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Research Article

Sensor Fault Diagnosis and Fault-Tolerant Control for Non-Gaussian Stochastic Distribution Systems

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A sensor fault diagnosis method based on learning observer is proposed for non-Gaussian stochastic distribution control (SDC) systems. First, the system is modeled, and the linear B-spline is used to approximate the probability density function (PDF) of the system output. Then a new state variable is introduced, and the original system is transformed to an augmentation system. The observer is designed for the augmented system to estimate the fault. The observer gain and unknown parameters can be obtained by solving the linear matrix inequality (LMI). The fault influence can be compensated by the fault estimation information to achieve fault-tolerant control. Sliding mode control is used to make the PDF of the system output to track the desired distribution. MATLAB is used to verify the fault diagnosis and fault-tolerant control results.

1. Introduction

With the rapid development of modern science and technology, control systems have become more complex, large-scale, and intelligent, which puts more demands on the control of engineering control systems [1, 2]. The safety and reliability of the system must be ensured during the operation of the system. Otherwise, huge personal and property losses will be caused. Luckily, fault diagnosis and fault-tolerant control play an important role in detecting and avoiding such accidents. It is essential to carry out the research of controlling stochastic distribution systems in the field of modern control theory, which is widely used in controlling of the ore particle distribution in the grinding process, controlling of pulp uniformity and controlling of particle uniformity in the papermaking process, and the polymer in the process of the chemical reaction as well as the flame distribution control during the combustion process of the boiler.

In reality, there are many production sites that are out of reach and are accompanied by high temperature, high pressure, and toxic environments. However, the fault is inevitable. If it fails to be dealt with in time, it will waste precious time and may lead to extremely serious consequences. Therefore, it is of great significance to carry out research on fault diagnosis and fault-tolerant control of stochastic distribution systems to improve its reliability and safety and avoid loss of personnel and property. Stochastic distribution control has been regarded an important topic in the control field in recent decades. In most control projects, practical systems are subjected to stochastic input. These inputs may be derived from noise, stochastic disturbances, or stochastic parameter changes. Professor Hong Wang put forward to the idea of stochastic distribution control [3]. Different from the model in the existing control systems, the overall shape of the probability density function of the output of the stochastic system is considered. The goal of the controller design is to select a good rigid control input so that the probability density function shape of the system output can track the given distribution.

At present, there are many research results for fault diagnosis and fault-tolerant control of actuator fault in non-Gaussian stochastic distribution systems. For fault diagnosis, fault diagnosis observers or filters based methods are usually used. In literatures [4–6], the adaptive fault diagnosis observer is used to diagnose the actuators fault, and the fault amplitude is accurately estimated. In literatures [7, 8], fault reconstruction based on learning observer is recorded. In literatures [9–11], sensor fault diagnosis and output feedback control are described. In literature [12], the filter based method has been shown to be effective method for fault diagnosis. For fault-tolerant control of non-Gaussian stochastic distribution systems, two situations are considered: (1) the desired PDF is known; (2) the desired PDF is not known in advance. An adaptive PI tracking fault-tolerant controller is designed to track the desired PDF in the literature [13]. In literatures [14, 15], the minimum entropy fault-tolerant control algorithm is constructed based on the unknown PDF, and the performance index function is designed to find the control input to make the performance index function be minimized. In literature [16], collaborative system fault diagnosis and model prediction fault-tolerant control for the stochastic distribution system are described. In literature [17], actuator fault diagnosis and PID tracking fault-tolerant control for n subsystems collaboration are introduced.

The research of sensor fault diagnosis and fault-tolerant control of non-Gaussian stochastic distribution system is rarely documented; however, the sensor fault is inevitable. Thus, it is very meaningful for the work to be carried out in this paper. In this paper, the learning observer is used to diagnose the fault of the non-Gaussian stochastic distribution system, and the fault is compensated using the fault estimation information. The sliding mode control algorithm is used to make the system output PDF to track the expected PDF.

2. Model Description

The system output probability density function (PDF) $\gamma(y, u(t))$ is approximated by linear B-spline function. $\phi_1(y), \phi_2(y), \cdots, \phi_n(y)$ are basis functions defined in advance on the interval $[a, b]$. $\omega_1, \omega_2, \cdots, \omega_n$ are corresponding weights associated with the number of basis functions. $\gamma(y, u(t))$ can be expressed as

$$\gamma(y, u(t)) = \sum_{i=1}^{n} \omega_i(u(t))\phi_i(y) \tag{1}$$

Since the integral of $\gamma(y, u(t))$ on $[a, b]$ is equal to 1, the following equation holds:

$$\omega_1 b_1 + \omega_2 b_2 + \cdots + \omega_n b_n = 1 \tag{2}$$

where $b_i = \int_a^b \phi_i(y)dy, i = 1, 2, \cdots, n$. Therefore, only $n-1$ weights are independent of each other, and the linear B-spline model is specifically given as follows:

$$\gamma(y, u(t)) = C(y)V(t) + T(y) + e_0 \tag{3}$$

where $C(y) = [\phi_1(y) - \phi_n(y)b_1/b_n, \phi_2(y) - \phi_n(y)b_2/b_n, \cdots, \phi_{n-1}(y) - \phi_n(y)b_{n-1}/b_n]$, $T(y) = \phi_n(y)/b_n \in R^{1\times 1}$, $V(t) = [\omega_1, \omega_2, \cdots \omega_{n-1}]^T \in R^{(n-1)\times 1}$. e_0 is the PDF approximation error that can be ignored. The non-Gaussian stochastic distribution system model can be expressed as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ V(t) &= Dx(t) + Gf(t) \\ \gamma(y, u(t)) &= C(y)V(t) + T(y) \end{aligned} \tag{4}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^n$ is the control input vector, $f(t) \in R^r$, $V(t) \in R^p$ is the fault vector and the weight vector, respectively, $G \in R^{p\times r}$ is a full rank matrix. (A, D) is observable; A, B, D, G are known matrices with appropriate dimension. A new state variable is introduced for fault diagnosis [18].

$$\dot{\hbar}(t) = -A_s\hbar(t) + A_sV(t) \tag{5}$$

where $-A_s$ is a Hurwitz matrix, $\hbar(t) \in R^p$. Combined with (4) and (5), the augmented system model can be expressed as follows:

$$\begin{aligned} \dot{\overline{x}}(t) &= \overline{A}\overline{x}(t) + \overline{B}u(t) + \overline{G}f(t) \\ \overline{V}(t) &= \overline{D}\overline{x}(t) \\ \overline{\gamma}(y, u(t)) &= C(y)\overline{V}(t) + T(y) \end{aligned} \tag{6}$$

where $\overline{x}(t) = \begin{bmatrix} x(t) \\ \hbar(t) \end{bmatrix}$ $\overline{A} = \begin{bmatrix} A & 0 \\ A_sD & -A_s \end{bmatrix}$ $\overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$ $\overline{G} = \begin{bmatrix} 0 \\ A_sG \end{bmatrix}$ $\overline{D} = \begin{bmatrix} 0 & I_p \end{bmatrix}$

3. Fault Diagnosis

In order to estimate the size of the fault, the fault diagnosis observer is designed as follows:

$$\begin{aligned} \dot{\hat{\overline{x}}}(t) &= \overline{A}\hat{\overline{x}}(t) + \overline{B}u(t) + \overline{G}Z(t) + \overline{L}\varepsilon(t) \\ \hat{\overline{V}}(t) &= \overline{D}\hat{\overline{x}}(t) \\ \hat{\overline{\gamma}}(y, u(t)) &= C(y)\hat{\overline{V}}(t) + T(y) \\ Z(t) &= K_1 Z(t-\tau) + K_2\varepsilon(t-\tau) \\ \dot{\hat{f}}(t) &= WZ(t) \end{aligned} \tag{7}$$

where $\hat{\overline{x}}(t) \in R^{n+p}$ is the estimation of state, $\hat{\overline{V}}(t) \in R^p$ is the estimation of weight vector, $Z(t) \in R^r$ is a state variable, $\hat{f}(t)$ is the estimation of $f(t)$, and $\varepsilon(t)$ is the residual can be expressed as

$$\begin{aligned} \varepsilon(t) &= \int_a^b \sigma(y)\left[\overline{\gamma}(y, u(t)) - \hat{\overline{\gamma}}(y, u(t))\right]dy \\ &= \int_a^b \sigma(y)C(y)\left[\overline{D}\left(\overline{x}(t) - \hat{\overline{x}}(t)\right]dy \\ &= \sum \overline{D}e_x(t) \end{aligned} \tag{8}$$

$\sum = \int_a^b \sigma(y)C(y)dy, \sigma(y) = y$, (A, D) is observable, and it is easy to know that $(\overline{A}, \overline{D})$ is observable. The parameter τ is defined as the learning interval, which can be taken as an integer multiple of the sampling period or sampling period [19]. $e_x(t)$ represents the state observation error. $\overline{L}$, K_1, K_2, and W are gain matrices with appropriate dimension that need to be determined.

Lemma 1. *For two matrices X and Y with approximate dimension, the following inequality holds [20]:*

$$2X^T \cdot Y \leq X^T \cdot X + Y^T \cdot Y \tag{9}$$

Assumption 2. $\|f(t)\| \leq K_f$, where K_f is a given positive constant.

$$e_x(t) = \bar{x}(t) - \hat{\bar{x}}(t) \tag{10}$$

The observation error dynamic system obtained by (6), (7) and (8) is formulated as follows:

$$\begin{aligned} \dot{e}_x(t) &= \dot{\bar{x}}(t) - \dot{\hat{\bar{x}}}(t) \\ &= \left(\bar{A} - \bar{L}\sum\bar{D}\right)e_x(t) + \bar{G}f(t) - \bar{G}Z(t) \end{aligned} \tag{11}$$

Theorem 3. *It is supposed that Assumption 2 holds. If there exist positive-definite symmetric matrices $P_1 \in R^{(n+p)\times(n+p)}$, $R_1 \in R^{(n+p)\times(n+p)}$, $Q_1 \in R^{(n+p)\times(n+p)}$, and matrices $K_1 \in R^{r\times r}$, $K_2 \in R^{r\times r}$, $Y \in R^{n+p}$, $\bar{L} \in R^{n+p}$, the following inequalities and equation hold:*

$$\begin{aligned} &P_1\bar{A} + \bar{A}^T P_1 - Y\sum\bar{D} - \left(\sum\bar{D}\right)^T Y^T + R_1 \\ &\quad + P_1\bar{G}\bar{G}^T P_1 = -Q_1 \end{aligned} \tag{12}$$

$$0 < (6+3\sigma)\left(K_2\sum\bar{D}\right)^T\left(K_2\sum\bar{D}\right) \leq R_1 \tag{13}$$

$$0 < (6+3\sigma)K_1^{\ T}K_1 \leq I \tag{14}$$

where σ is a given positive constant. The observer gain can be obtained from $\bar{L} = P_1^{-1}Y$. Equation (12) can be converted into the following LMI:

$$\begin{bmatrix} \left(\bar{A} - \bar{L}\sum\bar{D}\right)^T P_1 + P_1\left(\bar{A} - \bar{L}\sum\bar{D}\right) + R_1 + Q_1 & P_1\bar{G} \\ \bar{G}^T P_1 & -\gamma_1 I \end{bmatrix} < 0 \tag{15}$$

where γ_1 is a small positive number. Lemma 1 can ensure that the following inequalities hold:

$$\begin{aligned} &Z^T(t)Z(t) \\ &\quad \leq 3Z^T(t-\tau)K_1^{\ T}K_1Z(t-\tau) \\ &\qquad + 3e^T_{\ x}(t-\tau)\left(K_2\sum\bar{D}\right)^T\left(K_2\sum\bar{D}\right)e_x(t-\tau) \end{aligned} \tag{16}$$

$$\begin{aligned} &2\left\|e_x(t)P_1\bar{G}\right\|\|Z(t)\| \\ &\quad \leq e^T_{\ x}(t)P_1\bar{G}\bar{G}^T P_1 e_x(t) + Z^T(t)Z(t) \end{aligned} \tag{17}$$

In order to prove the stability of the system (11), the following Lyapunov function is selected as

$$\begin{aligned} V_1(t) &= e_x^{\ T}(t)P_1e_x(t) + \int_{t-\tau}^{t} e_x^{\ T}(s)R_1e_x(s)\,ds \\ &\quad + \int_{t-\tau}^{t} Z^T(s)Z(s)\,ds \end{aligned} \tag{18}$$

The derivative can be obtained as follows:

$$\begin{aligned} \dot{V}_1(t) &= e_x^{\ T}(t)\left[P_1\left(\bar{A} - \bar{L}\sum\bar{D}\right) + \left(\bar{A} - \bar{L}\sum\bar{D}\right)^T P_1\right] \\ &\quad \cdot e_x(t) + 2e_x^{\ T}(t)P_1\bar{G}f(t) - 2e_x^{\ T}(t) \\ &\quad \cdot P_1\bar{G}Z(t) + e_x^{\ T}(t)R_1e_x(t) - e_x^{\ T}(t-\tau) \\ &\quad \cdot R_1e_x(t-\tau) + Z^T(t)Z(t) - Z^T(t-\tau) \\ &\quad \cdot Z(t-\tau) \end{aligned} \tag{19}$$

From (14) and (15), the following inequality can be obtained:

$$\begin{aligned} \dot{V}_1(t) &\leq e_x^{\ T}(t)\left[\prod + R_1\right]e_x(t) + 2K_f\left\|P_1\bar{G}\right\|\|e_x(t)\| \\ &\quad + e^T_{\ x}(t)P_1\bar{G}\bar{G}^T P_1e_x(t) + \sigma Z^T(t)Z(t) + 2Z^T(t) \\ &\quad \cdot Z(t) - \sigma Z^T(t)Z(t) - e^T_{\ x}(t-\tau)R_1e_x(t-\tau) \\ &\quad - Z^T(t-\tau)Z(t-\tau) \leq e_x^{\ T}(t) \\ &\quad \cdot\left[\prod + R_1 + P_1\bar{G}\bar{G}^T P_1\right]e_x(t) + 2K_f\left\|P_1\bar{G}\right\|\|e_x(t)\| \\ &\quad - Z^T(t-\tau)Z(t-\tau) + (6+3\sigma)Z^T(t-\tau) \\ &\quad \cdot K_1^{\ T}K_1Z(t-\tau) + (6+3\sigma)e^T_{\ x}(t-\tau)\left(K_2\sum\bar{D}\right)^T \\ &\quad \cdot\left(K_2\sum\bar{D}\right)e_x(t-\tau) - e^T_{\ x}(t-\tau)R_1e_x(t-\tau) \\ &\quad - \sigma Z^T(t)Z(t) = e_x^{\ T}(t)\left[\prod + R_1 + P_1\bar{G}\bar{G}^T P_1\right] \\ &\quad \cdot e_x(t) + 2K_f\left\|P_1\bar{G}\right\|\|e_x(t)\| - \sigma Z^T(t)Z(t) \\ &\quad + Z^T(t-\tau)\left((6+3\sigma)K_1^{\ T}K_1 - I\right)Z(t-\tau) \\ &\quad + e^T_{\ x}(t-\tau) \\ &\quad \cdot\left((6+3\sigma)\left(K_2\sum\bar{D}\right)^T\left(K_2\sum\bar{D}\right) - R_1\right)e_x(t-\tau) \end{aligned} \tag{20}$$

For (20), when inequalities (12), (13), and (14) are satisfied, the following inequality can be further obtained:

$$\begin{aligned} \dot{V}_1(t) &\leq -\lambda_{\min}(Q_1)\|e_x(t)\|^2 + 2K_f\left\|P\bar{G}\right\|\|e_x(t)\| \\ &\quad - \sigma Z^T(t)Z(t) \end{aligned} \tag{21}$$

where $\prod = P_1(\bar{A} - \bar{L}\sum\bar{D}) + (\bar{A} - \bar{L}\sum\bar{D})^T P_1$. The proof is completed.

After diagnosing the system sensor fault, the original system is rebuilt to observe the original system PDF after the fault occurs and the subsequent fault-tolerant control is prepared. The original system state, weights, and output PDF and their observations can be obtained by the following coordinate transformation.

$$\begin{aligned} \hat{x}(t) &= \left[I_n \quad 0\right]\hat{\bar{x}}(t) \\ \widehat{V}(t) &= D\hat{x}(t) + G\hat{f}(t) \\ \hat{\gamma}(y, u(t)) &= C(y)\widehat{V}(t) + T(y) \end{aligned} \tag{22}$$

Inequality (16) can be proved as follows:

$$\begin{aligned} &Z^T(t)Z(t) \\ &\quad = Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + Z^T(t-\tau)K_1{}^T K_2 \sum \overline{D} e_x(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T K_1 Z(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \end{aligned} \tag{23}$$

It can be formulated that

$$\begin{aligned} &2Z^T(t)Z(t) \\ &\quad \le 2Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \\ &\qquad + Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \\ &\qquad + Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + 2e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \\ &\qquad + Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \\ &\quad = 6Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + 6e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \end{aligned} \tag{24}$$

It can be further obtained that

$$\begin{aligned} &Z^T(t)Z(t) \\ &\quad \le 3Z^T(t-\tau)K_1{}^T K_1 Z(t-\tau) \\ &\qquad + 3e^T{}_x(t-\tau)\left(K_2 \sum \overline{D}\right)^T \left(K_2 \sum \overline{D}\right) e_x(t-\tau) \end{aligned} \tag{25}$$

4. Fault-Tolerant Control

The basic idea of fault-tolerant control (FTC) is to compensate the influence of fault on the system performance. Sensor fault may exist in many forms, such as constant, time-varying, or even unbounded. When the system sensor fault occurs, it usually does not work properly. A simple fault compensation scheme is designed in this paper. $V(t)$ is the weight vector of the system output, which needs to be compensated in order to ensure the performance of the system after the fault occurs. The fault estimation $\hat{f}(t)$ can be obtained by the observer. The compensated weight $Vc(t)$ can be expressed as follows:

$$Vc(t) = V(t) - G\hat{f}(t) \tag{26}$$

Using a compensated sensor output $Vc(t)$, control algorithms are given to ensure that whether fault occurs or not, sensor fault compensation can be performed using fault-tolerant operation. To make sure that the probability density function (PDF) of the system can still track a given probability density function after a fault occurs, a fault-tolerant controller needs to be designed. A new weight error vector is defined as $e_V(t) = Vc(t) - Vg$, where Vg is the weight of the expected output. The following weight error dynamic system can be obtained.

$$\begin{aligned} \dot{e}_V(t) &= \dot{V}c(t) - \dot{V}g = D\dot{x}(t) + G\dot{e}_f(t) \\ &= D[Ax(t) + Bu(t)] + G\dot{e}_f(t) \\ &= DAx(t) + DBu(t) + G\dot{e}_f(t) \\ &= DAD^{-1}Dx(t) + DAD^{-1}Ge_f(t) - DAD^{-1}Vg \\ &\quad - DAD^{-1}Ge_f(t) + DAD^{-1}Vg + G\dot{e}_f(t) \\ &\quad + DBu(t) \\ &= A_1 e_V(t) - A_1 Ge_f(t) + A_1 Vg + G\dot{e}_f(t) \\ &\quad + Mu(t) \end{aligned} \tag{27}$$

where $A_1 = DAD^{-1}$ and $M = DB$.

With the sliding mode control method, two necessary conditions should be satisfied: the accessibility of the system state and the asymptotic stability of the sliding mode dynamic process. The sliding mode control law is designed so that the state trajectory at any moment can reach the sliding surface during a limited time. The switching function is designed as follows:

$$S(t) = He_V(t) - \int_0^t H\left(A_1 + MK_3\right) e_V(\tau)\,d\tau \tag{28}$$

where the matrix K_3 is selected such that $A_1 + MK_3$ is a Hurwitz matrix. The matrix H is chosen to make HM be a nonsingular matrix. After the state trajectory of the weight error dynamic system reaches the sliding surface, $S(t) = 0$ and $\dot{S}(t) = 0$ should be satisfied simultaneously. The equivalent control law is shown as follows:

$$u_{eq} = (HM)^{-1} H\left[MK_3 e_V(t) - A_1 Vg\right] \tag{29}$$

Equation (29) is substituted into (27), and (27) can be further obtained as

$$\begin{aligned} \dot{e}_V(t) &= \left[A_1 + M(HM)^{-1} HMK_3\right] e_V(t) \\ &\quad + \left[I - M(HM)^{-1} H\right] A_1 Vg - A_1 Ge_f(t) \\ &\quad + G\dot{e}_f(t) \end{aligned} \tag{30}$$

Theorem 4. *If both positive-definite symmetric matrices P_2 and Q_2 exist, the following inequality holds:*

$$P_2\left(A_1+MK_3\right)+\left(A_1+MK_3\right)^T P_2+Q_2 \leq 0 \tag{31}$$

Then the sliding mode dynamic system is stable. The following Lyapunov function is selected as

$$V_2(t)=e_V{}^T(t)\,P_2 e_V(t) \tag{32}$$

The first-order derivative of $V_2(t)$ is formulated as

$$\begin{aligned}\dot{V}_2(t)&=e_V{}^T(t)\left[P_2\left(A_1+MK_3\right)+\left(A_1+MK_3\right)^T P_2\right]\\&\cdot e_V(t)+2e_V{}^T(t)\,P_2\left(\left[I-M(HM)^{-1}H\right]A_1Vg\right.\\&\left.+G\dot{e}_f(t)-A_1Ge_f(t)\right)\leq-\lambda_{\min}\left(Q_2\right)\left\|e_V(t)\right\|^2\\&+2\left\|e_V(t)\right\|\left\|P_2\right\|\left(\left\|I-M(HM)^{-1}H\right\|\left\|A_1Vg\right\|\right.\\&\left.-\left\|A_1G\right\|\left\|e_f(t)\right\|+\|G\|\left\|\dot{e}_f(t)\right\|\right)=-\omega_1\left\|e_V(t)\right\|^2\\&+2\omega_2\left\|e_V(t)\right\|\end{aligned} \tag{33}$$

where $\omega_1 = \lambda_{\min}(Q_2)$ and $\omega_2 = \|P_2\|(\|I - M(HM)^{-1}H\|\|A_1Vg\| - \|A_1G\|\|e_f(t)\| + \|G\|\|\dot{e}_f(t)\|)$. Using (33), the following inequality can be obtained as

$$\dot{V}_2(t)\leq-\omega_1\left(\left\|e_V(t)\right\|-\frac{\omega_2}{\omega_1}\right)^2+\frac{\omega_2{}^2}{\omega_1} \tag{34}$$

when $\|e_V(t)\| \geq 2\omega_2/\omega_1$, $\dot{V}_2 \leq 0$, and the sliding mode dynamic system is stable. In order to ensure that the state trajectory starting at any position can reach the sliding surface during a limited time, the following sliding mode control law is designed:

$$u_n(t)=\begin{cases}-\alpha\dfrac{S(t)}{\|S(t)\|^2} & S(t)\neq 0\\ 0 & S(t)=0\end{cases} \tag{35}$$

where $\alpha > 0$. The sliding mode control law can ensure that the state trajectory reaches the sliding surface during a limited time, $S(t) = 0$.

The following Lyapunov function is chosen as

$$V_3(t)=\frac{1}{2}S^T(t)\,(HM)^{-1}S(t) \tag{36}$$

$$\begin{aligned}\dot{V}_3(t)&=S^T(t)\,(HM)^{-1}\dot{S}(t)=S^T(t)\,(HM)^{-1}\left[H\dot{e}_V(t)\right.\\&\left.-H\left(A+MK_3\right)e_V(t)\right]=S^T(t)\,(HM)^{-1}\\&\cdot H\left[A_1e_V(t)-A_1Ge_f(t)+A_1Vg+G\dot{e}_f(t)\right.\\&\left.+Mu(t)\right]-S^T(t)\,(HM)^{-1}H\left(A+BK_3\right)e_V(t)\\&=S^T(t)\,(HM)^{-1}HMu_n(t)+S^T(t)\,(HM)^{-1}\\&\cdot H\left[G\dot{e}_f(t)-A_1Ge_f(t)\right)\right]=-\alpha\frac{S^T(t)\,S(t)}{\|S(t)\|^2}+\omega_3\\&=-\alpha+\omega_3<0\end{aligned} \tag{37}$$

where α is selected as $\alpha > \omega_3$, $\omega_3 = \|S^T(t)\|\|(HM)^{-1}\|\|H\|[\|G\|\|\dot{e}_f(t)\| - \|A_1G\|\|e_f(t)\|]$. The state trajectory reaches the sliding surface during a limited time. The available fault-tolerant control law is shown as follows:

$$\begin{aligned}u(t)&=u_{eq}(t)+u_n(t)\\&=(HM)^{-1}H\left[MK_3\left(V(t)-G\hat{f}(t)-Vg\right)-A_1Vg\right]\\&-\alpha\frac{S(t)}{\|S(t)\|^2}\end{aligned} \tag{38}$$

5. A Simulation Example

In order to verify the effectiveness of the algorithm, the proposed method is applied to the process of molecular weight distribution (MWD) dynamic modeling and control, and a continuous stirring reactor (CSTR) is considered as an example. The closed-loop control diagram of the polymerization process is shown in Figure 1. The specific mathematical model is shown as follows:

$$\begin{aligned}\dot{I}(t)&=\frac{I_0-I(t)}{\theta}-K_dI(t)+K_Iu(t)\\\dot{M}(t)&=\frac{M_0-M(t)}{\theta}-2K_iI(t)+K_Mu(t)\\&-\left(K_p+K_{trm}\right)M(t)\,R_i\end{aligned} \tag{39}$$

where $\theta = V/F$ is the average residence time of reactants (s), I_0 is the initial concentration of initiator ($mol \cdot ml^{-1}$); I is the initiator concentration ($mol \cdot ml^{-1}$); M_0 is the initial concentration of monomer ($mol \cdot ml^{-1}$); M is the monomer concentration ($mol \cdot ml^{-1}$); K_i, K_p, K_{trm} are the reaction rate constants; K_I, K_M are constants associated with the control input; $R_i (i = 1, 2, \ldots, q)$ are the free radical. The above specific physical meaning can be referred to the literature [21].

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)\\V(t)&=Dx(t)+Gf(t)\\\gamma(y,u(t))&=C(y)\,V(t)+T(y)\end{aligned} \tag{40}$$

For this stochastic distribution system, the output probability density function (PDF) can be approximated by a linear B-spline basis function of the form:

$$\begin{aligned}\phi_1(y)&=\frac{1}{2}(y-2)^2I_1+\left(-y^2+7y-11.5\right)I_2\\&+\frac{1}{2}(y-5)^2I_3\end{aligned}$$

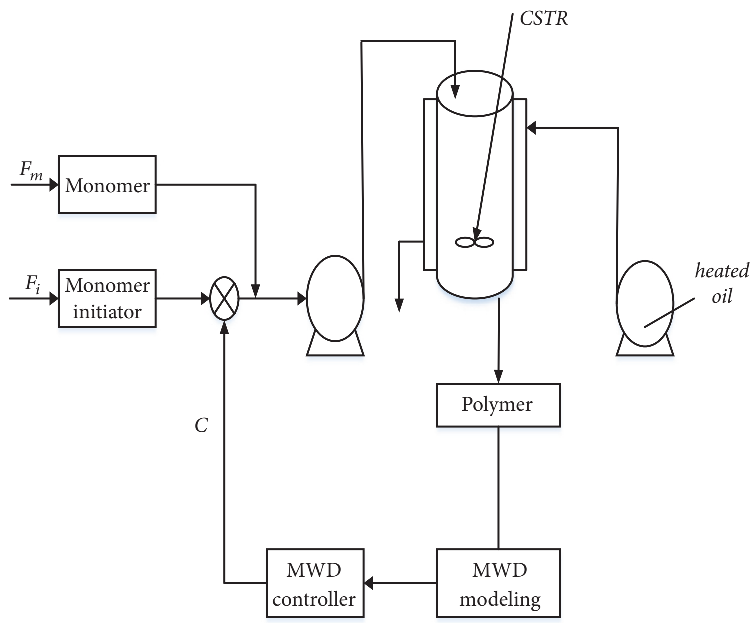

Figure 1: Schematic diagram of a continuous stirred reactor.

$$\begin{aligned}\phi_2(y) &= \frac{1}{2}(y-3)^2 I_2 + \left(-y^2+9y-19.5\right) I_3 \\ &\quad + \frac{1}{2}(y-6)^2 I_4 \\ \phi_3(y) &= \frac{1}{2}(y-4)^2 I_3 + \left(-y^2+11y-29.5\right) I_4 \\ &\quad + \frac{1}{2}(y-7)^2 I_5\end{aligned} \tag{41}$$

$$I_i(y) = \begin{cases} 1 & y \in [i+1, i+2] \\ 0 & otherwise, \end{cases} \quad (i = 1,2,3,4,5) \tag{42}$$

The system parameter matrices and vectors are given as follows:

$$\begin{aligned}A &= \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \\ B &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ D &= \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix} \\ G &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ A_s &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \overline{A} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0.5 & 1 & 0 & -1 \end{bmatrix} \\ \overline{B} &= \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\ \overline{G} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\ \overline{D} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned} \tag{43}$$

matrices $\overline{L}$, $K_i (i = 1,2,3)$, W and matrix H are selected as follows: $\overline{L} = \begin{bmatrix} -0.5059 \\ -0.0599 \\ 0.9326 \\ -2.9999 \end{bmatrix}$, $K_1 = 0.057$, $K_2 = -0.5$, $K_3 = \begin{bmatrix} 12.1635 & 0.0389 \end{bmatrix}$, $W = 22.5$, $H = \begin{bmatrix} 0.4 & -0.1 \end{bmatrix}$, $\sum = \begin{bmatrix} -2 & -1 \end{bmatrix}$, and $\tau = 0.01$. It is assumed that the fault is constructed as follows:

$$f(t) = \begin{cases} 0 & 0 \le t < 4 \\ 0.5 & 4 \le t < 40 \\ 1.6 - e^{-0.16(t-40)} & t \ge 40 \end{cases} \tag{44}$$

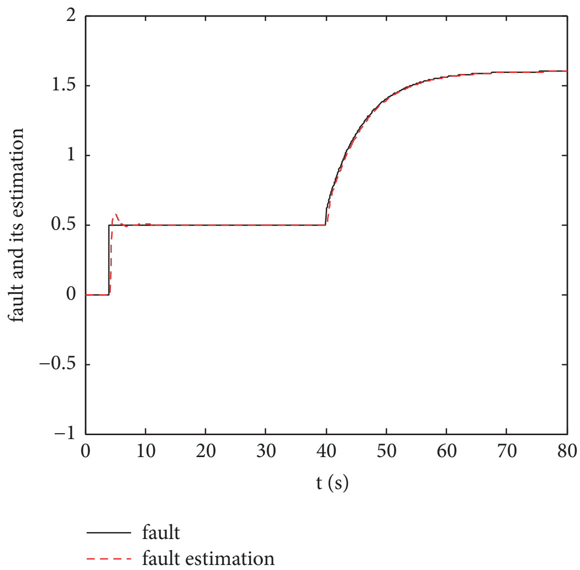

Figure 2: Fault and fault estimation.

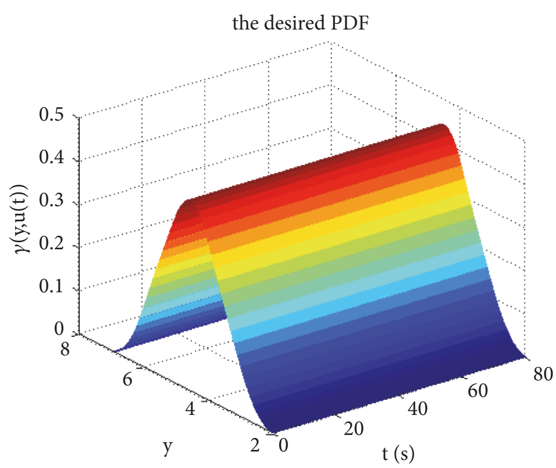

Figure 3: The system expectation output PDF.

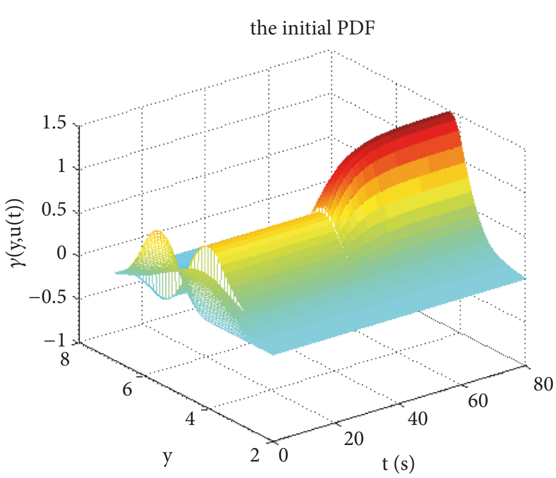

Figure 4: The system output PDF without fault-tolerant control.

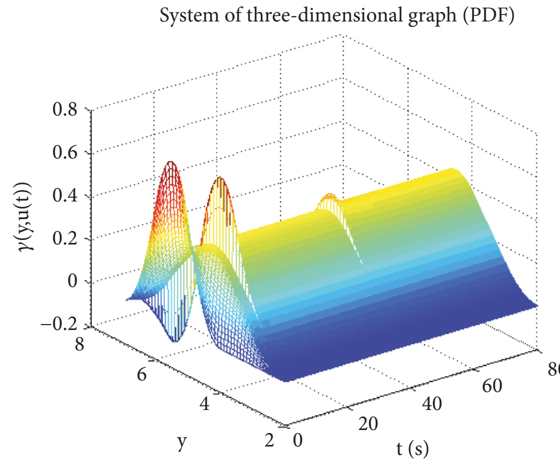

Figure 5: The system output PDF with fault-tolerant control.

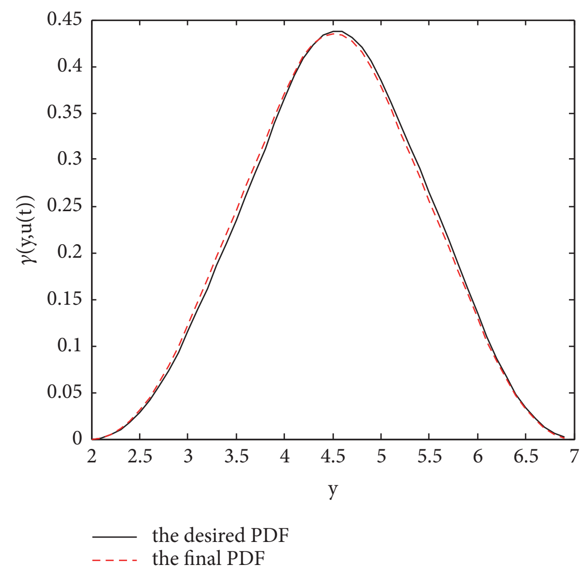

Figure 6: The system final and desired PDF.

From Figure 2, it can be seen that the fault occurs at t = 4s, and the fault estimation can track the change of fault after short transition. The desired output PDF can be seen in Figure 3. Figure 4 shows that the system actual output PDF cannot track the desired PDF without FTC when the fault occurs. The system postfault output PDF with FTC can track the desired PDF, which is shown in Figures 5 and 6, the validity of the FTC algorithm is verified.

6. Conclusions

In this paper, the problem of sensor fault diagnosis and fault-tolerant control for non-Gaussian stochastic distribution systems is studied. The learning observer is used to diagnose the sensor fault. The fault is compensated by the fault estimation information, and the sliding mode control algorithm is utilized to make the PDF of the system output to track the desired distribution. The Lyapunov stability theorem is applied to prove the stability of the dynamic system of the observation error and the whole control process. Finally, the mathematical model is built by the actual industrial control process and the computer simulation further verifies the effectiveness of the algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] X. Meng and T. Chen, "Event detection and control co-design of sampled-data systems," *International Journal of Control*, vol. 87, no. 4, pp. 777–786, 2014.

[2] X. Meng and T. Chen, "Event triggered robust filter design for discrete-time systems," *IET Control Theory & Applications*, vol. 8, no. 2, pp. 104–113, 2014.

[3] H. Wang, *Bounded Dynamic Stochastic Systems: Modeling and Control*, Springer, London, UK, 2000.

[4] L. N. Yao and B. Peng, "Fault diagnosis and fault tolerant control for the non-Gaussian time-delayed stochastic distribution control system," *Journal of The Franklin Institute*, vol. 351, no. 3, pp. 1577–1595, 2014.

[5] L. N. Yao, J. F. Qin, H. Wang, and B. Jiang, "Design of new fault diagnosis and fault tolerant control scheme for non-Gaussian singular stochastic distribution systems," *Automatica*, vol. 48, no. 9, pp. 2305–2313, 2012.

[6] L. Yao and L. Feng, "Fault diagnosis and fault tolerant tracking control for the non-Gaussian singular time-delayed stochastic distribution system with PDF approximation error," *International Journal of Control Automation and Systems*, vol. 14, no. 2, pp. 435–442, 2014.

[7] Q. X. Jia, W. Chen, Y. C. Zhang, and X. Q. Chen, "Robust fault reconstruction via learning observers in linear parameter-varying systems subject to loss of actuator effectiveness," *IET Control Theory & Applications*, vol. 8, no. 1, pp. 42–50, 2014.

[8] Q. Jia, W. Chen, Y. Zhang, and H. Li, "Integrated design of fault reconstruction and fault-tolerant control against actuator faults using learning observers," *International Journal of Systems Science*, vol. 47, no. 16, pp. 3749–3761, 2016.

[9] Y. Tan, D. Du, and S. Fei, "Co-design of event generator and quantized fault detection for time-delayed networked systems with sensor saturations," *Journal of The Franklin Institute*, vol. 354, no. 15, pp. 6914–6937, 2017.

[10] M. Liu and P. Shi, "Sensor fault estimation and tolerant control for Itô stochastic systems with a descriptor sliding mode approach," *Automatica*, vol. 49, no. 5, pp. 1242–1250, 2013.

[11] D. Du and V. Cocquempot, "Fault diagnosis and fault tolerant control for discrete-time linear systems with sensor fault," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 15754–15759, 2017.

[12] S. Xu, "Robust H∞ filtering for a class of discrete-time uncertain nonlinear systems with state delay," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, no. 12, pp. 1853–1859, 2003.

[13] J. Zhou, G. Li, and H. Wang, "Robust tracking controller design for non-Gaussian singular uncertainty stochastic distribution systems," *Automatica*, vol. 50, no. 4, pp. 1296–1303, 2014.

[14] H. Jin, Y. Guan, and L. Yao, "Minimum entropy active fault tolerant control of the non-Gaussian stochastic distribution system subjected to mean constraint," *Entropy*, vol. 19, no. 5, p. 218, 2017.

[15] D. Du, B. Jiang, and P. Shi, "Sensor fault estimation and compensation for time-delay switched systems," *International Journal of Systems Science*, vol. 43, no. 4, pp. 629–640, 2012.

[16] Y. Ren, A. Wang, and H. Wang, "Fault diagnosis and tolerant control for discrete stochastic distribution collaborative control systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 3, pp. 462–471, 2015.

[17] Y. Ren, Y. Fang, A. Wang, H. Zhang, and H. Wang, "Collaborative operational fault tolerant control for stochastic distribution control system," *Automatica*, vol. 9, no. 8, pp. 141–149, 2018.

[18] C. Edwards and C. P. Tan, "A comparison of sliding mode and unknown input observers for fault reconstruction," *European Journal of Control*, vol. 12, no. 3, pp. 245–260, 2006.

[19] Q. Jia, W. Chen, Y. Zhang, and H. Li, "Fault reconstruction for continuous-time systems via learning observers," *Asian Journal of Control*, vol. 18, no. 1, pp. 1–13, 2016.

[20] T. Youssef, M. Chadli, H. R. Karimi, and R. Wang, "Actuator and sensor faults estimation based on proportional integral observer for T-S fuzzy model," *Journal of the Franklin Institute*, vol. 354, no. 6, pp. 2524–2542, 2017.

[21] L. Yao, W. Cao, and H. Wang, "Minimum entropy fault-tolerant control of the non-gaussian stochastic distribution system," in *Proceedings of the International Conference on Information Engineering and Applications (IEA) 2012*, Springer, London, UK, 2013.