Research Article

Analysis of the Profits of Banks and Supply Chain Enterprises under Noncollaborative and Collaborative Financing

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In order to solve the problem of how to choose a financing strategy for supply chain enterprises with financial constraints, including the manufacturer and the retailer, this paper puts forward two financing strategies in the bilateral supply chain with uncertain output and certain demand. The two financing strategies are the noncollaborative financing (finance from a bank separately (FBS)) and the collaborative financing (finance from a bank uniformly (FBU)). It derives the production order formula of the supply chain enterprises under financial constraints. Under the complete information, according to this formula, it analyzes how the bank prices the loan interest rates and finds the optimal decisions under the two financing strategies. The following results are found: (1) The manufacturer’s planned output is negatively correlated with the bank’s loan interest rate. The increased interest rates do not necessarily lead to the increased bank’s loan profits. (2) The bank’s loan profit is higher, when the supply chain enterprises choose the FBS strategy. (3) The FBU strategy does not necessarily make the profits of the manufacturer and the retailer better. It is affirmative only if some parameters in the supply chain meet certain conditions. The above-mentioned conclusions supply a policy guiding the supply chain enterprises with financial constraints to make a choice of the financing strategy.

1. Introduction

In recent years, the research on supply chain operation and coordination in an uncertain environment has attracted more and more attention from academic and business circles. There is more concern about supply chain operation under the condition of uncertain demand [1–3], but in a customer-centric market economy, there are still many enterprises that take order-based production [4]. In the order-based production, the enterprises arrange production in accordance with the customer’s demand. Since the production is affected by various factors such as raw materials, equipment, processes, and employees, the number of qualified products is uncertain, but the demand is determined [5, 6]. For example, in the contract farming, there is a definite demand related to the order quantity in the downstream enterprises of the supply chain. But the farmers in production have uncertain output because of the limitation of natural conditions and other factors of the farmers.

For another example, in the semiconductor and electronics industries, where the production process and product quality requirements are more stringent, the actual output of manufacturers is often very different from the expected output. After an order is placed by a downstream enterprise, the manufacturer cannot guarantee that the output will match the order volume. Therefore, in reality, the supply chain with output uncertainty and demand determination is still widespread, and it often faces the problem of capital constraints. When a member of the supply chain has funds constraints, it is often unable to implement the optimal decision-making, thus “pulling the whole body” [7]. Zhang [8] first proposed that the operation of the supply chain and financing issues are very important to supply chain enterprises. In view of the fact that most scholars study the uncertainty of the output in the supply
chain, it is based on the bilateral supply chain [9], so this paper still analyzes the supply chain financing under the uncertainty of the output and the determination of demand on this basis.

Generally speaking, financing channels for supply chain enterprises are divided into internal financing and external financing. Internal financing refers to the sources of funds within the supply chain, mainly including the internal retained earnings, the trade credits of supply chain members, and the supply chain members’ funds. External financing refers to the sources of funds outside the supply chain, mainly including the stocks, the bonds, and the bank loans issued by the supply chain enterprises. By the end of 2018, 540,000 SMEs in China had obtained the bank loans. Bank loans, as the main external financing channel for the supply chain financing enterprises, played an important role in solving the funding problems of the supply chain enterprises. Therefore, this paper mainly studies the bank loan problem of external financing in the supply chain.

Supply chain financing enterprises have different options for bank loans. The traditional programs include mortgages and pledge financing. With the development of supply chain financing, the new programs rely more on the credit level of the core enterprises in the supply chain to provide loan services to supply chain SMEs. The typical model is the credit guarantee financing for the core enterprises in the supply chain. Whether it is traditional mortgage, pledge financing, or supply chain financing, for financing enterprises in the supply chain, there are two main ways to connect with the banks. First, the financing enterprise and the bank are directly connected to each other. For example, the enterprise obtains the bank loans through their own asset mortgage, pledge, and other ways. Because of different credit rating and the enterprise scale of different supply chain financing enterprises, different enterprises obtain different loan pricing from the banks. Second, the financing enterprise does not directly connect with the bank but obtains bank loans relying on the supply chain core enterprise or the third-party logistics enterprise to connect with the bank indirectly. The bank conducts loan pricing according to the credit level of the core enterprise or the third-party logistics enterprise, and the financing enterprises obtain the loan interest rate consistent with the pricing.

Therefore, according to different connection ways between the financing supply chain enterprises and the banks, the bank loan strategies of the bilateral supply chain are divided into noncollaborative financing (finance from a bank separately (FBS)) and collaborative financing (finance from a bank uniformly (FBU)) in this paper. Under the FBS strategy, all the financing enterprises in the bilateral supply chain are directly connected to the bank. The typical FBS strategy includes the traditional mortgage, the pledge financing model, the warehouse receipt pledge financing model, and the order financing model in supply chain financing. Under the FBU strategy, all financing enterprises in the bilateral supply chain rely on the overall strength and credit level of the core enterprises or third-party logistics enterprises in the supply chain to obtain bank loans. The typical FBU strategy includes the dealer, the supplier network financing model (using the credit introduction of core enterprises, a kind of financial service provided by banks to dealers and suppliers related to core enterprises), and the bank and logistics cooperative financing model (a financial service in which a bank cooperates with a third-party logistics company to provide customers with the credit through the logistics supervision or the credit guarantee). A typical case of the FBU strategy is as follows.

As a professional sports brand manufacturer, Li Ning Company has become a core enterprise in the supply chain with its own advantages. In the supply chain, a pattern has emerged in which the upstream and downstream enterprises jointly supply service for Li Ning. Obviously, the survival status of upstream and downstream enterprises in the supply chain has a great impact on the development of Li Ning. These upstream and downstream enterprises often find it difficult to directly obtain bank loans because of the scale of enterprises and credit standing level. The problem of financial constraints will not only affect the technological innovation and the product upgrading of the upstream and downstream enterprises but also affect the development of Li Ning indirectly. In order to achieve the stable development of Li Ning Company, Li Ning Company becomes aware of the importance of providing financial support for its upstream and downstream enterprises. Therefore, Li Ning Company and Standard Chartered Bank jointly, relying on Li Ning’s overall strength, the credit level, and the business transactions between Li Ning’s business and the upstream and downstream companies, provide financing services for upstream and downstream enterprises. Standard Chartered Bank only cares about Li Ning’s credit standing, strictly controls the coproduction process in the supply chain, and prices the loan interest rates for the chain financing enterprises according to Li Ning’s credit standing. Obviously, upstream and downstream financing enterprises in the supply chain have obtained the bank loans through the FBU strategy. However, in a bilateral supply chain with the uncertain output and the certain demand, the financing enterprises could choose either the FBS strategy or the FBU strategy to get the bank loans under the financial constraints. Which financing strategy is better for supply chain enterprises? How do the banks make loan decisions with the two choices of the financing strategies? How do the two financing strategies affect bank profit?

In order to solve the above problems, this paper studies the bilateral supply chain with the uncertain output and certain demand. This supply chain consists of a manufacturer and a retailer. In an order, both of them are under the financing constraints, and both of them can choose the FBS or the FBU strategy for the bank loans. This paper analyzes the interdependency between the bank’s loan interest rate and the supply chain enterprise operation decision and compares the profit of the bank and the supply chain enterprises in the two strategies. The main results of this paper are as follows.

First, under the condition of both the manufacturer and the retailer facing the financial constraints, the manufacturer’s optimal production formula is proved, which indicates that the manufacturer’s planned output and its own
loan interest rate are inversely correlated. It is proved that the optimal order volume of the retailer is the demand of the market. Simply put, under the certain market demand, although the manufacturer’s output is uncertain, the order quantity of the retailer is still the demand of the market.

Second, this paper puts forward the formula of the loan interest rate under different financing strategies in the supply chain and points out that the maximization of the bank’s loan interest rate does not necessarily make the maximum profit. On the contrary, under certain conditions, the bank’s loan profit is negatively correlated with the loan interest rate, which is contrary to our usual belief that the bank’s maximizing loan interest rate will boost the return on loans.

Third, through comparing the profit of the bank under the two financing strategies and the profit of the supply chain enterprises, this study finds that the bank has higher profit under the FBS strategy in the supply chain, and the FBU strategy is not necessarily more beneficial to supply chain enterprises. In the case that the bank’s optimal loan interest rate to the manufacturer is at its maximum, and some parameters in the supply chain meet a certain relationship, the FBU strategy is more beneficial to the supply chain. This is contrary to our usual intuition that the FBU strategy is more beneficial to the supply chain enterprises.

2. Literature Review

To solve the above-mentioned problems, this study examines two types of literature: (1) the interface between operations and finance and (2) the comparison of supply chain financing strategies.

2.1. Integration of Operations and Finance. Some early papers studied the relationship between the financing and the operational decisions by putting the condition of capital constraints into traditional newsvendor models. For example, Xu and Birge [10] explained the impact of corporate capital constraints and capital structure on inventory decisions. Zhang [8] analyzed the financing decisions of the banks and retailers at different capital levels and found that the asset-based financing can increase retailers’ profits. Kouvelis and Zhao [11] studied the impact of bankruptcy costs on the retailers’ order quantities. Later, some studies further analyzed the impact of other factors (agent costs and asymmetric information) on operations and finances. For example, Boyabath and Toktay [12] studied the impact of imperfect capital markets on technology choices for the budget-constrained companies. Alan and Gaur [13] showed that the borrowers’ investment decisions may be affected by bankruptcy costs and asymmetric demand information. Recently, some studies found that SMEs’ stronger partners could help SMEs improve their financing capabilities. For example, Sun et al. [14] analyzed the interdependence between business decisions and financial decisions under a credit guarantee contract. Bi et al. [15] pointed out that the pledge of the manufacturer’s products would increase the order volume of retailers and enhance the level of cooperation in the supply chain. Chen and Zhou [16] pointed out that repurchase guarantee financing can bring more benefits to the supply chain. The above study focuses on supply chain financing and operation issues for a single capital-constrained company. In contrast, our focus is on both the manufacturer and the retailer who are financially constrained. We pay attention to the impact of bank financial behavior on supply chain business operation decisions, that is, the dependence of bank financial behavior and supply chain operation decisions.

2.2. Comparison of Different Financing Options. Recently, scholars have gradually begun to pay attention to the choice of different financing methods in the supply chain [7, 17–23]. For example, Kouvelis and Zhao [7] emphasized that retailers will always prefer bank financing if the best structured solution is provided. Jing et al. [20] pointed out that trade credit financing is more beneficial to enterprises at lower production costs when both banks and trade credit are viable. Yang and Birge [21] pointed out that capital-constrained retailers can choose a combination of bank financing and trade credit financing to ease the financial pressures. Jin et al. [24] proposed that a collaborative financing strategy is beneficial to suppliers and the entire supply chain, while for retailers, noncooperative financing strategies are preferable. Our research is related to the studies by Kouvelis et al. [7], Jing et al. [20], and Jin et al. [24], but there are significant differences. First, Kouvelis and Zhao [7] and Jing et al. [20] considered the advantages and disadvantages of bank financing and trade credit financing under different conditions. The two financing models belong to the internal financing and external financing of the supply chain. We consider the advantages and disadvantages of the FBS strategy and FBU strategy from the perspective of the loan interest rates. The FBS strategy is a bank-led financing, and the FBU strategy is a combination of bank financing and trade credit financing. In fact, the two financing strategies we compare are the bank loans. Second, Jin et al. [24] pointed out that financing strategies are beneficial to suppliers and the entire supply chain, but not to retailers. We compare the difference between the FBS strategy and the FBU strategy of the supply chain, but the result is different from that of Jin et al. [24], because we consider the differences in the profit of the supply chain from the perspective of the financing interest rate. At the same time, drawing on the research of Grosfeld-Nir and Ronen [5] and Yano and Lee [6], we assume that the output is uncertain and the external market demand is determined.

3. Problem Description and Assumptions

3.1. Operating Environment and Product Market. In a supply chain consisting of a manufacturer and a retailer, the retailer places orders to the manufacturer based on external market demand, and the manufacturer arranges production plans based on the retailer’s needs and their own production capacity. The external market demand is determined. The manufacturer’s production volume is uncertain, that is,
there is a certain deviation between the manufacturer's planned output and the actual production. Because of the uncertainty of production, there is a compensation factor in the supply chain. When the actual production of the manufacturer is higher than the order quantity of the retailer, the retailer gives the manufacturer some compensation. Otherwise, the manufacturer gives the retailer some compensation. The parameter settings in this study are shown in Table 1.

The assumptions for the supply chain operating environment and product market are as follows: (1) The actual production of the manufacturer is subject to a uniform distribution with the planned output as expected. (2) Supply chain enterprises are risk-neutral. (3) Information is symmetrical, regardless of the cost of obtaining information from banks and supply chain enterprises. (4) The JIT method is adopted for the delivery, and the manufacturer's delivery quantity is not greater than the order quantity.

The manufacturer's planned output $q_0$ is deterministic, but because of the uncertainty in actual production, the actual output of the manufacturer is uncertain. The random factor $x$ is used to represent the uncertainty of the manufacturer’s actual output [25], where $x \sim U[1 - n, 1 + n]$ and $0 < n < 1$. Thus, the actual output of the manufacturer is $q_0x$ According to assumption (1), the actual output of the manufacturer is subject to a uniform distribution with the planned output as expected, so $E(q_0x) = q_0$. From assumption (2), we have $w < l_1 < p_1$, that is, the unsalable loss of the manufacturer’s unit product is between the compensation and the product price. The unsalable loss of the retailer’s unit product is between the purchase price and the selling price, that is, $p_1 < l_2 < p_2$.

From assumption (4), we know that the manufacturer delivers $\min\{q_0x, q_1\}$ to the retailer, and the retailer’s actual sales volume is $\min\{\min\{q_0x, q_1\}, D\}$. The supply chain operation structure under the uncertainty of production is shown in Figure 1.

### 3.2. Capital Structure and Financial Market

The manufacturer and retailer have full financial constraints in one order and need to lend to banks. The retailer is required to pay an advance payment when placing an order to the manufacturer. The prepayment amount is proportional to the order quantity, which is $kp_1q_1$. The amount that the manufacturer needs to make a loan to the bank is the difference between its production cost and the receipt of the advance payment $q_0c_0 - kp_1q_1$.

The manufacturer and retailer can choose to make loans to the bank in two strategies. One is the FBS strategy, in which the manufacturer and retailer make separate contacts with the bank, and the bank gives the manufacturer and retailer a loan interest rate that does not exceed the maximum acceptable loan rate for each enterprise ($r_1 \leq r_{10}$ and $r_2 \leq r_{20}$). The other is the FBU strategy, where the manufacturer or retailer interfaces with the bank as the core enterprise of the supply chain. The enterprise will give the loan to another enterprise at the same interest rate. The bank’s loan interest rate to the manufacturer and retailer does not exceed the minimum of the two enterprises’ acceptable loan interest rate ($r_3 \leq \min\{r_{10}, r_{20}\}$).

We do not consider the risk of loans because the bankruptcy of the supply chain enterprises is mainly due to commercial credit loans, while in mortgages and pledge loans, the collateral or pledge is often sufficient to repay their liabilities. Sufi [26] pointed out that, under the credit loan, the bank will determine the credit line of the financing enterprise. Therefore, the probability of default of the financing enterprise’s loan within the credit line is acceptable by the bank. On the contrary, if the financing company needs more than the credit line, it is obvious that the actual output of the supply chain cannot meet the demand $D$. At this time, the retailer and the manufacturer will moderately reduce the order quantity and output, which can be regarded as the external market demand $D’$. Therefore, without losing the generality, it is assumed that the loan amount of the financing enterprise is within the credit line. At this time, the default risk of the financing enterprise is within the controllable range of the bank. That is, under both financing strategies, the funds required by the manufacturer and retailer can be met by banks, and the manufacturer and retailer can repay in full and on time. At the same time, we assume that the bank’s capital cost rate remains unchanged. Two financing strategies for capital-constrained supply chains are shown in Figure 2.

<table>
<thead>
<tr>
<th>Indices</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$ ($i=1,2$)</td>
<td>Index for the manufacturer and retailer</td>
</tr>
<tr>
<td>$j$ ($j=S,U$)</td>
<td>Index for the optimal value of FBS and FBU strategies, respectively</td>
</tr>
<tr>
<td>Decision variables</td>
<td></td>
</tr>
<tr>
<td>$q_0$</td>
<td>Manufacturer’s planned output</td>
</tr>
<tr>
<td>$q_1$</td>
<td>Retailer’s order quantity</td>
</tr>
<tr>
<td>$r_1$</td>
<td>Bank’s loan interest rate to the enterprise $i$ under the FBS strategy</td>
</tr>
<tr>
<td>$r_{10}$</td>
<td>Bank’s interest rate to the core enterprise under the FBU strategy</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Bank’s loan interest rate to the enterprise $i$ under the FBU strategy</td>
</tr>
<tr>
<td>$r_{20}$</td>
<td></td>
</tr>
</tbody>
</table>

| Model parameters | |
| $c_0$ | Unit production cost of the manufacturer |
| $l_0$ | Loss of unit unsalable product of the enterprise $i$ |
| $p_i$ | Price of unit product of the enterprise $i$ |
| $w$ | Coefficient of compensation price |
| $D$ | Market demand for products |
| $r_0$ | Bank’s capital cost rate |
| $r_{i0}$ | The maximum loan interest rate acceptable by the enterprise $i$ |
| $k$ | The prepayment amount of the retailer’s unit product |
| $L_i$ | Loan amount of the enterprise $i$ |
| $\pi_i$ | Profit of the enterprise $i$ |
| $\pi_{i,2}$ | Bank’s loan profit from the enterprise $i$ |
| $x$ | Random factor of product production |

**Table 1:** Decision variables and model parameters.

\[
\begin{align*}
\text{Indices} & \quad i = (1, 2) \\
\text{Decision variables} & \quad q_0, q_1, r_i, r_{i0} \\
\text{Model parameters} & \quad c_0, l_0, p_i, w, D, r_0, r_{i0}, k, L_i, \pi_i, \pi_{i,2}, x
\end{align*}
\]

\[
\begin{align*}
\text{Table 1: Decision variables and model parameters.}
\end{align*}
\]
4. Model Construction and Analysis

4.1. Operational Decisions of Supply Chain Enterprises under Two Financing Strategies

4.1.1. Operational Decisions of Supply Chain Enterprises under FBS Strategy. According to Section 1, we can know that the profit of the manufacturer is

$$ \pi_1 = p_1 \min \{q_0 x, q_1\} - c_0 q_0 x + \left( w - l_i \right) \max \{q_0 x - q_1, 0\} - w \max \{q_1 - q_0 x, 0\} - \left( q_0 c_0 - k q_1 p_1 \right) r_1, $$

where the first item is the manufacturer’s sales revenue, the second item is the manufacturer’s production cost, the third item is the difference between the compensation received and the slow-moving loss, the fourth item is the compensation given to the retailer, and the fifth item is the manufacturer’s financing cost. The manufacturer expects the profit to be

$$ E \pi_1 = p_1 \left[ \int_0^{q_0 / q_1} q_0 x f (x) dx + \int_{q_1 / q_0}^{\infty} q_1 f (x) dx \right] - c_0 q_0 - l_i \int_{q_1 / q_0}^{\infty} (q_0 x - q_1) f (x) dx + w (q_0 - q_1) - \left( q_0 c_0 - k q_1 p_1 \right) r_1. $$

**Proposition 1.** Under the FBS strategy, the manufacturer’s expected profit is a convex function, and its optimal planned output satisfies the following formula:

$$ \int_0^{q_0 / q_1} x f (x) dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_i} $$

Equation (3) shows that the manufacturer’s planned output is negatively correlated with the bank’s loan interest rate. Excessively high bank loan pricing during the manufacturer loan phase will cause manufacturers to reduce the planned output to moderately reduce loan amounts, which indicates that excessive financing costs will affect the enthusiasm of financing companies. Obviously, the above conclusions are consistent with the reality. Conversely, if the manufacturer’s planned output is positively related to the bank’s loan interest rate, raising the loan interest rate will increase the manufacturer’s planned output, thereby indirectly increasing the loan amount and ultimately increasing the bank’s loan profits. Therefore, under the conclusion that the manufacturer’s planned output is positively related to the bank’s loan interest rate, the bank will always raise the loan interest rate, which is obviously inconsistent with the actual situation.

At this time, when formula (2) satisfies formula (3), the manufacturer’s expected profit $E \pi_1$ is the largest.

The manufacturer’s expected loan amount $L_1$ is

$$ L_1 = q_0 c_0 - k q_1 p_1. $$

The bank’s expected profit $E \pi_3$ is

$$ E \pi_3 = L_1 (r_1 - r_0) = (q_0 c_0 - k p_1 q_1) (r_1 - r_0). $$

$q_0$ in formulas (4) and (5) satisfies formula (3).
The expected profit $\pi_2$ of the retailer is

\[
\pi_2 = \begin{cases} 
(p_2 - p_1) \min[q_0, q_1] + w \max[q_1 - q_0, 0] - w \max[q_0 - q_1, 0] - kp_1 q_1 r_2, & q_1 < D, \\
(p_2 - p_1) \min[q_0, q_1, D] + w \max[q_1 - q_0, 0] - kp_1 q_1 r_2 - w \max[q_0 - q_1, 0] - l_2 \max[q_0, q_1] - D, 0], & q_1 \geq D.
\end{cases}
\]  

(6)

When $q_1 \geq D$, the first item is the difference between the retailer’s sales revenue and the ordering cost. The second item is the compensation that the retailer receives from the manufacturer because of insufficient manufacturer output. The third item is the financing cost of the retailer. The fourth item is the compensation that the manufacturer receives from the retailer when the manufacturer’s output is too large. The fifth item is the loss of the retailer’s products. The retailer expects the profit to be

\[
E\pi_2 = \begin{cases} 
(p_2 - p_1) \int_0^{q_0/q_0} x f(x)dx + q_1 \int_{q_0/q_0}^{q_1/q_0} f(x)dx + w(q_1 - q_0) - kp_1 q_1 r_2, & q_1 < D, \\
- l_2 \int_0^{q_1/q_0} x f(x)dx + q_1 \int_{q_0/q_0}^{q_1/q_0} f(x)dx + w(q_1 - q_0) - kp_1 q_1 r_2 \\
+ (p_2 - p_1 + l_2) D \int_{q_0/q_0}^{\infty} f(x)dx + (p_2 - p_1 + l_2) q_0 \int_{q_0/q_0}^{\infty} x f(x)dx, & q_1 \geq D.
\end{cases}
\]  

(7)

The retailer arranges the purchase according to the market demand $D$.

**Proposition 2.** Under the FBS strategy, the retailer expects the maximum profit $E\pi_2$ when $q_1 = D$.

This proposition shows that although the manufacturer has uncertainties in production, the optimal order quantity of the retailer is still the expected market demand since the actual production of the manufacturer is subject to the uniform distribution of the planned output. The reason is that when the order quantity is greater than the expected market demand, the retailer has a higher risk of slow sales. When the order quantity is less than the expected market demand, the retailer is at risk of being out of stock.

The maximum expected profit of the retailer is

\[
E\pi_2 = (p_2 - p_1) \int_0^{D/q_0} x f(x)dx + D \int_{D/q_0}^{\infty} f(x)dx \\
+ w(D - q_0) - kp_1 Dr_2.
\]  

(8)

The expected loan amount $L_2$ of the retailer is

\[
L_2 = kp_1 D,
\]  

(9)

where $k$ is the ratio of the prepayment of the retailer’s unit product to $p_0$.

The expected profit of the bank is

\[
E\pi_4 = L_2(r_2 - r_0) = kp_1 D (r_2 - r_0).
\]  

(10)

### 4.1.2. Production and Order Decision of Supply Chain Enterprises under FBU Strategy

Supply chain enterprises can choose either the FBS strategy or the FBU strategy. The FBU strategy refers to the process in which the supply chain lends loans to banks at all levels in the production process of the order. At this time, the bank provides a unified loan interest rate to the supply chain enterprises.

From Proposition 1 and Proposition 2, Proposition 3 and Proposition 4 are obtained.

**Proposition 3.** Under the FBU strategy, the manufacturer’s expected profit $E\pi_1$ is a convex function of the manufacturer’s planned output $q_0$, and its optimal planned yield $q_0$ satisfies the following formula:

\[
\int_0^{q_1/q_0} x f(x)dx = 1 - \frac{p_1 + w - c_0(1 + r_3)}{p_1 + l_1}.
\]  

(11)

**Proof.** At this point, the maximum profit of the manufacturer is

\[
E\pi_1 = p_1 \left( \int_0^{q_1/q_0} q_0 x f(x)dx + \int_{q_1/q_0}^{\infty} q_1 f(x)dx \right) \\
- c_0 q_0 - l_1 \int_{q_1/q_0}^{\infty} (q_0 x - q_1) f(x)dx + w(q_0 - q_1) \\
- (q_0 c_0 - kp_1 q_1) r_3.
\]  

(12)

The manufacturer’s expected loan amount $L_1$ is

\[
L_1 = q_0 c_0 - kp_1 q_1.
\]  

(13)

The expected profit of the bank is

\[
E\pi_5 = L_1(r_2 - r_0) = (q_0 c_0 - kp_1 q_1) (r_2 - r_0).
\]  

(14)

$q_0$ in equations (12)–(14) satisfies equation (11).
Proposition 4. Under the FBU strategy, the retailer expects the maximum profit $E\pi_2$ when $q_1 = D$.

Proof. At this point, the maximum profit of the retailer is

$$E\pi_2 = (p_2 - p_1)\left[q_0 + \int_0^{D-q_0} f(x)dx + D \int_{D-q_0}^{\infty} f(x)dx\right] + w(D - q_0) - kp_1 D r_3.$$  

(15)

The retailer expects the loan amount $L_2$ to be

$$L_2 = kp_1 D,$$  

(16)

where $k$ is the ratio of the prepayment of the retailer’s unit product to $p_1$.

The expected profit of the bank is

$$E\pi_4 = L_2(r_3 - r_0) = kp_1 D (r_3 - r_0).$$  

(17)

4.2. Bank Loan Interest Rate Setting under Two Financing Strategies

4.2.1. Bank Loan Interest Rate Setting under FBS Strategy. According to equation (11), the bank expects the manufacturer’s loan profit $E\pi_3$ to be related to $q_0$ and $r_1$. According to formula (3), the manufacturer’s optimal planned output $q_0$ is negatively correlated with the bank’s loan interest rate $r_1$. Therefore, the bank needs to set its loan interest rate $r_1$ to the manufacturer so that the bank’s expected profit for the manufacturer’s loan is the largest under the premise of satisfying the manufacturer’s maximum profit formula (3).

According to formula (10), the bank’s expected loan profit $E\pi_4$ to the retailer is only positively correlated with the variable $r_2$, so the bank needs to increase its loan interest rate to the retailer as much as possible.

Therefore, this study proposes an interest rate setting model under the bank’s expectation of loan profit maximization:

$$\text{max } \left\{ (q_0c_0 - kp_1 q_1)(r_1 - r_0) + kp_1 q_1 (r_2 - r_0) \right\},$$

s.t.

$$\int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0(1 + r_1)}{p_1 + l_1},$$

$$q_1 = D, \ r_0 \leq r_1 \leq r_{10}, \ r_0 \leq r_2 \leq r_{20}.$$  

(18)

Proposition 5. Under the FBS strategy, when the bank expects the maximum loan profit, the following are satisfied:

(1) The bank’s optimal loan interest rate $r_2$ to the retailer must be the maximum acceptable loan interest rate of the retailer. That is, when $r_2 = r_{20}$, $r_2$ is the optimal solution of equation (18).

(2) The bank’s loan interest rate $r_1$ to the manufacturer is not necessarily the maximum value $r_{10}$ that the manufacturer can accept.

This proposition points out the bank’s loan interest rate setting method under the FBS strategy. The method indicates that the bank should give the retailer a loan interest rate that should take the maximum value while giving the manufacturer a loan interest rate that does not necessarily have to take the maximum value. The reason is that the information is symmetrical. The bank knows that the bank’s loan interest rate to the retailer is more favorable to the bank’s loan profit under the condition that the retailer determines the order quantity. In contrast, the bank knows that the manufacturer’s planned output is inversely proportional to the loan interest rate, and the bank needs to consider the impact of the loan interest rate on the manufacturer’s planned output. Thus, under the FBS strategy, the bank’s optimal loan interest rate to the manufacturer is not always the maximum.

4.2.2. Bank Loan Interest Rate Setting under FBU Strategy. Under the FBU strategy, the bank gives a uniform loan interest rate to the supply chain. The interest rate is not greater than the maximum acceptable interest rate for the manufacturer and the retailer. The bank’s total profit from the manufacturer and retailer is

$$E\pi_3 + E\pi_4 = (q_0c_0 - kp_1 q_1)(r_3 - r_0) + kp_1 q_1 (r_3 - r_0) = q_0c_0(r_3 - r_0).$$  

(19)

According to formula (12), the manufacturer’s optimal planned output $q_0$ is negatively correlated with the bank’s loan interest rate $r_3$. Therefore, this study proposes that the loan interest rate setting model under the bank’s expected loan profit maximization is as follows:

$$\text{max } q_0c_0(r_3 - r_0),$$

s.t.

$$\int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0(1 + r_1)}{p_1 + l_1},$$

$$q_1 = D, \ r_3 < r_{10}, \ r_3 < r_{20}.$$  

(20)

Proposition 6. Under the FBS strategy, when the bank’s supply chain loan interest rate is the maximum value acceptable by the supply chain enterprise, the bank’s expected loan profit is the largest. That is, when $r_{10} = \min(r_{10}, r_{20})$, there is

$$\text{max}[q_0c_0(r_3 - r_0)] = q_0c_0(r^*_3 - r_0).$$  

(21)

This proposition points out the bank’s loan interest rate setting method under the FBU strategy. The method states that the bank’s loan interest rate should be the maximum under the FBU strategy. The reason is that the information is
symmetrical. The bank knows that the loan interest rate is inversely proportional to the manufacturer’s planned output, and the loan interest rate is directly proportional to the bank’s loan profit. Therefore, under the FBU strategy, the bank chooses to increase the loan interest rate to be more beneficial to itself.

4.3. Analysis of the Profit of the Bank and Supply Chain Enterprises under FBS Strategy and FBU Strategy

4.3.1. Comparison of the Bank’s Maximum Expected Loan Profit under the FBS Strategy and FBU Strategy. Based on formula (18) under the FBS strategy and formula (20) under the FBU strategy, the following propositions are proposed.

Proposition 7. The maximum loan profit of the bank under the FBS strategy is not less than the maximum loan profit of the bank under the FBU strategy. That is, the objective function value of formula (18) is not smaller than the objective function value of formula (20).

This proposition indicates that bank’s loan profits are greater under the FBS strategy. The reason is that, at this time, the bank can separately set loan interest rates to make its own profit even greater. Under the FBU strategy, the bank can only consider lending at a uniform loan interest rate. At this time, the loan profit from the manufacturer and the retailer is no higher than the loan profit under the FBS strategy.

4.3.2. Comparison of Maximum Expected Profit of Supply Chain Enterprises under FBS Strategy and FBU Strategy. Because of complete information, supply chain companies understand the maximization formulas (18) and (20) of the bank’s loan profit and know the optimal loan interest rate of the bank under the two financing strategies before the loan. Supply chain companies establish a better loan method based on their expected profit formulas to maximize their expected profits. Therefore, the following proposition is proposed.

Proposition 8. Under the FBS strategy, when the bank’s loan interest rate to the manufacturer is its acceptable maximum, that is, the optimal solution of equation (18) is $r_1^S = r_{10}$, the following are satisfied:

1. If $r_{10} \leq r_{20}$, the maximum expected profit of the supply chain under the FBU strategy is not less than its maximum expected profit under the FBS strategy.
2. If $r_{10} > r_{20}$, under $q_S^c_0 - k p_1 D > 0$ and $(p_2 - p_1)$ $\int_0^{D/D_S} x f(x)dx - w > 0$, the FBS strategy is more favorable to both enterprises in the supply chain; under $q_S^c_0 - k p_1 D > 0$ and $(p_2 - p_1)$ $\int_0^{D/D_S} x f(x)dx - w < 0$, the FBU strategy is more favorable to the manufacturer and not good for the retailer.

This proposition finds out what kind of financing strategy the supply chain enterprise chooses to be more favorable to itself. When the bank’s loan interest rate to the manufacturer under the FBS strategy is the maximum value acceptable by the manufacturer and less than the maximum value of the retailer’s acceptable loan interest rate, it is more advantageous for supply chain enterprises to choose the FBU strategy. When the bank’s loan interest rate to the manufacturer under the FBS strategy is the maximum value acceptable by the manufacturer and greater than the maximum loan interest rate acceptable by the retailer, we find two sets of conditions under which the manufacturer’s expected profit under the FBU strategy is higher than the expected profit under the FBS strategy, but the retailer does not fully earn more under the FBU strategy. Thus, Proposition 8 shows that the FBU strategy is beneficial to supply chain enterprises under certain conditions, but it is not always beneficial to supply chain enterprises.

5. Numerical Analysis

The setting of parameters is done according to the study of Jin et al. [24]. We assume $x \sim U[1 - n, 1 + n]$ and give the four types of supply chain parameters in Table 2. The bank’s optimal interest rate and the expected profits of banks and supply chain enterprises are shown in Table 3. Figures 3 and 4 show the comparisons of the supply chain enterprises’ profits under two different financing strategies in groups 2 and 3 in Table 3, respectively.

From Table 3 and Figures 3 and 4, we get the following results.

First, Proposition 5 is confirmed. As can be seen from Table 3 under the FBS strategy, the bank’s loan interest rate to the retailer is the maximum value of the retailer’s acceptable loan interest rate. However, it can be seen from the results of group 1 in Table 3 that, under this set of supply chain parameters, when the bank’s expected profit is the largest, the loan interest rate given to the manufacturer is 0.062 < 0.1. From the results, the bank’s maximum loan profit from the manufacturer is not necessarily obtained at the maximum loan interest rate, and the largest loan profit for the retailer is obtained at the maximum loan interest rate.

Second, Proposition 6 is confirmed. As can be seen from the five sets of data in Table 2, under the FBU strategy, the maximum bank’s loan interest rate acceptable by the supply chain is $min[0.09, 0.1] = 0.09$. Obviously, in the five sets of results in Table 3, the bank’s optimal lending rate under the FBU strategy is 0.09. From the results, when the supply chain enterprises adopt the FBU strategy, the bank’s largest loan profit is obtained at the largest loan interest rate.

Third, Proposition 7 is confirmed. The results of the five groups in Table 3 are tested as $814 > 633, 769 > 581$, and $807 > 581$. It is clear that the bank’s maximum expected profit is higher under the FBS strategy.

Fourth, Proposition 8 is confirmed. From the results of the data of groups 4 and 5 in Table 3, under $r_1^S = r_{10}$ and $r_{10} \leq r_{20}$, both sets of results indicate that the maximum expected profit of the manufacturer and the retailer under
the FBU strategy is not less than the maximum expected profit under the FBS strategy. This confirms the first conclusion of Proposition 8. From the results of the data of groups 2 and 3 in Table 3, under \( r_1 = r_{10} \) and \( r_{10} > r_{20} \), both sets of results indicate that the maximum expected profit of the manufacturer under the FBU strategy is greater than that under the FBS strategy. According to Figure 3 and group 2 in Table 3, the retailer’s maximum expected profit is larger under the FBU strategy than the FBS strategy. In contrast, in Figure 4 and group 3 in Table 3, the conclusion is reversed. The reason is that the second set of data satisfies \( (p_2 - p_1) \int_0^{D_1} x f_1(x)dx - w > 0 \) and the third set of data satisfies

### Table 2: Supply chain parameter settings.

<table>
<thead>
<tr>
<th>Types</th>
<th>( c_0 )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( D )</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( w )</th>
<th>( r_0 )</th>
<th>( k )</th>
<th>( r_{10} )</th>
<th>( r_{20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>15</td>
<td>20</td>
<td>3000</td>
<td>10</td>
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<td>4</td>
<td>0.02</td>
<td>0.2</td>
<td>0.1</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>10</td>
<td>20</td>
<td>3000</td>
<td>10</td>
<td>14</td>
<td>4</td>
<td>0.02</td>
<td>0.2</td>
<td>0.1</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>10</td>
<td>15</td>
<td>3000</td>
<td>10</td>
<td>14</td>
<td>4</td>
<td>0.02</td>
<td>0.2</td>
<td>0.1</td>
<td>0.09</td>
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<tr>
<td>4</td>
<td>3</td>
<td>10</td>
<td>20</td>
<td>3000</td>
<td>10</td>
<td>14</td>
<td>4</td>
<td>0.02</td>
<td>0.2</td>
<td>0.09</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>10</td>
<td>15</td>
<td>3000</td>
<td>10</td>
<td>14</td>
<td>4</td>
<td>0.02</td>
<td>0.2</td>
<td>0.09</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### Table 3: Profits of enterprises and banks under the FBS strategy and FBU strategy.

<table>
<thead>
<tr>
<th>Types</th>
<th>Financing strategies</th>
<th>Optimal loan interest rate</th>
<th>Optimal production and ordering decisions</th>
<th>Bank’s expected profit</th>
<th>Manufacturer’s expected profit</th>
<th>Retailer’s expected profit</th>
<th>Total expected profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( r_1 = 0.062 )</td>
<td>( r_2 = 0.09 )</td>
<td>( q_0 = 3023 )</td>
<td>3000</td>
<td>814</td>
<td>26627</td>
</tr>
<tr>
<td></td>
<td>FBS strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12266</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>FBS strategy</td>
<td>( r_1 = 0.09 )</td>
<td>( r_2 = 0.09 )</td>
<td>( q_1 = 3013 )</td>
<td>3000</td>
<td>633</td>
<td>26626</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12287</td>
</tr>
<tr>
<td>3</td>
<td>FBS strategy</td>
<td>( r_1 = 0.1 )</td>
<td>( r_2 = 0.09 )</td>
<td>( q_1 = 2762 )</td>
<td>3000</td>
<td>769</td>
<td>13423</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25667</td>
</tr>
<tr>
<td>4</td>
<td>FBS strategy</td>
<td>( r_1 = 0.09 )</td>
<td>( r_2 = 0.09 )</td>
<td>( q_1 = 2765 )</td>
<td>3000</td>
<td>581</td>
<td>13446</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25669</td>
</tr>
<tr>
<td>5</td>
<td>FBS strategy</td>
<td>( r_1 = 0.09 )</td>
<td>( r_2 = 0.1 )</td>
<td>( q_1 = 2765 )</td>
<td>3000</td>
<td>581</td>
<td>13446</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25669</td>
</tr>
<tr>
<td>6</td>
<td>FBS strategy</td>
<td>( r_1 = 0.09 )</td>
<td>( r_2 = 0.09 )</td>
<td>( q_1 = 2765 )</td>
<td>3000</td>
<td>581</td>
<td>13446</td>
</tr>
<tr>
<td></td>
<td>FBU strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25669</td>
</tr>
</tbody>
</table>

### Figure 3: Profit comparison of the supply chain enterprises under the two financing strategies in group 2 in Table 3.
\[(p_2 - p_1) \int_0^{D[|q_0]} x f_1(x) dx - w < 0, \] thus confirming the second conclusion of Proposition 8.

### 6. Conclusion

This paper studies the external financing problems of the bilateral supply chain with financial constraints. It starts with taking the manufacturer’s planned output and the retailer’s planned order quantity as variables, builds the expected profit model of the manufacturer and the retailer under the FBS strategy and FBU strategy, and analyzes the manufacturer’s optimal planned output and the retailer’s optimal order quantity under the two financing strategies. Then, it states, on the condition of symmetry of information, how could the bank determine the loan interest rate to maximize the expected loan profit, when the bank has obtained the information of the planned production and planned order quantity of the manufacturer and the retailer under the two financing strategies. At last, it proposes the maximum expected loan profit model of the bank under the FBS strategy and FBU strategy, to determine the optimal loan interest rate of the bank under the two financing strategies. And it compares the profit of the bank and the profit of the supply chain under the two financing strategies.

The research shows the following: (1) Under the two financing strategies, the manufacturer’s optimal planned output is inversely proportional to the loan interest rate, and the retailer’s optimal order quantity is the market demand. (2) Under the FBS strategy, it is more beneficial for the bank to get more expected profit when it raises the loan interest rates for the retailer, but it does not apply to the manufacturer. (3) Under the FBU strategy, when the bank sets the loan interest rate at the upper limit of the acceptable line, the bank’s expected loan profit is the largest, but it is less than the bank’s expected loan profit under the FBS strategy. (4) The FBU strategy is more profitable for both the manufacturer and the retailer when the bank’s loan interest rate to the manufacturer is its acceptable maximum and the loan interest rate acceptable by the manufacturer is less than that by the retailer.

The above-mentioned conclusions supply a policy guiding the supply chain enterprises with financial constraints to make a choice of the financing strategy. For the bank, (1) when the bilateral supply chain enterprises with uncertain output and certain demand choose the FBS strategy, the bank should give the retailer the maximum loan interest rate and give the manufacturer a loan interest rate according to formula (18); (2) when they choose the FBU strategy, the bank should give the supply chain enterprises the largest unified loan interest rate; and (3) when supply chain enterprises choose the FBS strategy, the bank make more profits, so the bank should encourage them to conduct the FBS strategy as much as possible. For supply chain enterprises, the optimal loan strategies of financing enterprises under different conditions are different, so they should reasonably choose the optimal financing strategy based on the bank’s quotation and their own acceptable loan interest rate.

For the bilateral supply chain, under the FBS and FBU financing strategies, all the financing modes are distinguished from the perspective of obtaining the bank’s loan interest rates, so there is no financing strategy superior to the FBU strategy. There are various modes under the FBU strategy, and each has its own advantages, such as the dealers and supplier network financing modes and the bank logistics cooperative financing modes. Therefore, various financing modes within the FBU strategy should be studied and analyzed in the future.

### Appendix

**Proof of Proposition 1.** Taking the first-order and second-order derivatives of \( E_{\pi_1} \) with respect to \( q_0 \), it follows that

![Figure 4: Profit comparison of the supply chain enterprises under the two financing strategies in group 3 in Table 3.](image-url)
\[
\frac{dE_{p_1}}{dq_0} = (p_1 + l_1) \int_0^{q_1/q_0} xf(x)dx + w - c_0 (1 + r_1) - l_1,
\]
(A.1)

\[
\frac{d^2E_{p_1}}{dq_0^2} = -\frac{q_1^2}{q_0^3} (p_1 + l_1) f\left(\frac{q_1}{q_0}\right) \leq 0.
\]
(A.2)

When \((q_1/q_0) > 1 + n\), that is, \(q_0 < (q_1/(1 + n))\), we have \(f(q_1/q_0) = 0\). Thus, \((d^2E_{p_1}/dq_0^2) = 0\). We obtain
\[
\frac{dE_{p_1}}{dq_0} = p_1 + w - c_0 (1 + r_1) > 0.
\]
(A.3)

So when \(q_0 < (q_1/(1 + n))\), \(E_{p_1}\) is a monotonically increasing function with a fixed slope.

When \((q_1/q_0) < 1 - n\), that is, \(q_0 > (q_1/(1 - n))\), we have \(f(q_1/q_0) = 0\). Thus, \((d^2E_{p_1}/dq_0^2) = 0\). We obtain
\[
\frac{dE_{p_1}}{dq_0} = w - c_0 (1 + r_1) - l_1 < 0.
\]
(A.4)

So when \((q_1/(1 - n)) < q_0\), \(E_{p_1}\) is a monotonically decreasing function with a fixed slope.

When \(1 - n \leq (q_1/q_0) \leq 1 + n\), we have \((d^2E_{p_1}/dq_0^2) < 0\). Therefore, \(E_{p_1}\) is a convex function about the variable \(q_0\).

\[
\frac{dE_{p_1}}{dq_0} = \int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}.
\]
(A.5)

and \(q_0 \in (0, +\infty)\), \(q_0\) is the only absolute maximum point of \(E_{p_1}\).

Proof of Proposition 2. Taking the first-order derivative of \(E_{p_2}\) with respect to \(q_1 \geq D\), it follows that
\[
\frac{dE_{p_2}}{dq_1} = \begin{cases} 
(p_2 - p_1) \int_{q_1/q_0}^{\infty} f(x)dx + w - kp_1 r_2, & q_1 < D, \\
-l_2 \int_{q_1/q_0}^{\infty} f(x)dx + w - kp_1 r_2, & q_1 \geq D.
\end{cases}
\]
(A.6)

(1) \(q_1 \geq D\)

According to Proposition 1, we have \((q_1/q_0) \in [1 - n, 1 + n]\). Because of \(xf(x) \geq 0\), we can obtain
\[
\int_0^{q_1/q_0} xf(x)dx > 0.
\]

Let \(u = (q_1/q_0) > 0\) and define the function
\[
\phi_1(u) = \int_0^u xf(x)dx.
\]
(A.7)

The first-order derivative of \(\phi_1(u)\) follows that
\[
\phi_1'(u) = uf(u) \geq 0.
\]
(A.8)

Because of \(u \in [1 - n, 1 + n]\), we have \(\phi_1'(u) > 0\). Thus, \(\phi_1(u)\) increases in \(u\), that is, \(\int_0^{q_1/q_0} xf(x)dx\) increases in \((q_1/q_0)\). Hence, there exists a unique \((q_1/q_0)\) satisfying
\[
\int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}.
\]
(A.9)

At this time, \((q_1/q_0)\) is a unique constant related to the coefficients and \(f(x)\). Hence, when \(q_1 \geq D\), \((dE_{p_2}/dq_1)\) is a unique constant. If \((dE_{p_2}/dq_1) > 0\), the retailer’s planned order quantity will be infinite, which contradicts the risk-neutral assumption. Hence, we have \((dE_{p_2}/dq_1) < 0\). Therefore, \(E_{p_2}\) decreases in \(q_1\) in the region \(q_1 \in [D, +\infty)\) and attains the maximum at the point \(q_1 = D\).

(2) \(q_1 < D\)

Because \((dE_{p_2}/dq_1)\) is a unique constant, if
\[
\frac{dE_{p_2}}{dq_1} = (p_2 - p_1) \int_{q_1/q_0}^{\infty} f(x)dx + w - kp_1 r_2 < 0,
\]
(A.10)

then \(E_{p_2}\) decreases in \(q_1\) in the region \(q_1 \in [0, D)\) and attains the local maximum at the point \(q_1 = 0\). Thus, \(E_{p_2}\) decreases in \(q_1\) when \(q_1 \geq 0\). \(E_{p_2}\) attains the absolute maximum at the point \(q_1 = 0\). At this point, the supply chain is in a state of no order. We will not discuss this situation.

In fact, according to the previous literature and the actual situation of the supply chain, there is often \(w - kp_1 r_2 > 0\). Hence, when \(q_1 < D\), we can obtain \((dE_{p_2}/dq_1) > 0\) and that \(E_{p_2}\) increases in \(q_1\).

In summary, \(E_{p_2}\) attains the maximum at the point \(q_1 = D\).

Proof of Proposition 3. It follows the proof of Proposition 1, so we omit it.

Proof of Proposition 4. It follows the proof of Proposition 2, so we omit it.

Proof of Proposition 5

(1) Obviously, nonlinear programming (18) can be divided into the following two planning problems:
\[ \max (q_0 c_0 - kp_1 q_1) (r_1 - r_0) \]
\[
\begin{aligned}
\text{s.t.} & \quad \int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}, \\
q_1 & = D, r_0 \leq r_1 \leq r_{10}.
\end{aligned}
\] (A.11)

\[ \max kp_1 q_1 (r_2 - r_0) \]
\[
\begin{aligned}
\text{s.t.} & \quad r_0 \leq r_2 \leq r_{20}.
\end{aligned}
\] (A.12)

\[ m_1(q_0) = (q_0 c_0 - kp_1 D) (r_1 - r_0), \]
\[
\begin{aligned}
\text{s.t.} & \quad \int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}, \\
m_2(q_1) & = (q_0 c_0 - kp_1 D) (r_1 - r_0), \quad \text{s.t.} \quad \int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}\] (A.13)

According to
\[
\int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1} = 0. \] (A.14)

We obtain that \( r_1 \) decreases in \( q_0 \). Thus, when \((dm_1/dq_0) > 0\), we have \((dm_2/dr_1) < 0\). When \((dm_1/dq_0) < 0\), we have \((dm_2/dr_1) > 0\).

Taking the first-order derivative of \( m_1 \) with respect to \( q_0 \), it follows that
\[
\frac{dm_1}{dq_0} = (p_1 + l_1) \]
\[
\begin{aligned}
& \cdot \int_0^{q_1/q_0} xf(x)dx - (p_1 + l_1) (q_0 c_0 - kp_1 D) q_1^2 f(q_1/q_0) \\
& - r_0 c_0 - l_1 - c_0 + w.
\end{aligned}
\] (A.15)

Under different parameters of the supply chain, formula (A.15) can satisfy both \((dm_1/dq_0) > 0\) and \((dm_1/dq_0) < 0\). Therefore, there is not always \((dm_2/dr_1) > 0\), that is, the optimal of \( r_1 \) is not always \( r_{10} \). In numerical analysis, we give examples to support this conclusion.

**Proof of Proposition 6.** Let
\[ g(q_0, r_3) = q_0 c_0 (r_3 - r_0). \] (A.16)

In linear programming (A.12), the objective function increases in \( r_2 \) in the region \( r_2 \in [r_0, r_{20}] \). Thus, we attain the optimum of equation (A.12) at the point \( r_2 = r_{20} \). It is also the optimum of nonlinear programming (18).

(2) In nonlinear programming (A.11), let
\[
m_1(q_0) = (q_0 c_0 - kp_1 D) (r_1 - r_0), \quad m_2(q_1) = (q_0 c_0 - kp_1 D) (r_1 - r_0), \]
\[
\begin{aligned}
\text{s.t.} & \quad \int_0^{q_1/q_0} xf(x)dx = 1 - \frac{p_1 + w - c_0 (1 + r_1)}{p_1 + l_1}
\end{aligned}
\] (A.13)

Since the relationship between \( q_0 \) and \( r_3 \) satisfies the constraint of (20), equation (A.16) can also express equation (21.2) as follows:
\[
g_1(q_0) = q_0 \left[ (p_1 + l_1) \int_0^{q_1/q_0} xf(x)dx + w - l_1 - c_0 (1 + r_0) \right]. \] (A.17)

Taking the first-order and second-order derivatives of \( g_1(q_0) \) with respect to \( q_0 \), it follows that
\[
g_1'(q_0) = \left[ (p_1 + l_1) \int_0^{q_1/q_0} xf(x)dx + w - l_1 - c_0 (1 + r_0) \right]
\[
- \frac{q_1^2}{q_0} \int_0^{q_1/q_0} f(q_1/q_0)\left( p_1 + l_1 \right).
\] (A.18)

Obviously, when \((q_1/q_0) \in [1 - n, 1 + n] \), that is, \( q_0 \in \left[ (q_1/(1 + n)), (q_1/(1 - n)) \right] \), \( g_1'(q_0) \) increases in \( q_0 \). According to the proof of Proposition 1, the manufacturer attains the optimum in the region \( q_0 \in \left[ (q_1/(1 + n)), (q_1/(1 - n)) \right] \). When \( q_0 = (q_1/(1 - n)) \), we can obtain
\[
g_1'(q_0) = w - l_1 - c_0 (1 + r_0) - \frac{(1 - n)^2}{2n} (p_1 + l_1) < 0. \] (A.19)
Thus, $g_1'(q_0)$ is negative in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$. $g_1(q_0)$ decreases in $q_0$ in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$. Since
\[
\int_0^{q_0/\eta_0} x f_1(x) dx = 1 - \frac{p_1 + w - c_0(1 + r_3)}{p_1 + l_1},
\]
(A.20)
s.t. $q_0 \in \left\{ \frac{q_1}{1 + n}, \frac{q_1}{1 - n} \right\}$,
we obtain that $r_3$ decreases in $q_0$ in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$. Hence, $g_1(q_0)$ increases in $r_3$ in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$.

According to formulas (A.3) and (A.4) and the conclusions of Proposition 1, when $r_1 \in \{(w - l_1 - c_0)/c_0\}, (p_1 + w - c_0)/c_0\}$, we have $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$. Since we assume that the manufacturer makes decisions based on the maximum expected profit, the must be $r_{10} \in \{(w - l_1 - c_0)/c_0\}, (p_1 + w - c_0)/c_0\}$ which maximizes the expected profit of the manufacturer under the FBS strategy. Similarly, there must be $q_0 \in [(q_1/(1 + n)), \min\{r_{10}, r_{20}\}]$ when $r_2 \in \{r_{10}, \min\{r_{10}, r_{20}\}\}$. Since $g_1(q_0)$ decreases in $q_0$ in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$, $g_1(q_0)$ increases in $r_3$ in the region $r_3 \in \{r_{10}, \min\{r_{10}, r_{20}\}\}$. Therefore, $g_1(q_0)$ attains the maximum at the point $r_3 = \min\{r_{10}, r_{20}\}$. We can obtain max\{$q_0c_0(r_3 - r_0)\} = q_0c_0(r_{10} - r_0)$.

**Proof of Proposition 7.** If formula (20) attains the optimum at the point $q_0 = q_0^*$ and $r_3^* = r_3^*$, let $q_0 = q_0^*$, $r_1 = r_1^*$, and $r_3 = r_3^*$ in formula (18). Obviously, they satisfy the constraint of formula (18). At this time, the objective function of formula (18) is equal to the maximum of formula (20), which means that the optimum of formula (20) must satisfy formula (18). Therefore, the maximum of formula (18) is not less than the maximum of formula (20).

**Proof of Proposition 8.**

1. When $r_{10} \leq r_{20}$, we have $r_1^* = r_{10} = \min\{r_{10}, r_{20}\} = r_1^*$ according to Proposition 5 and $r_3^* = r_{10}$. Thus, when $q_1 = D$ and $r_1 = r_2$, it is obvious that $q_0$ is equal in the two equations according to equations (3) and (11). Hence, under $q_0 = q_0^*$, $q_1 = q_1^*$, and $r_1 = r_2$, we can obtain that $E_{\Pi_1}^U > E_{\Pi_1}^U$ according to equations (2) and (12).

2. When $r_{10} \leq r_{20}$, we have $r_1^* = r_{10} \leq r_{20} = r_1^*$ according to Proposition 5 and $r_3^* = r_{10}$. Hence, under $q_0 = q_0^*$, $q_1 = q_1^*$, and $r_2 \geq r_3$, we can obtain that $E_{\Pi_2}^U \leq E_{\Pi_2}^U$ according to equations (8) and (15).

Therefore, when $r_1^* = r_{10}$ and $r_{10} \leq r_{20}$, we obtain that $E_{\Pi_1}^U = E_{\Pi_1}^U$ and $E_{\Pi_2}^U \leq E_{\Pi_2}^U$, which means the expected profit of the manufacturer and retailer under the FBS strategy is not less than the profit under the FBS strategy.

We know that $r_1$ decreases in $q_0$ in the region $q_0 \in [(q_1/(1 + n)), (q_1/(1 - n))]$. Since $r_1^* = r_{10}$ and $r_{10} > r_{20}$, we have $q_0 \in [q_0^*, +\infty)$. Substituting equation (3) in (2) and taking the first-order derivative of $\Pi\Pi_1$ with respect to $q_0$, the following can be obtained:
\[
\frac{d\Pi\Pi_1}{dq_0} = \left(p_1 + l_1\right)q_1^2 f\left(q_1\right)\left(c_0q_0 - kp_1D\right).
\]
(A.21)

Since $q_0^*c_0 - kD > 0$ and $q_0 \in [q_0^*, +\infty)$, we have $(d\Pi\Pi_1/dq_0)_0 > 0$. When the supply chain is under the FBU strategy, we have $r_1^U = r_{10}^U = \min\{r_{10}, r_{20}\} = r_{20}$ according to Proposition 6 and $r_{10} > r_{20}$. Thus, we have $r_3^U < r_{10}^U < r_1^U$ and $q_0^* > q_0^*$. Therefore, we can obtain that $E_{\Pi_1}^U > E_{\Pi_1}^U$ when $(d\Pi\Pi_1/dq_0)_0 > 0$ and $q_0^* > q_0^*$. In other words, we have $E_{\Pi_1}^U > E_{\Pi_1}^U$ when $r_1^U = r_{10}^U$, $q_0^*c_0 - kD > 0$, and $r_{10} > r_{20}$.

Taking the first-order and second-order derivatives of $\Pi\Pi_2$ with respect to $q_0$, the following can be obtained:
\[
\frac{d\Pi\Pi_2}{dq_0} = \left(p_2 - p_1\right) \int_0^{D_q} x f(x) dx - w_0,
\]
\[
\frac{d^2\Pi\Pi_2}{dq_0^2} = -\left(p_2 - p_1\right) \int_0^{D_q} x f(x) dx - w_0 < 0.
\]
(A.22)

When $r_1 = r_{10}$ and $r_{10} > r_{20}$, we have $r_3^U < r_1^U$ and $q_0^* > q_0^*$. Since $(d^2\Pi\Pi_2/dq_0^2)_0 < 0$, we obtain that $(d\Pi\Pi_2/dq_0)_0$ decreases in $q_0$. When $q_0 \in [q_0^*, +\infty)$, we have
\[
\min \frac{d\Pi\Pi_2}{dq_0} = \left(p_2 - p_1\right) \int_0^{D_q} x f(x) dx - w_0.
\]
(A.23)

When $(p_2 - p_1) \int_0^{D_q} x f(x) dx - w_0 > 0$ and $q_0 \in [q_0^*, q_0^*]$, we have $(d\Pi\Pi_2/dq_0)_0 > 0$ and $E_{\Pi_2}^U > E_{\Pi_2}^U$. When $(p_2 - p_1) \int_0^{D_q} x f(x) dx - w_0 < 0$ and $q_0 \in [q_0^*, q_0^*]$, we have $(d\Pi\Pi_2/dq_0)_0 < 0$ and $E_{\Pi_2}^U > E_{\Pi_2}^U$.

In summary, when $(p_2 - p_1) \int_0^{D_q} x f(x) dx - w_0 > 0$ and $q_0^*c_0 - kD > 0$, we have $E_{\Pi_1}^U > E_{\Pi_1}^U$ and $E_{\Pi_2}^U > E_{\Pi_2}^U$. When $(p_2 - p_1) \int_0^{D_q} x f(x) dx - w_0 < 0$ and $q_0^*c_0 - kD > 0$, we have $E_{\Pi_1}^U > E_{\Pi_1}^U$ and $E_{\Pi_2}^U > E_{\Pi_2}^U$.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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