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Research Article

Valuation of Swing Options under a Regime-Switching Mean-Reverting Model

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In this paper, we study the valuation of swing options on electricity in a model where the underlying spot price is set to be the product of a deterministic seasonal pattern and Ornstein-Uhlenbeck process with Markov-modulated parameters. Under this setting, the difficulties of pricing swing options come from the various constraints embedded in contracts, e.g., the total number of rights constraint, the refraction time constraint, the local volume constraint, and the global volume constraint. Here we propose a framework for the valuation of the swing option on the condition that all the above constraints are nontrivial. To be specific, we formulate the pricing problem as an optimal stochastic control problem, which can be solved by the trinomial forest dynamic programming approach. Besides, empirical analysis is carried out on the model. We collect historical data in Nord Pool electricity market, extract the seasonal pattern, calibrate the Ornstein-Uhlenbeck process parameters in each regime, and also get market price of risk. Finally, on the basis of calibration results, a specific numerical example concerning all typical constraints is presented to demonstrate the valuation procedure.

1. Introduction

In the long run, energy liberalisation in electricity markets improves economic benefits, since that it overcomes the overcapacity problem in regulated markets and improves efficiency in the operation of networks and transport services (see [1] for more detail). However, in the short term, it exposes participants to various risks, e.g., price risks and quantity risks, because that electricity energy is hard to store and agents usually can not control their consumption. The emergence of swing options brings participants opportunities to hedge these risks according to [2]. Swings provide the option holder with flexibilities in deciding when to exercise and choosing the trading volume of each exercise, with strike price predetermined. However, these flexibilities usually are limited for the benefit of the option writer. Firstly, the chosen exercise times should be in the range of a series of predetermined days. Secondly, the total number of exercise rights is limited. Thirdly, each time the holder chooses to exercise, they face the upper and lower bounds for the delivery volume. Fourthly, the total amount of delivery volume is limited. Fifthly, the time span between the two consecutive exercises must be longer than the refraction time. All the details of these constraints are presented in [3]. Comparing to vanilla options, all of these above features add complexity to the valuation of swings.

For the valuation of option, we need to model the underling spot price. According to [4], electricity price in real markets usually has three main characteristics, including seasonal pattern, mean-reversion and spikes. We refer the reader to [5] for various models used in electricity market. Among them, jump models and regime-switching models are popular. In particular, if the real data shows that the price remained high for some time after a jump, then regime-switching models are better to depict this phenomenon [6]. For example, different types of regime-switching models were selected in [6–10]. Besides, various kinds of jump-diffusion models were applied in [11–13].

Adopting a specific setting on swings, Carmona and Touzi [14] applied the Snell envelope theory to formulate the pricing of swings as an optimal multiple-stopping problem, which can be reduced to compound single stopping problem. And

they analyzed this problem in the Black-Scholes framework and use Monte-Carlo simulation to construct a numerical example. To tackle the optimal multiple-stopping problem in the form of several variational inequalities, finite element method was used by M. Wilhelm and C. Winter [15], and dual method was used in [16–18]. Moreover, optimal stopping boundaries associated with swings' pricing problems were studied in [19–21].

Besides, the valuation problem can also be formulated as an optimal stochastic control problem subject to constraints. Facing this problem, many researchers employed the dynamic programming method, which lead to the Bellman's equation for the value function. And to deal with the conditional expectations in the Bellman's equation, different kinds of numerical methods were applied; e.g., tree methods were used in [3, 22, 23], Monte-Carlo method was used in [24, 25], and the finite difference method was used in [26, 27]. Besides the dynamic programming method, the method of discretizing the underlying probability space was used in [28] to overcome the difficulty of facing infinitely many cases.

In this paper, to characterize the electricity price, we use a seasonal regime-switching mean-reverting model with n states, after adding the mean-reverting characteristic into the model adopted in [22]. The motivation to add this modification comes from empirical researches of [6], in which electricity spot prices have been verified to have three basic features, including seasonal pattern, mean-reversion, and spikes. As for the constraints of option, besides usual ones, we take refraction time constraint and a penalty function of more general form into consideration based on [22]. Under these settings, we formulate the pricing of swings as an optimal stochastic control problem subject to several constraints, including the refraction time, the local volume, and the global volume. Then a trinomial forest dynamic programming approach is adopted, and more applications of this approach can be found in [3, 22, 23], etc. Finally, based on the daily average data in Nord Pool, we calibrate the model with two regimes and construct a concrete example concerning all nontrivial constraints and a penalty function of general form.

The structure of this paper is as follows. Section 2 introduces the model for the underlying electricity price and characterizes the swing option. Section 3 presents the procedure of valuing swing options with the trinomial forest dynamic programming approach. Section 4 concentrates on the empirical calibration and finding the market price of risk when the number of regime is two. Section 5 gives a numerical example based on the results in Section 4 for the valuation of the swing option with all nontrivial constrains. Section 6 concludes the paper.

2. Electricity Price Model and Characteristics of Swing Options

2.1. Characteristics of Electricity Spot Prices. Our modelling is based on the electricity spot price data of the day-ahead market in Nord Pool, and we focus on the stochastic component of the deseasonalized prices. Firstly, the stochastic component has the mean-reverting feature captured by Lucia and Schwartz [29], which shows that the electricity prices tend to revert to the mean value over time. Secondly, the time series of electricity prices show spikes, which can sometimes be depicted by jump models or regime-switching models (see [30]). All these two characteristics of the deseasonalized prices are the basis of modelling in this paper.

2.2. Models for Electricity Spot Prices. Throughout this paper, for a fixed maturity $T > 0$, we work on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\le t\le T}, \mathbb{P})$, where $\{\mathcal{F}_t\}_{0\le t\le T}$ is the natural filtration generated by Brownian Motion $\{B_t\}_{0\le t\le T}$ in physical measure $\mathbb{P}$ and an independent observable continuous-time Markov chain $\{\alpha_t\}_{0\le t\le T}$. Assume that $\{\alpha_t\}_{0\le t\le T}$ takes values in a finite state space $\mathcal{M} := \{1, 2, \cdots, m_0\}$ and is generated by $Q = \{q_{ij}\}$, where $q_{ij} \ge 0$ for $i, j \in \mathcal{M}$, $j \ne i$, and $\sum_{j=1}^{m_0} q_{ij} = 0$ for each $i \in \mathcal{M}$. The transitional probability is given by

$$P_{ij}(\Delta) := P\{\alpha_{t+\Delta} = j \mid \alpha_t = i\} = \begin{cases} q_{ij}\Delta + o(\Delta), & j \ne i, \\ 1 + q_{ij}\Delta + o(\Delta), & j = i, \end{cases} \tag{1}$$

for a small time interval $\Delta > 0$. Besides, we denote by ρ_i the probability of being in state i at the initial time $t = 0$, i.e.,

$$\rho_i := P\{\alpha_0 = i\}, \quad i \in \mathcal{M}. \tag{2}$$

We start our modelling for the spot price from the one-factor model used in [29], which captures seasonal and mean-reversion feature. The model is

$$S_t = \exp\{f(t) + X_t\}, \tag{3}$$

$$dX_t = -\beta X_t dt + \sigma dB_t^{\mathbb{P}}. \tag{4}$$

In (3), the logarithm of the electricity price S_t is divided into two parts, one of which is the seasonal part $f(t)$, and the other is the deseasonalized part X_t. Equation (4) describes the dynamic of the deseasonalized part by Vasicek model, where the long-term mean is set to be zero, and the speed of reversion together with instantaneous volatility is denoted by β and σ respectively, both of which are positive constants.

In this paper, our model is

$$S_t = f(t) \cdot X_t, \tag{5}$$

$$dX_t = \beta_{\alpha_t}\left(\xi_{\alpha_t} - X_t\right) dt + \sigma_{\alpha_t} dB_t^{\mathbb{P}}. \tag{6}$$

In (5), the modification is based on the fact that the electricity price can be negative (see [5]). And in (6), to capture the feature that electricity price show spikes, we replace (4) with a n-state regime-switching mean-reverting model, which was proposed in [8, 10]. Specifically, $\beta(\cdot) : \mathcal{M} \longrightarrow \mathbb{R}^+$, $\xi(\cdot) : \mathcal{M} \longrightarrow \mathbb{R}$, and $\sigma(\cdot) : \mathcal{M} \longrightarrow \mathbb{R}^+$ represent the speed of adjustment, the long-term mean level and the volatility, respectively. Note that, for simplicity, we denote $\beta_{\alpha_t} := \beta(\alpha_t)$, $\xi_{\alpha_t} := \xi(\alpha_t), \sigma_{\alpha_t} := \sigma(\alpha_t)$.

2.3. Swings and Pricing Problem. In this subsection, we introduce the settings on swing options and formulate the corresponding pricing problem. To begin with, we introduce mathematical notations and use them to sketch the details of a swing option from the holder's view.

Given a time interval $[0, T]$, n points are selected from it to be the possible exercise times, which offer opportunities for the holder to exercise the swing option. We denote these days by $\tau_0, \tau_1 \cdots \tau_{n-1}$, where $0 < \tau_0 < \cdots < \tau_{n-1} = T < \infty$. Let $\Delta\tau_l := \tau_{l+1} - \tau_l (l = 0, 1, \cdots, n-2)$, and for simplicity we set $\Delta\tau_l$ to be equal, i.e., $\Delta\tau_k = \Delta\tau$ for $l = 0, 1, \cdots, n-2$. For convenience, we use k instead of τ_k to be the subscript of all variables in this paper.

At time τ_k, the holder has one right to trade u_k volume(s) of electricity with the strike price E_k, where u_k usually is limited. We define

$$N_k := \sum_{i=0}^{k} \mathbf{1}_{\{u_i > 0\}},$$
$$U_k := \sum_{i=0}^{k} u_i, \qquad k = 0, 1, \cdots, n-1, \tag{7}$$

which denote the number of rights used and the total volume delivered respectively up to τ_k^+. In particularly, we let $N_{-1} = 0$ and $U_{-1} = 0$, which means that the swing option has not been exercised before time τ_0.

Now we can introduce the main constraints in the contract of the swing option. In general, there are four kinds as follows:

(1) **Total number of rights constraint**. The total number of rights for holder to exercise is limited to $N^A \in \mathbb{N}^+$, and it should be less than or equal to the number of possible exercise days, i.e., $N^A \le n$.

(2) **Refraction time constraint**. Right after exercising, the holder should wait for at least the time period Δt_R(we call it refraction time) to exercise another right again. Here, we assume that $\Delta t_R = \kappa\Delta\tau$, where κ is some predetermined positive integer.

(3) **Local volume constraint**. For $k = 0, \cdots, n-1$, the delivery volume $u_k \in \mathscr{A} := \{-m_1, -m_1 + 1, \cdots, 0, \cdots, m_2 - 1, m_2\}$, where $m_1, m_2 \in \mathbb{N}^+$.

(4) **Global volume constraint**. Once the holder's total delivery volume U_{n-1} is beyond the global volume constraint (V_{min} and V_{max}), she will be penalized. And we adopt the general type penalty function introduced in [3], which is defined as

$$\psi(U_{n-1}) = \begin{cases} -C_1 & \text{if } U_{n-1} < V_{min}, \\ 0 & \text{if } V_{min} \le U_{n-1} \le V_{max}, \\ -C_2(U - V_{max}) & \text{if } U_{n-1} > V_{max}, \end{cases} \tag{8}$$

where C_1 and C_2 are positive constants.

In this paper, we consider the valuation problem of the swing option with model (5) and (6) under all the above constraints, e.g., the total number of rights constraint, the refraction time constraint, the local volume constraint, and the global volume constraint. Now we formulate the pricing problem. Firstly, we introduce the strategy of exercising swing option in Definition 1.

Definition 1. The sequence $\{u_k\}_{k=0}^{n-1}$ is defined as a strategy if it is $\mathscr{F}_k$-adapted and satisfies the following conditions:

(1) $N_k \le n, k = 0, 1, \cdots, n-1$.

(2) If $u_i, u_j > 0$, then $|\tau_i - \tau_j| \ge \kappa\Delta\tau, i \ne j$.

(3) $u_k \in \mathscr{A} := \{-m_1, -m_1 + 1, \cdots, 0, \cdots, m_2 - 1, m_2\}$, where $m_1, m_2 \in \mathbb{N}^+, k = 0, 1, \cdots, n-1$.

Moreover, the set of all admissible strategies is called admissible set and is denoted by $\mathscr{D}$.

Secondly, we define the value function in Definition 2, which is widely used in stochastic control theory.

Definition 2. Given the risk-neutral probability measure $\mathbb{Q}$, at time τ_j^- ($j \in \{1, 2, \cdots, n-1\}$), the value of swing option with $\overline{N}_{j-1}$ right(s) used and $\overline{U}_{j-1}$ volume(s) totally delivered is given by

$$V_j^{\overline{N}_{j-1}, \overline{U}_{j-1}}(s, i) = \sup_{\{u_k\}_{k=j}^{n-1} \in \mathscr{D}} \mathbb{E}^{\mathbb{Q}} \left\{ \sum_{k=j}^{n-1} e^{-r_f(k-j)\Delta\tau} \cdot u_k \phi(S_k, E_k) + e^{-r_f(n-1-j)\Delta\tau} \cdot \psi(U_{n-1}) \mid S_j = s, \alpha_j = i, N_{j-1} = \overline{N}_{j-1}, U_{j-1} = \overline{U}_{j-1} \right\}, \tag{9}$$

where $s \in \mathbb{R}$, $i \in \mathscr{M}$, $\overline{N}_{j-1} \in \{0, 1, \cdots, N^A\}$, $\overline{U}_{j-1} \in \mathbb{N}$, r_f represents the risk-free rate, and $\phi(S_k, E_k) := \max(S_k - E_k, E_k - S_k, 0)$.

It is pointed out that our final goal is to solve this primal optimal stochastic control problem; that is, we need to find $V_0^{N_{-1}, U_{-1}}(s, i)$ and the optimal strategy $\{u_k\}_{k=0}^{n-1}$. Note that if we delete the local constraint and global constraint and set $N^A = 1$, then this problem is pricing a Bermuda option. In addition, if we set $N^A = n$, then it becomes the problem of pricing a strip of European options, and the differences of these European option are only embodied in expiry times and strike prices.

We deal with the primal optimal stochastic control problem via the method of backward induction as follows:

(i) If $N_{k-1} = N^A$ for $k \in \{n-1, n-2, \cdots, 0\}$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = e^{-r_f(n-1-k)\Delta\tau} \cdot \psi(U_{k-1}); \tag{10}$$

(ii) If $N_{k-1} \in \{N^A - 1, N^A - 2, \cdots, 0\}$ for $k = n-1$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = \sup_{u_k \in \mathscr{A}} \left(u_k \cdot \phi(S_k, E_k) + \psi(U_{k-1} + u_k) \right); \tag{11}$$

(iii) If $N_{k-1} \in \{N^A-1, N^A-2, \cdots, 0\}$ for $k \in \{n-2, \cdots, n-\kappa\}$, then

$$\begin{aligned} V_k^{N_{k-1},U_{k-1}}(S_k,\alpha_k) = \sup_{u_k\in\mathscr{A}} \Big(& u_k \cdot \phi(S_k,E_k) + \mathbf{1}_{\{u_k=0\}} \\ & \cdot e^{-r_f\Delta\tau}\mathbb{E}^{\mathbb{Q}}\left[V_{k+1}^{N_{k-1},U_{k-1}}(S_{k+1},\alpha_{k+1}) \mid \mathscr{F}_k\right] + \mathbf{1}_{\{u_k>0\}} \\ & \cdot e^{-r_f(n-1-k)\Delta\tau}\psi(U_{k-1}+u_k)\Big); \end{aligned} \tag{12}$$

(iv) If $N_{k-1} \in \{N^A-1, N^A-2, \cdots, 0\}$ for $k \in \{n-\kappa-1, \cdots, 0\}$, then

$$\begin{aligned} V_k^{N_{k-1},U_{k-1}}(S_k,\alpha_k) = \sup_{u_k\in\mathscr{A}} \Big(& u_k \cdot \phi(S_k,E_k) \\ & + e^{-r_f(\widetilde{k}-k)\Delta\tau}\mathbb{E}^{\mathbb{Q}}\left[V_{\widetilde{k}}^{N_{k-1}+\mathbf{1}_{\{u_k>0\}},U_{k-1}+u_k}(S_{\widetilde{k}},\alpha_{\widetilde{k}}) \mid \mathscr{F}_k\right]\Big), \end{aligned} \tag{13}$$

where

$$\widetilde{k} = \begin{cases} k+1 & \text{if } u_k = 0, \\ k+\kappa & \text{if } u_k > 0. \end{cases} \tag{14}$$

With (10)-(13), we can adopt tree method to fulfill our goal of pricing. Therefore, our next step is to build trinomial tree for S_t.

3. Valuation of Swing Options in Trinomial Forest

Since that the trinomial forest method for pricing strong path-depend options is simple and fast even when the number of regime state is large (see [31]), we adopt it to calculate the conditional expectation in inductive equations. Note that in (10)-(13), expectations are taken under risk-neutral measure $\mathbb{Q}$. Therefore, we start from finding the risk-neutral measures in each regime. Let

$$B_t^{\mathbb{Q}} = B_t^{\mathbb{P}} + \frac{\mu}{\sigma_i}t, \quad i \in \mathscr{M},\ t \in [0,T], \tag{15}$$

where μ is an unknown constant, which can be calibrated on market data. By Girsanov Theorem, we know that $B_t^{\mathbb{Q}}$ is Brownian Motion under the risk-neutral measure $\mathbb{Q}$, which satisfies

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\int_0^T \frac{\mu}{\sigma_i}dB_s^{\mathbb{Q}} - \frac{1}{2}\int_0^T\left(\frac{\mu}{\sigma_i}\right)^2 ds\right), \quad i \in \mathscr{M}. \tag{16}$$

Then we can rewrite SDE (6) as

$$dX_t = \beta_{\alpha_t}\left(\xi_{\alpha_t}^* - X_t\right)dt + \sigma_{\alpha_t}dB_t^{\mathbb{Q}}, \tag{17}$$

with $X_0 = x_0$ and $\xi_{\alpha_t}^* := \xi_{\alpha_t} - \mu/\beta_{\alpha_t}$.

3.1. Tree-Building Procedure for Deseasonalized Prices. In order to build a tree for X_t based on the SDE (17), we discretize the time and the space. Given time interval $[0,T]$, we select a time sequence $0 < t_0 < t_1 < \cdots < t_{n-1} = T$ with equal step $\Delta t = (T-t_0)/\widetilde{n}$. Similarly, we let the space size be positive constant $\overline{\sigma} > 0$. Both Δt and $\overline{\sigma}$ are to be chosen later.

For each node in the tree, we need to know the value and regime state of this node. Hence, we introduce the pair $(X_k,\alpha_k) := (X_{t_k}, \alpha_{t_k})$, $k = 0,1,\cdots,n-1$ to convey the necessary information on all of the nodes in the tree.

We now show the tree construction, which is initially designed by Liu [32]. For the node with $(X_k,\alpha_k) = (x,i)$, we consider how it evolves in the next time. Firstly, the Markov chain evolves from $\alpha_k = i$ to $\alpha_{k+1} = j$ basing on the transitional probability defined in (1). Secondly, X_{k+1} may take three values denoted by x_{k+1}^t, x_{k+1}^m, and x_{k+1}^b, where the superscripts t, m, and b are short for "top", "middle", and "bottom". Hence, there are three branches emanating from $X_k = x$. Besides, Liu [32] designed three structures of the branches, and in different structures x_{k+1}^t, x_{k+1}^m, and x_{k+1}^b are also different. Choosing which structure depends on the comparison result between x and the two threshold values $x^{i,1}$ and $x^{i,2}$ as follows:

Structure(a): If $x^{i,1} \le x \le x^{i,2}$, then the possible values of X_{k+1} in regime α_{k+1} are up to $x + l_i\overline{\sigma}\sqrt{\Delta t}$,remaining at x, and down to $x - l_i\overline{\sigma}\sqrt{\Delta t}$;

Structure(b): If $x < x^{i,1}$, then the possible values of X_{k+1} in regime α_{k+1} are up to $x + 2l_i\overline{\sigma}\sqrt{\Delta t}$, up to $x + l_i\overline{\sigma}\sqrt{\Delta t}$, and remaining at x;

Structure(c): If $x > x^{i,2}$, then the possible values of X_{k+1} in regime α_{k+1} are remaining at x, down to $x - l_i\overline{\sigma}\sqrt{\Delta t}$, and down to $x - 2l_i\overline{\sigma}\sqrt{\Delta t}$.

Here l_i depending on current state i is constant to be chosen later. And $x^{i,1}, x^{i,2}$ are defined as

$$\begin{aligned} x^{i,1} &= \xi_i^* - \frac{l_i\overline{\sigma} - \sqrt{(l_i\overline{\sigma})^2 - \sigma_i^2}}{\beta_i\sqrt{\Delta t}}, \\ x^{i,2} &= \xi_i^* + \frac{l_i\overline{\sigma} - \sqrt{(l_i\overline{\sigma})^2 - \sigma_i^2}}{\beta_i\sqrt{\Delta t}}, \\ & \qquad i \in \mathscr{M}, \end{aligned} \tag{18}$$

where the parameters are as before. It is emphasized that $x^{i,1}$ and $x^{i,2}$ change with the state of regime i. More details on this mechanism of the tree designing can be found in [32].

In different structures, we introduce the notations $P_{x,i}^{t,j}$, $P_{x,i}^{m,j}$, and $P_{x,i}^{b,j}$, which denote the conditional probabilities transforming from node $(X_k,\alpha_k) = (x,i)$ to the nodes $(X_{k+1},\alpha_{k+1}) = (x_{k+1}^t, j)$, $(X_{k+1},\alpha_{k+1}) = (x_{k+1}^m, j)$, and $(X_{k+1}, \alpha_{k+1}) = (x_{k+1}^b, j)$, respectively. We can calculate them by matching the mean $E[X_{k+1} - X_k \mid X_k = x]$ and the variance $E[(X_{k+1} - X_k)^2 \mid X_k = x]$ derived from tree with their counterpart derived from SDE (6). The explicit formula for $P_{x,i}^{t,j}$, $P_{x,i}^{m,j}$, and $P_{x,i}^{b,j}$ in different structures is as follows, and for simplicity we write $P_{ij} = P_{ij}(\Delta t)$.

Structure (a)

$$P_{x,i}^{t,j} = P_{ij} \cdot \frac{\sigma_i^2 + \lambda_i(x) l_i \overline{\sigma} \sqrt{\Delta t} + \lambda_i^2(x) \Delta t}{2(l_i \overline{\sigma})^2},$$
$$P_{x,i}^{m,j} = P_{ij} \cdot \left(1 - \frac{\sigma_i^2 + \lambda_i^2(x) \Delta t}{(l_i \overline{\sigma})^2}\right), \tag{19}$$
$$P_{x,i}^{b,j} = P_{ij} - P_{x,i}^{t,j} - P_{x,i}^{m,j},$$

Structure (b)

$$P_{x,i}^{t,j} = P_{ij} \cdot \frac{\sigma_i^2 - \lambda_i(x) l_i \overline{\sigma} \sqrt{\Delta t} + \lambda_i^2(x)}{2(l_i \overline{\sigma})^2},$$
$$P_{x,i}^{m,j} = P_{ij} \cdot \left(-\frac{\sigma_i^2 - 2\lambda_i(x) l_i \overline{\sigma} \sqrt{\Delta t} + \lambda_i^2(x) \Delta t}{(l_i \overline{\sigma})^2}\right), \tag{20}$$
$$P_{x,i}^{b,j} = P_{ij} - P_{x,i}^{t,j} - P_{x,i}^{m,j},$$

Structure (c)

$$P_{x,i}^{t,j} = P_{ij} \cdot \left(1 + \frac{\sigma_i^2 + 3\lambda_i(x) l_i \overline{\sigma} \sqrt{\Delta t} + \lambda_i^2(x) \Delta t}{2(l_i \overline{\sigma})^2}\right),$$
$$P_{x,i}^{m,j} = P_{ij} \cdot \left(-\frac{\sigma_i^2 + 2\lambda_i(x) l_i \overline{\sigma} \sqrt{\Delta t} + \lambda_i^2(x) \Delta t}{(l_i \overline{\sigma})^2}\right), \tag{21}$$
$$P_{x,i}^{b,j} = P_{ij} - P_{x,i}^{t,j} - P_{x,i}^{m,j},$$

where $\lambda_i(x) = \beta_i(\xi_i^* - x)$. To keep these probabilities positive, it is required that the parameters $\Delta t, \overline{\sigma}, l_i$ $(i \in \mathscr{M})$ satisfy the following inequalities:

$$\frac{2\sigma_i}{\sqrt{3}} \le l_i \overline{\sigma} \le 2\sigma_i, \quad i \in \mathscr{M}, \tag{22}$$

$$\Delta t \le \min_{1 \le i \le m_0} \frac{2\sqrt{1 - (\sigma_i / l_i \overline{\sigma})^2}}{\beta_i}. \tag{23}$$

3.2. The Valuation Formula of the Swing Option in Trinomial Forest. Note that $\{\tau_k\}_{k=0}^{n-1}$ and $\{t_k\}_{k=0}^{n-1}$ are two partitions of $[0, T]$, but for simplicity, we let $t_k = \tau_k$ for $k = 0, \cdots, n-1$ and assume that $\Delta t = \Delta\tau$, and $\Delta\tau$ satisfies (22) and (23).

Now we can write the recursive value functions with the trinomial tree model for X_t as follows:

(i) If $N_{k-1} = N^A$ for $k \in \{n-1, n-2, \cdots, 0\}$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = e^{-r_f (n-1-k)\Delta\tau} \cdot \psi(U_{k-1}); \tag{24}$$

(ii) If $N_{k-1} \in \{N^A - 1, N^A - 2, \cdots, 0\}$ for $k = n-1$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = \sup_{u_k \in \mathscr{A}} \left(u_k \cdot \phi(S_k, E_k) + \psi(U_{k-1} + u_k)\right); \tag{25}$$

(iii) If $N_{k-1} \in \{N^A - 1, N^A - 2, \cdots, 0\}$ for $k \in \{n-2, \cdots, n-\kappa\}$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = \sup_{u_k \in \mathscr{A}} \left\{ u_k \cdot \phi(S_k, E_k) + \mathbf{1}_{\{u_k = 0\}} \cdot e^{-r_f \Delta\tau} \sum_{\gamma_k = t,m,b} \sum_{\alpha_{k+1}=1}^{m_0} \left(P_{x_k, \alpha_k}^{\gamma_k, \alpha_{k+1}} \cdot V_{k+1}^{N_{k-1}, U_{k-1}}(\overline{S}_{k+1}, \alpha_{k+1})\right) + \mathbf{1}_{\{u_k > 0\}} \cdot e^{-r_f (n-1-k)\Delta\tau} \psi(U_{k-1} + u_k) \right\}; \tag{26}$$

(iv) If $N_{k-1} \in \{N^A - 1, N^A - 2, \cdots, 0\}$ for $k \in \{n-\kappa-1, \cdots, 0\}$, then

$$V_k^{N_{k-1}, U_{k-1}}(S_k, \alpha_k) = \sup_{u_k \in \mathscr{A}} \left\{ u_k \cdot \phi(S_k, E_k) + e^{-r_f (\widetilde{k}-k)\Delta\tau} \sum_{\gamma_k = t,m,b} \sum_{\alpha_{k+1}=1}^{m_0} \cdots \sum_{\gamma_{\widetilde{k}-1} = t,m,b} \sum_{\alpha_{\widetilde{k}}=1}^{m_0} \left(\prod_{\omega=k}^{\widetilde{k}-1} P_{x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}}, \alpha_\omega}^{\gamma_\omega, \alpha_{\omega+1}} \cdot V_{\widetilde{k}}^{N_{k-1} + \mathbf{1}_{\{u_k > 0\}}, U_{k-1} + u_k}(\overline{S}_{\widetilde{k}}, \alpha_{\widetilde{k}}) \right) \right\}, \tag{27}$$

where

$$\widetilde{k} = \begin{cases} k+1 & \text{if } u_k = 0, \\ k+\kappa & \text{if } u_k > 0. \end{cases} \tag{28}$$

We need to explain $\overline{S}_{k+1}$ in (26), $x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}}$ for $\omega \in \{k, \cdots, \widetilde{k}-1\}$ in (27), and $\overline{S}_{\widetilde{k}}$ in (27). Note that $\overline{S}_{k+1}$ is a special case of $\overline{S}_{\widetilde{k}}$; hence we firstly consider $x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}}$ for $\omega \in \{k, \cdots, \widetilde{k}-1\}$. When $\omega = k$, we define $x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}} := X_k$, which is known at time τ_k. When $\omega > k$, we define $x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}}$ as follows:

(i) if $x^{\alpha_{\omega-1}, 1} \le x_{\omega-1}^{\gamma_{\omega-2}, \alpha_{\omega-2}} \le x^{\alpha_{\omega-1}, 2}$, then

$$x_\omega^{\gamma_{\omega-1}, \alpha_{\omega-1}} := \begin{cases} x_{\omega-1}^{\gamma_{\omega-2}, \alpha_{\omega-2}} + l_{\alpha_{\omega-1}} \overline{\sigma} \sqrt{\Delta\tau}, & \gamma_{\omega-1} = t, \\ x_{\omega-1}^{\gamma_{\omega-2}, \alpha_{\omega-2}}, & \gamma_{\omega-1} = m, \\ x_{\omega-1}^{\gamma_{\omega-2}, \alpha_{\omega-2}} - l_{\alpha_{\omega-1}} \overline{\sigma} \sqrt{\Delta\tau}, & \gamma_{\omega-1} = b, \end{cases} \tag{29}$$

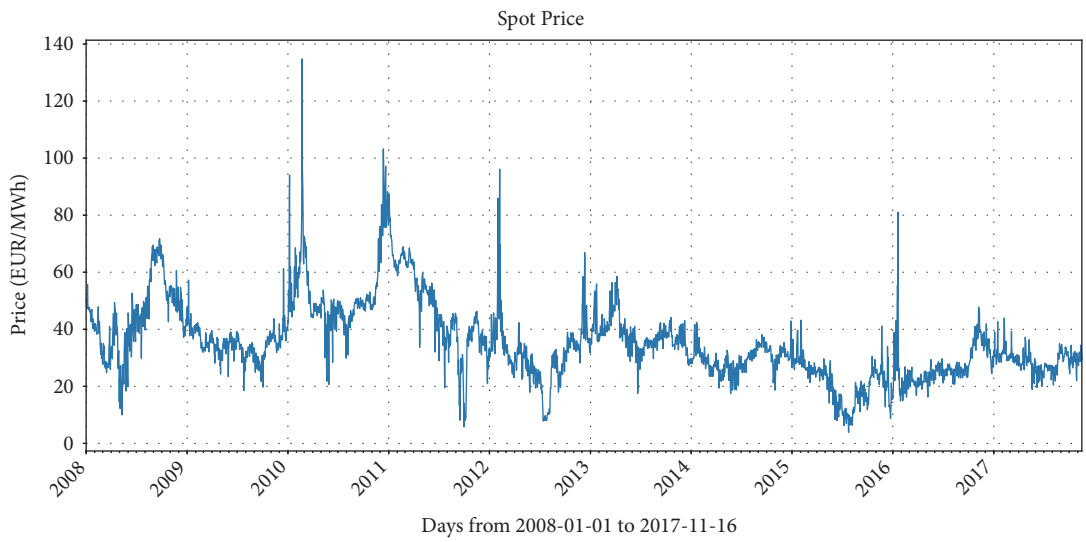


Figure 1: Daily average system electricity price.

(ii) if $x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} < x^{\alpha_{\omega-1},1}$, then

$$x_{\omega}^{\gamma_{\omega-1},\alpha_{\omega-1}} := \begin{cases} x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} + 2l_{\alpha_{\omega-1}}\overline{\sigma}\sqrt{\Delta\tau}, & \gamma_{\omega-1} = t, \\ x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} + l_{\alpha_{\omega-1}}\overline{\sigma}\sqrt{\Delta\tau}, & \gamma_{\omega-1} = m, \\ x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}}, & \gamma_{\omega-1} = b, \end{cases} \quad (30)$$

(iii) if $x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} > x^{\alpha_{\omega-1},2}$, then

$$x_{\omega}^{\gamma_{\omega-1},\alpha_{\omega-1}} := \begin{cases} x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}}, & \gamma_{\omega-1} = t, \\ x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} - l_{\alpha_{\omega-1}}\overline{\sigma}\sqrt{\Delta\tau}, & \gamma_{\omega-1} = m, \\ x_{\omega-1}^{\gamma_{\omega-2},\alpha_{\omega-2}} - 2l_{\alpha_{\omega-1}}\overline{\sigma}\sqrt{\Delta\tau}, & \gamma_{\omega-1} = b. \end{cases} \quad (31)$$

Secondly, we explain $\overline{S}_{\tilde{k}}$ as follows:

(i) if $x^{\alpha_{\tilde{k}-1},1} \le x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} \le x^{\alpha_{\tilde{k}-1},2}$, then

$$\overline{S}_{\tilde{k}} := \begin{cases} \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} + l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = t, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = m, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} - l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = b, \end{cases} \quad (32)$$

(ii) if $x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2}\alpha_{\tilde{k}-2}} < x^{\alpha_{\tilde{k}-1},1}$, then

$$\overline{S}_{\tilde{k}} := \begin{cases} \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} + 2l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = t, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} + l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = m, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = b, \end{cases} \quad (33)$$

(iii) if $x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2}\alpha_{\tilde{k}-2}} > x^{\alpha_{\tilde{k}-1},2}$, then

$$\overline{S}_{\tilde{k}} := \begin{cases} \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2}\alpha_{\tilde{k}-2}}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = t, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} - l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = m, \\ \left(x_{\tilde{k}-1}^{\gamma_{\tilde{k}-2},\alpha_{\tilde{k}-2}} - 2l_{\alpha_{\tilde{k}-1}}\overline{\sigma}\sqrt{\Delta\tau}\right) f(\tau_{\tilde{k}}), & \gamma_{\tilde{k}-1} = b. \end{cases} \quad (34)$$

Table 1: $f_{we}(t)$ of one loop cycle.

t	0	1	2	3	4	5	6
$f_{we}(t)$	1.0405	1.0334	1.0140	0.9351	0.9018	1.0350	1.0402

In Figure 4, we illustrate the pricing procedure depicted by formulas (24)-(27).

4. Empirical Calibration and the Market Price of Risk

Before implementing numerical examples of pricing swing options with the above trinomial forest approach, we do calibration for model (5) and (6). Our dataset covers the period from 2008/1/1 to 2017/11/16; therefore in this section, we let $t \in \{0, 1, \cdots, 3607\}$, where 0 denotes 2008/1/1, 3597 denotes 2017/11/16, and the time step 1 denotes one day. We set the states of the regime to be 2, and what we need to calibrate are $f(t)$ and $\theta := (\beta_i, \xi_i, \sigma_i, P_{ii}, \rho_i)$, $i = 1, 2$. After getting calibration results, we compare the forecasting and actual data. In addition, we also estimate the market price of risk.

4.1. Empirical Calibration. In this subsection we carry out calibration. Firstly, we obtain the system daily average electricity price in Nord Pool market from the Bloomberg service, which is presented in Figure 1. Then we deal with the seasonal pattern $f(t)$ by using the moving average and least squares fit techniques (see [4] for more details). We assume that $f(t)$ has the weekly part $f_{we}(t)$ together with the annually part $f_{an}(t)$ and that $f(t) = f_{we}(t) \cdot f_{an}(t)$. Then we apply moving average technique to $f_{we}(t)$; that is, we let $f_{we}(t) = f_{we}(t-7)$ for $t > 7$. And the annual cycle can be estimated with the sinusoidal form $f_{an}(t) = A\sin(2\pi(t+B)/365) + Ct + D$ via least squares fit. The results of $f_{we}(t)$ and $f_{an}(t)$ are presented in Table 1 and (35), respectively.

Table 2: Calibration results for two-state regime-switching model.

Regime	$\widehat{\beta}_i$	$\widehat{\xi}_i$	$\widehat{\sigma}_i$	$\widehat{P}_{ii}$	$\widehat{P}(\alpha_0 = i)$
1	0.1244	1.0460	0.1875	0.7739	1
2	0.0181	1.0195	0.0517	0.9574	0

Table 3: Errors for one-step ahead forecast.

Forecast error measures	MSE	MAE	MAPE	MdAPE
Values	0.0081	0.0560	0.0617	0.0378

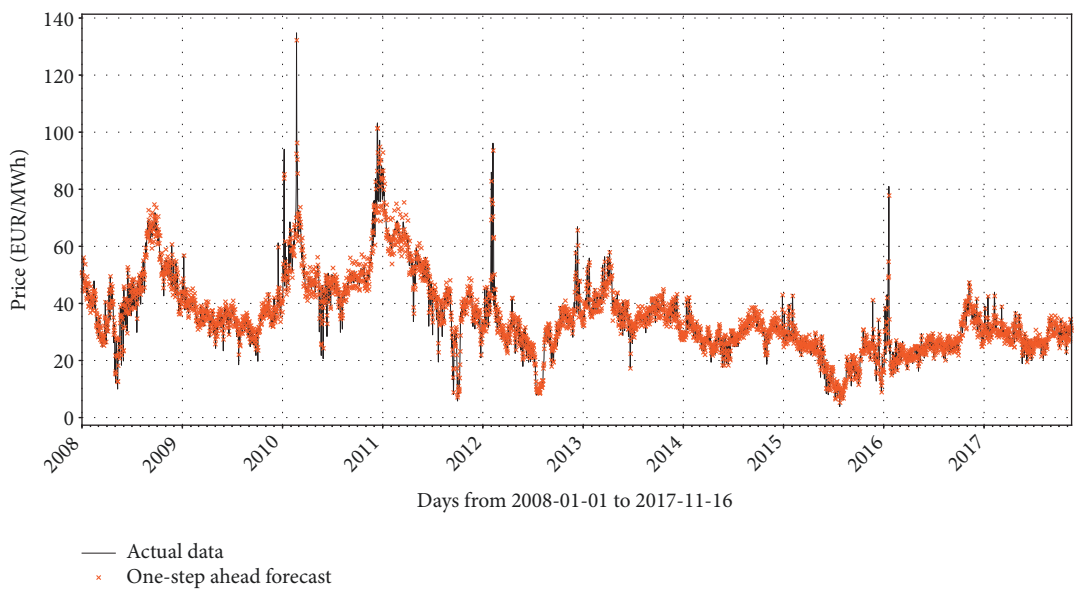


Figure 2: Actual data and one-step ahead forecast of X_t.

$$f_{an}(t) = -4.7728 \sin\left(\frac{2\pi}{365}(t + 275.7260)\right) - 0.0067t + 47.8687 \tag{35}$$

After removing the seasonal part $f(t)$, we focus on the estimation of $\theta = (\beta_i, \xi_i, \sigma_i, P_{ii}, \rho_i)$ for $i = 1, 2$. To obtain $\widehat{\theta} = (\widehat{\beta}_i, \widehat{\xi}_i, \widehat{\sigma}_i, \widehat{P}_{ii}, \widehat{\rho}_i)$, $i = 1, 2$, we discretize (6) to have that

$$X_{t+1} = X_t + \beta_{\alpha_t}\left(\xi_{\alpha_t} - X_t\right) + \sigma_{\alpha_t}\epsilon_t, \tag{36}$$

where $\epsilon_t \sim N(0, 1)$. Then by employing Expectation-Maximization (EM) algorithm (see [33–35]), we conduct estimation. And values of $\widehat{\theta}$ are shown in Table 2.

4.2. Assessment of Forecasts. In this part, we assess the predicted yields by calculating the differences between the one-step ahead forecasts and the actual data, as conducted in [36]. In particular, we choose to use the mean square error (MSE), the mean absolute Error (MAE), the mean absolute percent error (MAPE) and the median relative absolute percentage error (MdAPE) as indicators. Firstly, the one-step ahead predicted X_{t+1} are calculated by

$$\begin{aligned} &E^{\mathbb{P}}\left[X_{t+1} \mid \mathcal{F}_t\right] \\ &\quad = E^{\mathbb{P}}\left[\widehat{\beta}_{\alpha_t}\widehat{\xi}_{\alpha_t} + \left(1 - \widehat{\beta}_{\alpha_t}\right)X_t + \widehat{\sigma}_{\alpha_t}\epsilon_t \mid \mathcal{F}_t\right] \\ &\quad = \sum_{i=1}^{2}\widehat{P}(\alpha_t = i)\cdot\left(\widehat{\beta}_i\widehat{\xi}_i + \left(1 - \widehat{\beta}_i\right)X_t\right). \end{aligned} \tag{37}$$

With the calibrated parameters in Tables 1 and 2 and (35), we can calculate MSE, MAE, MAPE, and MdAPE, and results of these errors are presented in Table 3. Moreover, the actual data and the one-step ahead forecast are plotted in Figure 2, and we find that curves of them are very close to each other.

4.3. The Market Price of Risk. Note that the fair price of the swing option in Definition 2 is the discounted expected future payoff under a martingale measure $\mathbb{Q}$. In order to find $\mathbb{Q}$, we apply the risk premium approach proposed in [37, 38]. Define the risk premium as

$$RP\left(\widehat{T}\right) := E^{\mathbb{P}}\left(S_{\widehat{T}} \mid \mathcal{F}_0\right) - F_0^{\widehat{T}}, \tag{38}$$

Table 4: Option values when **2** rights are used (no rights left).

When **2** volume delivered				When **3** volume delivered				When **4** volume delivered			
τ_0	τ_1	τ_2	τ_3	τ_0	τ_1	τ_2	τ_3	τ_0	τ_1	τ_2	τ_3
0.0, 0.0	0.0, 0.0	0.0, 0.0	0.0, 0.0	0.0, 0.0	0.0, 0.0	0.0, 0.0	0.0, 0.0	**−2.7**, −2.7	−2.8, −2.8	−2.9, −2.9	−3.0, −3.0
	0.0, 0.0	0.0, 0.0	0.0, 0.0		0.0, 0.0	0.0, 0.0	0.0, 0.0		−2.8, −2.8	−2.9, −2.9	−3.0, −3.0
	0.0, 0.0	0.0, 0.0	0.0, 0.0		0.0, 0.0	0.0, 0.0	0.0, 0.0		−2.8, −2.8	−2.9, −2.9	−3.0, −3.0
	0.0, 0.0	0.0, 0.0	0.0, 0.0		0.0, 0.0	0.0, 0.0	0.0, 0.0		−2.8, −2.8	−2.9, −2.9	−3.0, −3.0
	0.0, 0.0	0.0, 0.0	0.0, 0.0		0.0, 0.0	0.0, 0.0	0.0, 0.0		−2.8, −2.8	−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0			−2.9, −2.9	−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0
			0.0, 0.0				0.0, 0.0				−3.0, −3.0

where $E^{\mathbb{P}}$ is the expectation under the physical measure $\mathbb{P}$ and $F_0^{\widehat{T}}$ is the price of future contract at present time 0 with the delivery time $\widehat{T}$. With an equation connecting the risk premium $RP(\widehat{T})$ and the market price of risk μ emerging in (15), we can get an estimation of μ. All the calculation details are presented in Appendix A. Here we just show the result

$$\widehat{\mu} = -0.0032. \quad (39)$$

5. Numerical Example: All Constraints Are Nontrivial

In this section, with all the results obtained in Section 4, we implement numerical examples in the case that all constrains are nontrivial.

The settings on swing option are as follows. The holding period is $[0, T]$, where 0 represents the date 2008/1/1 and $T = 1$ represents 2008/12/31. Set $\Delta\tau = 1/3$ (120 days), which implies that τ_0, τ_1, τ_2, and τ_3 representing 2008/1/6, 2008/5/5, 2008/9/2, and 2008/12/31, respectively. The refraction time ($2\Delta\tau$) is 240 days, i.e., $\kappa = 2$. The corresponding strike prices E_k ($k = 0, \cdots n-1$) are chosen to be 45 euros. The possible delivery volume(s) each time are 0 MWh, 1 MWh, or 2 MWh, respectively. The total number of rights N^A is set to be 2. As for the penalty function, we adopt a specific form of function (8) as

$$\psi\left(U_{n-1}\right) = \begin{cases} 0 & \text{if } 0 \le U_{n-1} \le 3, \\ -3\left(U_{n-1} - 3\right) & \text{if } U_{n-1} > 3. \end{cases} \quad (40)$$

For the tree-building part, we set $\overline{\sigma} = 0.1$, $l_1 = 3$, and $l_2 = 1$ and use the information of Table 2. Firstly, by the method introduced in Section 3.1, we build a tree for X_t, which is presented in Figure 3(a). Secondly, we calculate the values of $f(t)$ at 4 possible exercise days, which are approximately equal to 54.42, 46.05, 45.88, and 51.85, respectively. Thirdly, we use the relationship

$$S_t = X_t \cdot f(t) \quad (41)$$

to get a trinomial tree for S_t, which is shown in Figure 3(b).

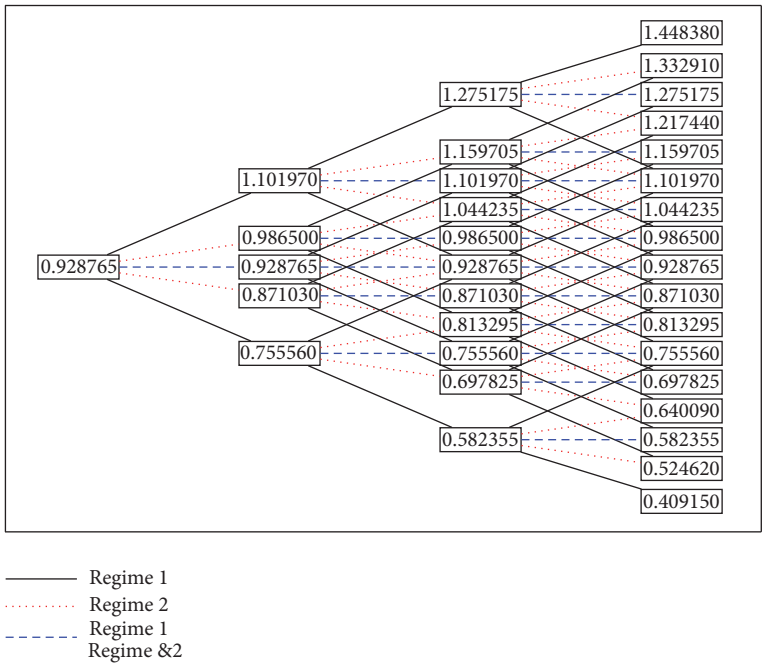
Besides, we set $r_f = 0.1$. Under all these above settings and with the information listed in Table 2, we carry out a numerical example. The results are as follows.

Firstly, we present the values of the swing option under different cases. To be specific, we present the case where the swing option has been used all its rights (2 rights) in Table 4, the case where the swing option has been used 1 right in Table 5 and the case where the swing option has not been used any right in Table 6. Secondly, we see from Table 6 the values of the swing option at time τ_0 in two regimes, respectively, which are 19.2 under state 1 and 15.2 under state 2. Thirdly, we show details of calculating the bolded numbers in these tables. In Table 4,

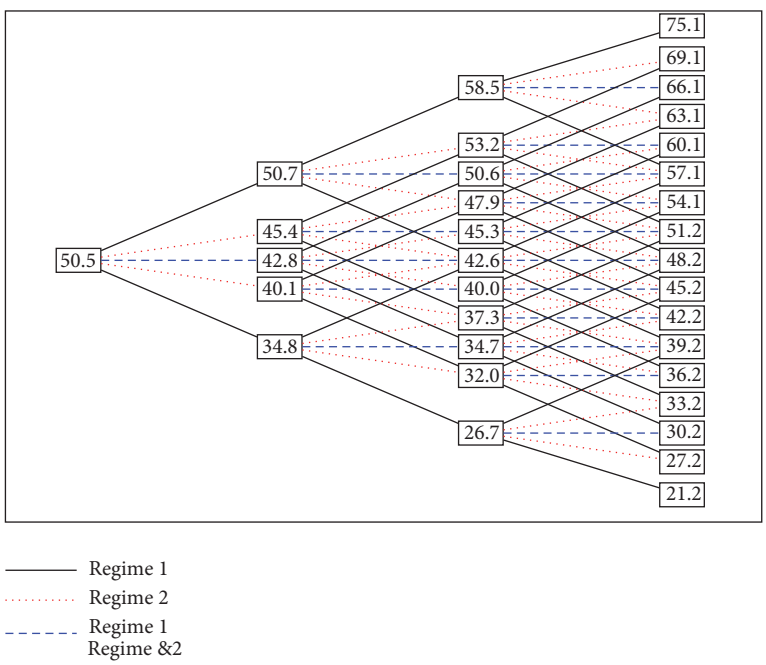
$$\mathbf{-2.7} \overset{(24)}{=} e^{-r_f \cdot 3\Delta\tau} \cdot \psi(4) = e^{-0.1\cdot 3\cdot 1/3} \cdot (-3) \cdot (4-3). \quad (42)$$

In Table 5,

$$\begin{aligned} \mathbf{60.2} &\overset{(25)}{=} \max\{\psi(1), 1\cdot(75.1-45)+\psi(1+1), 2 \\ &\quad \cdot (75.1-45)+\psi(1+2)\}, \\ \mathbf{37.2} &\overset{(26)}{=} \max\{e^{-r_f\Delta\tau}(0.13\times 57.2 + 0.47\times 39.2 \\ &\quad + 0.18\times 21.2 + 0.04\times 57.2 + 0.14\times 39.2 + 0.05 \\ &\quad \times 21.2), 1\cdot\max(58.5-45, 0) + e^{-r_f\Delta\tau}\cdot\psi(2+1), 2 \end{aligned}$$



(a) Trinomial Tree for X_t



(b) Trinomial Tree for S_t

Figure 3: Trinomial tree for underlying variable.

$$\cdot \max(58.5-45,0)+e^{-r_f\Delta\tau}\cdot\psi(2+2)\Big\}$$
$$=\max\{37.2,13.5,24.1\}. \tag{43}$$

In Table 6,

$$\mathbf{19.2} \overset{(27)}{=} \max\Big\{e^{-r_f\Delta\tau}(0.16\times 30.9+0.47\times 9.7$$
$$+0.14\times 2.5+0.05\times 31.4+0.14\times 6.1+0.04$$
$$\times 0.1),1\cdot\max(50.5-45,0)+e^{-r_f\cdot 2\Delta\tau}\sum_{i=1}^{36}a_ib_ic_i,2$$
$$\cdot\max(50.5-45,0)+e^{-r_f\cdot 2\Delta\tau}\sum_{i=1}^{36}d_ie_if_i\Big\},$$

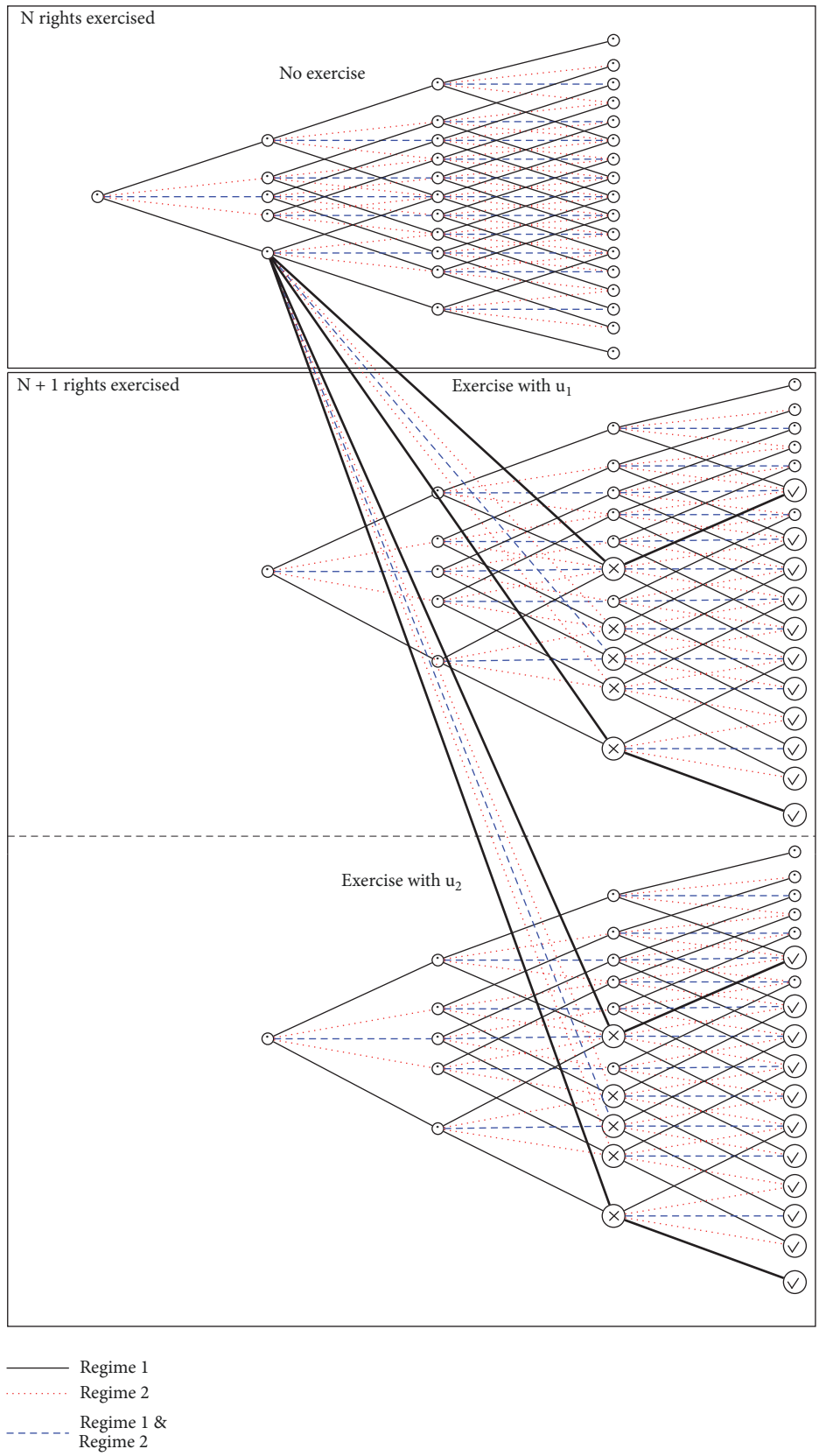


Figure 4: Connection Between the level with N rights exercised and the level with $N + 1$ rights exercised in the Swing forest.

Table 5: Option values when **1** right is used (1 right left).

When **1** volume delivered				When **2** volume delivered			
τ_0	τ_1	τ_2	τ_3	τ_0	τ_1	τ_2	τ_3
11.0, 11.0	22.2, 22.8	40.2, 40.5	**60.2**, 60.2	8.3, 8.3	19.5, 20.0	**37.2**, 37.6	57.2, 57.2
	13.6, 11.6	28.7, 29.0	48.2, 48.2		11.4, 8.8	25.8, 26.1	45.2, 45.2
	9.7, 6.1	22.9, 23.5	42.2, 42.2		7.6, 3.9	20.0, 20.6	39.2, 39.2
	5.8, 1.8	17.5, 17.7	36.2, 36.2		4.8, 1.0	15.1, 14.8	33.2, 33.2
	2.4, 0.1	13.2, 12.0	30.2, 30.2		1.6, 0.0	10.8, 9.0	27.2, 27.2
		8.7, 6.1	24.2, 24.2			6.3, 3.6	21.2, 21.2
		4.1, 1.1	18.2, 18.2			3.4, 0.6	15.2, 15.2
		2.6, 0.1	12.4, 12.4			2.0, 0.8	9.4, 9.4
		1.4, 0.0	6.4, 6.4			0.8, 0.0	3.4, 3.4
		0.1, 0.0	0.4, 0.4			0.0, 0.0	0.2, 0.2
		0.0, 0.0	0.0, 0.0			0.0, 0.0	0.0, 0.0
			0.0, 0.0				0.0, 0.0
			0.0, 0.0				0.0, 0.0
			0.0, 0.0				0.0, 0.0
			0.0, 0.0				0.0, 0.0
			0.0, 0.0				0.0, 0.0

Table 6: Option values when **0** rights are used (2 rights left).

When **0** volume delivered			
τ_0	τ_1	τ_2	τ_3
19.2, 15.2	30.9, 31.4	40.2, 40.5	60.2, 60.2
	14.0, 12.0	28.7, 29.0	48.2, 48.2
	9.7, 6.1	22.9, 23.5	42.2, 42.2
	5.8, 1.8	17.5, 17.7	36.2, 36.2
	2.5, 0.1	13.2, 12.0	30.2, 30.2
		8.7, 6.1	24.2, 24.2
		4.1, 1.1	18.2, 18.2
		2.6, 0.1	12.4, 12.4
		1.4, 0.0	6.4, 6.4
		0.1, 0.0	0.4, 0.4
		0.0, 0.0	0.0, 0.0
			0.0, 0.0
			0.0, 0.0
			0.0, 0.0
			0.0, 0.0
			0.0, 0.0
			0.0, 0.0

$$= \max\{11.9, 15.6, 19.2\}, \tag{44}$$

where sequences $\{a_i\}$, $\{b_i\}$, $\{c_i\}$, $\{d_i\}$, $\{e_i\}$, and $\{f_i\}$ are presented in Appendix B.

6. Conclusions

Under the settings that the underlying asset follows a seasonal mean-reverting regime-switching model with n regimes and that some typical constraints are nontrivial, we have proposed a trinomial forest framework for the valuation of the swing option, which is a multiple-layer tree extension of the traditional trinomial tree approach. It is known that tree approach is based on the risk-neutral pricing theory. Thus we have introduced the risk premium to transform the physical measure into the risk-neutral one. Then the problem of pricing swing options has been stated as an optimal stochastic control problem, where complex constraints on options are inherited by admissible control. In light of stochastic control theory, we have figured out that finding the option price is

equivalent to obtaining the associated value function, which satisfies a Bellman equation. On the basis of the above work, we have succeeded in stating pricing formula for swing options via backward induction.

To show the details of pricing, we have conducted a numerical example based on the calibration results. Comparing to previous work, in our example we have added the refraction time constraint and a more general form global constraint to make the valuation more complicated but closer to the real trading of swing options. This example has also shown that the trinomial forest method does work when both the underlying asset process and the constraints embedded in swing contract are complex.

Appendix

A. Details of Finding the Market Price of Risk

From Table 2, we see that the estimated value of $P(\alpha_0 = 2)$ is 1. And we know that $x_0 = 0.928765$ from Figure 3. Set $\widehat{T}$ to be the day 2008/07/08. Now we try to find the market price of risk μ.

With the fact

$$E^{\mathbb{P}}\left(X_{\widehat{T}} \mid \alpha_{\widehat{T}} = i, \mathscr{F}_0\right) = e^{-\beta_i \widehat{T}} x_0 + \xi_i \left(1 - e^{-\beta_i \widehat{T}}\right), \tag{A.1}$$

we can calculate

$$\begin{aligned} &E^{\mathbb{P}}\left(S_{\widehat{T}} \mid \mathscr{F}_0\right) \\ &\quad = P_{21}^{(\widehat{T})} E^{\mathbb{P}}\left(S_{\widehat{T}} \mid \alpha_{\widehat{T}} = 1, \mathscr{F}_0\right) \\ &\qquad + P_{22}^{(\widehat{T})} E^{\mathbb{P}}\left(S_{\widehat{T}} \mid \alpha_{\widehat{T}} = 2, \mathscr{F}_0\right) \\ &\quad = f\left(\widehat{T}\right) \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} E^{\mathbb{P}}\left(X_{\widehat{T}} \mid \alpha_{\widehat{T}} = i, \mathscr{F}_0\right) \\ &\quad = f\left(\widehat{T}\right) \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} \left(e^{-\beta_i \widehat{T}} x_0 + \xi_i \left(1 - e^{-\beta_i \widehat{T}}\right)\right), \end{aligned} \tag{A.2}$$

where $P_{2j}^{(\widehat{T})} := P(\alpha_{\widehat{T}} = j \mid \alpha_0 = 2)$.

Similar to [38], we assume that

$$\begin{aligned} F_0^{\widehat{T}} := {} & P_{21}^{(\widehat{T})} E^{\mathbb{Q}}\left(S_{\widehat{T}} \mid \alpha_{\widehat{T}} = 1, \mathscr{F}_0\right) \\ & + P_{22}^{(\widehat{T})} E^{\mathbb{Q}}\left(S_{\widehat{T}} \mid \alpha_{\widehat{T}} = 2, \mathscr{F}_0\right). \end{aligned} \tag{A.3}$$

Then with the fact

$$\begin{aligned} &E^{\mathbb{Q}}\left(X_{\widehat{T}} \mid \alpha_{\widehat{T}} = i, \mathscr{F}_0\right) \\ &\quad = e^{-\beta_i \widehat{T}} x_0 + \xi_i \left(1 - e^{-\beta_i \widehat{T}}\right) - \frac{\left(1 - e^{-\beta_i \widehat{T}}\right)\mu}{\beta_i}, \end{aligned} \tag{A.4}$$

we find that (A.3) becomes

$$\begin{aligned} F_0^{\widehat{T}} &= f\left(\widehat{T}\right) \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} E^{\mathbb{Q}}\left(e^{X_{\widehat{T}}} \mid \alpha_{\widehat{T}} = i, \mathscr{F}_0\right) = f\left(\widehat{T}\right) \\ &\quad \cdot \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} \left(e^{-\beta_i \widehat{T}} x_0 + \xi_i \left(1 - e^{-\beta_i \widehat{T}}\right)\right. \\ &\quad \left. - \frac{\left(1 - e^{-\beta_i \widehat{T}}\right)\mu}{\beta_i}\right). \end{aligned} \tag{A.5}$$

By substitute (A.2) and (A.5) into the expression of the risk premium defined in (38), we get an equation connecting $RP(\widehat{T})$ and μ, which is given by

$$\begin{aligned} RP\left(\widehat{T}\right) &= f\left(\widehat{T}\right) \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} \left(E^{\mathbb{P}}\left(X_{\widehat{T}} \mid \alpha_{\widehat{T}} = i, \mathscr{F}_0\right)\right. \\ &\quad \left. - E^{\mathbb{Q}}\left(X_{\widehat{T}} \mid \alpha_{\widehat{T}} = i, 11\mathscr{F}_0\right)\right) = f\left(\widehat{T}\right) \sum_{i=1}^{2} P_{2i}^{(\widehat{T})} \\ &\quad \cdot \frac{\left(1 - e^{-\beta_i \widehat{T}}\right)\mu}{\beta_i}. \end{aligned} \tag{A.6}$$

According to [38], we should rewrite (38) and (A.6) in discrete time scale to define

$$\begin{aligned} &RP^{data}\left(T_1, T_2\right) \\ &\quad = \frac{1}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} E^{\mathbb{P}}\left(S_t \mid \mathscr{F}_0\right) - F_0^{[T_1, T_2]} \\ &\quad = \frac{\sum_{t=T_1}^{T_2} \left[f(t) \sum_{i=1}^{2} P_{2i}^{(t)} \left(e^{-\beta_i t} x_0 + \xi_i \left(1 - e^{-\beta_i t}\right)\right)\right]}{T_2 - T_1 + 1} \\ &\qquad - F_0^{[T_1, T_2]}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} &RP\left(T_1, T_2\right) \\ &\quad = \frac{\sum_{t=T_1}^{T_2} \left[f(t) \sum_{i=1}^{2} P_{2i}^{(t)} \cdot \left(1 - e^{-\beta_i t}\right)\mu/\beta_i\right]}{T_2 - T_1 + 1} \end{aligned} \tag{A.8}$$

where $P_{2i}^{(t)} := P(\alpha_t = i \mid \alpha_0 = 2)$ and $[T_1, T_2]$ represents the futures delivery period.

Finally, with the data listed in Table 7, we find $\widehat{\mu} \approx -0.0032$ by the least square estimation.

B. Value of Sequences $\{a\}$-$\{f\}$

$$\begin{aligned} a = \{&0.16, 0.16, 0.16, 0.16, 0.16, 0.16, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.47, 0.47, 0.47, 0.47, 0.47, 0.47, \\ &0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04\} \end{aligned}$$

Table 7: Information of future contracts in Nord Pool market from Bloomberg on 2008/01/01.

Name	Settlement price(euros)	T_1	T_2	Time index of T_1, T_2
FEB-08	51.11	2008/02/01	2008/02/29	31, 59
MAR-08	52.95	2008/03/03	2008/03/31	62, 90
APR-08	49.49	2008/04/01	2008/04/30	91, 120
MAY-08	49.60	2008/05/02	2008/05/30	122, 150
JUN-08	47.60	2008/06/02	2008/06/30	153, 181
JUL-08	47.98	2008/07/01	2008/07/31	182, 212

$$\begin{aligned}
b &= \{0.14, 0.04, 0.47, 0.14, 0.16, 0.05, 0.01, 0.12, 0.03, 0.7, 0.01, 0.14, 0.16, 0.05, 0.47, 0.14, 0.14, 0.04,\\
&\quad 0.01, 0.12, 0.03, 0.7, 0.01, 0.13, 0.18, 0.05, 0.47, 0.14, 0.13, 0.04, 0.01, 0.13, 0.03, 0.7, 0.01, 0.12\}\\
c &= \{40.2, 40.5, 22.9, 23.5, 8.7, 6.1, 28.7, 29.0, 22.9, 23.5, 17.5, 17.7, 22.9, 23.5, 8.7, 6.1, 1.4, 0.0, 13.2, 12.0,\\
&\quad 8.7, 6.1, 4.1, 1.1, 8.7, 6.1, 1.4, 0.0, 0.0, 0.0, 2.6, 0.1, 1.4, 0.0, 0.1, 0.0\}\\
d &= \{0.16, 0.16, 0.16, 0.16, 0.16, 0.16, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.47, 0.47, 0.47, 0.47, 0.47, 0.47,\\
&\quad 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04\}\\
e &= \{0.14, 0.04, 0.47, 0.14, 0.16, 0.05, 0.01, 0.12, 0.03, 0.7, 0.01, 0.14, 0.16, 0.05, 0.47, 0.14, 0.14, 0.04,\\
&\quad 0.01, 0.12, 0.03, 0.7, 0.01, 0.13, 0.18, 0.05, 0.47, 0.14, 0.13, 0.04, 0.01, 0.13, 0.03, 0.7, 0.01, 0.12\}\\
f &= \{37.2, 37.6, 20.0, 20.6, 6.3, 3.6, 25.8, 26.1, 20.0, 20.6, 15.1, 14.8, 20.0, 20.6, 6.3, 3.6, 0.8, 0.0, 10.8, 9.0,\\
&\quad 6.3, 3.6, 3.4, 0.6, 6.3, 3.6, 0.8, 0.0, 0.0, 0.0, 2.0, 0.0, 0.8, 0.0, 0.0, 0.0\}
\end{aligned} \tag{B.1}$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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