Research Article

In Situ Determination of Density Profiles in Complex Strata Using the Nuclear Density Cone Penetrometer

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The nuclear density cone penetrometer (ND-CP) is an *in situ* testing device that can provide continuous data on soil density, but it measures the composite density (ρ_c) of the soil within a spheroid centered at the midpoint between the gamma ray source and the detector. A theoretical model for predicting ρ_c of the ND-CP is proposed, and equations for calculating ρ_c are derived when the ND-CP penetrates into strata with different functions of density distributions. The calculated ρ_c profiles provide a good fit to the laboratory-measured ρ_c profiles by the ND-CP, indicating that the proposed theoretical model can be used to calculate the ρ_c within the spheroid. The comparisons of the density (ρ) and ρ_c values show that the ρ_c profiles are considerably different from the actual ρ profiles at the boundaries of each stratum where density changes suddenly. Therefore, a method for deducing the actual ρ profiles from the measured ρ_c profiles in complex strata is proposed, and this method is used to determine the actual ρ profiles from the ND-CP measured ρ_c profiles in the field. The research results would be beneficial for the *in situ* determination of the density profile in complex strata using the ND-CP.

1. Introduction

Soil density is defined as the mass of the soil per unit of volume. It depends on the density of the solid constituents, the porosity of the aggregate, and the degree of saturation [1, 2]. Soil density measurement is important because it determines the degree of compactness as a measure of soil structure and is used for calculating the pore space of soils [3]. The density of soil is an indispensable parameter used for the classification of stratigraphy, identification of soft interlayers, determination of the relative density of sand, assessment of the effect of ground improvement, and calculation of a series of soil physical indexes [4, 5]. Furthermore, the density of bed sediments is an important parameter used for predicting sediment transport, determining nautical depth, and planning dredging projects [6].

There are mainly two types of methods available for soil density measurement. One is the long-established direct method, which involves the measurement of the sample mass

and volume. The other is the indirect nondestructive method, which involves measuring the attenuation or scattering of gamma radiation by the soil and determining the soil density from the intensities of the original and the attenuated gamma radiation. The nondestructive method is equal in accuracy to the direct method of density determination and is simpler and quicker to use, especially where measurements at different depths are required [7]. It can be classified into the backscatter type and direct transmission type. In backscatter gauges, the gamma radiation source and detector are lead-shielded and located on the soil surface or are lowered into an access hole in the soil. The gamma rays emitted from the source enter into and interact with the soil and attenuate by scattering. In this process, some of the rays that are scattered towards the detector are counted. In transmission gauges, the gamma radiation source emits rays that enter into and interact with the soil, and some of the rays are scattered away from the detector. The rays passing through the soil are detected and counted by the detector [8].

The use of gamma radiation to measure density is a geophysical technique that was first applied in oil well logging [9]. Subsequently, this technique has been used for a number of years in geotechnical engineering to measure the wet density of soils [10–14]. With recent advances in electronics and the miniaturization of various electronic sensors and devices, the original bulky nuclear density gauge has been developed and incorporated into the piezocone penetrometer, which is the most versatile tool currently available for *in situ* soil investigation [15, 16]; thus, the nuclear density cone penetrometer (ND-CP) was developed [17–19]. The ND-CP has been used in soil investigations, and it can provide reliable soil density measurements [20–23].

In the ND-CP, the density is calculated by the count rate ratio (R_p) of the gamma rays detected by the detector to the gamma rays emitted from the source. Thus, the ND-CP measures the composite density (ρ_c) of the soil within a spheroid centered at the midpoint between the gamma ray source and the detector [24]. Therefore, the measured ρ_c is not the actual ρ of the soil at a certain depth in the ground. There may be a considerable difference between the measured ρ_c profile and the actual ρ profile at the boundaries of each stratum, where the density changes suddenly, e.g., in layered strata, strata with soft interlayers, and bed sediments. Therefore, it is necessary to study how to determine the actual ρ profiles in complex strata from the measured ρ_c profiles by the ND-CP.

In this study, the density measurement principle and the density measurement volume of the ND-CP are described first. Then, a theoretical model for predicting ρ_c is proposed when the ND-CP penetrates into strata with different density distribution functions. The theoretical model is verified by comparing the calculated ρ_c profiles by the theoretical model with the measured ρ_c profiles by the ND-CP in the laboratory. Then, a method for deducing the actual ρ profile from the measured ρ_c profile is proposed based on the difference between the ρ profiles and the ρ_c profiles in complex strata. Finally, the proposed method is used to determine the actual ρ profiles from ND-CP measured ρ_c profiles in the field.

2. Measurement Principle of Nuclear Density Cone Penetrometer

2.1. Nuclear Density Cone Penetrometer. Figure 1 shows the ND-CP. The ND-CP mainly consists of a standard piezocone penetrometer and a nuclear densitometer. The piezocone penetrometer is identical to the standard electronic cone. The nuclear densitometer mainly consists of a gamma ray source and a gamma ray detector. The diameters of the piezocone penetrometer and the nuclear densitometer are 35.7 mm and 48.6 mm, respectively. The distances of the cone resistance sensor, the pore pressure sensor, the sleeve friction sensor, the inclinometer sensor, the gamma ray source, the density count gamma ray detector, and the background count gamma ray detector from the apex of the cone are 0 m, 0.04 m, 0.11 m, 0.5 m, 0.721 m, 0.986 m, and 1.4535 m, respectively [24].

2.2. Density Measurement Principle. Gamma rays are a by-product of the natural decay of some radionuclides. The density measurement principle of the ND-CP is based on the absorption of gamma rays (interaction between the rays and the soil that reduces the intensity of the rays). The photons of the gamma rays can interact with the electrons of the soil. Three different interaction processes, namely, the photoelectric effect, Compton scattering, and pair production, are observed. The photoelectric effect is the interaction of a gamma photon with an electron in the inner shell of the atom. The gamma photon is completely absorbed, and the electron uses the additional energy to leave its atom as a free electron. The Compton effect is the scattering of a gamma photon by an inner shell electron, whereby only part of the photon energy is transferred to the electron. The energy and direction of the gamma photon thus change, and the electron leaves its electronic shell. Pair production is a process in which a gamma ray of sufficient energy is converted into an electron and a positron. Energies of gamma rays range from approximately 10 keV to 10 MeV. The photoelectric effect generally predominates at lower ray energies, the Compton effect predominates when the energy of the ray ranges from 1 MeV to 5 MeV, and pair production occurs at ray energies of several MeVs [25].

The principle of the soil density measurement in the ND-CP utilizes the Compton scattering of gamma rays. When the ND-CP penetrates the ground, the gamma rays emitted from the source pass through the soil particles in the ground, interact with the SiO_2 in the soil particles, collide with the electrons in the outer shell, and scatter repeatedly with the atomic electrons in the soil (Figure 2). In this process, some of the gamma rays are absorbed by the soil, while others reach the detector. The absorption depends on the density of the soil. The higher the density is, the more the electrons are available within a given volume to interact with the gamma rays, and the more gamma rays are absorbed. Hence, the soil density can be determined from the intensities of gamma rays emitted from the source and detected by the detector.

The count rate cannot be directly used to calculate density because the radioactive materials decay exponentially with time. The decay rate is characterized by the half-life of a radioisotope, which is defined as the time taken for half the original number of radioactive nuclei to decay. The gamma ray source used in the ND-CP is the radioactive isotope cesium-137, which has a half-life of approximately 30 years. Therefore, the measured count rate must be normalized by the standard count rate to account for the radioactive decay. In addition, to obtain the accurate absorption of gamma radiation in the soil, the amount of natural gamma rays in the ground must be measured and subtracted from the total count rate detected by the detector. The count rate ratio (R_p), which is used to calculate the soil density, is defined as follows [24]:

$$R_p = \frac{DCR - BCR}{SCR} \quad (1)$$

where DCR is the density count rate, which is measured by the density count gamma ray detector; BCR is the background count rate, which is measured by the background count

FIGURE 1: Nuclear density cone penetrometer.

gamma ray detector; and SCR is the standard count rate, which is measured in an inactive material under controlled conditions before each *in situ* investigation [26]. The calibration equation between soil density and R_p is as follows [24]:

$$R_p = A\rho^2 - B\rho + C \quad (2)$$

where ρ is density and A , B , and C are 0.6264, 4.0954, and 6.8422, respectively, for density values ranging from 1.0 g/cm^3 to 2.2 g/cm^3 . Theoretically, the ND-CP can be used to measure the density of soil up to 2.2 g/cm^3 . However, similar to the piezocone penetrometer, the ND-CP is applicable to general clay, silt, and sand, especially to loose soil and mucky soil below the groundwater level, for which undisturbed samples are difficult to obtain.

2.3. Density Measurement Volume. The absorption of gamma rays and the measured DCR depend on the density of the soil within a spheroid between the gamma ray source and

the detector, as shown in Figure 1. The measurement volume is an oblate spheroid centered at the midpoint between the gamma ray source and detector (Figure 3). The length of semiaxis b , which is the distance from the center to the pole along the symmetry axis, is half of the gamma ray source-detector separation distance ($2b$) of the ND-CP. In this study, the value of $2b$ is 26.5 cm. The length of semiaxis a , which is the equatorial radius of the spheroid, decreases with an increasing density of the surrounding soil [10]. The lengths of a are approximately 24 cm, 20 cm, and 17 cm in water, clay, and sand, respectively [24]. Notably, the measured DCR is more sensitive to the density of the soil closer to the center of the spheroid.

The density measured by the ND-CP is the composite density (ρ_c) of the soil within a spheroid, as shown in Figure 3. Thus, the measured density (ρ_c) is not the actual density (ρ) at a certain depth. If the density changes suddenly with depth, e.g., as in layered soils, the measured ρ_c profile will be different from the actual ρ profile.

FIGURE 2: Schematic diagram of Compton scattering of gamma rays in the soil (modified from <http://www.soilandrock.co.jp/>).

FIGURE 3: Density measurement volume of the ND-CP.

3. Theoretical Model for Calculating Composite Density

3.1. Theoretical Model for Predicting Count Rate Ratio. A mathematical model is proposed for the ND-CP to predict the count rate ratio (R_p) when the ND-CP spans two or more strata. Then, the composite density (ρ_c) within the measurement spheroid can be calculated by (2). Figure 4 shows the spheroid coordinates for the theoretical model used to calculate R_p [24]. The adopted assumptions are as follows.

(1) It is assumed that the scattering and attenuation of gamma rays occur over a spheroid volume; thus, the volume affecting the measured R_p is a spheroid [27]. The center of the spheroid is at the midpoint between the source and the detector. The vertical section of the spheroid is an ellipse, and the horizontal section of the spheroid is a circle.

(2) The spheroid volume is constant. The length of semiaxis b is equal to half of the gamma ray source-detector separation distance (26.5 cm) of the ND-CP. The length of semiaxis a decreases with increasing density of the surrounding soil [10]. For simplification, it is assumed to be a constant.

FIGURE 4: Spheroid coordinates for the theoretical model to calculate the count rate ratio.

(3) All parts of the spheroid contribute equally to the scattering and attenuation of radiation, and the contribution of the count rate ratio from a small volume (ΔR_ρ) in the spheroid to the total count rate ratio (R_ρ) is proportional to its volume. This assumption is not strictly true but does not lead to a large error [28].

(4) The calibration equation between R_ρ and density is defined by (2) for the soils with densities ranging from 1.0 g/cm³ to 2.2 g/cm³.

The volume of the spheroid is as follows:

$$V = \frac{4}{3}\pi a^2 b \quad (3)$$

The vertical cross-section of the spheroid in the x - z plane is an ellipse, and its equation is

$$\frac{x^2}{a^2} + \frac{(z - z_i)^2}{b^2} = 1 \quad (4)$$

where a is the length of the major semiaxis, b is the length of the minor semiaxis, z is the depth, and z_i is the depth corresponding to the midpoint of the ellipse. The horizontal cross-section of the spheroid at any point is a circle of radius x , where $0 < x < a$. A small volume of the spheroid ∂V is

$$\partial V = \pi x^2 \delta z \quad (5)$$

where δz is the differential of z . The x calculated by (4) can be substituted into (5) to give

$$\partial V = \frac{\pi a^2}{b^2} [b^2 - (z - z_i)^2] \delta z \quad (6)$$

The assumption is that the contribution of the small volume ∂R_ρ to R_ρ is proportional to its volume:

$$\frac{\partial R_\rho}{R_\rho} = \frac{\partial V}{V} \quad (7)$$

Substituting (2), (3), and (6) into (7) gives

$$\partial R_\rho = \frac{3(A\rho^2 - B\rho + C)}{4b^3} [b^2 - (z - z_i)^2] \delta z \quad (8)$$

Assuming the density ρ is a function of depth $\rho = \rho(z)$, integrating (8) over the spheroid volume gives R_ρ :

$$R_\rho = \frac{3}{4b^3} \int (A\rho^2(z) - B\rho(z) + C) [b^2 - (z - z_i)^2] \delta z \quad (9)$$

3.2. Equations for Calculating the Count Rate Ratio. If the density function $\rho(z)$ of the strata is known, (9) can be used to calculate the count rate ratio (R_ρ) when the ND-CP penetrates into the strata. The probable density profiles in the ground are described by constant, linear, parabolic, and square root functions (Figure 5), the equations of which are as follows:

$$\rho(z) = \rho_0 \quad (10)$$

$$\rho(z) = \rho_1 + \beta_1(z - z_1) \quad (11)$$

$$\rho(z) = \alpha_1(z - z_1)^2 + \rho_1 \quad (12)$$

$$\rho(z) = \alpha_2(z - z_1)^{1/2} + \rho_1 \quad (13)$$

If the density and depth at the top of the stratum (ρ_1, z_1) and at the bottom of the stratum (ρ_2, z_2) are known, the values

FIGURE 5: Assumed density profiles in the ground: (a) constant; (b) linear; (c) parabolic; (d) square root.

of β_1 , α_1 , and α_2 in (11), (12), and (13) can be determined as follows:

$$\beta_1 = \frac{(\rho_2 - \rho_1)}{(z_2 - z_1)} \quad (14)$$

$$\alpha_1 = \frac{(\rho_2 - \rho_1)}{(z_2 - z_1)^2} \quad (15)$$

$$\alpha_2 = \frac{(\rho_2 - \rho_1)}{(z_2 - z_1)^{0.5}} \quad (16)$$

Eqs. (17), (18), (19), and (20) can be used to calculate R_ρ when the density functions are constant, linear, parabolic, and square root, respectively.

$$R_\rho = \frac{3(A\rho_0^2 - B\rho_0 + C)}{4b^3} \int [b^2 - (z - z_i)^2] \delta z \quad (17)$$

$$R_\rho = \frac{3}{4b^3} \int \left\{ A[\rho_1 + \beta(z - z_1)]^2 - B[\rho_1 + \beta(z - z_1)] + C \right\} [b^2 - (z - z_i)^2] \delta z \quad (18)$$

$$R_\rho = \frac{3}{4b^3} \int \left\{ A[\alpha_1(z - z_1)^2 + \rho_1]^2 - B[\alpha_1(z - z_1)^2 + \rho_1] + C \right\} [b^2 - (z - z_i)^2] \delta z \quad (19)$$

$$R_\rho = \frac{3}{4b^3} \int \left\{ A[\alpha_2(z - z_1)^{1/2} + \rho_1]^2 - B[\alpha_2(z - z_1)^{1/2} + \rho_1] + C \right\} [b^2 - (z - z_i)^2] \delta z \quad (20)$$

The solutions for (17) to (20) are derived in (21) to (24), respectively.

$$R_\rho = \frac{3}{4b^3} (A\rho_0^2 - B\rho_0 + C) \left\{ -\frac{z^3}{3} + z_i z^2 + (b^2 - z_i^2) z \right\} \Big|_{z_1}^{z_2} \quad (21)$$

$$R_\rho = \frac{3}{4b^3} \left\{ \begin{aligned} & \frac{-A\beta^2 z^5}{5} + \frac{(-2A\rho_1\beta + 2Az_i\beta^2 + 2A\beta^2 z_1 + B\beta) z^4}{4} \\ & + \left(\frac{b^2 A\beta^2 - A\rho_1^2 + 4Az_i\rho_1\beta - Az_i^2\beta^2 + 2A\rho_1\beta z_1 - A\beta^2 z_1^2 - 4Az_i\beta^2 z_1}{+B\rho_1 - B\beta z_1 - 2Bz_i\beta - C} \right) z^3 \\ & + \left(\frac{2b^2 A\beta\rho_1 - 2b^2 A\beta^2 z_1 - 2A\rho_1\beta z_i^2 + 2Az_i\rho_1^2 - 4Az_i\rho_1\beta z_1 + 2Az_i\beta^2 z_1^2}{+2Az_i^2\beta^2 z_1 - b^2 B\beta + Bz_i^2\beta - 2Bz_i\rho_1 + 2Bz_i\beta z_1 + 2Cz_i} \right) z^2 \\ & + \left(\frac{b^2 A\rho_1^2 - 2b^2 A\rho_1\beta z_1 + b^2 A\beta^2 z_1^2 - Az_i^2\rho_1^2 + 2A\rho_1\beta z_1 z_i^2 - Az_i^2\beta^2 z_1^2}{-b^2 B\rho_1 + b^2 B\beta z_1 + Bz_i^2\rho_1 - Bz_i^2\beta z_1 + Cb^2 - Cz_i^2} \right) z \end{aligned} \right\} \Big|_{z_1}^{z_2} \quad (22)$$

$$R_\rho = \frac{3}{4b^3} \left\{ \begin{aligned} & \left(\frac{(-A\alpha_1^2)z^7}{7} + \frac{(4A\alpha_1^2z_1 + 2Az_i\alpha_1^2)z^6}{6} \right. \\ & + \frac{(b^2A\alpha_1^2 - 6A\alpha_1^2z_1^2 - 2A\alpha_1\rho_1 - 8Az_i\alpha_1^2z_1 - A\alpha_1^2z_i^2 + B\alpha_1)z^5}{5} \\ & + \frac{\left(\frac{-4b^2A\alpha_1^2z_1 + 4A\alpha_1^2z_1^3 + 4A\alpha_1\rho_1z_1 + 12Az_i\alpha_1^2z_1^2 + 4Az_i\alpha_1\rho_1 + 4Az_i^2\alpha_1^2z_1}{-2Bz_1\alpha_1 - 2Bz_i\alpha_1} \right)z^4}{4} \\ & + \frac{\left(\frac{6b^2A\alpha_1^2z_1^2 + 2b^2A\alpha_1\rho_1 - A\alpha_1^2z_1^4 - 2A\alpha_1\rho_1z_1^2 - A\rho_1^2 - 8Az_i\alpha_1^2z_1^3 - 8Az_i\alpha_1\rho_1z_1}{-6Az_i^2\alpha_1^2z_1^2 - 2Az_i^2\alpha_1\rho_1 - b^2B\alpha_1 + Bz_1^2\alpha_1 + B\rho_1 + 4Bz_i\alpha_1 + Bz_i^2\alpha_1 - C} \right)z^3}{3} \\ & + \frac{\left(\frac{-4b^2A\alpha_1\rho_1z_1 - 4b^2A\alpha_1^2z_1^3 + 2Az_i\alpha_1^2z_1^4 + 2Az_i\rho_1^2 + 4Az_i\alpha_1\rho_1z_1^2 + 4Az_i^2\alpha_1^2z_1^3}{+4Az_i^2\alpha_1\rho_1z_1 + 2b^2Bz_1\alpha_1 - 2Bz_i\alpha_1^2 - 2Bz_i\rho_1 - 2Bz_i^2z_1\alpha_1 + 2Cz_i} \right)z^2}{2} \\ & \left. + \left(\frac{2b^2A\alpha_1\rho_1z_1^2 + b^2A\rho_1^2 + b^2A\alpha_1^2z_1^4 - Az_i^2\alpha_1^2z_1^4 - 2Az_i^2\alpha_1\rho_1z_1^2 - Az_i^2\rho_1^2 - b^2Bz_1^2\alpha_1}{-b^2B\rho_1 + Bz_i^2z_1^2\alpha_1 + Bz_i^2\rho_1 + Cb^2 - Cz_i^2} \right)z \right\} \Bigg|_{z_1}^{z_2} \quad (23) \end{aligned} \right.$$

$$R_\rho = \frac{3}{4b^3} \left\{ \begin{aligned} & -\frac{1}{4}A\alpha_2^2(z-z_1)^4 + \frac{2}{7}(B\alpha_2 - 2A\rho_1\alpha_2)(z-z_1)^{7/2} \\ & + \frac{1}{3} \left[-2A\alpha_2^2(z_1-z_i) \right] (z-z_1)^3 - \frac{1}{3}(A\rho_1^2 - B\rho_1)(z-z_i)^3 - \frac{1}{3}Cz^3 \\ & + \frac{2}{5} [2B\alpha_2(z_1-z_i) - 4A\rho_1\alpha_2(z_1-z_i)] (z-z_1)^{5/2} \\ & + \frac{1}{2} [b^2A\alpha_2^2 - A\alpha_2^2(z_1-z_i)^2] (z-z_1)^2 + Cz_i z^2 \\ & + \frac{2}{3} [2b^2A\rho_1\alpha_2 - 2A\rho_1\alpha_2(z_1-z_i)^2 - b^2B\alpha_2 + B\alpha_2(z_1-z_i)^2] (z-z_1)^{3/2} \\ & + [b^2A\rho_1^2 - b^2B\rho_1 + C(b^2 - z_i^2)] z \end{aligned} \right\} \Bigg|_{z_1}^{z_2} \quad (24)$$

If the gamma ray source and detector are in strata that can be described by one function of density, R_ρ is calculated as an integral of that function. If the gamma ray source and detector are in strata described by two or more functions, R_ρ is calculated as a piecewise integration of those functions. Then, the composite density (ρ_c), which is referenced to the midpoint of the spheroid, can be calculated by (25). Finally, the ρ_c profile can be obtained as the midpoint of the spheroid moves with depth along with the penetration of the ND-CP.

$$\rho_c = \frac{4.0954 - \sqrt{2.5056R_\rho - 0.3715}}{1.2528} \quad (25)$$

3.3. Verification of the Theoretical Model. The ND-CP was used to measure the density profiles of two kinds of layered soils by Karthikeyan and Tan in the laboratory [29]. The experimental setup consists of a loading frame and a cubic steel tank with dimensions of 1.5 m × 1.5 m × 1.5 m. The loading frame is placed on top of the steel tank to push the ND-CP using a hydraulic piston. The ND-CP measurement and the depth meter readings were recorded through a data logger. Figure 6 shows schematic configurations for layered soils in the steel tank. One configuration is with a top layer of water and bottom layer of soil (Figure 6(a)), and the other configuration is with a top layer of kaolin clay slurry and bottom layer of soil (Figure 6(b)). The measured ρ_c profiles do not show a sharp transition at the interface of the water and soil layers (Figure 6(a)) or at the interface of the slurry and soil layers (Figure 6(b)). The change in density from the

water/slurry to the soil is extended over a distance equal to the source-detector separation (2b). This transition occurs as the gamma ray source enters the soil layer and continues until the gamma ray detector enters the soil layer. The actual ρ profile shows a step change in density, while the measured ρ_c profile shows a continuous change in density. The change pattern in the measured ρ_c profile indicates that the ND-CP is penetrating the interface of two layers with different density values. The ρ_c profiles predicted by the theoretical model are also shown in the figure, and these predicted results show a close agreement with the measured ρ_c profiles, suggesting that the theoretical model is able to describe the signature of the ρ_c profiles measured by the ND-CP. The theoretical model will be used to investigate the differences between the actual ρ profiles in the ground and the measured ρ_c profiles by the ND-CP.

4. Method for Deducing Actual ρ Profile

4.1. Comparison of ρ and ρ_c Profiles. The density profiles of complex strata, which are common in engineering practice, are assumed in Figure 7. Figure 7(a) shows the density profiles of layered strata. The density of the first layer is 1.6 g/cm³, and the densities of the second layer are 1.5 g/cm³, 1.7 g/cm³, or 1.8 g/cm³. Figure 7(b) shows the density profiles of strata with soft interlayers. The density of the strata is 1.7 g/cm³. Three different soft interlayers are assumed. For the first case, the soft interlayer has a density of 1.3 g/cm³ and a thickness of 0.15 m; for the second case, the density is 1.5 g/cm³ and

FIGURE 6: Verification of the theoretical model with laboratory test results.

the thickness is 0.15 m; for the third case, the density is 1.3 g/cm^3 and the thickness is 0.3 m. Figure 7(c) shows the density profiles of bed sediments. The density of the water is 1.0 g/cm^3 , and the density of the bed mud increases with depth in a linear fashion. Three different layers of fluid mud are assumed. For the first case, the fluid mud has a maximum density of 1.2 g/cm^3 and thickness of 0.2 m; for the second case, the maximum density is 1.3 g/cm^3 and the thickness is 0.2 m; for the third case, the maximum density is 1.15 g/cm^3 and the thickness is 0.25 m.

The ρ_c profiles are calculated by the theoretical model when the ND-CP penetrates into the assumed complex strata (Figure 7), and the results are compared with the ρ profiles. The ρ_c profiles are clearly different from the ρ profiles near the interface of the layered strata, near the soft interlayer, and near the upper and lower boundaries of the fluid mud. The differences between ρ_c and ρ near the interface of the layered strata in Figure 7(a) are explained as follows. (1) When both the gamma ray source and detector are in the top layer, ρ_c is equal to the density of the top layer. (2) Once the gamma ray source enters the bottom layer, ρ_c will be the weighted average density of the two layers, while ρ is still the density of the first layer. (3) When the midpoint of the gamma ray source and detector enters the bottom layer, ρ will change to the density of the bottom layer, while ρ_c is still the weighted average density of the two layers because the gamma ray detector is still in the first layer. (4) When the gamma ray detector also enters the bottom layer, ρ_c is equal to ρ of the bottom layer. Similar to the profiles for the case in which the ND-CP passes through the interface of layered strata, the ρ_c profiles are different from the ρ profiles when the ND-CP

passes through a soft interlayer and the fluid mud. Therefore, the ρ_c profile measured by the ND-CP must be interpreted, and the actual ρ profile must be deduced from the measured ρ_c profile.

4.2. Method for Deducing Actual ρ Profile from Measured ρ_c Profile. A method for deducing the actual ρ profile from the measured ρ_c profile is proposed as follows:

(1) Determining the depths of and densities at the boundaries of each stratum based on the features of the measured ρ_c profile. (2) Assuming the probable density function of each stratum and the probable ρ profile of the strata. (3) Back-calculating the ρ_c profile by using the theoretical model for the case of the ND-CP passing through layered strata. (4) Comparing the calculated ρ_c profile with the measured ρ_c profile and adjusting the assumed ρ profile until the calculated ρ_c profile matches the measured ρ_c profile well. (5) Determining the actual ρ profile to be the assumed ρ profile when the calculated ρ_c profile fits the measured ρ_c profile.

For the layered strata, as shown in Figure 7(a), the measured ρ_c profile can be divided into three sections, namely, the first constant section, the second curved section, and the third constant section. The measured ρ_c values are different from the actual ρ values in the range of $2b$, which is the distance between the gamma ray source and detector. From the ρ_c profile, the boundary between layered strata can be determined, i.e., the midpoint of the second curved section of the ρ_c profile. The actual ρ value of the top layer is assumed to be the measured ρ_c value of the first constant section, and the actual ρ value of the bottom layer is assumed to be the measured ρ_c value of the third constant section. The ρ_c profile

FIGURE 7: Comparison of assumed ρ and calculated ρ_c profiles.

is back-calculated assuming that the ND-CP passes through the layered strata. If the calculated ρ_c profile matches well with the measured ρ_c profile, the actual ρ profile of the layered strata was determined.

For the strata with a soft interlayer, as shown in Figure 7(b), the measured ρ_c profile can be divided into three sections, namely, the first constant section, the second curved section, and the third constant section. The measured ρ_c values are different from the actual ρ values near the soft interlayer ($2b$ plus the thickness of the soft interlayer). From

the ρ_c profile, the upper and lower boundaries of the soft interlayer can be determined, that is, the depth of the start point of the second curved section plus b and the depth of the end point of the second curved section minus b . The actual ρ of the top layer is assumed to be the measured ρ_c of the first constant section, and the actual ρ of the bottom layer is assumed to be the measured ρ_c of the third constant section. The actual ρ of the soft interlayer can be assumed based on the minimum ρ_c in the second curved section. The ρ_c profile is back-calculated assuming that the ND-CP passes through

FIGURE 8: Investigation locations.

FIGURE 9: Investigation results: (a) at location A1; (b) at location A2.

the strata with a soft interlayer. The actual ρ profile of strata with a soft interlayer is determined when the calculated ρ_c profile matches the measured ρ_c profile well.

For bed sediments, as shown in Figure 7(c), the measured ρ_c profile can be divided into three sections, namely, the first constant section, the second curved section, and the third linear section. The measured ρ_c values are different from the actual ρ values near the upper and lower boundaries of the fluid mud. From the ρ_c profile, the upper and lower boundaries of the fluid mud can be determined; that is, the depth of the point from which density increases plus b and the depth of the point from which density slightly increases linearly minus b . The actual ρ of the water layer is assumed to be the measured ρ_c of the first constant section. The slope of the actual ρ profile of the bed mud layer is assumed to be the slope of the measured ρ_c profile of the third linear section. The actual ρ distribution of the fluid mud layer can be assumed to

be a parabolic, linear, or square root function. The ρ_c profile is back-calculated assuming that the ND-CP passes through bed sediments. The actual ρ profile of bed sediments can be determined when the calculated ρ_c profile fits the measured ρ_c profile.

5. An Example of Practical Application

5.1. ND-CP Measured ρ_c Profile. The density profiles of bed sediments at Isahaya Bay, Japan, were investigated using the ND-CP. The details on the equipment and methods used in this investigation can be found in a prior publication [23]. The ND-CP measured density profiles at two locations (Figure 8) are used to illustrate how to use the proposed method to determine the actual ρ profile from the measured ρ_c profile.

The measured BCR and DCR and the calculated ρ_c values at locations A1 and A2 are shown in Figure 9. BCR

FIGURE 10: Determination of the actual density profile of bed sediments at location A1: (a) measured ρ_c profile; (b) assumed ρ profile; (c) calculated ρ_c profile; (d) deduced ρ profile.

FIGURE 11: Determination of the actual density profile of bed sediments at location A2: (a) measured ρ_c profile; (b) assumed ρ profile; (c) calculated ρ_c profile; (d) deduced ρ profile.

is approximately zero in water and approximately 50 cps in bed mud. DCR in bed sediments is lower than that in water and decreases with increasing density. R_p can be calculated according to the measured BCR and DCR by using (1), and then the density can be calculated by using (25). The density profiles show similar transitions and vary significantly around the water/mud interface. The measured ρ_c profiles can be divided into three sections, namely, an initial constant section, a middle curved section, and a final linear section.

5.2. Determination of the Actual ρ Profile. There is an obvious difference between the actual ρ and the measured ρ_c profiles near the upper and lower boundaries of the fluid mud layer, as shown in Figure 7(c). The theoretical model proposed in Section 3 and the method proposed in Section 4 were used to deduce the actual ρ profiles from the measured ρ_c profiles as follows.

(1) For the measured ρ_c profile at location A1 (Figure 10(a)), the ρ_c at and depth of the intersection of the initial constant and middle curved sections (point A) are 1.000 g/cm^3 and 1.83 m , respectively. The ρ_c at and depth of the intersection of the middle curved and final linear sections (point B) are 1.287 g/cm^3 and 2.31 m , respectively. For the measured ρ_c profile at location A2 (Figure 11(a)), ρ_c and depth at point A are 1.025 g/cm^3 and 10.45 m , respectively. ρ_c and depth at point B are 1.233 g/cm^3 and 10.83 m , respectively.

(2) For the assumed ρ profile at location A1 (Figure 10(b)), the depth of the upper boundary of the fluid mud at point C is 1.9625 m , which is equal to the depth at point A (1.83 m) plus b (0.1325 m). The depth of the lower boundary of the fluid mud at point D is 2.1775 m , which is equal to the depth at point B (2.31 m) minus b (0.1325 m). ρ at point C is 1.000 g/cm^3 , which is equal to ρ_c at point A. ρ at point D is 1.286 g/cm^3 , which is equal to ρ_c at point B minus kb (where k is the slope of the final linear section). For the assumed ρ profile at location A2

(Figure 11(b)), the depth at point C is 10.5825 m, and the depth at point D is 10.6975 m. ρ at point C is 1.025 g/cm^3 , and ρ at point D is 1.222 g/cm^3 .

(3) The ρ function of the first section (water layer) is assumed to be constant, the ρ function of the second section (fluid mud) is assumed to be parabolic, linear, or square root, and the ρ function of the third section (bed mud) is assumed to be linear. The ρ of the first section is assumed to be 1.000 g/cm^3 (freshwater) and 1.025 g/cm^3 (seawater) for ρ profiles at locations A1 and A2, respectively. The parabolic, linear, and square root density functions for the second section can be determined according to the density and depth values at point C and point D by (11) to (16). The slope of the third section is assumed to be 0.005 and 0.08, respectively, for the ρ profiles at locations A1 and A2 based on the measured ρ_c values at the final linear sections.

(4) The ρ_c profiles are calculated by (21) to (25), assuming the ND-CP passes through the strata with the assumed ρ profiles in Figures 10(b) and 11(b). The calculated ρ_c profiles at locations A1 and A2 are shown in Figures 10(c) and 11(c), respectively. The calculated ρ_c profiles clearly vary when the assumed ρ function of the second section varies (among parabolic, linear, or square root functions). Therefore, the calculated ρ_c profile can be changed by adjusting the assumed ρ profile.

(5) The calculated ρ_c profiles for the different assumed ρ profiles are compared with the measured ρ_c profiles. The calculated ρ_c profiles match the measured ρ_c profiles well if the density function of the second section is assumed to be a square root function [24]. The deduced ρ profiles are also shown in Figures 10(d) and 11(d). The ρ values are clearly different from the ρ_c values around the fluid mud layer. Therefore, it is suggested that the deduced ρ profile from the measured ρ_c profile should be adopted to predict sediment transport, determine nautical depth, and plan dredging projects.

6. Summary and Conclusions

The nuclear density cone penetrometer (ND-CP) can be used for *in situ* investigation on the soil density, while it measures the composite density (ρ_c) of the soil within a spheroid centered at the midpoint between the gamma ray source and the detector. A theoretical model for calculating ρ_c and a method for deducing the ρ profile from the measured ρ_c profile were proposed in this study. The following conclusions can be drawn from this study.

(1) A theoretical model for predicting the count rate ratio (R_p) of the ND-CP is proposed, and equations for calculating R_p are derived for the case of the ND-CP penetrating into strata with different density distribution functions. The calculated ρ_c profiles exhibit a good fit to the laboratory-measured ρ_c profiles by the ND-CP, indicating that the proposed theoretical model can be used to calculate the ρ_c within the spheroid. A comparison of the ρ and ρ_c values shows that the ρ_c profiles are considerably different from the actual ρ profiles at the boundaries of each stratum where the density changes suddenly.

(2) A method for deducing the actual ρ profile from the measured ρ_c profile is proposed according to the differences between the ρ and ρ_c profiles. The depth of and density at the boundaries of each stratum are first determined based on the features of the measured ρ_c profile. Then, the ρ profile of the strata is assumed, and the ρ_c profile is back-calculated. Finally, the calculated ρ_c profile is compared with the measured ρ_c profile, and the actual ρ profile is identified when the calculated ρ_c profile fits the measured ρ_c profile.

(3) The proposed model and method are used to determine the actual ρ profiles from measured ρ_c profiles of bed sediments in the field. The actual ρ values are clearly different from the ρ_c values around the fluid mud layer. It is suggested that the deduced ρ profile should be adopted to predict sediment transport, determine nautical depth, and plan dredging projects. The findings in this study would be beneficial for the *in situ* determination of density profiles using the ND-CP. Further research should be conducted to verify the proposed model and method in determining the actual ρ profiles from the measured ρ_c profiles by the ND-CP in the field of geotechnical and pavement engineering.

Acronyms

a :	Equatorial radius of the spheroid
$2b$:	Gamma ray source-detector separation distance
A :	Constant 0.6264
B :	Constant 4.0954
C :	Constant 6.8422
BCR :	Background count rate
DCR :	Density count rate
SCR :	Standard count rate
k :	Slope of the final linear region
ND-CP:	Nuclear density cone penetrometer
R_p :	Count rate ratio
V :	Volume of spheroid
z :	Depth
z_i :	Depth corresponding to the midpoint of the ellipse
z_1 :	Depth at the top of stratum
z_2 :	Depth at the bottom of stratum
α_1 :	Fitting parameter for parabolic density distribution
α_2 :	Fitting parameter for square root density distribution
β_1 :	Fitting parameter for linear density distribution
ρ :	Density
ρ_c :	Composite density
ρ_0 :	Density value of constant density distribution
ρ_1 :	Density value at the top of stratum
ρ_2 :	Density value at the bottom of stratum
ΔR_p :	Count rate ratio from a small volume
δz :	Differential of z
∂V :	Differential of V
∂R_p :	Differential of count rate ratio.

Data Availability

The data used to support the findings of this article are available from the authors upon request.

Conflicts of Interest

The authors declare that they have no competing interests.

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