate and efficient, which is suitable for the optimal calculation.

The total orientation workspace defined by the set of geometric parameters of the Stewart parallel manipulator and the desired total orientation workspace shown in Table 1 are shown in Figure 9 [26]. It can be observed that the total orientation workspace contains the desired total orientation workspace.

6. Conclusions

A hybrid method for solving the maximal and minimal leg lengths of 6-DOF parallel manipulator with desired total orientation workspace is proposed. The maximal and minimal leg lengths are obtained by the hybrid method, which consists of the numerical method used for orientation variables and algebraic method used for translation variables. The extremal points can be obtained by the hybrid method based on the three-element function without solving six-element function. And compared with the numerical method, it is more accurate and efficient.

The actuator stroke length is obtained to be the constraints, which is calculated by the maximal and minimal length of the legs, which enlarges the optimal range without the predefined maximal and minimal length of the legs.

Aiming at the physical size of the parallel manipulator and the proposed DWCI, the optimization of the geometric parameters of the parallel manipulator is conducted based on the multiobjective optimization algorithm (NSGA-II), which is subject to the actuator stroke length. Stewart platform is set as the example; the geometric parameters of the platform whose workspace contains the desired total orientation workspace are optimized and the hybrid method is proved to be more accurate and efficient compared to the traditional numerical method. This method provides the optimization guidance for engineering designers to design the parallel manipulator for desired total orientation workspace.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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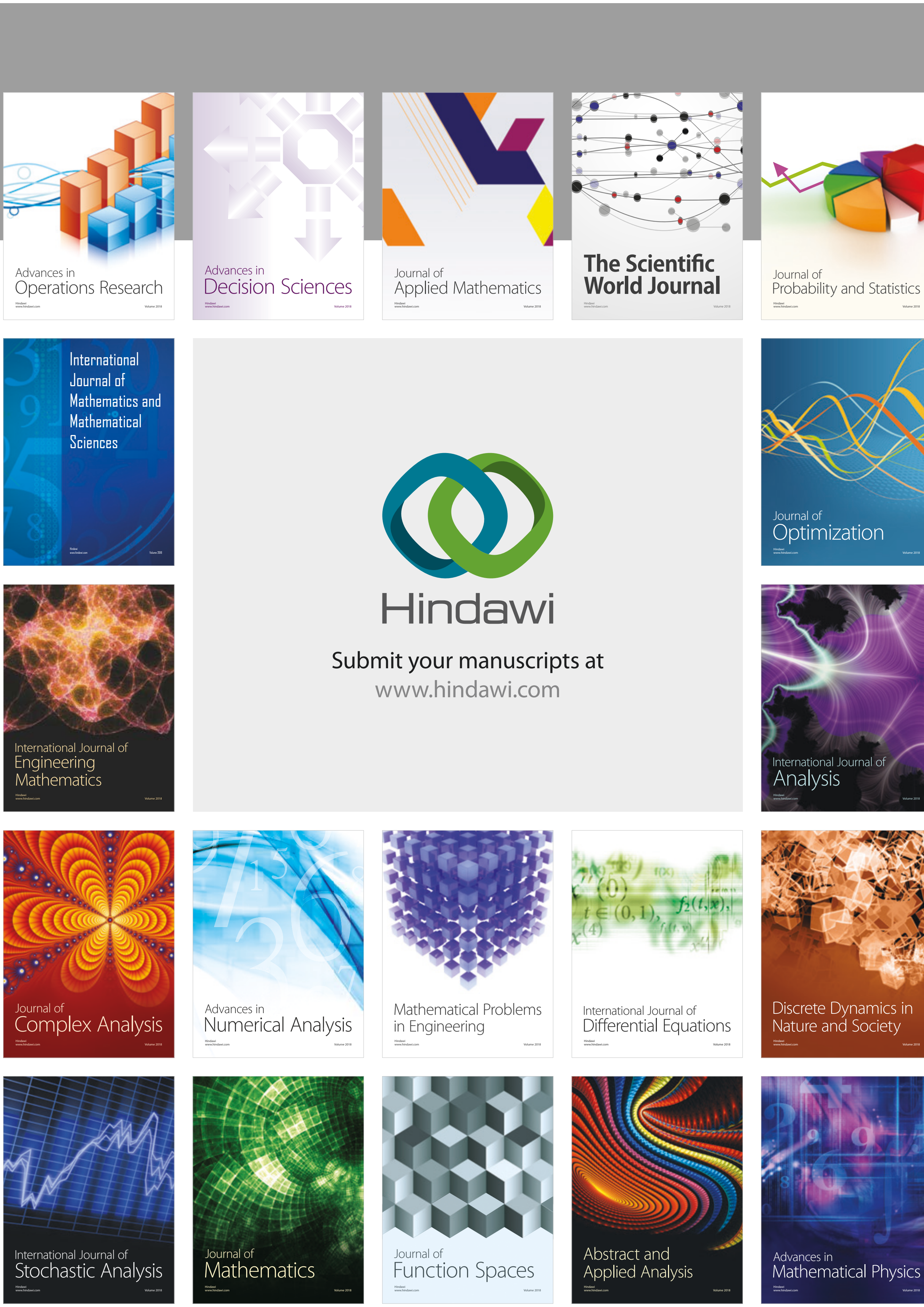
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