Research Article

Research on Exact Thresholds for ARAIM
MHSS Fault Monitoring

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Multiple Hypothesis Solution Separation (MHSS) is the baseline algorithm for Advanced Receiver Autonomous Integrity Monitoring (ARAIM), and it detects faults by comparing the test statistic with a threshold. However, the cuboid threshold structure of the MHSS fault monitoring baseline algorithm lacks omnidirectionality, which leads to low conformity between the threshold and the spatial distribution of the test statistic and to low fault monitoring accuracy. To resolve these problems, we analyzed the distribution of a test statistic for single-, double-, and triple-fault hypotheses. By extracting the eigenvectors and eigenvalues of the solution separating variance, we designed an omnidirectional threshold structure. The simulation verifies the effectiveness of the fault detection method by detecting faults from noise. The results show that the proposed method is more exact, stable, and applicable than the MHSS fault detection baseline.

1. Introduction

Receiver Autonomous Integrity Monitoring (RAIM) is an important method of ensuring integrity. RAIM is a sensor-level integrity monitoring system that uses a self-consistency check to detect and exclude potential excessive ranging errors [1]. With the modernization of GPS and GLONASS and the development of Galileo and BDS, the number of GNSS satellites is increasing rapidly [2], and the requirements of higher accuracy and integrity are achieved using dual frequencies and multiple constellations. The future mult constellation global navigation satellite system (GNSS) will provide a large number of redundant ranging signals, which will improve the Receiver Autonomous Integrity Monitor (RAIM) performance but will also increase the probability of satellite faults [3]. Therefore, the Advanced Receiver Autonomous Integrity Monitor (ARAIM), which is expected to provide vertical guidance for LPV-200, has been proposed by the Federal Aviation Administration. Compared with RAIM, ARAIM can support multiconstellation and dual-frequency [4] signals. Additionally, the ARAIM nominal performance and fault probability can be updated by integrity support messages to ensure the integrity of navigation services [5].

As an important part of ARAIM, fault detection serves to avoid the use of excessive errors in positioning. Through GPS measurements and the application of the RAIM, the current study illustrates the performance of the proposed fault detection algorithm (MHSS FD) [6]. It is often only necessary to consider a single fault in the Receiver Autonomous Integrity Monitoring (RAIM) procedure, so it would be ideal if a fault could be correctly identified [7]. Compared with RAIM, ARAIM must consider more types of threats to meet the higher requirements [8]. Therefore, the fault detection must be more accurate.

The main method of fault detection focuses on the pseudorange domain and positioning domain [9, 10]. The Working Group C ARAIM Technical subgroup defined Multiple Hypothesis Solution Separation (MHSS) as the ARAIM baseline algorithm [11]. This baseline algorithm included the computation of the protection levels, the effective monitor threshold, the accuracy, and a preliminary description of an exclusion algorithm [12]. This article focuses on the effective monitoring threshold. In addition, the MHSS fault detection (FD) baseline detects faults by checking the consistency of the solutions from different subsets with the all-in-view set. The solution separation statistic is closely linked to the
optimal detection statistics. In an example based on an ARAIM scenario, a detection region based on the solution separation statistics gets within an order of magnitude of the lower bound (as opposed to the chi-square statistic, which is, in most cases, several orders of magnitude above) [13]. Specifically, faults are identified by comparing the values of a test statistic and a predetermined threshold [14, 15]. There are many articles that focus on the MHSS fault detection of the baseline. The Chi2 ARAIM approach can provide a tighter integrity risk bound than the MHSS baseline algorithm. An expansion of the baseline ARAIM MHSS algorithm is termed Q*-MHSS. In the ARAIM baseline algorithm, the fault mode determination is an independently sequential structure [16, 17]. Therefore, the threshold settings are particularly critical.

However, the threshold structure of the MHSS fault monitoring baseline algorithm is cuboid, which means that the MHSS FD baseline compares a test statistic with a predetermined threshold between the threshold and the spatial distribution of the test statistic. The simulation verifies the effectiveness of the fault detection method by detecting faults from the noise. The results show that the proposed method is more exact, stable, and applicable than the MHSS FD baseline.

### 2. MHSS FD Baseline

In ARAIM, the MHSS FD baseline is used to monitor faults. A brief description of MHSS follows.

The MHSS FD baseline compares the difference values between a subset solution and the all-in-view solution with the thresholds in three directions: east, north, and up. For every fault hypothesis, we proceed with the solution separation threshold test. Here, the fault k=0 means the all-in-view solution. Thus, according to the basic satellite positioning algorithm [19], we have

\[
\Delta x^{(k)} = \tilde{x}^{(k)} - \tilde{x}^{(0)} = \left( \zeta^{(k)} - \zeta^{(0)} \right) y
\]

\[
S^{(k)} = \left( G^T W^{(k)} G \right)^{-1} G^T W^{(k)}
\]

where the geometry matrix G is an \(N_{sat} \times (3 + N_{const})\) matrix, \(N_{sat}\) is the number of satellites, and \(N_{const}\) is the number of constellations. The first 3 columns of matrix G are identical to those of the basic positioning method, the other columns are set according to the reference clock of each constellation, and \(W\) is a weighting matrix. The solution separation variance between the subset solution and the all-in-view solution is

\[
\sigma_{ss}^{(k)2} = \left( S^{(k)} - S^{(0)} \right) C_{acc} \left( S^{(k)} - S^{(0)} \right)^T, \quad \text{(2)}
\]

\[
\sigma_{ss}^{(k)2} = \tilde{e}_{ENU}^T \tilde{e}_{ENU} \quad \text{(3)}
\]

where \(C_{acc}\) is the pseudo-range error diagonal covariance matrices used for accuracy and continuity and \(e_{ENU}\) is the coordinate standard vector, which represents the 3 directions, east, north, and up. The variance can be observed in the 3 directions, and the threshold in the direction of ENU is \(T_{k, ENU}\). Thus,

\[
T_{k, ENU} = K_{fa, ENU} \sigma_{ss, ENU}^{(k)} \quad \text{(4)}
\]

and

\[
K_{fa, E} = K_{fa, N} = Q^{-1} \left( \frac{P_{fa, HOR}}{4 N_{fault \ modes}} \right)
\]

\[
K_{fa, U} = Q^{-1} \left( \frac{P_{fa, VERT}}{2 N_{fault \ modes}} \right)
\]

where \(Q^{-1}(p)\) is the inverse of the \(Q'\) function and \(Q'\) is the tail probability of a zero mean standard normal distribution:

\[
Q'(u) = \frac{1}{\sqrt{2\pi}} \int_{u}^{+\infty} e^{-t^2/2} dt.
\]

where \(u\) is the quartile of a zero mean standard normal distribution and \(t\) is a random variable of a zero mean standard normal distribution.

\(P_{fa}\) is the false alarm rate and \(N_{fault \ modes}\) is the number of fault hypotheses. A fault-free case is observed when the following conditions are met:

\[
\tau_{k, ENU} = \frac{\left| \tilde{x}_{ENU}^{(k)} - \tilde{x}_{ENU}^{(0)} \right|}{T_{k, ENU}} \leq 1. \quad \text{(7)}
\]

Considering that \(N_{fault \ modes}\) and \(P_{fa}\) are constant during a single positioning, if the above algorithm is employed to calculate the threshold, then \(\sigma_{ss, ENU}^{(k)}\) must be determined because \(T_{k, ENU}\) is determined by \(\sigma_{ss, ENU}^{(k)}\).

Simulations based on a practical positioning example that use all-visible satellites for positioning can be used to obtain the all-in-view solution. By excluding the nth satellite, we can obtain the subset solution with supposed fault \(n\) and calculate the test statistic \(\Delta \tilde{x}^{(k)}\), which is the difference between the subset solution and the all-in-view solution. If we add noise \(\sigma_{noise}\) to each measurement, then we can obtain the distribution of the all-in-view solution, the subset solution, and the test statistic of the fault-free case as shown in Figure 1. The test statistic is the difference between the subset solution and the all-in-view solution, and thus the points in Figures 1(a)–1(c) are one-to-one matched. Because of the errors intentionally added to the satellite measurements, the all-in-view solutions, subset solutions, and test statistics finally
result in the point cloud. In Figure 1(c), the test statistics are distributed as a point cloud with an obvious inclined direction, and the cloud is dense in the middle, sparse around the edges, and completely distributed as an ellipsoid-like point cloud.

Observing the ellipsoidal point cloud is difficult, and it can be better expressed by simplifying the three-directional (east, north, and up) threshold construction to two directions (east and north), as shown in Figure 2.

As in Figure 1, the distribution of the test statistic in Figure 2 is an elliptical point cloud.

To monitor the test statistic points, thresholds were set in the x and y coordinates and are shown as shadows in Figure 2. It is worth mentioning that we set a small threshold in the figure to better express the main idea. According to the MHSS FD baseline, a point in the white part in the middle means that it is fault-free, whereas the shadowed parts are outside the thresholds. If a point appears in the right diagonal lines shadow, then it is outside the threshold in the x direction. If a point falls within the left diagonal lines shadow, then it is outside the threshold in the y direction. If a point falls within the crossed lines shadow, then it is outside the thresholds in both x and y directions.

In a 2D plane, because of the elliptical distribution of the test statistic, the thresholds should be rectangular. However, in 3D space, the thresholds should be cubes because of the ellipsoidal distribution of the test statistic. Considering the thresholds and the distribution of the test statistic together, we make the following conclusions:

(1) The cube threshold construction of the MHSS FD baseline lacks omnidirectionality.

The purpose of the error monitoring is to distinguish whether a test statistic is in the normal area or not, and the thresholds are determined by the distribution of the test statistics. In the case of MHSS, the distribution of the test statistic is an ellipsoid-like point cloud with a clear inclined direction. However, the MHSS FD baseline only considers the east, north, and up directions, which generates a type of cubical threshold structure. This type of threshold lacks omnidirectionality, which will lead to low conformity between the threshold and the spatial distribution of the test statistic and to a low fault monitoring accuracy.

(2) The MHSS FD baseline should not monitor the test statistics for different fault hypotheses using the threshold of the same structure.

The weighted matrices varied according to the fault hypothesis. For the single-fault hypothesis, the row corresponding to the faulty satellite in the matrix was set to 0. For the double-fault hypothesis, two rows in the matrix were set to 0 and so on. The weighted matrices will influence the S matrix, the distribution of the subset solution, and then the distribution of the test statistics. Therefore, the choice of fault hypothesis can seriously influence the distribution of the test statistics, and the fixed thresholds in the horizontal and vertical directions for each situation are not appropriate.

(3) The allocation of the false alarm rate is repeated. According to the calculation of threshold $T_{f,ENU}$, $K_{f,ENU}$ is a quantile determined by the false alarm rates in the horizontal and vertical directions. According to the allocation of the false alarm rate,

$$P_{FA} = P_{FA,VERT} + P_{FA,HOR} + P_{FA,HIL}. \tag{8}$$

The quantities in the east, north, and up directions are determined by the false alarm rates in the three directions. Regarding the 2D situation shown in Figure 3, the right diagonal lines and left diagonal lines shadow areas were calculated twice in crossed lines. Thus, the MHSS FD baseline problematically allocates the false alarm rates, which may be allocated repeatedly.

### 3. Proposed Threshold Method

To resolve the problem noted in Section 2, we propose an improved threshold method.

The maximum number of faulty satellites $s$ is determined by $P_{sat}$, $P_{const}$, and $P_{SAT,THRES}$. According to the milestone report [21], the typical value of $P_{sat}$ for GPS is regarded to be $10^{-5}$. If the value of $P_{sat}$ is set to $10^{-4}$, then the maximum number of faults that should be considered is 2, which is
considered the 1- and 2-fault hypotheses. Because we consider a fault hypothesis of 3 satellites. Therefore, we mainly consider the 3-fault hypothesis method.

Compared to the MHSS FD baseline, this method considered the spatial features of the test statistic and solved the problem of allocating the false alarm rate.

3.1. Proposed Threshold for the Single-Fault Hypothesis. For the fault hypothesis that excludes one satellite, if we only add biases on the pseudorange measurements of the satellite excluded by the subset, then the point cloud of the test statistics will be distributed along a line, as shown in Figure 3. The red cube is the threshold of the MHSS FD baseline.

Figure 3 shows the distribution of the test statistics when adding biases to the pseudorange measurements of the satellite excluded by the subset.

In the ideal case, the pseudorange measurements have no errors and should gather at the position of the all-visible satellite solution. However, if one of the satellites develops a fault, then the positioning solution, including the faulty measurements, should be pushed or pulled in the direction of the faulty satellite and result in an excursion on 1 degree of freedom (DOF). In the 3D space, an excursion of a DOF will result in a linear distribution of the positioning solutions. We call this direction "the main direction."

Thus, in Figure 3, errors in the measurements of the satellite with a supposed fault result in a linear distribution of the test statistics. Furthermore, if we add noise to the other satellites, then the point cloud will have excursions in several DOFs. Based on the linear distribution, there will be an ellipsoid distribution because of the smaller excursion compared to the faulty measurement. The direction of the semimajor axis of the ellipsoid is still the main direction. In addition, the main direction is the eigenvector of the test statistic covariance.

Considering that the threshold for monitoring a fault should cater to the distribution of the test statistics and the supposed fault is used to monitor whether the supposed satellite suffers the fault, the test statistics would be distributed in one DOF, resulting in a linear distributed point cloud. Thus, a threshold should be set on the line.

We can map the test statistics to the main direction and set the threshold to monitor the measurement of the supposed satellite. The main direction can be obtained from the eigenvector of the test statistic covariance, which is the first eigenvector of $\sigma_{s,s}^{(k,2)}$.

Therefore, for the single-fault $k$, the threshold can be determined by the direction of the eigenvector. We can calculate the first unit eigenvector and the eigenvalue of $\sigma_{s,s}^{(k,2)}$:

$$
\lambda' = e_{\text{value}}(\sigma_{s,s}^{(k,2)})
$$

$$
\tilde{e}_k' = e_{\text{vector}}(\sigma_{s,s}^{(k,2)}).
$$

$e_{\text{value}}$ is the unit eigenvector and $e_{\text{vector}}$ is the eigenvalue, and they are related by

$$
\lambda' = \tilde{e}_k'^T \sigma_{s,s}^{(k,2)} \tilde{e}_k'.
$$

Calculate the quantile of the false alarm rate:

$$
K_{fa,k}' = Q^{-1} \left( \frac{P_{fa}}{2N_{\text{fault modes}}} \right).
$$

It is regarded to be fault-free when the following conditions are met:

$$
(\Delta x_{k}^{(k,2)} \cdot \tilde{e}_k')^2 \leq K_{fa,k}' \lambda'.
$$

According to (7), the calculations of (13) are given in Figure 4.

<table>
<thead>
<tr>
<th>Number of visible satellites</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{sat}=10^{-5}$</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$P_{sat}=10^{-4}$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$P_{sat}=5\times10^{-4}$</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
3.2. Proposed Threshold for the Double-Fault Hypothesis. For the fault hypothesis that excludes two satellites, if we add biases on the pseudorange measurements of the two satellites excluded by the subset, the point cloud of the test statistics will be distributed in a plane, as shown in Figure 5. The red cube is the threshold of the MHSS FD baseline.

Figure 5 shows the distribution of the test statistics when adding errors on the pseudorange measurements of the two satellites excluded by the subset.

Similarly, if two of the satellites experience faults, the positioning solution, including the faulty measurements, will be influenced in two directions, resulting in excursions in two DOFs. In the 3D space, the excursion of the two DOFs will result in a planar distribution of the existing positioning solutions. We call this direction “the main plane.”

Thus, Figure 5 only adds biases to the measurements of the satellites with supposed faults, the test statistics will have excursions in two DOFs, and the distribution of the point cloud will be elliptical. We can map the test statistics onto the main plane and set the monitoring thresholds to monitor the measurements of the hypothetical two satellites. We can describe an ellipse by its semimajor and semiminor axes since they are uncorrelated. Similarly, an axis is determined by the first and second eigenvectors of $\sigma_{ss}^{(k)2}$. Therefore, for double faults $k$, the thresholds can be determined by the directions of the eigenvectors. We can calculate the first and second unit eigenvectors and the eigenvalues of $\sigma_{ss}^{(k)2}$:

$$
\begin{aligned}
\lambda_1 &= e_i g_{\text{value}}(\sigma_{ss}^{(k)2}), \\
\lambda_2 &= e_i g_{\text{vector}}(\sigma_{ss}^{(k)2}).
\end{aligned}
$$  

Figure 4: 2D Illustration of the test statistics on the single-fault hypothesis.
Calculate the semimajor and semiminor axes of the ellipse:

\[
\begin{align*}
\bar{a}'' &= \sqrt{\lambda_1'' \hat{e}_{\sigma_1''}}; \\
\bar{b}'' &= \sqrt{\lambda_2'' \hat{e}_{\sigma_2''}}.
\end{align*}
\]  (16)

In 2D space, the quantile of the false alarm rate can be obtained from the joint Gauss distribution function; it is explicitly calculated as

\[
K_{fa,q''} = Q''^{-1}\left(\frac{P_{FA}}{N_{fa\text{u}l\text{t}\_m\text{odels}}}\right)
= \sqrt{-2 \ln \left(\frac{P_{FA}}{N_{fa\text{u}l\text{t}\_m\textodels}}\right)}.
\]  (17)

The two-dimensional Gauss distribution is

\[Q''(u) = e^{-u^2/2}.\]  (18)

It is regarded as fault-free when the following conditions are met:

\[
\frac{(\Delta \tilde{e}_{\sigma_1''}^T \cdot \hat{e}_{\sigma_1''})^2}{\lambda_1''} + \frac{(\Delta \tilde{e}_{\sigma_2''}^T \cdot \hat{e}_{\sigma_2''})^2}{\lambda_2''} \leq K_{fa,q''}^2
\]  (19)

The calculations of (13) are given in Figure 6 by substituting the test statistics into the elliptic equation \(x^2/a''^2 + y^2/b''^2 = 1\).

3.3. Proposed Threshold for Triple-Fault Hypothesis. According to the previous analysis, the possibility of 3 faulty satellites is small. Nevertheless, we will briefly introduce the triple-fault case.

Repeat the simulation process for the previous section. If we add biases to the pseudorange measurements of the three satellites excluded by the subset, then the test statistics will be distributed in an ellipsoid-like point cloud, as shown in Figure 7. The red cubes still represent the thresholds of the MHSS FD baseline.

The triple-fault case is similar to the double-fault hypothesis case described in the last section; therefore, we will not reiterate the description here. In this case, the test statistics are distributed in three DOFs. The threshold should be set as an ellipsoid, which can be described by the first, second, and third eigenvectors of \(\sigma_{ss}^{(k)2}\) since they are uncorrelated.

Calculate the unit eigenvector and the eigenvalue of \(\sigma_{ss}^{(k)2}\):

\[
\begin{align*}
\hat{e}_{\sigma_1''} &= \text{eig}_1\text{value}(\sigma_{ss}^{(k)2}), \\
\hat{e}_{\sigma_2''} &= \text{eig}_2\text{vector}(\sigma_{ss}^{(k)2}), \\
\hat{e}_{\sigma_3''} &= \text{eig}_3\text{vector}(\sigma_{ss}^{(k)2}).
\end{align*}
\]  (20)

The lengths of the semiaxes are

\[
\begin{align*}
\bar{a}''' &= \sqrt{\lambda_1''' \hat{e}_{\sigma_1'''}}; \\
\bar{b}''' &= \sqrt{\lambda_2''' \hat{e}_{\sigma_2'''}}; \\
\bar{c}''' &= \sqrt{\lambda_3''' \hat{e}_{\sigma_3'''}}.
\end{align*}
\]  (22)

Calculate the quantile of the false alarm rate:

\[K_{fa,q'''} = Q'''^{-1}\left(\frac{P_{FA}}{N_{fa\text{u}l\text{t}\_m\textodels}}\right).\]  (23)

The three-dimensional Gauss distribution is

\[Q'''(u) = \sqrt{\frac{2}{\pi}} u e^{-u^2/2} + 2Q'(u).\]  (24)
Substituting the test statistics into the ellipsoid equation
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \]
the case is regarded as fault-free when the following conditions are met:
\[
\begin{align*}
\frac{(\Delta x^{[k]}T \cdot \vec{e}_{q''1})^2}{\lambda_1^{[q'')}} + \frac{(\Delta x^{[k]}T \cdot \vec{e}_{q''2})^2}{\lambda_2^{[q'')}} \\
+ \frac{(\Delta x^{[k]}T \cdot \vec{e}_{q''3})^2}{\lambda_3^{[q'')}} \leq K_{fa,q''}^{(k)}
\end{align*}
\] (25)

4. Simulation and Analysis

To verify the effectiveness and superiority of the algorithm, we detected faults in several positioning simulations based on MAAST from the Stanford GPS Lab. We obtained the ideal measurements from the presupposed satellite positions and user position and added artificial normally distributed noise to simulate the practical fault-free situation. In this case, the algorithm should not trigger an alarm. If we intentionally add large biases to the measurements of the supposed faulty satellite, then the algorithm should trigger an alarm. However, the greatest difficulty is distinguishing errors from noise. For a small noise case, a good threshold should be stable and not affected by the noise, and whether faults occur in the supposed measurement must be determined. In contrast, for a large noise, a weak algorithm would trigger an alarm regardless of the occurrence of a fault. Therefore, the most important point is how to set a proper threshold that can more effectively find the fault and not be affected by the noise. This chapter will verify the MHSS FD baseline and the proposed threshold for the three types of fault hypotheses by intentionally adding different sizes of errors and noise.

4.1. Single-Fault Hypothesis Case. We meshed the Asian-Pacific region within latitudes N55° - S55° and longitudes E70° - E150° and introduced a deviation in the pseudorange in the measurements of the supposed faulty satellite at each gridpoint.

Table 2 shows the parameters of the simulation for the single-fault hypothesis case. It is worthy of note that this work aims to detect the fault measurements. For simulation, if the proposed threshold could detect the artificial biases well, we consider that the threshold is effective. Thus, the artificial biases are quite important. The biases in the paper are the manifestation of an artificial fault in the pseudorange domain and are different from the nominal errors and biases.

Figure 8 shows the resulting comprehensive statistics from the simulation for GPS and BDS.

Figure 8 provides the alarm rates of the MHSS FD baseline and proposed method in the case of a single fault, whose measurement pseudorange deviations have been artificially added. The MHSS FD baseline is tighter than the proposed method in this case. This finding is related to the repeating distribution of the probability of false alarm.
Table 2: Simulation parameters for the single-fault hypothesis case.

<table>
<thead>
<tr>
<th>Constellations</th>
<th>GPS+BDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of SVs</td>
<td>GPS: 24, BDS: 14</td>
</tr>
<tr>
<td>$P_{\text{detection}}$</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>$P_{\text{FA}}$</td>
<td>4x10^{-6}</td>
</tr>
<tr>
<td>$P_{\text{FA,VERT}}$</td>
<td>3.9x10^{-6}</td>
</tr>
<tr>
<td>$P_{\text{FA,HOR}}$</td>
<td>1x10^{-7}</td>
</tr>
<tr>
<td>URA</td>
<td>1 m</td>
</tr>
<tr>
<td>URE</td>
<td>0.66 m</td>
</tr>
<tr>
<td>User grid</td>
<td>55° S-55° N, 70° E-150° E</td>
</tr>
<tr>
<td>Time</td>
<td>86400 s, interval of 300 s</td>
</tr>
<tr>
<td>Mask angle</td>
<td>3°</td>
</tr>
<tr>
<td>Artificial bias</td>
<td>1 m-15 m</td>
</tr>
<tr>
<td>Repetitions</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 8: Comparison of the alarm rates for the MHSS FD baseline and proposed method in the case of a single-fault hypothesis.

alarms and the structure of the proposed threshold method. However, regardless of whether the detection is strict, the availability of the algorithm cannot be determined because the threshold depends on the repeating distribution of the probability of false alarms. A detailed analysis is needed to determine whether the algorithm can exactly detect the faults of supposed faulty satellites to judge its availability.

Since Figure 8 is based on comprehensive comparisons of results observed over a long duration (86400 s) and between many locations (55° S-55° N, 70° E-150° E), after the average calculations for large volumes of data, the detection rate curves were relatively smooth. However, for the detection of specific single points in time and in a single location, the differences in the results were generally large, as discussed in detail below.

Figures 9–12 show the alarm rates for PRN1 satellite faults and analyses of the performances of the two algorithms under other satellite noises at given times and places that are described in Table 3. In fact, the detection rate curves were similar over the simulation experiments. As examples, we chose several typical locations in China and two simulation times when the PRNI satellite was visible, as shown in Table 3. The other parameters are the same as those in Table 2. In Figures 9–12, each curve represents an alarm rate at a time and place.

Figures 9–12 show the alarm rates with varying degrees of noise. Under an ideal condition, the detection of faults is relatively easy because there are no deviations in the pseudorange for any satellite. However, noise is observed in the pseudoranges of all satellites, and it will impact the fault detection results. The pseudorange noises of all satellites may increase in the actual satellite navigation operations and, in that moment, if the stability of the fault detection algorithm is poor, then false alarms will occur. We determined the pseudorange of a supposed faulty satellite with a fault and added 1-2.5 $\sigma$ noise to the pseudoranges of the other satellites, and the simulation results in Figures 9–12 show that the proposed threshold method can guarantee greater detection stability for 1-2.5 $\sigma$ noise. Indeed, this solution still achieved good detection with 2 $\sigma$ noise interference, whereas the MHSS FD baseline was unstable.

The analysis of the alarm rate at a given time and place has been provided above. The MHSS FD baseline will produce a false alarm when the error for the supposed faulty satellite is small, and the reasons are discussed below by increasing the error in the supposed faulty satellite gradually and analyzing the effect of the other satellites’ noise on the alarm rate.

In Figure 8, two solutions have similar detection curves when the other satellites had normal noise less than 1$\sigma$, and the average detection rate remained low from long-term observations when we added artificial errors of 1-4.5 meters to the supposed faulty satellite. However, if we increase the other satellites' noise, then the alarm rate results from the MHSS FD baseline were badly affected, as shown in Figure 13.

In Figure 13, the x-coordinate represents the other satellites’ noise, the y-coordinate is the alarm rate, and each line was calculated from the errors introduced artificially to the supposed faulty satellite. If we increase the other satellites' noise, then the detection results will be greatly affected because the addition of the other satellites' noise will lead to a serious false alarm condition. However, the proposed method has little influence and is strongly stable.

The reason why the proposed threshold method can improve the detection performance under the hypothesis of a single fault is discussed in detail. Figure 14 shows an illustration of the single-fault detection results in 2D. The blue points represent the distribution of the test statistics, and the red rectangle (a cube in 3D space) represents the MHSS FD baseline threshold. $\Delta \bar{x}^{(k)}$ located outside the rectangle indicates a fault via the MHSS FD baseline, and $\Delta \bar{x}^{(k)}$ inside the green zone indicates a fault via the proposed threshold method.

As shown in Figure 14, the threshold of a single-fault hypothesis should be tangent to the MHSS FD baseline threshold in 2D. The distribution of the test statistics is
### Table 3: Simulation locations and time.

<table>
<thead>
<tr>
<th>No</th>
<th>Simulation point</th>
<th>Location</th>
<th>Time point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Urumchi</td>
<td>E87.474 N43.907</td>
<td>0 s, 3600 s</td>
</tr>
<tr>
<td>2</td>
<td>Harbin</td>
<td>E126.250 N45.623</td>
<td>0 s, 3600 s</td>
</tr>
<tr>
<td>3</td>
<td>Sanya</td>
<td>E109.412 N18.302</td>
<td>0 s, 3600 s</td>
</tr>
<tr>
<td>4</td>
<td>Shanghai</td>
<td>E121.792 N31.143</td>
<td>0 s, 3600 s</td>
</tr>
<tr>
<td>5</td>
<td>Xi'an</td>
<td>E108.752 N34.446</td>
<td>0 s, 3600 s</td>
</tr>
</tbody>
</table>

**Figure 9:** Alarm rates for the two methods with 1σ noise in the single-fault case.

**Figure 10:** Alarm rates for the two methods with 1.5σ noise in the single-fault case.
ellipse-like and has an obvious distribution direction, and the red rectangle of the MHSS FD baseline should not be used. Based on the analysis in Section 3.1, the supposed faulty satellite fault will lead to the extension of the ellipse-like distribution in the main direction. Setting a threshold in the main direction can allow the detection of the supposed faulty satellite, especially without the interference of other noise.

4.2. Double-Fault Hypothesis Case. Most of the double-fault hypothesis simulation parameters were the same as those listed in Table 2. We introduce deviations of the pseudoranges in the measurements for the two supposed faulty satellites at each grid point. Figures 15(a) and 15(b) show the resulting comprehensive statistics from the simulation and compare the detection rates of the proposed method and the MHSS
Figure 13: Alarm rates of the two methods with other satellite noise.

(a) MHSS FD baseline

(b) Proposed threshold method

Figure 14: Illustration of the fault monitoring results in the case of the single-fault hypothesis.

FD baseline for a case with two satellites with different faults. The x-coordinate and y-coordinate represent the deviations of the faulty satellites, and the z-coordinate is the alarm rate. Figure 15(c) shows the difference between the detection results of the MHSS FD baseline and the proposed method.

Figure 16 shows a diagonal section of Figure 15, and the conclusion is similar to that for the case of a single faulty satellite.

Similar to the case of a single fault, Figures 17–20 show the alarm rates of the PRN1 and PRN2 satellite faults and analyze the performances of the two algorithms under other satellite noises at a given time and place. Each curve represents a detection rate at a particular time and place.

The MHSS FD baseline was unstable under the interference of 2σ noise, whereas the proposed method still achieved good detection. In addition, the solution maintained good stability until the interference of 2.5σ noise, under which the MHSS FD baseline was almost unusable.

Figure 21 shows an illustration of the double-fault detection results in 2D. The blue points represent the distribution of the test statistics. The threshold for the MHSS FD baseline is shown as a red rectangle. \( \Delta x^{(k)} \) located outside of the rectangle indicates a fault by the MHSS FD baseline, and \( \Delta x^{(k)} \) located outside of the purple ellipse indicates a fault by the proposed threshold method. In Figure 21, the orange points represent the standard deviations of the blue points in all directions of the unit vector.

The MHSS FD baseline only selects \( c_{k,i}^{(k)} \) on the x- and y-coordinate axes, which is the red rectangle shown in Figure 21. Therefore, the proposed threshold is an omnidirectional extension of the MHSS FD baseline in the case of double faults.

The single-fault threshold should be tangent to the MHSS FD baseline threshold in 2D, as shown in Figure 21. The distribution of the test statistics was ellipse-like and had an obvious distribution direction, and the red rectangle of the MHSS FD baseline should not be used. Based on the analysis in Section 3.2, the supposed faulty satellite will lead to the extension of the ellipse-like distribution in the main plane.
The supposed faulty satellite can therefore be detected by setting a threshold in the main plane, especially without the interference of other noise.

4.3. Triple-Fault Hypothesis Case. For the case of the triple-fault hypothesis, Figure 22 illustrates the double-fault detection results. The blue points represent the distribution of the test statistics. The threshold of the MHSS FD baseline is shown as a red cube. $\Delta x^{(k)}$ located outside of the cube indicates a fault by the MHSS FD baseline, and $\Delta x^{(k)}$ located outside of the ellipsoid indicates a fault by the proposed threshold method.

As in the analysis in Sections 4.1 and 4.2, the cubic threshold did not fit the distribution of the test statistics. Setting an ellipsoid-like threshold calculated from the distribution will be more suitable for fault detection.

We also simulated the alarm rates under noise. The parameters were identical to those of the single-fault hypothesis, and they do not need to be reiterated here. The results are shown in Figures 23 and 24. Each curve represents a detection rate at a time and place.

From Figures 23 and 24, we can also conclude that the proposed threshold method is more stable than the MHSS FD baseline.

It is worth noting that the triple-fault detection followed the single-fault and double-fault detections.

4.4. Comparison between the Proposed Method and MHSS FD Baseline. The MHSS FD baseline is the most authoritative, most commonly used algorithm, and few people have studied the MHSS FD threshold. In fact, though, the MHSS FD baseline is not the most suitable method for fault detection and still needs to be improved. Based on this, we analyzed the distributions of the test statistics under single-, double-, and triple-fault hypotheses. By extracting the eigenvectors and eigenvalues of the solution separating variance, we were able to design an omnidirectional threshold structure.

A comparison of the proposed threshold and MHSS FD baseline shows that the new threshold is more suitable than the MHSS FD baseline given the distribution of the test statistics. First, we analyzed the distributions of the test statistics under single-, double-, and triple-fault hypotheses and proposed an elliptical threshold structure instead of the original cubic threshold. The single-fault threshold should be tangent to the MHSS FD baseline threshold for any fault mode, as indicated by the analysis in this chapter. By analyzing the simulation data, we can determine that the detection area of the proposed threshold method is reduced by 25% compared to the MHSS FD baseline. Then, we compare the alarm rates of the two methods under different conditions, and the two solutions have similar detection curves when the other satellites had normal noise less than $1\sigma$. The average detection rate remained low from long-term observations when we added artificial errors of 1-4.5 meters to the supposed faulty satellite. In summary, the proposed method can achieve good detection with $2.5\sigma$.
noise interference, while the MHSS FD baseline starts to be unstable at 2σ noise interference.

The appearance of $\Delta \hat{x}^{(k)}$ in the space between the proposed method and the MHSS FD baseline threshold is caused by the supposed faulty satellite being fault-free; however, the deviations of the other satellites’ pseudoranges are large. If the MHSS FD baseline directly identified a fault, then the significance of the multiple hypotheses cannot be embodied.

The single-fault threshold should be tangent to the MHSS FD baseline threshold for any fault mode, as indicated by the analysis in this chapter, because the MHSS FD baseline can only calculate the projections on the x-coordinate, y-coordinate, and z-coordinate. If the standard deviations of the test statistics are processed omnidirectionally, then we can obtain a more accurate method. The solution is an omnidirectional generalization of the MHSS FD baseline, and the MHSS FD baseline is a special situation of the proposed method on the x-coordinate, y-coordinate, and z-coordinate.

A single fault is a deviation on 1 degree of freedom and a double fault is a deviation on 2 degrees of freedom.
and so on. The threshold model is the fault detection for the corresponding degrees of freedom. Therefore, the single-fault model degenerates from the double-fault model, which degenerates from the triple-fault model. An ellipsoid in 3D therefore degrades into an ellipse in 2D and a line in 1D.

5. Results Discussion

In the case of a single fault, Figure 3 shows that we just need to set the threshold at the main direction of the fault satellite and do not need to detect the faults in x, y, and z, respectively, which shows that the proposed threshold reduces the area of detection and simplifies the inspection process. From Figure 8, the alarm rate of the MHSS FD baseline is higher than that of the proposed method with the same bias and even almost up to 40% higher when the bias is lower than 6 m, which means that the MHSS FD baseline is more conservative and sacrifices availability for a lower alarm rate. Figures 9–12 show the results of a simulation experiment carried out under the single algorithm and the curves of the alarm rate are similar at each time point and position, but comparing the two methods, the proposed method is more
stable than the MHSS FD baseline. When the bias is less than 6 m, the proposed method can maintain a much lower alarm rate. When we added $2\sigma$ errors artificially and set PRN1 errors of 3 m and 5 m, the proposed method reduced the alarm rate by 5% and 40%, respectively.

In the case of two or three faults, Figures 17–20 and Figures 23 and 24 also showed that, in the same configuration, the MHSS FD baseline was unstable with $2\sigma$ noise interference, while the proposed method showed low but acceptable stability with $2.5\sigma$ noise interference. Figures 21 and 22 show a comparison of the threshold of the two methods. In short, the proposed threshold method can guarantee greater detection stability for $1-2.5\sigma$ noise under one, two, and three faults. Indeed, this solution still achieved good detection with $2\sigma$ noise interference, whereas the MHSS FD baseline was unstable. We can conclude that the detection area of the proposed threshold method was reduced by 25% compared to the MHSS FD baseline.

6. Conclusions

The MHSS FD baseline detects faults by checking the consistency of the solutions from different subsets of the all-in-view set. However, the threshold structure of the MHSS fault monitoring baseline algorithm is cuboid, which means that the MHSS FD baseline compares the test statistics with a predetermined threshold in the east, north, and up directions.

By analyzing the distributions of the test statistics under different faults, we believe that the MHSS FD baseline is not the most suitable method for fault detection, and thus we consider that the MHSS FD baseline needs to be improved. (1) The cuboid threshold structure lacks omnidirectionality, which leads to a low conformity between the threshold and the spatial distribution of the test statistic. (2) The test statistic distributions of all types of fault hypotheses differ, and the spatial distributions of the test statistics of different faults are difficult to describe using one threshold. (3) In the threshold determination process, the distributions of the false alarm rate in the horizontal and vertical directions are counted twice. Therefore, the MHSS FD baseline leads to a low fault monitoring accuracy.

No one has studied this before, so we proposed a new method that is more consistent with the distribution of test statistics to resolve these problems; we analyzed the distributions of the test statistics under single-, double-, and triple-fault hypotheses. We added artificial errors of 2-5.5 meters to the supposed faulty satellite and can conclude that the alarm rate of the proposed threshold was reduced by 40%. By extracting the eigenvectors and eigenvalues of the solution separating variance, we were able to design an omnidirectional threshold structure.

For the single-fault hypothesis, we mapped the test statistics to the main direction and set the monitoring threshold to monitor the measurements of the supposed faulty satellite. For the double-fault hypothesis, the threshold is described by an ellipse, and the axis is determined by the first and second eigenvectors of the test statistics. For the triple-fault hypothesis, the threshold is described by an ellipsoid, and the axis is determined by the eigenvectors of the test statistics.

This manuscript proposed a method for a monitoring threshold and compared it with the MHSS FD baseline threshold and its alarm rate. The simulation results show that the proposed method is more exact, stable, and applicable than the MHSS FD baseline.

In 2018, the Beidou No. 3 system has been completely built and is going to provide global services. We will thus verify the new proposed method based on the actual data of BDS and utilize the system data to optimize our method, focusing on the accuracy of the threshold. In addition to verifying the reasonableness of the threshold setting, we will also analyze and optimize the performance of the method based on actual data, including increasing its availability, protection level, vertical accuracy, and integrity risk.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Figure 22: Illustration of the fault monitoring results in the case of the triple-fault hypothesis (observed from two directions).

Figure 23: Alarm rates of the two methods with 1.5σ noise in the triple-fault case.

Figure 24: Alarm rates of the two methods with 2σ noise in the triple-fault case.
References


