Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 5843751, 10 pages
https://doi.org/10.1155/2019/5843751

Research Article

Quadratic Programming Method for Cooperative Games with Coalition Values Expressed by Triangular Fuzzy Numbers and Its Application in the Profit Distribution of Logistics Coalition

Jia-Cai Liu,[1,2] **Yuan-Fei Zhu**,[2] **and Wen-Jian Zhao**[3]

[1]*College of Management & College of Tourism, Fujian Agriculture and Forestry University, Fuzhou 350002, China*
[2]*College of Transportation and Civil Engineering, Fujian Agriculture and Forestry University, Fuzhou 350002, China*
[3]*Jinshan College, Fujian Agriculture and Forestry University, Fuzhou 350002, China*

Correspondence should be addressed to Wen-Jian Zhao; wenjianzhao@fafu.edu.cn

Received 8 September 2018; Accepted 6 December 2018; Published 21 January 2019

Academic Editor: Bonifacio Llamazares

A quadratic programming model is constructed for solving the fuzzy cooperative games with coalition values expressed by triangular fuzzy numbers, which will be abbreviated to TFN-typed cooperative games from now on. Based on the concept of α-cut set and the representation theorem for the fuzzy set, the least square distance solution for solving TFN-typed cooperative games is proposed. The least square distance solution successfully avoids the subtraction operation of TFNs, which may inevitably lead to the amplification of uncertainty and the distortion of decision information. A calculating example related to the profit distribution of logistics coalition is illustrated to show the advantages, validity, and applicability of the proposed method. Besides, the least square distance solution for solving TFN-typed cooperative games satisfies many important properties of cooperative games, such as uniqueness, additivity, symmetry, and uniqueness.

1. Introduction

There exist many problems and phenomena related to fuzzy cooperative games in our daily lives. An increasing number of researchers turn their attention to the theory and application of fuzzy cooperative games. Generally speaking, we usually divide the fuzzy cooperative games into three different types as follows: cooperative games with fuzzy coalition values [1–3], cooperative games with fuzzy coalitions [4–6], and cooperative games with both fuzzy coalitions and fuzzy coalition values [7, 8], respectively. As for the three types of fuzzy cooperative games mentioned above, the cooperative games with fuzzy coalition values gradually become a research hotspot in recent years. Some researchers extend the common solutions of crisp cooperative games to fuzzy cooperative games and propose some corresponding solution methods and solution concepts for solving fuzzy cooperative games, such as fuzzy Shapley value [9–11], fuzzy set-valued solution [12], fuzzy least square prenucleolus and B-nucleolus [13–15], fuzzy bargaining sets [16, 17], and fuzzy equalizer and lexicographical solution [18].

However, many existing models and methods for solving fuzzy cooperative games inevitably use the subtraction operation of fuzzy numbers. As is known to all, some operations of fuzzy numbers especially the subtraction operation may easily result in the amplification of uncertainty and the distortion of decision information.

In this paper, we are absorbed in developing a new and intuitionistic method for the TFN-typed cooperative games, which can successfully avoid the subtraction operation of TFNs. The rest of the paper is arranged as follows. Section 2 briefly introduces some key concepts for constructing the quadratic programming model, such as the TFN, the α-cut set, and the representation theorem for the fuzzy set. Section 3 puts forward the least square distance solution for TFN-typed cooperative games based on the square distance and α-cut sets of TFNs. In order to show the superiority, advantages, and applicability of the proposed method, a calculating example about the profit distribution of logistics coalition is illustrated. Besides, the interval Shapley-like value proposed in [9] is used to redetermine the optimal allocation strategy and the results from the two methods are compared with each other. Conclusion is made in Section 5.

2. Preliminaries

2.1. The Definition of TFNs. The membership function of a random TFN $\overline{a} = (a_l, a_m, a_r)$ [19] can be shown as follows:

$$\mu_{\overline{a}}(x) = \begin{cases} \dfrac{(x - a_l)}{(a_m - a_l)} & \text{if } a_l \le x < a_m \\ 1 & \text{if } x = a_m \\ \dfrac{(a_r - x)}{(a_r - a_m)} & \text{if } a_m < x \le a_r \\ 0 & \text{else,} \end{cases} \tag{1}$$

and, in (1), a_l denotes the lower bound of $\overline{a}$, a_m denotes the mean of $\overline{a}$, and a_r denotes the upper bound of $\overline{a}$, respectively. It is not difficult to find that the TFN $\overline{a}$ will reduce to a real number if the three values a_l, a_m, and a_r are equal. In other words, an arbitrary real number a can be changed to the form of the corresponding TFN, which is shown as $\overline{a} = (a, a, a)$.

2.2. The α-Cut Set of the TFN and the Representation Theorem for the Fuzzy Set. As is known to all, the definition of the α-cut set of an arbitrary TFN $\overline{a}$ can be shown as $\overline{a}(\alpha) = \{x \mid \mu_{\overline{a}}(x) \ge \alpha\}$ $(\alpha \in [0,1])$. Therefore, the α-cut set of an arbitrary TFN $\overline{a}$ at any confidence level can be obtained, which is actually an interval expressed by $\overline{a}(\alpha) = [a_l(\alpha), a_r(\alpha)]$.

According to (1), we have

$$a_l(\alpha) = \alpha a_m + (1 - \alpha) a_l \tag{2}$$

and

$$a_r(\alpha) = \alpha a_m + (1 - \alpha) a_r. \tag{3}$$

That is to say,

$$\begin{aligned} \overline{a}(\alpha) &= [a_l(\alpha), a_r(\alpha)] \\ &= [\alpha a_m + (1 - \alpha) a_l, \alpha a_m + (1 - \alpha) a_r]. \end{aligned} \tag{4}$$

It is obvious that

$$\overline{a}(1) = [a_l(1), a_r(1)] = [a_m, a_m] \tag{5}$$

and

$$\overline{a}(0) = [a_l(0), a_r(0)] = [a_l, a_r]. \tag{6}$$

Through the further analysis based on the interval operations [20], we have

$$\begin{aligned} \overline{a}(\alpha) &= [\alpha a_m + (1 - \alpha) a_l, \alpha a_m + (1 - \alpha) a_r] \\ &= \alpha [a_m, a_m] + (1 - \alpha) [a_l, a_r] \\ &= \alpha \overline{a}(1) + (1 - \alpha) \overline{a}(0), \end{aligned} \tag{7}$$

which means as long as we know the 1-cut set and 0-cut set of an arbitrary TFN $\overline{a}$, then we can conveniently calculate its α-cut set at any confidence level.

According to the representation theorem for the fuzzy set [21], a random TFN $\overline{a}$ can be denoted as the following form:

$$\begin{aligned} \overline{a} &= \bigcup_{\alpha \in [0,1]} \{\alpha \otimes \overline{a}(\alpha)\} \\ &= \bigcup_{\alpha \in [0,1]} \{\alpha \otimes [\alpha \overline{a}(1) + (1 - \alpha) \overline{a}(0)]\}, \end{aligned} \tag{8}$$

3. The Least Square Distance Solution for TFN-Typed Cooperative Games Based on the Square Distance and α-Cut Sets

3.1. TFN-Typed Cooperative Games. In this section, we will demonstrate the mathematical representation of the TFN-typed cooperative games. A TFN-typed cooperative game in coalitional form can be shown as an ordered pair (N, v). $N = \{1, 2, \ldots, n\}$ is the set of all the players conducting cooperation. $v : 2^N \longrightarrow I(\mathrm{R})$ is the characteristic function of the coalition S $(S \subseteq N)$, which is a TFN expressed as $\overline{v}(S) = (v_l(S), v_m(S), v_r(S))$. For any coalition S $(S \subseteq N)$, $v_l(S)$ denotes the minimal profit, $v_m(S)$ denotes the mean profit, and $v_r(S)$ denotes the maximal profit the coalition S can realize if all of the players in S form a coalition and cooperate with each other. What needs illustration is that the memberships of the possible coalition values (i.e., profits) are usually different from one another. Particularly, $\varnothing$ is an empty coalition, which means nobody is willing to join the cooperation, so $\overline{v}(\varnothing) = (0, 0, 0)$. For the sake of convenient description, $\overline{v}(\{i\})$ $(i \in N)$ is abbreviated to $v(i)$ and $\overline{v}(\{ij \ldots\})$ $(i, j, \ldots \in N)$ is abbreviated to $\overline{v}(ij \ldots)$, respectively, in this paper.

3.2. Quadratic Programming Model for Solving the Least Square Distance Solution. As mentioned before, at any confidence level, the α-cut set of the characteristic function of the coalition S is an interval, which can be shown as follows:

$$\begin{aligned} \overline{v}(S)(\alpha) &= [v_l(S)(\alpha), v_r(S)(\alpha)] \\ &= [\alpha v_m(S) + (1 - \alpha) v_l(S), \alpha v_m(S) + (1 - \alpha) v_r(S)] \\ &= \alpha [v_m(S), v_m(S)] + (1 - \alpha) [v_l(S), v_r(S)] \\ &= \alpha \overline{v}(S)(1) + (1 - \alpha) \overline{v}(S)(0). \end{aligned} \tag{9}$$

For an arbitrary TFN-typed cooperative game, conclusion can be drawn that every player i $(i \in N)$ should obtain a TFN-typed payoff, which is described by $\overline{x}(i) = (x_l(i), x_m(i), x_r(i))$ because the characteristic function of the coalition S ($S \subseteq N$) is a TFN. Hence, $\bar{\boldsymbol{x}} = (\bar{x}(1), \bar{x}(2), \dots, \bar{x}(n))^{\mathrm{T}}$ denotes a payoff vector, where the payoff of every player i ($i \in N$) is expressed by a TFN. As is known to all, if a payoff vector $\bar{\boldsymbol{x}}$ satisfies the effectiveness of attribution (i.e., $\bar{x}(N) = \bar{v}(N)$), the payoff vector is regarded as a preimputation or an efficient payoff. Similarly, if a payoff vector $\bar{\boldsymbol{x}}$ simultaneously satisfies the effectiveness of attribution (i.e., $\bar{x}(N) = v(N)$) and the individual rationality (i.e., $\bar{x}(i) \geq \bar{v}(i)$ for all $i \in N$), the payoff vector is regarded as an imputation. Otherwise, the payoff vector may be unsatisfactory.

According to the definition and properties of the α-cut set, the α-cut set of the TFN-typed payoff of the player i ($i \in N$) can be written as follows:

$$\begin{aligned} \bar{x}(i)(\alpha) &= \left[x_l(i)(\alpha), x_r(i)(\alpha)\right] \\ &= \left[\alpha x_m(i) + (1-\alpha)x_l(i), \alpha x_m(i) + (1-\alpha)x_r(i)\right] \\ &= \alpha\left[x_m(i), x_m(i)\right] + (1-\alpha)\left[x_l(i), x_r(i)\right] \\ &= \alpha\bar{x}(i)(1) + (1-\alpha)\bar{x}(i)(0). \end{aligned} \tag{10}$$

$\bar{x}(S)(\alpha)$ denotes the sum of α-cut sets of the TFN-typed payoffs of all players in the coalition S ($S \subseteq N$). Therefore, we have

$$\bar{x}(S)(\alpha) = \sum_{i\in S}\bar{x}(i)(\alpha) = \left[\sum_{i\in S}x_l(i)(\alpha), \sum_{i\in S}x_r(i)(\alpha)\right], \tag{11}$$

which is also an interval.

In order to construct the quadratic programming model for solving the optimal attribution strategy of players, we use the square distance to measure the difference between $\bar{v}(S)(\alpha)$ and $\bar{x}(S)(\alpha)$ based on the least square method, which is shown as follows:

$$\begin{aligned} D(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) &= \left(v_l(S)(\alpha) - \sum_{i\in S}x_l(i)(\alpha)\right)^2 \\ &+ \left(v_r(S)(\alpha) - \sum_{i\in S}x_r(i)(\alpha)\right)^2. \end{aligned} \tag{12}$$

To some extent, the square distance between $\bar{v}(S)(\alpha)$ and $\bar{x}(S)(\alpha)$ can be regarded as a measure of the dissatisfaction of coalition S ($S \subseteq N$) once the payoff vector $\bar{\boldsymbol{x}} = (\bar{x}(1), \bar{x}(2), \dots, \bar{x}(n))^{\mathrm{T}}$ is advised to be the final attribution strategy.

For the sake of concise description, $v_l(S)(\alpha) - \sum_{i\in S}x_l(i)(\alpha)$ is replaced by $D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$ and $v_r(S)(\alpha) - \sum_{i\in S}x_r(i)(\alpha)$ is replaced by $D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$, respectively. Hence, the square distance between $\bar{v}(S)(\alpha)$ and $\bar{x}(S)(\alpha)$ can be rewritten as

$$\begin{aligned} D(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) &= D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))^2 \\ &+ D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))^2 \end{aligned} \tag{13}$$

and the sum of the dissatisfaction of coalition S can be shown as

$$\begin{aligned} \sum_{S\subseteq N} D(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) &= \sum_{S\subseteq N}\left[D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))^2\right. \\ &\left.+ D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))^2\right]. \end{aligned} \tag{14}$$

Generally speaking, $\overline{D}_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$ denotes the mean value of $\sum_{S\subseteq N} D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$ and $D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$ denotes the mean value of $\sum_{S\subseteq N} D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))$, respectively. Conclusions can be drawn that

$$\begin{aligned} \sum_{S\subseteq N} D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) &= \sum_{S\subseteq N} v_l(S)(\alpha) - 2^{n-1}v_l(N)(\alpha) \end{aligned} \tag{15}$$

and

$$\begin{aligned} \sum_{S\subseteq N} D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) &= \sum_{S\subseteq N} v_r(S)(\alpha) - 2^{n-1}v_r(N)(\alpha). \end{aligned} \tag{16}$$

Based on the principle of fairness, i.e., each player i ($i \in N$) would like to minimize its loss [20, 21], we construct the following quadratic programming model:

$$\begin{aligned} \min \quad & \sum_{S\subseteq N}\left[\left(D_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) - \overline{D}_l(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))\right)^2 + \left(D_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha)) - \overline{D}_r(\bar{v}(S)(\alpha), \bar{x}(S)(\alpha))\right)^2\right] \\ \text{s.t.} \quad & \begin{cases} \displaystyle\sum_{i=1}^{n} x_l(i)(\alpha) = v_l(N)(\alpha) \\ \displaystyle\sum_{i=1}^{n} x_r(i)(\alpha) = v_r(N)(\alpha). \end{cases} \end{aligned} \tag{17}$$

Because of

$$\begin{aligned}&\overline{D}_l\left(\overline{v}(S)(\alpha),\overline{x}(S)(\alpha)\right)\\&\quad=\frac{1}{2^n-1}\sum_{S\subseteq N}D_l\left(\overline{v}(S)(\alpha),\overline{x}(S)(\alpha)\right)\\&\quad=\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_l(S)(\alpha)-2^{n-1}v_l(N)(\alpha)\right)\end{aligned}\tag{18}$$

and

$$\begin{aligned}&\overline{D}_r\left(\overline{v}(S)(\alpha),\overline{x}(S)(\alpha)\right)\\&\quad=\frac{1}{2^n-1}\sum_{S\subseteq N}D_r\left(\overline{v}(S)(\alpha),\overline{x}(S)(\alpha)\right)\\&\quad=\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_r(S)(\alpha)-2^{n-1}v_r(N)(\alpha)\right),\end{aligned}\tag{19}$$

the quadratic programming model (17) can be rewritten as follows:

$$\begin{aligned}\min\ &\sum_{S\subseteq N}\left[\left(\left(v_l(S)(\alpha)-\sum_{i\in S}x_l(i)(\alpha)\right)-\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_l(S)(\alpha)-2^{n-1}v_l(N)(\alpha)\right)\right)^2+\left(\left(v_r(S)(\alpha)-\sum_{i\in S}x_r(i)(\alpha)\right)-\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_r(S)(\alpha)-2^{n-1}v_r(N)(\alpha)\right)\right)^2\right]\\ \text{s.t.}\ &\begin{cases}\sum_{i=1}^{n}x_l(i)(\alpha)=v_l(N)(\alpha)\\ \sum_{i=1}^{n}x_r(i)(\alpha)=v_r(N)(\alpha).\end{cases}\end{aligned}\tag{20}$$

According to the Lagrange multiplier method, the Lagrange function of the quadratic programming model (20) can be obtained as follows:

$$\begin{aligned}L(\boldsymbol{x},\lambda,\mu)={}&\sum_{S\subseteq N}\left[\left(\left(v_l(S)(\alpha)-\sum_{i\in S}x_l(i)(\alpha)\right)\right.\right.\\&\left.-\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_l(S)(\alpha)-2^{n-1}v_l(N)(\alpha)\right)\right)^2\\&+\left(\left(v_r(S)(\alpha)-\sum_{i\in S}x_r(i)(\alpha)\right)\right.\\&\left.\left.-\frac{1}{2^n-1}\left(\sum_{S\subseteq N}v_r(S)(\alpha)-2^{n-1}v_r(N)(\alpha)\right)\right)^2\right]\\&+\lambda\left(\sum_{i=1}^{n}x_l(i)(\alpha)-v_l(N)(\alpha)\right)+\mu\left(\sum_{i=1}^{n}x_r(i)(\alpha)\right.\\&\left.-v_r(N)(\alpha)\right).\end{aligned}\tag{21}$$

Take the solution process of $x_l(i)(\alpha)$ $(i=1,2,\dots,n)$ as an example. Let partial derivatives of $L(\boldsymbol{x},\lambda,\mu)$ with regard to the variables $x_l(j)(\alpha)$ and λ be equal to 0, respectively. So,

$$\begin{aligned}&-2\sum_{S:i\in S}\left[v_l(S)(\alpha)-\sum_{i\in S}x_l^*(S)(\alpha)\right.\\&\quad\left.-\frac{1}{2^n-1}\left(\sum_{S\subseteq N,S\neq\varnothing}v_l(S)(\alpha)-2^{n-1}v_l(N)(\alpha)\right)\right]\\&\quad+\lambda^*=0\end{aligned}\tag{22}$$

and

$$\sum_{i=1}^{n}x_l^*(i)(\alpha)=v_l(N)(\alpha).\tag{23}$$

The following result can be obtained through mathematical derivation that

$$\sum_{S:i\in S}x_l^*(S)(\alpha)=2^{n-1}x_l^*(i)(\alpha)+\sum_{j\in N\backslash i}2^{n-2}x_j^*(i)(\alpha)\qquad(i,j\in N).\tag{24}$$

Based on (22) and (24), we have

$$\begin{aligned}&-2\sum_{S:i\in S}v_l(S)(\alpha)+2\times 2^{n-1}x_l^*(i)(\alpha)+2\\&\quad\times\sum_{j\in N\backslash i}2^{n-2}x_l^*(j)(\alpha)+\frac{2}{2^n-1}\sum_{S\subseteq N,S\neq\varnothing}v_l(S)(\alpha)\\&\quad-\frac{2^n}{2^n-1}v_l(N)(\alpha)+\lambda^*=0.\end{aligned}\tag{25}$$

It is obvious to see that

$$x_l^*(i)(\alpha)+\sum_{j\in N\backslash i}x_l^*(j)(\alpha)=v_l(N)(\alpha)\quad(i,j\in N).\tag{26}$$

Therefore,

$$\begin{aligned}&-2\sum_{S:i\in S}v_l(S)(\alpha)+2^{n-1}x_l^*(i)(\alpha)\\&\quad+\left(2^{n-1}-\frac{2^n}{2^n-1}\right)v_l(N)(\alpha)\\&\quad+\frac{2}{2^n-1}\sum_{S\subseteq N,S\neq\varnothing}v_l(S)(\alpha)+\lambda^*=0.\end{aligned}\tag{27}$$

As a result,

$$x_l^*(i)(\alpha) = \frac{2\sum_{S:i\in S} v_l(S)(\alpha) - \left(2^{n-1} - 2^n/(2^n-1)\right) v_l(N)(\alpha) - \left(2/(2^n-1)\right)\sum_{S\subseteq N, S\neq\varnothing} v_l(S)(\alpha) - \lambda^*}{2^{n-1}}. \quad (28)$$

Based on (23) and (28), we can obtain

$$\sum_{i\in N} \frac{2\sum_{S:i\in S} v_l(S)(\alpha) - \left(2^{n-1} - 2^n/(2^n-1)\right) v_l(N)(\alpha) - \left(2/(2^n-1)\right)\sum_{S\subseteq N, S\neq\varnothing} v_l(S)(\alpha) - \lambda^*}{2^{n-1}} = v_l(N)(\alpha). \quad (29)$$

Hence,

$$\begin{aligned} \lambda^* &= \frac{2\sum_{S\subseteq N, S\neq\varnothing} s v_l(S)(\alpha)}{n} \\ &\quad - \left(2^{n-1} - \frac{2^n}{2^n-1}\right) v_l(N)(\alpha) \\ &\quad - \frac{2}{2^n-1} \sum_{S\subseteq N, S\neq\varnothing} v_l(S)(\alpha) - \frac{2^{n-1}}{n} v_l(N)(\alpha). \end{aligned} \quad (30)$$

where s denotes the number of players in coalition S.

Combining with (28) and (30), we can obtain the following analysis formula of $x_l^*(i)(\alpha)$:

$$\begin{aligned} x_l^*(i)(\alpha) &= \frac{v_l(N)(\alpha)}{n} \\ &\quad + \frac{1}{n2^{n-2}} \left(n \sum_{S:i\in S} v_l(S)(\alpha) - \sum_{S\subseteq N, S\neq\varnothing} s v_l(S)(\alpha) \right). \end{aligned} \quad (31)$$

In the similar way, we can finally obtain the following analysis formula of $x_r^*(i)(\alpha)$:

$$\begin{aligned} x_r^*(i)(\alpha) &= \frac{v_r(N)(\alpha)}{n} \\ &\quad + \frac{1}{n2^{n-2}} \left(n \sum_{S:i\in S} v_r(S)(\alpha) - \sum_{S\subseteq N, S\neq\varnothing} s v_r(S)(\alpha) \right). \end{aligned} \quad (32)$$

Until now, we have obtained the optimal solution of the quadratic programming model (17) (i.e., (20)). According to the representation theorem for the fuzzy set, the TFN-typed imputation of the player i ($i \in N$) can be expressed as

$$\begin{aligned} \overline{x}^*(i) &= \left(x_l^*(i), x_m^*(i), x_r^*(i)\right) \\ &= \left(x_l^*(i)(0), x_l^*(i)(1), x_r^*(i)(0)\right) \\ &= \left(x_l^*(i)(0), x_r^*(i)(1), x_r^*(i)(0)\right). \end{aligned} \quad (33)$$

4. A Numerical Example and Computational Result Analysis

In Section 3, we construct a quadratic programming model for solving the least square distance solution of the TFN-typed cooperative games. The method proposed in this paper can be applied to many fields, which may relate to cooperation and the distribution of profits such as supply chain management, logistics coalition, environmental collaboration, and strategic cooperation. We have elaborated with detail the process of the least square distance solution of the TFN-typed cooperative games and what is following is a calculating example about the profit distribution of logistics coalition to examine its practicability, rationality, and superiority.

Example 1. Considering a logistics coalition composed of three logistics enterprises, which are called player 1, player 2, and player 3, respectively, each logistics enterprise can operate alone with small profit. However, once all of them form a coalition and work together, the operational hazard will reduce, the market share will increase, and the divisible profit will rise. Owing to the fuzzy uncertainty in the freight transport market, the cooperative profits cannot be forecast accurately and just the value ranges, the profit, and the corresponding membership degrees can be estimated. As a result, we use the TFN $\overline{v}(S) = (v_l(S), v_m(S), v_r(S))$ to denote the characteristic function (i.e., profit) of the coalition S ($S \subseteq N$). Through the elaborated market research, we can know the coalition profits in different situations as follows: $\overline{v}(1) = (10, 15, 18)$, $\overline{v}(2) = (20, 29, 36)$, $\overline{v}(3) = (40, 60, 74)$, $\overline{v}(12) = (50, 78, 97)$, $\overline{v}(13) = (60, 90, 105)$, $\overline{v}(23) = (75, 100, 118)$, and $\overline{v}(123) = (300, 450, 560)$.

4.1. Computational Results Obtained by the Proposed Method. According to (31), for logistics enterprise (i.e., player) 1, we have

$$\begin{aligned} x_l^*(1)(\alpha) &= \frac{v_l(N)(\alpha)}{n} + \frac{1}{n2^{n-2}} \left(n \sum_{S:1\in S} v_l(S)(\alpha) \right. \\ &\quad \left. - \sum_{S\subseteq N, S\neq\varnothing} s v_l(S)(\alpha) \right) = \frac{\alpha v_m(N) + (1-\alpha) v_l(N)}{3} \end{aligned}$$

$$+\frac{1}{3\times 2}\times 3\left[\left(\alpha v_m(1)+(1-\alpha)v_l(1)\right)+\alpha v_m(12)+(1-\alpha)v_l(12))+\alpha v_m(13)+(1-\alpha)v_l(13))+\alpha v_m(123)+(1-\alpha)v_l(123))\right]-\frac{1}{3\times 2}\left[\left(\alpha v_m(1)+(1-\alpha)v_l(1)\right)+\left(\alpha v_m(2)+(1-\alpha)v_l(2)\right)+\left(\alpha v_m(3)+(1-\alpha)v_l(3)\right)+2\left(\alpha v_m(12)+(1-\alpha)v_l(12)\right)+2\left(\alpha v_m(13)+(1-\alpha)v_l(13)\right)+2\left(\alpha v_m(23)+(1-\alpha)v_l(23)\right)+3\left(\alpha v_m(123)+(1-\alpha)v_l(123)\right)\right]=\frac{450\alpha+300(1-\alpha)}{3}+\frac{1}{3\times 2}\times 3[15\alpha+10(1-\alpha)+78\alpha+50(1-\alpha)+90\alpha+60(1-\alpha)+450\alpha+300(1-\alpha)]-\frac{1}{3\times 2}[15\alpha+10(1-\alpha)+29\alpha+20(1-\alpha)+60\alpha+40(1-\alpha)+2(78\alpha+50(1-\alpha))+2(90\alpha+60(1-\alpha))+2(100\alpha+75(1-\alpha))+3(450\alpha+300(1-\alpha))]=50\alpha+100+\frac{1}{2}(213\alpha+420)-\frac{1}{6}(650\alpha+1340)=\frac{289}{6}\alpha+\frac{260}{3}. \tag{34}$$

For logistics enterprise (i.e., player) 2, we have

$$x_l^*(2)(\alpha)=\frac{v_l(N)(\alpha)}{n}+\frac{1}{n2^{n-2}}\left(n\sum_{S:2\in S}v_l(S)(\alpha)-\sum_{S\subseteq N,S\neq\varnothing}sv_l(S)(\alpha)\right)=\frac{\alpha v_m(N)+(1-\alpha)v_l(N)}{3}+\frac{1}{3\times 2}\times 3\left[\left(\alpha v_m(2)+(1-\alpha)v_l(2)\right)+\alpha v_m(12)+(1-\alpha)v_l(12))+\alpha v_m(23)+(1-\alpha)v_l(23))+\alpha v_m(123)+(1-\alpha)v_l(123))\right]-\frac{1}{3\times 2}\left[\left(\alpha v_m(1)+(1-\alpha)v_l(1)\right)+\left(\alpha v_m(2)+(1-\alpha)v_l(2)\right)+\left(\alpha v_m(3)+(1-\alpha)v_l(3)\right)+2\alpha v_m(12)+(1-\alpha)v_l(12))+2\alpha v_m(13)+(1-\alpha)v_l(13))+2\alpha v_m(23)+(1-\alpha)v_l(23))+\alpha v_m(123)+(1-\alpha)v_l(123))\right]=\frac{450\alpha+300(1-\alpha)}{3}+\frac{1}{3\times 2}\times 3[29\alpha+20(1-\alpha)+78\alpha+50(1-\alpha)+100\alpha+75(1-\alpha)+450\alpha+300(1-\alpha)]-\frac{1}{3\times 2}[15\alpha+10(1-\alpha)+29\alpha+20(1-\alpha)+60\alpha+40(1-\alpha)+2(78\alpha+50(1-\alpha))+2(90\alpha+60(1-\alpha))+2(100\alpha+75(1-\alpha))+3(450\alpha+300(1-\alpha))]=50\alpha+100+\frac{1}{2}(212\alpha+445)-\frac{1}{6}(650\alpha+1340)=\frac{143}{3}\alpha+\frac{595}{6}. \tag{35}$$

For logistics enterprise (i.e., player) 3, we have

$$x_l^*(3)(\alpha)=\frac{v_l(N)(\alpha)}{n}+\frac{1}{n2^{n-2}}\left(n\sum_{S:3\in S}v_l(S)(\alpha)-\sum_{S\subseteq N,S\neq\varnothing}sv_l(S)(\alpha)\right)=\frac{\alpha v_m(N)+(1-\alpha)v_l(N)}{3}+\frac{1}{3\times 2}\times 3\left[\left(\alpha v_m(3)+(1-\alpha)v_l(1)\right)+\alpha v_m(13)+(1-\alpha)v_l(13))+\alpha v_m(23)+(1-\alpha)v_l(23))+\alpha v_m(123)+(1-\alpha)v_l(123))\right]-\frac{1}{3\times 2}\left[\left(\alpha v_m(1)+(1-\alpha)v_l(1)\right)+\left(\alpha v_m(2)+(1-\alpha)v_l(2)\right)+\left(\alpha v_m(3)+(1-\alpha)v_l(3)\right)+2\alpha v_m(12)+(1-\alpha)v_l(12))+2\alpha v_m(13)+(1-\alpha)v_l(13))+2\alpha v_m(23)+(1-\alpha)v_l(23))+\alpha v_m(123)+(1-\alpha)v_l(123))\right]=\frac{450\alpha+300(1-\alpha)}{3}+\frac{1}{3\times 2}\times 3[60\alpha+40(1-\alpha)+90\alpha+60(1-\alpha)+100\alpha+75(1-\alpha)+450\alpha+300(1-\alpha)]-\frac{1}{3\times 2}[15\alpha+10(1-\alpha)+29\alpha+20(1-\alpha)+60\alpha+40(1-\alpha)+2(78\alpha+50(1-\alpha))+2(90\alpha+60(1-\alpha))+2(100\alpha+75(1-\alpha))+3(450\alpha+300(1-\alpha))]=50\alpha+100+\frac{1}{2}(225\alpha+475)-\frac{1}{6}(650\alpha+1340)=\frac{325}{6}\alpha+\frac{685}{6}. \tag{36}$$

In the similar way, according to (32), for logistics enterprise (i.e., player) 1, we have

$$\begin{aligned}
x_r^*(1)(\alpha) &= \frac{v_r(N)(\alpha)}{n} \\
&+ \frac{1}{n2^{n-2}}\left(n\sum_{S:1\in S} v_r(S)(\alpha) - \sum_{S\subseteq N, S\neq\varnothing} s v_r(S)(\alpha)\right) \\
&= \frac{\alpha v_m(N) + (1-\alpha) v_r(N)}{3} + \frac{1}{3\times 2} \\
&\times 3\left[\left(\alpha v_m(1) + (1-\alpha) v_r(1)\right) + \alpha v_m(12)\right. \\
&+ (1-\alpha) v_r(12)) + \alpha v_m(13) + (1-\alpha) v_r(13)) \\
&\left.+ \alpha v_m(123) + (1-\alpha) v_r(123))\right] \\
&- \frac{1}{3\times 2}\left[\left(\alpha v_m(1) + (1-\alpha) v_r(1)\right)\right. \\
&+ \left(\alpha v_m(2) + (1-\alpha) v_r(2)\right) \\
&+ \left(\alpha v_m(3) + (1-\alpha) v_r(3)\right) \\
&+ 2\left(\alpha v_m(12) + (1-\alpha) v_r(12)\right) \\
&+ 2\left(\alpha v_m(13) + (1-\alpha) v_r(13)\right) \\
&+ 2\left(\alpha v_m(23) + (1-\alpha) v_r(23)\right) \\
&\left.+ 3\left(\alpha v_m(123) + (1-\alpha) v_r(123)\right)\right] \\
&= \frac{450\alpha + 560(1-\alpha)}{3} + \frac{1}{3\times 2}\times 3\left[15\alpha\right. \\
&+ 18(1-\alpha) + 78\alpha + 97(1-\alpha) + 90\alpha \\
&\left.+ 105(1-\alpha) + 450\alpha + 560(1-\alpha)\right] - \frac{1}{3\times 2}\left[15\alpha\right. \\
&+ 18(1-\alpha) + 29\alpha + 36(1-\alpha) + 60\alpha + 74(1-\alpha) \\
&+ 2(78\alpha + 97(1-\alpha)) + 2(90\alpha + 105(1-\alpha)) \\
&\left.+ 2(100\alpha + 118(1-\alpha)) + 3(450\alpha + 560(1-\alpha))\right] \\
&= \frac{-110\alpha + 560}{3} + \frac{1}{2}(-147\alpha + 780) - \frac{1}{6}(-458\alpha \\
&+ 2448) = -\frac{203}{6}\alpha + \frac{506}{3}
\end{aligned} \tag{37}$$

For logistics enterprise (i.e., player) 2, we have

$$\begin{aligned}
x_r^*(2)(\alpha) &= \frac{v_r(N)(\alpha)}{n} \\
&+ \frac{1}{n2^{n-2}}\left(n\sum_{S:1\in S} v_r(S)(\alpha) - \sum_{S\subseteq N, S\neq\varnothing} s v_r(S)(\alpha)\right) \\
&= \frac{\alpha v_m(N) + (1-\alpha) v_r(N)}{3} + \frac{1}{3\times 2} \\
&\times 3\left[\left(\alpha v_m(2) + (1-\alpha) v_r(2)\right) + \alpha v_m(12)\right. \\
&+ (1-\alpha) v_r(12)) + \alpha v_m(23) + (1-\alpha) v_r(23)) \\
&\left.+ \alpha v_m(123) + (1-\alpha) v_r(123))\right] \\
&- \frac{1}{3\times 2}\left[\left(\alpha v_m(1) + (1-\alpha) v_r(1)\right)\right. \\
&+ \left(\alpha v_m(2) + (1-\alpha) v_r(2)\right) \\
&+ \left(\alpha v_m(3) + (1-\alpha) v_r(3)\right) \\
&+ 2\left(\alpha v_m(12) + (1-\alpha) v_r(12)\right) \\
&+ 2\left(\alpha v_m(13) + (1-\alpha) v_r(13)\right) \\
&+ 2\left(\alpha v_m(23) + (1-\alpha) v_r(23)\right) \\
&\left.+ 3\left(\alpha v_m(123) + (1-\alpha) v_r(123)\right)\right] \\
&= \frac{450\alpha + 560(1-\alpha)}{3} + \frac{1}{3\times 2}\times 3\left[29\alpha\right. \\
&+ 36(1-\alpha) + 78\alpha + 97(1-\alpha) + 100\alpha \\
&\left.+ 118(1-\alpha) + 450\alpha + 560(1-\alpha)\right] - \frac{1}{3\times 2}\left[15\alpha\right. \\
&+ 18(1-\alpha) + 29\alpha + 36(1-\alpha) + 60\alpha + 74(1-\alpha) \\
&+ 2(78\alpha + 97(1-\alpha)) + 2(90\alpha + 105(1-\alpha)) \\
&\left.+ 2(100\alpha + 118(1-\alpha)) + 3(450\alpha + 560(1-\alpha))\right] \\
&= \frac{-110\alpha + 560}{3} + \frac{1}{2}(-154\alpha + 811) - \frac{1}{6}(-458\alpha \\
&+ 2448) = -\frac{112}{3}\alpha + \frac{1105}{6}
\end{aligned} \tag{38}$$

For logistics enterprise (i.e., player) 3, we have

$$\begin{aligned}
x_r^*(3)(\alpha) &= \frac{v_r(N)(\alpha)}{n} + \frac{1}{n2^{n-2}}\left(n\sum_{S:1\in S} v_r(S)(\alpha)\right. \\
&\left.- \sum_{S\subseteq N, S\neq\varnothing} s v_r(S)(\alpha)\right) = \frac{\alpha v_m(N) + (1-\alpha) v_r(N)}{3} \\
&+ \frac{1}{3\times 2}\times 3\left[\left(\alpha v_m(3) + (1-\alpha) v_r(3)\right) + \alpha v_m(13)\right. \\
&+ (1-\alpha) v_r(13)) + \alpha v_m(23) + (1-\alpha) v_r(23)) \\
&\left.+ \alpha v_m(123) + (1-\alpha) v_r(123))\right] \\
&- \frac{1}{3\times 2}\left[\left(\alpha v_m(1) + (1-\alpha) v_r(1)\right)\right. \\
&+ \left(\alpha v_m(2) + (1-\alpha) v_r(2)\right) \\
&+ \left(\alpha v_m(3) + (1-\alpha) v_r(3)\right)
\end{aligned}$$

$$+ 2\left(\alpha v_m(12) + (1-\alpha) v_r(12)\right)$$
$$+ 2\left(\alpha v_m(13) + (1-\alpha) v_r(13)\right)$$
$$+ 2\left(\alpha v_m(23) + (1-\alpha) v_r(23)\right)$$
$$+ 3\left(\alpha v_m(123) + (1-\alpha) v_r(123)\right)\Big]$$
$$= \frac{450\alpha + 560(1-\alpha)}{3} + \frac{1}{3\times 2} \times 3\,[60\alpha$$
$$+ 74(1-\alpha) + 90\alpha + 105(1-\alpha) + 100\alpha$$
$$+ 118(1-\alpha) + 450\alpha + 560(1-\alpha)] - \frac{1}{3\times 2}\,[15\alpha$$
$$+ 18(1-\alpha) + 29\alpha + 36(1-\alpha) + 60\alpha + 74(1-\alpha)$$
$$+ 2(78\alpha + 97(1-\alpha)) + 2(90\alpha + 105(1-\alpha))$$
$$+ 2(100\alpha + 118(1-\alpha)) + 3(450\alpha + 560(1-\alpha))]$$
$$= \frac{-110\alpha + 560}{3} + \frac{1}{2}(-157\alpha + 857) - \frac{1}{6}(-458\alpha$$
$$+ 2448) = -\frac{233}{6}\alpha + \frac{1243}{6} \tag{39}$$

According to (33), we can calculate the optimal distribution strategy of the three logistics enterprises when they form a cooperative coalition and work together, which are shown as follows:

$$\begin{aligned}\overline{x}^*(1) &= \left(x_l^*(1)(0), x_l^*(1)(1), x_r^*(1)(0)\right)\\ &= \left(x_l^*(1)(0), x_r^*(1)(1), x_r^*(1)(0)\right)\\ &= (86.7, 134.8, 168.7),\\ \overline{x}^*(2) &= \left(x_l^*(2)(0), x_l^*(2)(1), x_r^*(2)(0)\right)\\ &= \left(x_l^*(2)(0), x_r^*(2)(1), x_r^*(2)(0)\right)\\ &= (99.2, 146.8, 184.2)\end{aligned} \tag{40}$$

and

$$\begin{aligned}\overline{x}^*(3) &= \left(x_l^*(3)(0), x_l^*(3)(1), x_r^*(3)(0)\right)\\ &= \left(x_l^*(3)(0), x_r^*(3)(1), x_r^*(3)(0)\right)\\ &= (114.2, 168.3, 207.2),\end{aligned} \tag{41}$$

respectively.

4.2. Results Analysis and Comparison. By watching the distribution results carefully, we see that the sum of the lower bounds of the three logistics enterprises' distribution can be shown as $86.7 + 99.2 + 114.2 = 300$, which is equal to the lower bound of the characteristic function of the grand coalition N. The similar conclusions apply to logistics enterprise 2 and logistics enterprise 3. All of the profits created by the cooperative coalition N have been absolutely distributed to the three logistics enterprises. In other words, the optimal distribution strategy based on the least square distance solution satisfies the property of efficiency. What is more, each logistics enterprise obtains more profit than they operate business alone. Therefore, all of the three logistics enterprises are willing to work together to make more profits.

In order to show more intuitively the superiority of the least square distance solution proposed in this paper, we redetermine the allocation strategy according to the interval Shapley-like value $\overline{\phi}^*(v)$ [9].

Based on (9), the α-cut sets of the TFN-typed profits of coalition S ($S \subseteq N$) (i.e., $\overline{v}(S)$) in Example 1 can be written as the following intervals:

$$\begin{aligned}\overline{v}(1)(\alpha) &= [15\alpha + 10(1-\alpha), 15\alpha + 18(1-\alpha)]\\ &= [10 + 5\alpha, 18 - 3\alpha],\\ \overline{v}(2)(\alpha) &= [29\alpha + 20(1-\alpha), 29\alpha + 36(1-\alpha)]\\ &= [20 + 9\alpha, 36 - 7\alpha],\\ \overline{v}(3)(\alpha) &= [60\alpha + 40(1-\alpha), 60\alpha + 74(1-\alpha)]\\ &= [40 + 20\alpha, 74 - 14\alpha],\\ \overline{v}(12)(\alpha) &= [78\alpha + 50(1-\alpha), 78\alpha + 97(1-\alpha)]\\ &= [50 + 28\alpha, 97 - 19\alpha],\\ \overline{v}(13)(\alpha) &= [90\alpha + 60(1-\alpha), 90\alpha + 105(1-\alpha)]\\ &= [60 + 30\alpha, 105 - 15\alpha],\\ \overline{v}(23)(\alpha) &= [100\alpha + 75(1-\alpha), 100\alpha + 118(1-\alpha)]\\ &= [75 + 25\alpha, 118 - 18\alpha]\end{aligned} \tag{42}$$

and

$$\begin{aligned}&\overline{v}(123)(\alpha)\\ &\quad = [450\alpha + 300(1-\alpha), 450\alpha + 560(1-\alpha)]\\ &\quad = [300 + 150\alpha, 560 - 110\alpha]\end{aligned} \tag{43}$$

Therefore, the α-cut sets of the TFN-typed Shapley-like values $\overline{\phi}_i^*(v)$ ($i \in N$) can be shown as follows:

$$\begin{aligned}\overline{\phi}_1^*(v)(\alpha) &= \left[\phi_{1l}^*(v)(\alpha), \phi_{1r}^*(v)(\alpha)\right]\\ &= \sum_{S\subseteq N\setminus 1} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!}\\ &\quad\cdot \left(v(S\cup\{1\})(\alpha) - v(S)(\alpha)\right)\\ &= [64 + 70.8\alpha, 191.3 - 56.5\alpha],\\ \overline{\phi}_2^*(v)(\alpha) &= \left[\phi_{2l}^*(v)(\alpha), \phi_{2r}^*(v)(\alpha)\right]\\ &= \sum_{S\subseteq N\setminus 2} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!}\\ &\quad\cdot \left(v(S\cup\{2\})(\alpha) - v(S)(\alpha)\right)\\ &= [77.2 + 69.7\alpha, 206.2 - 59.3\alpha]\end{aligned} \tag{44}$$

and

$$\overline{\phi}_3^*(v)(\alpha) = \left[\phi_{3l}^*(v)(\alpha), \phi_{3r}^*(v)(\alpha)\right] = \sum_{S \subseteq N \backslash 3} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} \cdot (v(S \cup \{3\})(\alpha) - v(S)(\alpha)) = [94.5 + 73.8\alpha, 226.8 - 58.5\alpha] \quad (45)$$

According to the representation theorem for the fuzzy set, we can obtain the following results:

$$\begin{aligned} \overline{\phi}_1^*(v) &= \left(\phi_{1l}^*(v)(0), \phi_{1l}^*(v)(1), \phi_{1r}^*(v)(0)\right) \\ &= \left(\phi_{1l}^*(v)(0), \phi_{1r}^*(v)(1), \phi_{1r}^*(v)(0)\right) \\ &= (64, 134.8, 191.3), \\ \overline{\phi}_2^*(v) &= \left(\phi_{2l}^*(v)(0), \phi_{2l}^*(v)(1), \phi_{2r}^*(v)(0)\right) \\ &= \left(\phi_{2l}^*(v)(0), \phi_{2r}^*(v)(1), \phi_{2r}^*(v)(0)\right) \\ &= (77.2, 146.9, 206.2) \end{aligned} \quad (46)$$

and

$$\begin{aligned} \overline{\phi}_3^*(v) &= \left(\phi_{3l}^*(v)(0), \phi_{3l}^*(v)(1), \phi_{3r}^*(v)(0)\right) \\ &= \left(\phi_{3l}^*(v)(0), \phi_{3r}^*(v)(1), \phi_{3r}^*(v)(0)\right) \\ &= (94.5, 168.3, 226.8) \end{aligned} \quad (47)$$

Obviously, the sum of the means of the three logistics enterprises' distribution (i.e., $134.8 + 146.9 + 168.3$) is equal to the mean of the characteristic function of the grand coalition N. However, the sum of the lower bounds of the three logistics enterprises' distribution is $64 + 77.2 + 94.5 = 235.7$ and the sum of the upper bounds of them is $191.3 + 206.2 + 226.8 = 624.3$. Therefore, the former one is less than 300 and the latter one is more than 560, respectively. That is to say, the optimal allocation strategy according to the interval Shapley-like value does not satisfy the efficiency.

5. Conclusions

Based on the comparative analysis of the calculated results, conclusions can be drawn as follows:

(1) Through observing carefully the derivation process of the least square distance solution proposed in this paper, we can see that the least square distance solution proposed in this paper satisfies many important properties of cooperative games, such as uniqueness, additivity, symmetry, and uniqueness.

(2) The least square distance solution proposed in this paper successfully avoids the subtraction operation of TFNs, which may inevitably lead to the amplification of uncertainty and the distortion of decision information. However, many existing methods are proposed based on the subtraction operation of fuzzy numbers. The interval Shapley-like value [9] is a case in point which used the subtraction operation of intervals.

(3) We use the concept of α-cut sets to transform the operation between TFNs to intervals, which can effectively reduce the dimension of fuzzy number and improve the reliability of decision information.

Due to the vagueness of things themselves, the lack of expertise, and the imperfection of technical means, the problems of fuzzy cooperative games could be seen everywhere in our daily lives. There are many types of fuzzy data, such as intervals, triangular/trapezoid fuzzy number, and triangular/trapezoid intuitionistic fuzzy number. In the near future, we will try to extend the least square distance solution proposed in this paper to other types of fuzzy operative games and elaborate their good properties.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by the Special Foundation Program for Science and Technology Innovation of Fujian Agriculture and Forestry University of China (No. CXZX2018030), the Social Science Planning Program of Fujian Province of China (No. FJ2018B014) and the National Natural Science Foundation of China (No. 71572040).

References

[1] S. Borkotokey and R. Neog, "Dynamic resource allocation in fuzzy coalitions: a game theoretic model," *Fuzzy Optimization and Decision Making. A Journal of Modeling and Computation Under Uncertainty*, vol. 13, no. 2, pp. 211–230, 2014.

[2] Y. Azrieli and E. Lehrer, "On some families of cooperative fuzzy games," *International Journal of Game Theory*, vol. 36, no. 1, pp. 1–15, 2007.

[3] S. Li and Q. Zhang, "The measure of interaction among players in games with fuzzy coalitions," *Fuzzy Sets and Systems*, vol. 159, no. 2, pp. 119–137, 2008.

[4] L. Mallozzi, V. Scalzo, and S. Tijs, "Fuzzy interval cooperative games," *Fuzzy Sets and Systems*, vol. 165, pp. 98–105, 2011.

[5] X. Yu and Q. Zhang, "The fuzzy core in games with fuzzy coalitions," *Journal of Computational and Applied Mathematics*, vol. 230, no. 1, pp. 173–186, 2009.

[6] D.-F. Li and J.-C. Liu, "Quadratic programming models and method for interval-valued cooperative games with fuzzy coalitions," in *Game theory and applications*, vol. 758 of *Commun. Comput. Inf. Sci.*, pp. 318–336, Springer, Singapore, 2017.

[7] A. Abed-Elmdoust and R. Kerachian, "Water resources allocation using a cooperative game with fuzzy payoffs and fuzzy coalitions," *Water Resources Management*, vol. 26, no. 13, pp. 3961–3976, 2012.

[8] S. Borkotokey, "Cooperative games with fuzzy coalitions and fuzzy characteristic functions," *Fuzzy Sets and Systems*, vol. 159, no. 2, pp. 138–151, 2008.

[9] W. Han, H. Sun, and G. Xu, "A new approach of cooperative interval games: the interval core and Shapley value revisited," *Operations Research Letters*, vol. 40, no. 6, pp. 462–468, 2012.

[10] S. Borkotokey and R. Mesiar, "The Shapley value of cooperative games under fuzzy settings: a survey," *International Journal of General Systems*, vol. 43, no. 1, pp. 75–95, 2014.

[11] D. Butnariu, "Stability and Shapley value for an n-persons fuzzy game," *Fuzzy Sets and Systems*, vol. 4, no. 1, pp. 63–72, 1980.

[12] L. Monroy, M. A. Hinojosa, A. M. Marmol, and F. R. Fernandez, "Set-valued cooperative games with fuzzy payoffs. The fuzzy assignment game," *European Journal of Operational Research*, vol. 225, no. 1, pp. 85–90, 2013.

[13] J. Lin and Q. Zhang, "The least square B-nucleolus for fuzzy cooperative games," *Journal of Intelligent & Fuzzy Systems*, vol. 30, no. 1, pp. 279–289, 2015.

[14] J. Liu and W. Zhao, "Cost-sharing of ecological construction based on trapezoidal intuitionistic fuzzy cooperative games," *International Journal of Environmental Research and Public Health*, vol. 13, no. 11, article no. 1102, 2016.

[15] W. Zhao and J. Liu, "Interval-valued fuzzy cooperative games based on the least square excess and its application to the profit allocation of the road freight coalition," *Symmetry*, vol. 10, no. 12, p. 709, 2018.

[16] L. Liu and X. Liu, "Fuzzy extensions of bargaining sets and their existence in cooperative fuzzy games," *Fuzzy Sets and Systems*, vol. 188, pp. 88–101, 2012.

[17] E. Roszkowska and T. R. Burns, "Fuzzy bargaining games: Conditions of agreement, satisfaction, and equilibrium," *Group Decision and Negotiation*, vol. 19, no. 5, pp. 421–440, 2010.

[18] E. Molina and J. Tejada, "The equalizer and the lexicographical solutions for cooperative fuzzy games: characterization and properties," *Fuzzy Sets and Systems*, vol. 125, no. 3, pp. 369–387, 2002.

[19] D.-F. Li, "A fast approach to compute fuzzy values of matrix games with payoffs of triangular fuzzy numbers," *European Journal of Operational Research*, vol. 223, no. 2, pp. 421–429, 2012.

[20] R. E. Moore, *Methods and Applications of Interval Analysis*, SIAM, Philadelphia, Pa, USA, 1979.

[21] L. A. Zadeh, "Fuzzy sets," *Information Control*, vol. 8, pp. 338–356, 1965.