Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 5820394, 14 pages
https://doi.org/10.1155/2019/5820394

Research Article

Active Fault Tolerant Control Design for LPV Systems with Simultaneous Actuator and Sensor Faults

Raouaa Tayari, Ali Ben Brahim, Fayçal Ben Hmida, and Anis Sallami

Laboratory of Engineering of Industrial Systems and Renewable Energy (LISIER), National Higher Engineering School of Tunis (ENSIT), University of Tunis, 5 Avenue Taha Hussein, BP 56, 1008 Tunis, Tunisia

Correspondence should be addressed to Raouaa Tayari; tayari.raouaa@hotmail.com

Received 7 August 2018; Revised 18 December 2018; Accepted 9 January 2019; Published 23 January 2019

Academic Editor: Jean Jacques Loiseau

The present paper addresses the problem of robust active fault tolerant control (FTC) for uncertain linear parameter varying (LPV) systems with simultaneous actuator and sensor faults. First, fault estimation (FE) scheme is designed based on two adaptive sliding mode observers (SMO). Second, using the information of simultaneous system state, actuator, and sensor faults, two active FTC are conceived for LPV systems described with polytopic representation as state feedback control and sliding mode control. The stability of closed-loop systems is guaranteed by mean of H_∞ performance; sufficient conditions of the proposed methods are derived in LMIs formulation. The performance effectiveness of FTC design is illustrated using a VTOL aircraft system with both sensor and actuator faults as well as disturbances. In addition, comparative simulations are provided to verify the benefits of the proposed methods.

1. Introduction

In recent decades, the performance of industrial equipment has been increasing significantly. The gain in performance was accompanied by an increasing in the complexity of the installations causing a higher demand of strong availability and security. However, by realising some tasks a degradation of performances may occur; these degradations are caused by abnormal phenomena which are called faults. The faults may lead to performance deterioration or even instability of the system. To preserve robust system performance, fault-tolerant control designs have attracted significant attention like in [1–3].

In the literature, two types of FTC approaches have been distinguished: passive ([4, 5]) and active ([6–8]). The passive control uses a fixed controller to deal with any fault free and faulty cases, which is based on robust control techniques. Also, it consists of ensuring that the closed-loop system remains insensitive to the occurrence of certain defects. In the case of active approaches, depending on the severity of the fault impact on the system, a new control law is applied. Moreover, to ensure the highest possible performance of the controlled system in presence of a fault, an active FTC strategy is necessary. Thus, the FTC unit receives the signal from the FE module identifying the type of the fault; an appropriate decision must be made in order to preserve the system performance.

Compared with a linear system, FTC for nonlinear system is much more challenging because of its complexity. Linear parameter varying models offer a potent tool to research the nonlinear characteristics of systems, where the nonlinear system can be approximated by several local linear models and the global stability over the entire working space can also be easily guaranteed. Consequently, LPV systems have attracted considerable attention in [9–11].

Active FTC is often dedicated to linear systems or the linearization of nonlinear systems, but rarely to LPV systems. However, few studies take into account the FTC of LPV systems. In [12] an observer based FTC design for LPV descriptor systems with actuator faults is developed. A proportional derivative extended state observer (PDESO) based FE and FTC design is proposed in [13] for LPV descriptor systems with both actuator and sensor faults. In [14] a Low Order (LO) observer and FTC strategy for LPV systems subject to sensor faults are proposed. Finally, virtual actuator FE/FTC are developed in [15].

In the last decade, sliding mode (SM) concepts have been the focus of research because of their robustness. The SM control methodology has the advantage of producing low complexity control laws compared to other robust control approaches [16, 17]. One of the FTC approaches which has received increasing attention in recent years is SMC. Despite the popularity of SMC, there has been little work on LPV systems.

This active approach comprises FE observer and FTC control modules. The FE module determines the size or even the dynamic behavior of the fault. So, FE approach gives the detailed information of the fault signal which is necessary to be developed. In fact, many practical applications may have multiple single faults that appear simultaneously, which is known as "simultaneous faults" [18]. Several FE strategies based on LPV systems have been proposed, e.g., using learning observer [19], SMO [20, 21], and reduced-order observer [22]. These approaches are based on robustness concepts and are thus good candidates to include in active FE based FTC system analysis and design.

In this paper, the FE is obtained using a sliding mode observers-based actuator and sensor faults estimation for linear parameter varying systems by [23]. FTC is then realized through a state feedback and sliding mode controller, which can compensate for both actuator and sensor faults based on fault estimation.

The main contributions of this paper compared with the existing control based FTC work are summarized as follows:

(i) This paper investigates new methods based on a robust active FTC for polytopic LPV system with simultaneous actuator and sensor faults as well as disturbances. Hence, the FTC strategy is developed to avoid the effects of simultaneous actuator and sensor faults on polytopic LPV systems. In many research works, the FTC design is only used for polytopic LPV systems either for actuator faults or sensor faults, but does not consider simultaneous actuator and sensor faults. This paper deals with an FTC synthesis in the presence of simultaneous faults.

(ii) A novel FTC based fault estimation (FE) design using sliding mode for each one is studied for polytopic system. In the literature, the SM was only considered for either FTC or FE like in [24, 25]. The proposed method transforms the LPV system into two subsystems where the first one includes the effects of actuator faults but is free from sensor faults and the second subsystem has only sensor faults. Furthermore, two sliding mode observers are designed where each one estimate a class of faults. Based on the updated values of these estimations, the proposed sliding mode controller is constructed for polytopic LPV system with simultaneous faults.

(iii) A robust design of FTC is against unknown input disturbances, especially against measurement noise.

(iv) A comparative study is developed using the Integral Absolute Error (IAE) to measure the performance of fault tolerant sliding mode control which gives exact comparisons between different control schemes highlighting the effectiveness of the proposed sliding mode control.

The remaining part of this paper is organized as follows. Formulation of the problem is in Section 2. In Section 3, a simultaneous actuator and sensor faults estimation design are presented. FTC design using SMC and state feedback control are considered in Section 4. Section 5, provide an illustrative example. Finally, some concluding remarks are made in Section 6.

The following Lemma is presented, which is rather standard, to facilitate the proofs of the main results of this paper.

Lemma 1. *For matrices X and Y with appropriate dimensions, the following condition holds:*

$$X^T Y + Y^T X \leq \partial^{-1} X^T X + \partial Y^T Y \tag{1}$$

where ∂ is a positive scalar.

2. Problem Formulation

Consider an uncertain LPV system subject to both actuator and sensor faults as follows:

$$\begin{aligned} \dot{\chi}(t) &= A(\theta(t))\chi(t) + B(\theta(t))u(t) + M(\theta(t))f_a(t) \\ &\quad + D(\theta(t))\xi(t) \\ y(t) &= C(\theta(t))\chi(t) + N(\theta(t))f_s(t) \end{aligned} \tag{2}$$

where the parameter varying matrices $A(\theta(t)) \in R^{n\times n}$, $B(\theta(t)) \in R^{n\times m}$, $M(\theta(t)) \in R^{n\times q}$, $D(\theta(t)) \in R^{n\times l}$, $C(\theta(t)) \in R^{p\times n}$, and $N(\theta(t)) \in R^{p\times w}$. In (2), the state vectors $x(t) \in R^n$, $u(t) \in R^m$ represent the controlled input and $y(t) \in R^p$ is the output measurements. $f_a(t) \in R^q$ and $f_s(t) \in R^w$ denote actuators and sensors faults, respectively. The signal $\xi(t) \in R^l$ includes the unknown external disturbances vector.

In the following, the parameter vector $\theta(t)$ is considered bounded in the hypercube Σ such that

$$\Sigma = \left\{\theta(t) \mid \theta(t) \leq \theta(t) \leq \overline{\theta}(t)\right\}, \quad \forall t \geq 0 \tag{3}$$

The LPV system (2) can be written in a polytopic form:

$$\begin{aligned} \dot{\chi}(t) &= \sum_{i=1}^{h} \mu_i(\theta(t))\left\{A_i\chi(t) + B_i u(t) + D_i\xi(t) + M_i f_a(t)\right\} \\ y(t) &= \sum_{i=1}^{h} \mu_i(\theta(t))\left\{C_i\chi(t) + N_i f_s(t)\right\} \end{aligned} \tag{4}$$

where $A_i \in R^{n\times n}, B_i \in R^{n\times m}, C_i \in R^{p\times n}, N_i \in R^{p\times w}, M_i \in R^{n\times q}$, and $D_i \in R^{n\times l}$ are real time matrices.

$\mu_i(\theta(t))$ denotes the weighting function and satisfies the convex set properties:

$$\begin{aligned} &\mu_i(\theta(t)) \geq 0 \\ &\sum_{i=1}^{h} \mu_i(\theta(t)) = 1, \quad \forall i \in [1, \ldots, h] \end{aligned} \tag{5}$$

Before starting the main results of the present paper, we will make the following assumptions which are typically required in the sliding mode observer and control design for LPV systems.

Assumption 2. The actuator fault matrix is assumed to verify

$$rank\left(C_iM_i\right) = rank\left(M_i\right) = q, \quad \forall i \in [1,\ldots,h]. \tag{6}$$

Assumption 3. The invariant zeros of the system triplet (A_i, M_i, C_i), $\forall i = [1,\ldots,h]$ are all in the open left-hand complex plant, such that

$$rank\begin{bmatrix} sI_n - A_i & M_i \\ C_i & 0 \end{bmatrix} = n + q \tag{7}$$

holds for all complex number s where $\mathrm{Re}(s) \geq 0$.

Assumption 4. Faults and external disturbances are bounded by some constants such as $\|f_a\| \leq \rho_1, \|f_s\| \leq \rho_2$ and $\|\xi(t)\| \leq \xi_0$.

Assumption 5. The pair (A_i, B_i) are controllable.

Paper objective. Given the polytopic LPV system (4) subject to actuator faults $f_a(t)$, sensor faults $f_s(t)$, and external disturbances $\xi(t)$.

This paper gives a robust fault estimation based sliding mode control design, i.e, the main objective reside primarily to solve two problems as follows:

(i) Estimate simultaneous actuator and sensor faults as well as system states using sliding mode observers.

(ii) Conceive sliding mode controller to stabilize the closed-loop system after the occurrence of faults and disturbances.

In Section 5, simulation results are developed to show the effectiveness of fault estimation based sliding fault tolerant control design for the model of VTOL aircraft system.

3. Simultaneous Actuator and Sensor Faults Estimation Design

Based on the polytopic LPV system representation, the objective is to design a robust simultaneous actuator and sensor faults estimation. Transformation coordinate is introduced to decouple faults, and two polytopic subsystems are constructed. The first one is subject actuator faults and the second subsystem is affected only by sensor faults.

The present subsection is given to determinate the stability of sliding mode observers based faults estimation.

3.1. Transformation Coordinate System. Under Assumption 2, for the polytopic LPV system (4), there exists the following transformations $\forall i \in [1,\ldots,h]$:

$$\begin{aligned} T_iA_iT_i^{-1} &= \begin{bmatrix} A_{1,i} & A_{2,i} \\ A_{3,i} & A_{4,i} \end{bmatrix}, \\ T_iB_i &= \begin{bmatrix} B_{1,i} \\ B_{2,i} \end{bmatrix}, \\ T_iD_i &= \begin{bmatrix} D_{1,i} \\ D_{2,i} \end{bmatrix}, \\ T_iM_i &= \begin{bmatrix} M_{1,i} \\ 0 \end{bmatrix}, \\ S_iC_iT_i^{-1} &= \begin{bmatrix} C_{1,i} & 0 \\ 0 & C_{4,i} \end{bmatrix}, \\ S_iN_i &= \begin{bmatrix} 0 \\ N_{2,i} \end{bmatrix} \end{aligned} \tag{8}$$

where $T_i = \left[\begin{smallmatrix} T_{1,i} \\ T_{2,i} \end{smallmatrix}\right] \in R^{n\times n}$, $S_i = \left[\begin{smallmatrix} S_{1,i} \\ S_{2,i} \end{smallmatrix}\right] \in R^{p\times p}$, $A_{1,i} \in R^{q\times q}$, $B_{1,i} \in R^{q\times m}$, $D_{1,i} \in R^{q\times l}$, $M_{1,i} \in R^{q\times q}$, $C_{1,i} \in R^{q\times q}$, $C_{4,i} \in R^{(p-q)\times(n-q)}$, $N_{2,i} \in R^{(p-q)\times w}$, and $C_{1,i}$ are invertible.

Based on $\left[x_1^T(t) \;\; x_2^T(t)\right] = T_i\chi(t)$ and $\left[y_1^T(t) \;\; y_2^T(t)\right] = S_iy(t)$, system (4) is transformed into two subsystems [26]:

$$\begin{aligned} \dot{x}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{A_{1,i}x_1(t) + A_{2,i}x_2(t) \\ &\quad + B_{1,i}u(t) + D_{1,i}\xi(t) + M_{1,i}f_a(t)\} \\ y_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{1,i}x_1(t)\} \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{x}_2(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{A_{4,i}x_2(t) \\ &\quad + A_{3,i}x_1(t) + B_{2,i}u(t) + D_{2,i}\xi(t)\} \\ y_2(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{4,i}x_2(t) + N_{2,i}f_s(t)\} \end{aligned} \tag{10}$$

where (9) is affected by actuator faults and (10) is subject to only sensor faults.

Define a new state $x_3(t) = \int_0^t y_2(\tau)d\tau$ and introduce a coordinate transformation $z(t) = T_Lx_0(t)$, where

$$\begin{aligned} x_0(t) &= \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix}, \\ T_L &= \begin{bmatrix} I_{n-q} & L \\ 0 & I_{p-q} \end{bmatrix} \end{aligned} \tag{11}$$

with $L \in R^{(n-q)\times(p-q)}$.

Therefore, systems (9) and (10) can be rewritten as the following form:

$$\begin{aligned}\dot{x}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{A_{1,i}x_1(t) + A_{2,i}z_1(t) \\ &\quad - A_{2,i}Ly_0(t) + B_{1,i}u(t) + D_{1,i}\xi(t) + M_{1,i}f_a(t)\} \\ y_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{1,i}x_1(t)\}\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{z}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{(A_{4,i} + LC_{4,i})z_1(t) \\ &\quad - (A_{4,i} + LC_{4,i})Lz_2(t) + A_{3,i}x_1(t) + B_{2,i}u(t) \\ &\quad + D_{2,i}\xi(t)\} \\ \dot{z}_2(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{4,i}z_1(t) - C_{4,i}Lz_2(t) \\ &\quad + N_{2,i}f_s(t)\} \\ y_0(t) &= z_2(t)\end{aligned} \tag{13}$$

The following subsection is dedicated to conceive two sliding mode observers in order to estimate states and simultaneous actuator and sensor faults.

3.2. Sliding Mode Observers Design. For subsystems (12)-(13), we propose to conceive two polytopic sliding mode observers with the following structures:

$$\begin{aligned}\dot{\hat{x}}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{A_{1,i}\hat{x}_1(t) + A_{2,i}\hat{z}_1(t) \\ &\quad - A_{2,i}Ly_0(t) + B_{1,i}u(t) + M_{1,i}v_{1,i}(t) \\ &\quad + (A_{1,i} - A_{1,i}^s)C_{1,i}^{-1}(y_1(t) - \hat{y}_1(t))\} \\ \hat{y}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{1,i}\hat{x}_1(t)\}\end{aligned} \tag{14}$$

$$\begin{aligned}\dot{\hat{z}}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{(A_{4,i} + LC_{4,i})\hat{z}_1(t) \\ &\quad - (A_{4,i} + LC_{4,i})L\hat{z}_2(t) + A_{3,i}C_1^{-1}y_1(t) + B_{2,i}u(t)\} \\ \dot{\hat{z}}_2(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\{C_{4,i}\hat{z}_1(t) - C_{4,i}L\hat{z}_2(t) \\ &\quad - (C_{4,i}L + K_i)(y_0(t) - \hat{y}_0(t)) + N_{2,i}v_{2,i}(t)\} \\ \hat{y}_0(t) &= \hat{z}_2(t)\end{aligned} \tag{15}$$

$\hat{x}_1(t), \hat{z}_1(t), \hat{z}_2(t), \hat{y}_1(t), \hat{y}_0(t)$ denote, respectively, the estimation of $x_1(t), z_1(t), z_2(t), y_1(t), y_0(t)$. $A_{1,i}^s \in R^{q\times q}$ is chosen to be a stable matrix; $K_i \in R^{(p-q)\times(p-q)}$ is the SMO observers gains.

The discontinuous output error injection terms $v_{1,i}(t)$ and $v_{2,i}(t)$ are defined by

$$v_{1,i} = \begin{cases}(\hat{\rho}_1(t) + \alpha_{1,i})\dfrac{M_{1,i}^T P_1(C_{1,i}^T S_{1,i}y_1 - \hat{x}_1)}{\|M_{1,i}^T P_1(C_{1,i}^T S_{1,i}y_1 - \hat{x}_1)\|} & if\ (C_{1,i}^T S_{1,i}y_1 - \hat{x}_1) \neq 0 \\ 0 & otherwise\end{cases} \tag{16}$$

$$v_{2,i} = \begin{cases}(\hat{\rho}_2(t) + \alpha_{2,i})\dfrac{N_{2,i}^T P_{0,2}(y_0 - \hat{y}_0)}{\|N_{2,i}^T P_{0,2}(y_0 - \hat{y}_0)\|} & if\ (y_0 - \hat{y}_0) \neq 0 \\ 0 & otherwise\end{cases} \tag{17}$$

To know the bound of the faults ρ_1 and ρ_2, we use an adaptive algorithm described as

$$\dot{\hat{\rho}}_1(t) = \varepsilon_1\|M_{1,i}^T P_1 e_1\| \tag{18}$$

$$\dot{\hat{\rho}}_2(t) = \varepsilon_2\|N_{2,i}^T P_{0,2}e_3\| \tag{19}$$

where $P_1 \in R^{q\times q} > 0$, $P_{0,2} \in R^{(p-q)\times(p-q)} > 0$ are a symmetric definite Lyapunov matrix, $\alpha_{1,i}, \alpha_{2,i}, \varepsilon_1, \varepsilon_2 > 0$ and $\tilde{\rho} = \rho - \hat{\rho}$. With $e_1 = x_1 - \hat{x}_1, e_2 = z_1 - \hat{z}_1, e_3 = z_2 - \hat{z}_2$ are the state estimation errors.

3.3. Convergence Analysis. Consider Lyapunov candidate function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{20}$$

where $V_1(t) = e_1^T P_1 e_1 + \varepsilon_1^{-1}\tilde{\rho}_1^2$, $V_2(t) = e_2^T P_{0,1}e_2$, and $V_3(t) = e_3^T P_{0,2}e_3 + \varepsilon_2^{-1}\tilde{\rho}_2^2$. We define $\tilde{\rho}_1 = \rho_1 - \hat{\rho}_1$ and $\tilde{\rho}_2 = \rho_2 - \hat{\rho}_2$. The time derivative of $V_1(t)$ can be obtained as

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\Big\{e_1^T\left((A_{1,i}^s)^T P_1 + P_1 A_{1,i}^s\right)e_1(t) \\ &\quad + 2e_1^T P_1 A_{2,i}e_2(t) + 2e_1^T(t)P_1 D_{1,i}\xi(t) \\ &\quad + 2e_1^T P_1 M_{1,i}(f_a(t) - v_{1,i}(t)) - 2\varepsilon_1^{-1}\tilde{\rho}_1(t)(\dot{\hat{\rho}}_1(t))\Big\}\end{aligned} \tag{21}$$

From the fact that $f_a(t)$ is bounded, it follows from (16) and (18) that

$$\begin{aligned}
&e_1^T P_1 M_{1,i}\left(f_a - v_{1,i}\right) + \varepsilon_1^{-1}\tilde{\rho}_1\left(-\dot{\hat{\rho}}_1\right)\\
&\quad = e_1^T P_1 M_{1,i} f_a\\
&\qquad - \left(\hat{\rho}_1 + \alpha_{1,i}\right) e_1^T P_1 M_{1,i} \frac{M_{1,i}^T P_1\left(C_{1,i}^T S_{1,i} y_1 - \hat{x}_1\right)}{\left\|M_{1,i}^T P_1\left(C_{1,i}^T S_{1,i} y_1 - \hat{x}_1\right)\right\|}\\
&\qquad + \varepsilon_1^{-1}\tilde{\rho}_1\left(-\dot{\hat{\rho}}_1\right)\\
&\quad \le \left\|M_{1,i}^T P_1 e_1\right\| \rho_1 - \left(\rho_1 - \tilde{\rho}_1\right)\left\|M_{1,i}^T P_1 e_1\right\|\\
&\qquad - \alpha_{1,i}\left\|M_{1,i}^T P_1 e_1\right\| + \varepsilon_1^{-1}\tilde{\rho}_1\left(-\dot{\hat{\rho}}_1\right)\\
&\quad \le \tilde{\rho}_1\left\|M_{1,i}^T P_1 e_1\right\| - \alpha_{1,i}\left\|M_{1,i}^T P_1 e_1\right\| + \varepsilon_1^{-1}\tilde{\rho}_1\left(-\dot{\hat{\rho}}_1\right)\\
&\quad \le -\alpha_{1,i}\left\|M_{1,i}^T P_1 e_1\right\| < 0
\end{aligned} \tag{22}$$

It turns out that

$$\begin{aligned}
\dot{V}_1(t) &\le \sum_{i=1}^{h}\mu_i(\theta(t))\\
&\quad \cdot \left\{e_1^T(t)\left(\left(A_{1,i}^s\right)^T P_1 + P_1 A_{1,i}^s\right) e_1(t)\right.\\
&\quad \left. + 2e_1^T P_1 A_{2,i} e_2(t) + 2e_1^T(t) P_1 D_{1,i}\xi(t)\right\}
\end{aligned} \tag{23}$$

Likewise, the derivatives $V_2(t)$ and $V_3(t)$ with respect to time can be derived as

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^{h}\mu_i(\theta(t))\left\{e_2^T(t)\right.\\
&\quad \cdot \left[\left(A_{4,i} + LC_{4,i}\right)^T P_{0,1} + P_{0,1}\left(A_{4,i} + LC_{4,i}\right)\right] e_2(t)\\
&\quad \left. + 2e_2^T(t) P_{0,1} D_{2,i}\xi(t)\right\}
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
\dot{V}_3(t) &\le \sum_{i=1}^{h}\mu_i(\theta(t))\left\{e_3^T(t)\left(P_{0,2}K_i + K_i^T P_{0,2}\right) e_3(t)\right.\\
&\quad \left. + 2e_3^T(t) P_{0,2} C_{4,i} e_2(t)\right\}
\end{aligned} \tag{25}$$

Substituting (23), (24), and (25) into (20), it results that

$$\dot{V}(t) \le \sum_{i=1}^{h}\mu_i(\theta(t))\left\{\begin{bmatrix} e_1\\ e_2\\ e_3\end{bmatrix}^T \begin{bmatrix} Q_{1,i} & P_1 A_{2,i} & 0\\ A_{2,i}^T P_1 & Q_{2,i} & C_{4,i}^T P_{0,2}\\ 0 & P_{0,2}C_{4,i} & Q_{3,i}\end{bmatrix}\begin{bmatrix} e_1\\ e_2\\ e_3\end{bmatrix} + 2e_1^T P_1 D_{1,i}\xi(t) + 2e_2^T P_{0,1} D_{2,i}\xi(t)\right\} \tag{26}$$

To minimize the effect of disturbances, we use the H_∞ performances. The performance of H_∞ can be defined as

$$\|H\|_\infty = \sup_{\|\xi(t)\|_{L2}\neq 0}\frac{\|g(t)\|_{L2}^2}{\|\xi(t)\|_{L2}^2} \le \varsigma \tag{27}$$

where ς is a small positive constant; suppose that

$$g(t) = He(t) = H\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix} \tag{28}$$

is the controlled output for the error system. H is assumed to have full rank matrix, having the following structure:

$$H := \begin{bmatrix} H_1 & 0 & 0\\ 0 & H_2 & 0\\ 0 & 0 & H_3\end{bmatrix} \tag{29}$$

where $H_1 \in R^{q\times q}, H_2 \in R^{(n-q)\times(n-q)}, H_3 \in R^{(p-q)\times(p-q)}$.

In that order, small H_∞ gain of the transfer function from the external disturbances $\xi(t)$ to the estimation errors combination $g(t)$ means a small influence of $\xi(t)$ on $e(t)$.

To attain robustness to the disturbances $\xi(t)$ in the L_2 sense, we impose the following constraint on our stability criteria:

$$\dot{V}(t) + g^T(t)\,g(t) - \varsigma\xi^T(t)\,\xi(t) \le 0 \tag{30}$$

We conclude that

$$J(t) \le \sum_{i=1}^{h}\mu_i(\theta(t))\left\{\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\\ \xi(t)\end{bmatrix}^T \Delta_i \begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\\ \xi(t)\end{bmatrix}\right\} \tag{31}$$

with

$$\Delta_i = \begin{bmatrix} Q_{1,i} + H_1^T H_1 & P_1 A_{2,i} & 0 & P_1 D_{1,i}\\ A_{2,i}^T P_1 & Q_{2,i} + H_2^T H_2 & C_{4,i}^T P_{0,2} & P_{0,1} D_{2,i}\\ 0 & P_{0,2} C_{4,i} & Q_{3,i} + H_3^T H_3 & 0\\ D_{1,i}^T P_1 & D_{2,i}^T P_0^1 & 0 & -\varsigma I_l\end{bmatrix} \tag{32}$$

In this way, if $\Delta_i < 0$, for $(i = 1\ldots h)$, then $J(t) \le 0$ along the system trajectories with attenuation level $\sqrt{\varsigma}$.

Integrating the inequality in (30) from 0 to, ∞ we have

$$V(\infty) - V(0) + \|g\|_{L2}^2 - \varsigma \|\xi\|_{L2}^2 \leq 0 \tag{33}$$

With zero initial condition, we have

$$\begin{aligned} V(t=0) &= 0 \\ V(t \longrightarrow \infty) &= e_1^T(\infty) P_1 e_1(\infty) + \varepsilon_1^{-1} \tilde{\rho}_1^2(\infty) \\ &\quad + e_2^T(\infty) P_{0,1} e_2(\infty) \\ &\quad + e_3^T(\infty) P_{0,2} e_3(\infty) + \varepsilon_2^{-1} \tilde{\rho}_2^2(\infty) \\ &\geq 0 \end{aligned} \tag{34}$$

Thus, we obtain

$$\frac{\int_0^\infty g^T(t) g(t)}{\int_0^\infty \xi^T(t) \xi(t)} \leq \varsigma \tag{35}$$

So, if there exists the matrices $P_1 > 0$, $P_{0,1} > 0$, $P_{0,2} > 0$, and $X_{1,i} < 0$ the observer estimation errors are stable with attenuation level $\sqrt{\varsigma}$ subject to $\|g(t)\|_{L2} \leq \sqrt{\varsigma}\|\xi(t)\|_{L2}$.

The estimate actuator and sensor faults $f_a(t)$ and $f_s(t)$ will be obtained by scaling the so-called equivalent injection signals. Specifically

$$\begin{aligned} \hat{f}_a &\cong \sum_{i=1}^{h} \mu_i \theta(t) \\ &\quad \cdot \left\{ (\hat{\rho}_1 + \alpha_{1,i}) \frac{M_{1,i}^T P_1 \left(C_{1,i}^T S_{1,i} y_1 - \hat{x}_1 \right)}{\left\| M_{1,i}^T P_1 \left(C_{1,i}^T S_{1,i} y_1 - \hat{x}_1 \right) \right\| + \delta_1} \right\} \end{aligned} \tag{36}$$

$$\hat{f}_s \cong \sum_{i=1}^{h} \mu_i \theta(t) \left\{ (\hat{\rho}_2 + \alpha_{2,i}) \frac{N_{2,i}^T P_{0,2} e_3}{\left\| N_{2,i}^T P_{0,2} e_3 \right\| + \delta_2} \right\} \tag{37}$$

where δ_1 and δ_2 are two small positive scalars.

Based on Assumptions 2–5, the challenge of this paper is to construct a robust controller. Hence, the closed-loop system response converges to zero even in the case that actuator fault, sensor fault, and disturbance simultaneously have effects on the system dynamics.

4. Fault Tolerant Control Design

This section explores a fault tolerant control design for the polytopic LPV system (4) based on the provided information about simultaneous actuator and sensor faults estimation. In this way, we propose to conceive two FTC strategies, subject to provide a corrective action after the occurrence of faults and disturbances, as well as stabilize the LPV system, such that we have the following:

(i) State feedback fault tolerant control.

(ii) Sliding mode fault tolerant control.

4.1. State Feedback Fault Tolerant Control. Based on both actuator fault and system states estimation, we propose to conceive a robust controller as

$$u(t) = \sum_{i=1}^{h} \sum_{j=1}^{h} \mu_i \mu_j (\theta(t)) \left\{ -k_{1,j} \hat{\chi}(t) - q_{1,i} \hat{f}_a(t) \right\} \tag{38}$$

where $k_{1,j} \in R^{m \times n}$ and $q_{1,i} \in R^{m \times q}$ represent, respectively, the state-feedback control and fault compensation gains. We assume that $q_{1,i} = B_i^+ M_i$.

Substituting (38) into (4), it remains to develop the following model as

$$\begin{aligned} \dot{\chi}(t) &= \sum_{i=1}^{h} \sum_{j=1}^{h} \mu_i \mu_j (\theta(t)) \left\{ \left(A_i - B_i k_{1,j} \right) \chi(t) + \overline{B}_{i,j} \overline{d}(t) \right\} \\ y_c(t) &= \sum_{i=1}^{h} \mu_i (\theta(t)) \left\{ y(t) - N_i \hat{f}_s(t) \right\} \\ y_m(t) &= \sum_{i=1}^{h} \mu_i (\theta(t)) \left\{ C_i \chi(t) \right\} \end{aligned} \tag{39}$$

where $y_c(t)$ is the compensated output vector, $y_m(t)$ represent the measurement output, $\overline{B}_{i,j} = \left[B_i k_{1,j} \;\; M_i \;\; D_i \right]$, and $\overline{d} = \left[e_x^T(t) \;\; e_{f_a}^T(t) \;\; \xi^T(t) \right]^T$.

The active fault tolerant design strategy based on state feedback control is shown in Figure 1 for the polytopic LPV system.

Hence, the stability of the closed-loop systems (39) is given in the next theorem.

Theorem 6. *The closed-loop LPV system (39) is stable and the H_∞ performance is guaranteed with an attenuation level ς_x, if there exist symmetric positive definite matrices P_X and matrices Γ_j and scalar ς_x satisfying the following LMI optimization problem, $\forall i, j \in [1, \ldots, h]$,*

minimizes ς_x subject to

$$\begin{bmatrix} \Theta_{i,j} & B_i \Gamma_j & M_i & D_i & P_X C_i^T \\ * & -2P_X + \varsigma_x^{-1} I_n & 0 & 0 & 0 \\ * & * & -\varsigma_x I_q & 0 & 0 \\ * & * & * & -\varsigma_x I_l & 0 \\ * & * & * & * & -I_p \end{bmatrix} < 0 \tag{40}$$

where

$$\begin{aligned} P_X &= P_x^{-1} \\ \Gamma_j &= k_{1,j} P_X \\ \Theta_{i,j} &= P_X A_i^T - \Gamma_j^T B_i^T + A_i P_X - B_i \Gamma_j \end{aligned} \tag{41}$$

Proof. In order to prove the stability of the closed-loop system, we consider the following Lyapunov function:

$$V_x(t) = \chi^T(t) P_x \chi(t) \tag{42}$$

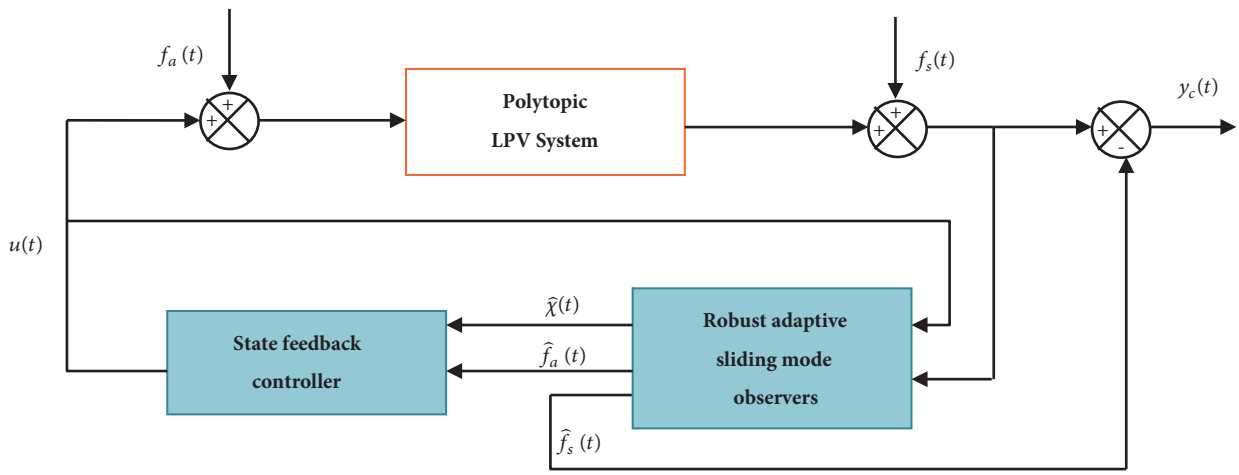

Figure 1: Structure of state feedback fault tolerant control for LPV system.

The time derivative of $V_x(t)$ can be obtained as

$$\begin{aligned}\dot{V}_x(t) = \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\Big\{&\chi^T(t)\\ &\cdot\left[\left(A_i - B_ik_{1,j}\right)^T P_x + P_x\left(A_i - B_ik_{1,j}\right)\right]\chi(t)\\ &+ 2\chi^T(t)P_x\overline{B}_{i,j}\overline{d}(t)\Big\}\end{aligned} \tag{43}$$

The objective is to guarantee the stability of the closed-loop system such that

$$\|y_m\|_{L2}^2 \le \varsigma_x\|\overline{d}\|_{L2}^2 \tag{44}$$

To attain the required performance (44), the following relation should hold:

$$J_x(t) = \dot{V}_x(t) + y_m^T(t)\,y_m(t) - \varsigma_x\overline{d}^T(t)\,\overline{d}(t) < 0 \tag{45}$$

Inequality (45) implies that

$$\begin{bmatrix}\pi_{i,j} + C_i^TC_i & P_x\overline{B}_{i,j}\\ * & -\varsigma_xI\end{bmatrix} < 0 \tag{46}$$

where $\pi_{i,j} = (A_i - B_ik_{1,j})^TP_x + P_x(A_i - B_ik_{1,j})$.

Before and after multiplying (46) by $\mathrm{diag}(P_X, P_X, I, I, I)$ and its transpose, respectively, and using the Schur complement, inequality (46) is rewritten as

$$\begin{bmatrix}P_x^{-1}\pi_{i,j}P_x^{-1} & B_ik_{1,j}P_x^{-1} & M_i & D_i & P_x^{-1}C_i^T\\ * & -P_x^{-1}\varsigma_xP_x^{-1} & 0 & 0 & 0\\ * & * & -\varsigma_xI_q & 0 & 0\\ * & * & * & -\varsigma_xI_l & 0\\ * & * & * & * & -I_p\end{bmatrix} < 0 \tag{47}$$

Using Lemma 1, we can express the following relation:

$$P_x^{-1} + P_x^{-1} \le \varsigma_xP_x^{-1}P_x^{-1} + \varsigma_x^{-1}I_n \tag{48}$$

or equivalently

$$-P_x^{-1}\varsigma_xP_x^{-1} \le -2P_x^{-1} + \varsigma_x^{-1}I_n \tag{49}$$

From this, inequality (47) directly leads to (40).

This completes the proof. □

Sliding mode control has been widely studied in the literature based on its computational simplicity and in particular strong robustness against uncertainties, disturbances, and measurement noise. The proposed sliding mode controller with adaptive law is assigned to provide a corrective action in order to compensate simultaneous actuator and sensor faults effects.

4.2. Sliding Mode Fault Tolerant Control Design. The objective is to force the controlled output for the closed-loop system to zero infinite time and induce a sliding motion on the surface, as

$$S = \{y(t)\in^P; S(t) = 0\} \tag{50}$$

We propose to conceive a sliding mode controller of the form

$$u(t) = u_l(t) + u_n(t) \tag{51}$$

$u_l(t)$ is a linear part such that

$$u_l(t) = \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\left\{-k_{2,j}\hat{\chi}(t) - q_{2,i}\hat{f}_a(t)\right\} \tag{52}$$

It is assumed that $k_{2_j}\in R^{m\times n}$ and $q_{2,i} = B_i^+M_i$.

The nonlinear component is assumed to have the form

$$u_n = \begin{cases}-\gamma(t)\dfrac{S(t)}{\|S(t)\|} & if\ S(t)\ne 0\\ 0 & if\ S(t)\ne 0\end{cases} \tag{53}$$

where $\gamma(t) = \hat{\eta} + \phi + \varphi$. The sliding surface $S(t)$ is designed as

$$S(t) = \sum_{i=1}^{h}\mu_i(\theta(t))\{N_{c,i}y_c(t)\} \tag{54}$$

with $N_{c,i} = (C_iB_i)^+ - \kappa(I_p - C_iB_i(C_iB_i)^+)$ such that $(C_iB_i)^+ = ((C_iB_i)^+C_iB_i)^{-1}(C_iB_i)^T$ and $\kappa \in R^{m\times p}$ is an arbitrary matrix.

In addition, $\phi > 0$ and $\varphi > 0$ are a small constant. $\hat{\eta}$ is used to determine the unknown scalar η that is defined by

$$\dot{\hat{\eta}} = \sigma \|S\| \tag{55}$$

and σ is a positive constant.

To analyze the sliding motion corresponding to the sliding surface S, we consider the Lyapunov function as

$$V_\alpha(t) = V_s(t) + \frac{1}{2\sigma}\tilde{\eta}^2(t) \tag{56}$$

with $\tilde{\eta} = \eta - \hat{\eta}$ and $V_s = (1/2)S^TS$.

Referring to the open-loop system (4), the time derivative of $V_s(t)$ with respect to time gives

$$\begin{aligned}\dot{V}_s(t) \le{}& \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j\theta(t)\\ &\cdot\left\{\left(\left(N_{c,i}C_iA_i - k_{2,j}\right)\|\chi\| + \eta - \gamma\right)\|S\|\right\}\end{aligned} \tag{57}$$

It follows from (56) and (57) that

$$\begin{aligned}\dot{V}_\alpha(t) \le{}& \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j\theta(t)\\ &\cdot\left\{\left(\left(N_{c,i}C_iA_i - k_{2,j}\right)\|\chi\| + \eta - \gamma\right)\|S\| - \hat{\eta}\|S\|\right\}\\ \le{}& \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j\theta(t)\\ &\cdot\left\{-\varphi\|S\| - \left[\phi - \left(N_{c,i}C_iA_i - K_{2,j}\right)\right]\|S\|\right\}\end{aligned} \tag{58}$$

It results from (58) that $\dot{V}_\alpha(t) \le 0$ in the subset $\Delta = \{\chi : \|\chi\| \le \lambda\}$ if the parameter $\phi > \|(N_{c,i}C_iA_i - K_{2,j})\|\,\lambda$, such that $\lambda > 0$.

In the following, we propose to analyze the system stability on the sliding mode; we give the equivalent control input as

$$\begin{aligned}&u_{eq}(t)\\ &= \sum_{i=1}^{h}\mu_i(\theta(t))\left\{-N_{c,i}\left(C_iA_i\chi(t) + C_iD_i\xi(t)\right) + u_l(t)\right\}\end{aligned} \tag{59}$$

Substituting (59) into (4), the closed-loop system state satisfies the differential equation as

$$\begin{aligned}\dot{\chi}(t) &= \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\left\{\left(\psi_iA_i - B_ik_{2,j}\right)\chi(t)\right.\\ &\quad\left. + B_ik_{2,j}e_x(t) + M_ie_{f_a}(t) + \psi_iD_i\xi(t)\right\}\\ &= \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\left\{\left(\psi_iA_i - B_ik_{2,j}\right)\chi(t)\right.\\ &\quad\left. + \overline{B}_{2_{i,j}}\overline{d}(t)\right\}\end{aligned} \tag{60}$$

where $\psi_i = (I_n - B_iN_{c,i}C_i)$, $\overline{B}_{2_{i,j}} = [B_ik_{2,j}\ \ M_i\ \ \psi_iD_i]$, and $\overline{d} = \left[e_x^T(t)\ \ e_{f_a}^T(t)\ \ \xi^T(t)\right]^T$

The following closed-loop polytopic LPV system is rewritten as follows:

$$\begin{aligned}&\dot{\chi}(t)\\ &= \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\left\{\left(\psi_iA_i - B_ik_{2,j}\right)\chi(t) + \overline{B}_{2_{i,j}}\overline{d}(t)\right\}\\ &y_c(t) = \sum_{i=1}^{h}\mu_i(\theta(t))\left\{y(t) - N_i\hat{f}_s(t)\right\}\\ &y_m(t) = \sum_{i=1}^{h}\mu_i(\theta(t))\left\{C_i\chi(t)\right\}\end{aligned} \tag{61}$$

The proposed design of sliding fault tolerant control based on output feedback is summarized in Figure 2.

A sufficient condition to obtain the stability of the closed-loop polytopic LPV system (61) on the sliding surface S despite the presence of uncertainties, actuator, and sensor faults is given in the next theorem.

Theorem 7. *Given positive scalar ς_s, the closed loop LPV system (61) is stable with H_∞ performance, if there exist symmetric positive definite matrices P_s and matrices Γ_{2_j} satisfying the following LMI optimization problem, $\forall i, j \in [1, \ldots, h]$,*

minimize ς_s subject to

$$\begin{bmatrix}\Theta_{s_{i,j}} & B_i\Gamma_{2_j} & M_i & \psi_iD_i & P_sC_i^T\\ * & -2P_s + \varsigma_s^{-1}I_n & 0 & 0 & 0\\ * & * & -\varsigma_sI_q & 0 & 0\\ * & * & * & -\varsigma_sI_l & 0\\ * & * & * & * & -I_p\end{bmatrix} < 0 \tag{62}$$

where

$$\begin{aligned}P_s &= P_\tau^{-1}\\ \Gamma_{2_j} &= k_{2_j}P_s\\ \Theta_{s_{i,j}} &= P_sA_i^T\psi_i - \Gamma_{2_j}^TB_i^T + \psi_iA_iP_s - B_i\Gamma_{2_j}\end{aligned} \tag{63}$$

Proof. In order to ensure the closed-loop stability, we started by considering the following Lyapunov function:

$$V_\tau(t) = \chi^T(t)P_\tau\chi(t) \tag{64}$$

where $P_\tau = P_\tau^T > 0$. By taking into account the closed-loop system (61), $\dot{V}_\tau(t)$ is handled as

$$\begin{aligned}\dot{V}_\tau(t) &= \sum_{i=1}^{h}\sum_{j=1}^{h}\mu_i\mu_j(\theta(t))\left\{\chi^T(t)\right.\\ &\quad\cdot\left[\left(\psi_iA_i - B_ik_{2,j}\right)^TP_\tau + P_\tau\left(\psi_iA_i - B_ik_{2,j}\right)\right]\chi(t)\\ &\quad\left. + 2\chi^T(t)P_\tau\overline{B}_{2_{i,j}}\overline{d}(t)\right\}\end{aligned} \tag{65}$$

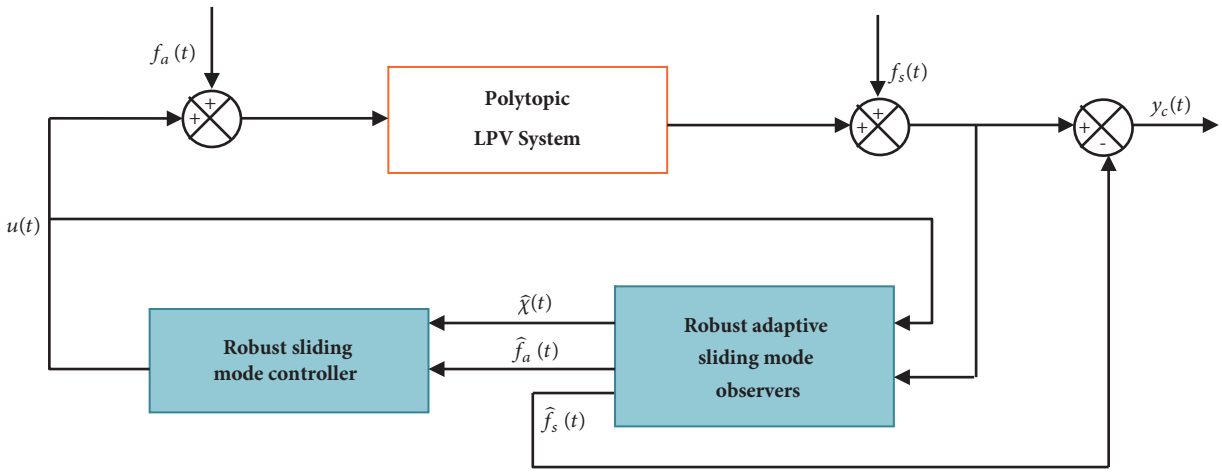

Figure 2: Frameworks of sliding fault tolerant control for LPV system.

The objective is to ensure the robustness against the additive term $\overline{d}(t)$. For this, we define

$$J_s(t) = \dot{V}_\tau(t) + y_m^T(t)\, y_m(t) - \varsigma_s \overline{d}^T(t)\, \overline{d}(t) < 0 \tag{66}$$

According to comparison principle and similarly to Theorem 6, it remains to prove that

$$\begin{bmatrix} \pi_{s_{i,j}} + C_i^T C_i & P_\tau \overline{B}_{2_{i,j}} \\ * & -\varsigma_s I \end{bmatrix} < 0 \tag{67}$$

where $\pi_{s_{i,j}} = (\psi_i A_i - B_i k_{2,j})^T P_\tau + P_\tau(\psi_i A_i - B_i k_{2,j})$.

If relation (67) is satisfied, which implies that $J_s(t) < 0$, it remains to conclude that the closed-loop polytopic LPV system is robustly stable with respecting the H_∞ attenuation level ς_s. □

5. Illustrative Example

In this section, the best performances of the proposed fault tolerant control are verified by considering the following:

(i) Simultaneous actuator and sensor faults estimation based state feedback controller.

(ii) Simultaneous actuator and sensor faults estimation based sliding mode controller.

5.1. Linear Parameter Varying Model. To demonstrate the effectiveness of the developed methods, we use a VTOL aircraft model taken from [27].

Consider the LPV system in the form of (2) and denote the state vector as $\chi(t) = [V_w \;\; V_v \;\; \Upsilon \;\; Z]^T$ where V_w, V_v, Υ, and Z represent, horizontal velocities, vertical velocities, pitch rate, and pitch angle, respectively. $u = [u_1 \;\; u_2]^T$ where u_1 and u_2 represent, respectively, collective pitch control and longitudinal cyclic pitch control.

The polytopic LPV system matrices are given as follows:

$$A_1 = \begin{bmatrix} -9.9477 & -0.74776 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.1361 & -4.1975 & -21.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -9.9477 & -0.74776 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.1361 & -4.1975 & -17.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -9.9477 & -0.74776 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 3.1361 & -4.1975 & -21.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -9.9477 & -0.74776 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 3.1361 & -4.1975 & -17.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.4422 & 0.1761 \\ 1.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.4422 & 0.1761 \\ 5.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.4422 & 0.1761 \\ 1.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0.4422 & 0.1761 \\ 5.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$M_1 = M_2 = M_3 = M_4 = \begin{bmatrix} 0.1761 \\ -7.5922 \\ 4.4900 \\ 0 \end{bmatrix},$$

$$D_1 = D_2 = D_3 = D_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$N_1 = N_2 = N_3 = N_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{68}$$

The weighting functions $\mu_i(\theta(t))$ are defined as

$$\begin{aligned} \mu_1(\theta(t)) &= \frac{(\theta_1(t)+0.5)(\theta_2(t)+2)}{4} \\ \mu_2(\theta(t)) &= \frac{(\theta_1(t)+0.5)(2-\theta_2(t))}{4} \\ \mu_3(\theta(t)) &= \frac{(0.5-\theta_1(t))(\theta_2(t)+2)}{4} \\ \mu_4(\theta(t)) &= \frac{(0.5-\theta_1(t))(2-\theta_2(t))}{4} \end{aligned} \tag{69}$$

where $\theta_1(t) = 0.5\sin(t)$ and $\theta_2(t) = \sin(t)$.

The initial value of original and estimated states are set as $\chi(0) = [0; 0; 0; 0]^T$ and $\hat{\chi}(0) = [0.1; 0.05; 0.02; 0.05]^T$.

We assume also that $\alpha_{1,i} = 10$, $\alpha_{2,i} = 12$, $\delta_1 = 0.0001$, $\delta_2 = 0.01$. The gain of the adaption law given in (18) and (19) is $\varepsilon_1 = 15$, $\varepsilon_2 = 16$.

Table 1: H_∞ attenuation level.

State feedback control	Sliding mode control
$\varsigma_{x\ \min} = 0.8277$	$\varsigma_{s\ \min} = 0.7936$

The simulation was accomplished with disturbances $\xi(t) = 0.1\sin(0.2t)$ and faults are

$$\begin{aligned} f_a(t) &= \begin{cases} 0, & 0 \le t \le 5 \\ \sin(t), & 5 < t \le 25 \\ 0, & t > 25, \end{cases} \\ f_s(t) &= \begin{cases} 0, & 0 \le t \le 10 \\ -1, & t > 10 \end{cases} \end{aligned} \tag{70}$$

5.2. Fault Tolerant Control Design

5.2.1. State Feedback Control Case. Assumptions 2–5 are satisfied; solving Theorem 6 with $\varsigma_x = 0.8277$ gives

$$\begin{aligned} P_x &= \begin{bmatrix} 40.348 & 5.058 & 5.134 & -19.904 \\ 5.058 & 1.063 & 0.609 & -2.543 \\ 5.134 & 0.609 & 0.854 & -2.114 \\ -19.904 & -2.543 & -2.114 & 13.749 \end{bmatrix}, \\ k_{11} &= k_{12} = k_{13} = k_{14} \\ &= \begin{bmatrix} 0.431 & -0.060 & -0.069 & -0.636 \\ -14.445 & -6.543 & -0.509 & 10.768 \end{bmatrix}. \end{aligned} \tag{71}$$

5.2.2. Sliding Mode Control Case. To illustrate the performance of the proposed FTC with the sliding mode control we set the scalars parameters, $\sigma = 0.01$, $\varphi = 0.002$, $\phi = 0.05$, $\lambda = 0.001$, and the parameter matrices, $\kappa = \begin{bmatrix} -1 & 1 & 0.2 \\ -1 & 2 & -0.1 \end{bmatrix}$.

Solving Theorem 7 with the chosen parameters gives the following controller gains:

$$P_\tau = \begin{bmatrix} 77.521 & 9.984 & 5.343 & -39.189 \\ 9.984 & 2.104 & 0.622 & -5.295 \\ 5.343 & 0.622 & 0.865 & -2.243 \\ -39.189 & -5.295 & -2.243 & 23.892 \end{bmatrix}, \tag{72}$$

$$\begin{aligned} k_{21} &= k_{22} = k_{23} = k_{24} \\ &= \begin{bmatrix} 20.8456 & 1.209 & -0.563 & -10.433 \\ -19.864 & -9.585 & -0.372 & 14.072 \end{bmatrix}. \end{aligned} \tag{73}$$

The H_∞ attenuation levels of the two proposed methods are listed in Table 1. Compared with the sliding mode control, the LMI optimization gain which refers to the state feedback control case loses a certain degree of FTC robustness. Obviously, Table 1 illustrates a better compensation using FE based sliding mode control.

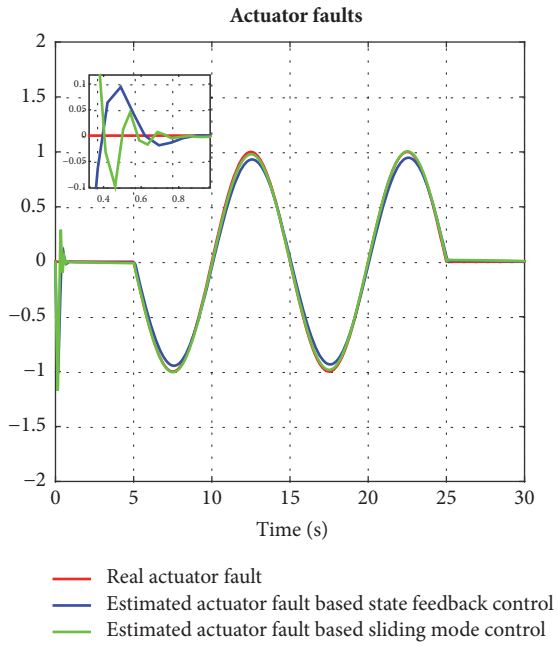

Figure 3: Estimation of actuator faults.

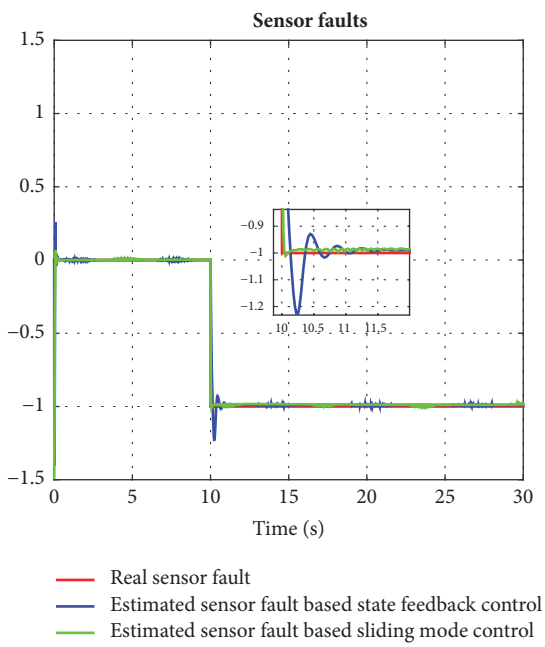

Figure 4: Estimation of sensor faults.

Figures 3 and 4 show the simulation results for the faults estimation of the closed loop system; they prove the ability of the polytopic LPV observers to provide a good estimation of simultaneous actuator and sensor faults, respectively, based on the sliding mode control and the state feedback control. Moreover, it is possible to conclude from zoomed versions that the FE in the case of presence of sliding mode control has quicker response and smaller overshoot than the state feedback control.

Figures 5, 6, and 7 show a comparative study between the FTC based sliding mode fault tolerant control and based state feedback control for the time response of the closed-loop system outputs.

We can conclude that the responses of output tend to zero when the system includes simultaneous faults. So, the controllers stabilize the closed loop systems as well as compensating the effect of the faults.

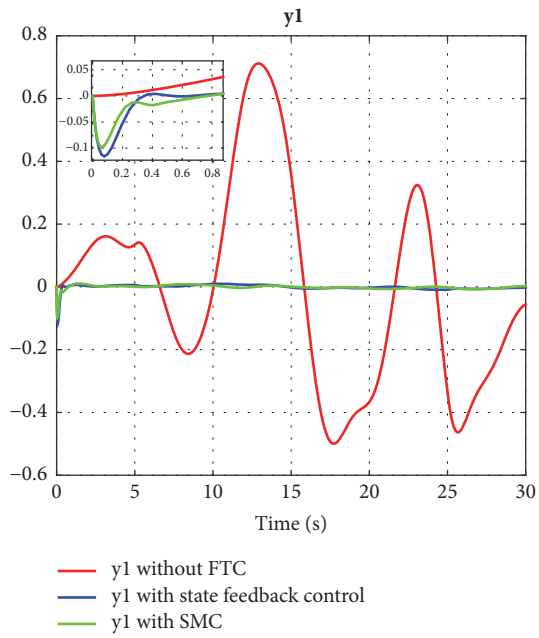

Figure 5: FTC performance for first output.

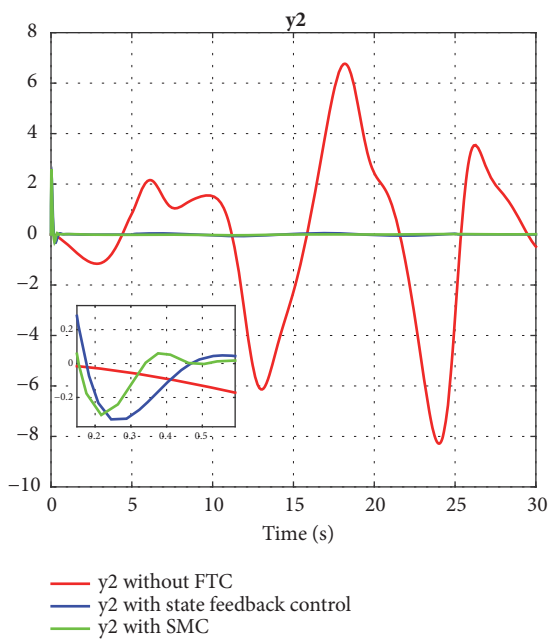

Figure 6: FTC performance for second output.

Furthermore, by the sliding mode fault tolerant control the system can quickly attain stability compared with the state feedback control.

5.3. FTC with Measurement Noise. In order to demonstrate the effectiveness and feasibility of the FTC with SMC designs and its superior performance compared with the state feedback control, the system is affected by an additive measurement noise with a variance of $10^{-5} \times I_3$ and with zero average.

Summarizing the results, Table 2 compares the two proposed methods by adding the measurement noise. Moreover, to evaluate the performance of the fault tolerant sliding mode control, the IAE in the responses of output is used and the comparison results demonstrate that the best IAE is obtained by fault tolerant sliding mode control method.

6. Conclusion

In this paper, a new approach of active FTC and FE for polytopic LPV systems has been proposed. The controllers

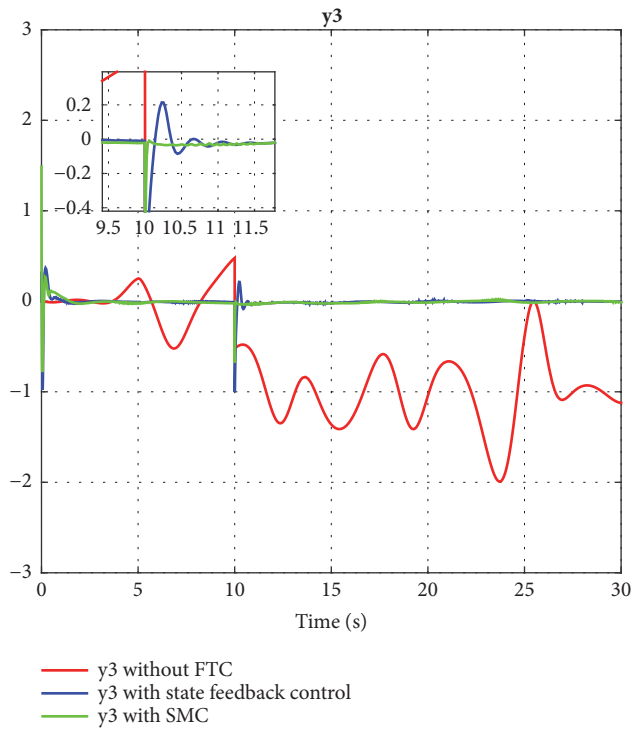

Figure 7: FTC performance for third output.

Table 2: IAE of outputs.

Outputs	State feedback control	Sliding mode control
$y_1(t)$	8.7×10^{-4}	6.18×10^{-4}
$y_2(t)$	0.010	0.0065
$y_3(t)$	0.010	0.0064

are built to reduce the effect of faults and the systems stability was ensured using a Lyapunov analysis. We note that the stability conditions are based on LMI techniques. The innovation of this paper consists of the use of both SMO and SMC which was not dealt with by researchers. The simulation and comparison of the proposed methods for a VTOL aircraft model show a good track performance of FTC with sliding mode control design compared with the state feedback control. Also, based on the IAE, a comparative study is established to valorize the fault tolerant sliding mode control despite the presence of measurement noise.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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