Research Article

Two-Period Dynamic versus Fixed-Ratio Pricing Policies under Duopoly Competition

Hao Li,1,2 Xi Yang,1 Yu Tu,1 and Ting Peng1

1School of Economics and Management, Chongqing Jiaotong University, Chongqing 400074, China
2Western China Transportation Economy-Society Development Studies Center, Chongqing 400074, China

Correspondence should be addressed to Xi Yang; 1903366491@qq.com

Received 16 November 2018; Accepted 4 March 2019; Published 28 March 2019

Academic Editor: Vyacheslav Kalashnikov

Copyright © 2019 Hao Li et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper introduces a two-period, pricing policy under duopoly competition between two firms offering an identical product to consumers who are intertemporal utility maximization. Firms have equal inventories of faultlessly replaceable and perishable products. The firms adjust prices to maximize profits and determine optimal pricing policies, choosing from dynamic pricing, fixed-ratio pricing, and elastic pricing policies. According to a duopoly competition model, the consumer is limited to a single firm visit per period. The consumer decides to purchase the product at current price from a firm and remain in the market to purchase product from the other firm in the next period or exit the market. The results offer three main conclusions. First, elastic pricing is consistent with dynamic pricing. Second, the more consumers visit the firm in the first period, the more profits the firm will make. Third, we explore the effectiveness of different pricing policies. The results show that although dynamic pricing is a more complex policy than fixed-ratio pricing, it may lead to decreased equilibrium profits when the firms sharply discounts prices and consumer rationality is unlimited.

1. Introduction

Revenue management is an important part of a firm’s operations, especially in the service industry such as the airline, cruise tourism, hotel and motel, music tickets, and apparel industries [1, 2]. Firms have several challenges, such as managing consumer’s valuations of their products and small marginal costs among product varieties. Additionally, managing perishable product inventory can be particularly challenging, as an expired product’s residual value is approximately zero. For example, airline tickets are worthless after the flights take off, and music tickets have no value after the concert begins.

Setting prices is one of the essential factors that affect revenue management, but it is also one of the most difficult decisions [3–5]. Common pricing policies include dynamic pricing and fixed-ratio pricing policies. For instance, airlines dynamically adjust prices to maximize profits within a selling season. In contrast, sellers of fresh products (e.g., breads, milk), are willing to preannounce future prices. Firms have the difficult task of adopting the optimal pricing policy to maximize their profits.

In this paper we evaluate three pricing policies, dynamic pricing, fixed-ratio pricing, and elastic pricing. Under dynamic pricing policy, firms announce their current price, simultaneously observe their current selling situation, and then determine whether to alter the price in the next period. If the sale goes well in the current period, firms may increase the price in later period to capture more sales from consumers who are willing to pay more to purchase the same product. However, if the sale is poor in the first period, firms may decrease the price to sell more and minimize losses because unsold products have no value. Under fixed-ratio pricing, firms make their prices for whole periods at the beginning. In fact, only one pricing decision can be made by firms in each selling season. Thus, it is a static pricing strategy compared with dynamic pricing. In reality, firms in the market do not choose the same pricing policy, and elastic pricing is more common than dynamic and fixed-ratio pricing strategies. Typically, one firm adopts dynamic pricing while another firm chooses fixed-ratio pricing.

This study aims to determine a firm’s optimal pricing policy in a duopolistic market. In our model, there are two
profit-maximizing firms, each firm's inventory can satisfy the whole market, and they compete over prices. The strategic consumers, who strategically attempt time product purchases with lower prices to maximize individual utility, visit each firm to gather price information. We neglect search costs but assume that, due to these costs, the consumer is limited to a single firm visit per period. We refer to this type of competition as zigzag competition and note that it exposes the two firms to the difficulties of pricing correctly in a competitive market [6].

Our analysis produced several meaningful results. First, the equilibrium state of static pricing is consistent with that of dynamic pricing. Second, given a finite number of consumers in the market, a firm's optimal pricing policy is the same as in the single consumer case under different pricing policies. Further, the more consumers who visit the firm in the first period, the more profits the firm will make. Third, although dynamic pricing is a more complex policy than fixed-ratio pricing, it may lead to decreased equilibrium profits. There are two factors that play an important role to this result: firms sharply discount prices, and consumers' rationality is unlimited. Otherwise, dynamic pricing policy is better.

This study is based on the following key concepts: strategic consumer behavior, pricing policy, and zigzag competition. We review the relevant literature related to these concepts and establish our study within the relevant literature.

In this study, we consider strategic consumer behavior, which has received considerable attention in the recent revenue management literatures. Coase [7] introduced the concept of strategic consumer behavior in revenue management. Besanko and Winston [8] introduced this concept in relation to dynamic pricing and studied the intertemporal pricing problem for a monopolistic firm marketing a new product. The researchers point out that if a monopolistic firm facing rational consumers implements the optimal myopic consumer pricing policy, profits can be significantly less than if the monopolistic firm follows the equilibrium pricing policy for rational consumers. Su [5] divided consumers into four types based on different degrees of valuation and waiting costs and showed that intertemporal pricing with strategic consumer behavior does not necessarily damage the profits of retailers. Prasad et al. [9] studied the two-period dynamic pricing of monopolistic manufacturers facing the market in the presence of myopic consumers and strategic consumers and found that the profits of manufacturers increase as the number of strategic consumers increase.

These studies mainly investigated strategic consumer behavior but ignored the influence of consumer behavior on the firms' pricing policies. This paper aims to fill this gap in the literature. Many papers deal with dynamic pricing in the presence of strategic customers. Levin and McGill [10] considered a dynamic pricing model for a monopolistic firm selling a perishable product to a finite population of strategic consumers and proved that dynamic price adjustments in the presence of strategic consumers will increase the firm's profits. Dasu and Tong [11] studied the dynamic pricing policy of a monopolistic firm selling perishable products to strategic consumers during a limited sales period and discussed the effect of price changes on a firm's profits. Zhao et al. [12] introduced a dynamic pricing policy for a monopolistic firm selling perishable goods to consumers who may be influenced by inertia. It is proved that consumers' inertia will have a negative impact on firm's expected profits.

Fixed-ratio pricing policy under strategic consumer behavior is also an important topic in marketing. Aviv and Pazgal [13] studied the problem of optimal pricing under contingencies and found a fixed-discount pricing policy in the presence of strategic consumers with a single seller. In addition, the researchers observed that an announced pricing policy can be advantageous to the seller, compared to contingent pricing schemes under strategic consumer behavior. Su and Zhang [14] found that fixed-ratio pricing, as an optimal pricing strategy, created the incentive to overcompensate consumers under the assumption of out-of-stock costs. Babaioff et al. [15] also considered the problem of maximizing revenue given limited supply. Moreover, Chen et al. [16] investigated monopolistic firms facing rational consumers and reached similar conclusions. Correa et al. [17] studied the fixed-ratio pricing policy of a monopolistic manufacturer under limited inventory; the results showed that the fixed-ratio pricing policy is more effective. Kim et al. [18] evaluated the uniqueness of the equilibrium behavior of strategic consumers making intertemporal purchasing decisions; they found that a unique symmetric equilibrium exists when there are no more than three buyers whose valuations follow a uniform distribution with fixed-ratio pricing. Li and Pu [19] considered a two-period model in which an e-retailer can use different pricing policies to manage consumers' price comparison behaviors: dynamic pricing or fixed-ratio pricing policy. They found that e-retailers prefer fixed-ratio pricing rather than dynamic pricing policy and determined the optimal price discount in the fixed-ratio pricing policy.

Most models analyzed in the aforementioned studies only consider scenarios of consumers with a single firm, and only a few papers examine competitive marketing with more than one firm facing strategic consumers. Lin and Sibdari [20] established a competition model among multiple manufacturers, discussed the price equilibrium with consumer strategic behavior, and proved the existence of Nash equilibrium. Zhang et al. [21] studied the dynamic pricing mechanism of perishable products that takes into account both the threat of competitors' entry and consumer behavior. The results showed that incumbent enterprises could adopt intelligent dynamic pricing mechanisms to maximize their own revenue according to the proportion of strategic consumers.

Several other papers consider two kinds of pricing policy in competitive markets. Dasci [22] analyzes the impact of dynamic and fixed-ratio pricing policies on firm profits and equilibrium prices under competition. The conclusion is that fixed-ratio pricing policy may lead to greater equilibrium profits, compared to dynamic pricing policy. Liu and Zhang [23] considered dynamic pricing competition between two firms offering vertically differentiated products and showed that there exists a Markov perfect equilibrium of mixed strategy in the game. They also considered the model where
Materials and Methods

2.1. The Model. Each of the two competing firms, A and B, holds equal inventory and satisfies the whole demand in the market. Each has in stock \( N \) units of inventory. The two firms are risk-neutral and pursue maximum profits. We assume that there are no vertical differentiations between products that are offered by the two firms. The products are perishable and will spoil and lose their value if not sold before a certain time. Some products may have some salvage value after they expire, but we ignore this and normalize the value to zero in the model.

Strategic consumers do not have the firms’ periodic market pricing information. To obtain that information, the consumers must visit all the firms in order to observe all the different prices. Due to distance and search costs, the consumer is limited to a single firm visit per period. We assume that the consumer first visits one firm to obtain their pricing information and then visits the other firm in the following period. Thus, the consumers visit the firms in a zigzag manner to maximize their net utility as shown in Figure 1.

In this paper, we focus on the impact of different pricing policies on firms’ profits under duopoly competition. It differs from the existing literature in that we examine three kinds of pricing policies: dynamic, fixed-ratio, and elastic pricing. Although they are similar in setting, we focus on a duopoly competition scenario and examine firms’ optimal profit equilibrium. The differences between this paper and the existing literature are shown in Table 1.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Market Circumstances</th>
<th>Dynamic Pricing</th>
<th>Fixed-Ratio Pricing</th>
<th>Strategic Consumer Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5, 8–12]</td>
<td>Monopoly</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>[20, 21]</td>
<td>Competition</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>[22, 23]</td>
<td>Competition</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>[6]</td>
<td>Zigzag Competition</td>
<td>✓</td>
<td>x</td>
<td>X</td>
</tr>
<tr>
<td>[24]</td>
<td>Zigzag Competition</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>This Paper</td>
<td>Zigzag Competition</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

2.2. Section 2.2, Section 2.3, and Section 2.4 evaluate the equilibrium results between two firms under dynamic, fixed-ratio pricing and elastic pricing strategies, respectively. Section 3 reports results and discussion. In Section 4 we conclude and briefly discuss managerial insights and implications for future research.
The firms can choose a dynamic pricing policy to sell the products during the period of sale. The firms can sell products through a fixed-ratio pricing policy. One firm chooses the dynamic pricing policy while another firm chooses the fixed-ratio pricing policy. Since the firms sell products over two periods and possibly change prices, the consumers strategically choose when to purchase the product based on their valuation and the pricing policy of firms. We differentiate the three pricing policies using the superscripts \( d \), \( f \), and \( e \) to represent dynamic, fixed-ratio, and elastic pricing, respectively.

2.2. Dynamic Pricing. In this section, we analyze optimal price and equilibrium profits in the dynamic pricing policy. The subsequent subsections are devoted to the analysis of two cases: (1) the one consumer and (2) \( N \) consumers. Two firms post prices for two periods with \((R^{d}_{1i}, R^{f}_{1j})\). The decision-making process is as follows.

2.2.1. One Consumer. In this subsection, we simplify the process and analyze the basic model with one consumer in the market. We refer to the firm who encounters the consumer in the first period as firm \( i \) and the other firm as firm \( j \). We further assume that when firm \( i \) encounters the consumer in the first period, the consumer will visit the competing firm \( j \) in the second period.

In each period the price of the firm not visited by the consumer is not relevant, and no assumption is needed for a firm’s knowledge about the other firm’s prices. However, since both firms solve the same model, firm \( i \) can compute firm \( j \)’s prices. The consumer needs to consider the intertemporal choice between two firms over two periods, where each consumer decides which firm to purchase from and when to purchase in order to maximize consumer surplus. A consumer can also decide to not purchase, in which case he/she receives zero surplus.

We denote the expected profit of firm \( i \) as \( \pi^{d}_{1i}(1,0) \), when there is one consumer in the market. When firm \( i \) is visited by the consumer in the first period, we have the following:

\[
\pi^{d}_{1i}(1,0) = \max_{R^{d}_{1i}} R^{d}_{1i} \left( 1 - \frac{R^{d}_{1i} - R^{d}_{2j}}{1 - \theta} \right)
\]

Similarly, the expected profit of firm \( j \) can be written as follows:

\[
\pi^{d}_{2j}(0,1) = \max_{R^{d}_{2j}} R^{d}_{2j} \left( 1 - \frac{R^{d}_{2j}}{1 - \theta} \right)
\]

According to the above analysis, the two firms’ equilibrium price and profits can be directly calculated when there is one consumer in the market, as shown in Theorem 1.

To simplify presentation, we define

\[
K = \sqrt{17\theta^2 - 12\theta + 4}, \quad i, j = A, B, i \neq j
\]

These notations will be used throughout the paper.

**Theorem 1.** In the dynamic pricing policy, when there is only one consumer in the market, the optimal price of two firms is as follows:

\[
R^{d}_{1i} = \frac{10 - 7\theta - K}{16}
\]

\[
R^{d}_{2j} = \frac{2 + \theta - K}{8}
\]

The expected profits of two firms are as follows:

\[
\pi^{d}_{1i}(1,0) = \frac{(K + 7\theta - 10)^2}{256(1 - \theta)}
\]

\[
\pi^{d}_{2j}(0,1) = \frac{(2 + \theta - K)(6 - 9\theta + K)(7\theta - 2 + K)}{1024\theta(1 - \theta)}
\]

The proof of Theorem 1 is in the Appendix. Theorem 1 has some important economic implications. First, the optimal price illustrates how the firms’ profits can be obtained directly when there are \( N(N > 1) \) consumers in the market, as shown below.

2.2.2. \( N \) Consumers. In this subsection we extend the model to \( N \) consumers. We assume the consumers necessarily zigzag as two groups. That is, in the first period \( N_{1i} \) consumers visit firm \( i \), and \( N_{1j} \) consumers visit firm \( j \), where \( N_{1i} + N_{1j} = N \). The consumers maintain the zigzagging search pattern over periods. We denote \( N_{1i} \) as the number of consumers who visit firm \( i \) in period \( t \). Let \( \pi^{d}_{1i}(N_{1i}, N_{1j}) \) denote the profit of firm \( i \), \( i = A, B \), when there are \( N_{1i} \) consumers visiting the firm \( i \) in the market, and there are \( N_{1j} \) consumers in the market in period \( t \).

Because the consumers’ valuations may be different, it is possible that some consumers will purchase products in the first period while the others will continue the zigzag search. Let \( P^{d}_{N_{1i}, n_{1i}} \) denote the probability that firm \( i \), who is visited by \( N_{1i} \) consumers and sells \( n_{1i} \) units in the first period. Thus, in the first period, \( P^{d}_{N_{1i}, n_{1i}} = (N_{1i}/n_{1i}!)(N_{1i} - n_{1i})!F((R^{d}_{1i} - R^{d}_{2j})/(1 - \theta))^{n_{1i}}. \) In the same way, \( P^{d}_{N_{1j}, n_{1j}} \) denotes the probability that firm \( i \), who is visited by \( N_{1j} - n_{1j} \) consumers and sells \( n_{1j} \) units in the second period.
So, in the second period the probability is $P_{N_{1j}}^{d} - n_{i1} n_{u} = ((N_{1j} - n_{i1})! n_{u}!(N_{1j} - n_{i1} - n_{u}!) F(R_{2i}^{d}/\theta) N_{1j} - n_{i1} - n_{u}! [1 - F(R_{2i}^{d}/\theta)] ) n_{u}$. The total expected profits of firm $i$ are given by

$$\pi_{i}^{d}(N_{1i}, N_{1j}) = \max_{R_{ii}} \left( \sum_{n_{t}=0}^{N_{1i}} P_{N_{1i}, n_{t}}^{d} n_{t} R_{ii}^{d} \right) \left( 1 - n_{t}/N_{1i} \right)$$

We can see that since the number of products sold is a variable in the first period; the number of consumers reaching the competing firm is difficult to be estimated in the second period. We simplify the calculation and discuss the equilibrium decision of the two firms.

**Theorem 2.** In the dynamic pricing policy, when there are $N(N > 1)$ consumers in the market, the expected profits of two firms are as follows:

$$\pi_{i}^{d}(N_{1i}, N_{1j}) = N_{i} R_{ii}^{d}(1, 0) + N_{j} R_{ij}^{d}(0, 1)$$

$$\pi_{j}^{d}(N_{1i}, N_{1j}) = N_{j} R_{jj}^{d}(1, 0) + N_{i} R_{ij}^{d}(0, 1)$$

$$\pi_{i}^{d}(1, 0) \text{ and } \pi_{j}^{d}(0, 1) \text{ are shown in (5) and (6); they are the firms' expected profits under the dynamic pricing policy, in the market with a single consumer, where } i, j = A, B \text{ and } i \neq j.$$

The proof of Theorem 2 is in the Appendix. Theorem 2 shows that the resulting pricing given $N$ consumers is the same given one consumer. The pricing policy is independent of the number of consumers in the market, as long as each of the firms can satisfy the whole market demand.

### 2.3. Fixed-Ratio Pricing

2.3.1. **One Consumer.** The following analysis is consistent with dynamic pricing. Firm $i$ is visited by a consumer in the first period, and we have the following equation:

$$\pi_{i}^{f}(1, 0) = \max_{R_{ii}} \left( R_{ii}^{f} \left( 1 - \frac{R_{ii}^{f} - \beta R_{ij}^{f}}{1 - \theta} \right) \right)$$

where $\pi_{i}^{f}(1, 0)$ is the expected profits from selling one unit in the first period when a consumer first visits firm $i$.

Similarly, the expected profit of firm $j$ can be written as follows:

$$\pi_{i}^{f}(0, 1) = \max_{R_{ij}} \left( R_{ij}^{f} \left( 1 - \frac{\beta R_{ij}^{f}}{1 - \theta} \right) \right)$$

According to the above analysis, the firms’ equilibrium prices and profits can be directly calculated when there is a consumer in the market, as shown in Theorem 3.

**Theorem 3.** In the fixed-ratio pricing policy, when there is only one consumer in the market, the optimal prices of two firms are as follows:

$$R_{ii}^{f} = R_{ij}^{f} = \frac{2 + \theta - K}{8\beta}$$

The expected profits of two firms are

$$\pi_{i}^{f}(1, 0) = \frac{(K - 2 - \theta)(\beta K + 7\beta \theta - K - 10\beta + \theta + 2)}{64\beta^{2}(1 - \theta)}$$

$$\pi_{i}^{f}(0, 1) = \frac{(2 + \theta - K)^{2} [K(\beta - 1) - (1 - \beta)(\theta + 2)](2 - 7\theta - K)}{512\beta(1 - \theta)}$$

According to Theorem 3, the simplified calculation of the firms’ profit can be obtained directly when there are $N(N > 1)$ consumers in the market, as shown below.

2.3.2. **$N$ Consumers.** Let $P_{N_{1i}, n_{i}}^{f}$ denote the probability that firm $i$ sells $n_{i}$ units, when $N_{1i}$ consumers visit the firm. Thus, in the first period, $P_{N_{1i}, n_{i}}^{f} = ((N_{1i} - n_{i})!/(N_{1i} - n_{i}!)) F((R_{ii}^{f} - \beta R_{ij}^{f})(1 - \theta) N_{1i} - n_{i}! [1 - F(R_{ii}^{f} - \beta R_{ij}^{f})(1 - \theta)]) n_{i}$. Similarly, let $P_{N_{1j}, n_{j}, n_{u}}^{f}$ denote the probability that firm $i$, who is visited by $N_{1i} - n_{i}$ consumers and sells $n_{u}$ units in the second period, $P_{N_{1j}, n_{j}, n_{u}}^{f} = ((N_{1j} - n_{j})!/(N_{1j} - n_{j}!)) F((R_{ij}^{f}/\theta)(1 - \theta) N_{1j} - n_{j}! [1 - F(R_{ij}^{f}/\theta)]) n_{u}$. The total expected profit of firm $i$ is given by the following:

$$\pi_{i}^{f}(N_{1i}, N_{1j}) = \max_{R_{ii}} \sum_{n_{i}=0}^{N_{1i}} P_{N_{1i}, n_{i}}^{f} \left( n_{i} R_{ii}^{f} + \sum_{n_{j}=0}^{N_{1j}} \sum_{n_{u}=0}^{N_{1i} - n_{i}} P_{N_{1j}, n_{j}, n_{u}}^{f} n_{j} R_{ij}^{f} + n_{u} R_{ii}^{f} \right)$$

As (14) shows, fixed-ratio pricing also has the problem of uncertainty in the decision-making process. The simplified calculation of the firms’ equilibrium profits given a single consumer is considered below.
Theorem 4. In the fixed-ratio pricing policy, when there are \(N(N > 1)\) consumers in the market, the firms’ expected profits are calculated as follows:

\[
\begin{align*}
\pi_i^f (N_{ij}, N_{ij}) &= N_{ij} \pi_{ij}^f (1, 0) + N_{ij} \pi_{2j}^f (0, 1), \\
\pi_j^f (N_{ij}, N_{ij}) &= N_{ij} \pi_{ij}^f (1, 0) + N_{ij} \pi_{2j}^f (0, 1),
\end{align*}
\]

(15)

where \(\pi_{ij}^f (1, 0)\) and \(\pi_{2j}^f (0, 1)\) are the equilibrium profits of two firms under fixed-ratio pricing policy in the market with a single consumer as shown in (12) and (13) with \(i, j = A, B\) and \(i \neq j\).

2.4. Elastic Pricing. Thus far, we have studied the model under assumptions that the firms choose the same pricing policy. Hereby, we relax the assumptions. In this subsection, we study the pricing policy of each firm assuming the pricing policy is not necessarily the same between firms. That is, firm \(i\) can choose dynamic pricing, while firm \(j\) commits to using fixed-ratio pricing, or firm \(j\) adopts dynamic pricing, while firm \(i\) uses fixed-ratio pricing. Because of symmetry, we only need to analyze the first scenario, and we also analyze optimal price and equilibrium profits in the elastic pricing policy scenario.

2.4.1. One Consumer. The order of consumers will affect the firms’ profits since the two firms use different pricing policies. Firm \(i\) is visited by the consumer in the first period, and we have the following:

\[
\pi_i^e (1, 0) = \max_{R_{ii}^e} \left[ R_{ii}^e \left( 1 - \frac{R_{ii}^e - \beta R_{ij}^e}{1 - \theta} \right) \right]
\]

(16)

The expected profit of firm \(j\) can be written as follows:

\[
\pi_j^e (0, 1) = \max_{R_{ij}^e} \beta R_{ij}^e \left( 1 - \frac{\beta R_{ij}^e}{\theta} \right)
\]

(17)

Similarly, Firm \(j\) is visited by the consumer in the first period, and we have the following:

\[
\pi_j^e (1, 0) = \max_{R_{ij}^e} \left[ R_{ij}^e \left( 1 - \frac{R_{ij}^e - R_{2j}^e}{1 - \theta} \right) \right]
\]

(18)

where the term in \(\pi_{ij}^e (1, 0)\) is the expected profit in the first period when a consumer first visits the firm \(j\).

The expected profits of firm \(i\), which does not encounter the consumer in the first period but may in the second period, can be calculated as follows:

\[
\pi_i^e (0, 1) = \max_{R_{ii}^e} \left[ R_{ii}^e \left( 1 - \frac{R_{ii}^e - R_{ij}^e}{1 - \theta} \right) \right]
\]

(19)

Theorem 5. In the elastic pricing policy, when there is only one consumer in the market, the firms’ expected prices are as follows:

\[
\begin{align*}
R_{1i}^e &= \frac{10 - 7\theta - K}{16}, \\
R_{2j}^e &= \frac{\theta + 2 - K}{8}, \\
R_{ij}^e &= \frac{10 - 7\theta - K}{16}, \\
R_{ji}^e &= \frac{2 + \theta - K}{8}
\end{align*}
\]

(20)

The expected profits are as follows:

\[
\begin{align*}
\pi_i^e (1, 0) &= \pi_{ij}^e (1, 0) = \frac{(K + 7\theta - 10)^2}{256 (1 - \theta)}, \\
\pi_j^e (0, 1) &= \pi_{2j}^e (0, 1) \\
&= \frac{(2 + \theta - K) (6 - 9\theta + K) (7\theta - 2 + K)}{1024\theta (1 - \theta)}
\end{align*}
\]

(21)

(22)

According to Theorem 5, the simplified calculation of the firms’ profit can be obtained directly when there are \(N(N > 1)\) consumers in the market, as shown below.

2.4.2. \(N\) Consumers. \(P_{N_i,n_{ij}}^e\) denotes the probability that firm \(i\), which is visited by \(N_{ij}\) consumers, sells \(n_{ij}\) units. Thus, in the first period, \(P_{N_i,n_{ij}}^e = (N_{ij}/n_{ij}!(N_{ij} - n_{ij})!))F((R_{ii}^e - \beta R_{ij}^e)/(1 - \theta))^{N_{ij} - n_{ij}}[1 - F(R_{ij}^e - \beta R_{ij}^e)/(1 - \theta))]^{n_{ij}}\). Let \(P_{N_i,n_{ij}}^e\) denote the probability that firm \(i\), which is visited by \(N_{ij} - n_{ij}\) consumers and sells \(n_{2i}\) units. Thus, in the second period, \(P_{N_i,n_{ij}}^e = ((N_{ij} - n_{ij})!/n_{2ij}!(N_{ij} - n_{ij} - n_{2i})!))F(R_{ij}^e/\theta)^{N_{ij} - n_{ij} - n_{2i}}[1 - F(R_{ij}^e/\theta)]^{n_{2i}}\). The total expected profits of firm \(i\) are given by the following:

\[
\pi_i^e (N_{ij}, N_{ij}) = \max_{R_{ii}^e} \left( \sum_{n_{ij}=0}^{N_{ij}} P_{N_i,n_{ij}}^e \left( n_{ij} R_{ii}^e + \sum_{n_{ij}=0}^{N_{ij}} P_{N_i,n_{ij}}^e \sum_{n_{2i}=0}^{N_{ij} - n_{ij}} n_{2i} P_{N_i,n_{ij}}^e \right) \right)
\]

(23)

Theorem 6. In the elastic pricing policy, when there are \(N(N > 1)\) consumers in the market, the expected profits of the two firms are as follows:

\[
\begin{align*}
\pi_i^e (N_{ij}, N_{ij}) &= N_{ij} \pi_{ij}^e (1, 0) + N_{ij} \pi_{2j}^e (0, 1), \\
\pi_j^e (N_{ij}, N_{ij}) &= N_{ij} \pi_{ij}^e (1, 0) + N_{ij} \pi_{2j}^e (0, 1),
\end{align*}
\]

(24)

where \(\pi_{ij}^e (1, 0), \pi_{ij}^e (1, 0), \pi_{2j}^e (0, 1), \pi_{2j}^e (0, 1)\) are shown in (21) and (22). They are the equilibrium profits of two firms under an elastic pricing policy in the market with a single consumer, where \(i, j = A, B\) and \(i \neq j\).
Let us examine the firms’ equilibrium profit over a range of consumer rationality under different pricing policies. The first factor is strategic consumer behavior, which denotes the difference in equilibrium profits of firm $i$ between dynamic pricing (elastic pricing) and a fixed-ratio pricing policy, where

\[ \Delta \pi_i(N_{ij}, N_{ij}) = \pi^d_i(N_{ij}, N_{ij}) - \pi^e_i(N_{ij}, N_{ij}) \]

which denotes the difference in equilibrium profits of firm $i$ between dynamic pricing (elastic pricing) and a fixed-ratio pricing policy, where $i, j = A, B$ and $i \neq j$. In addition, we consider different combinations of $N_{ij}$ and $N_{ij}$ while fixing $N_{ii} + N_{jj} = 100$. Under this fixed condition, $N_{ij}$ tends to increase the equilibrium profits of firm $i$ under different pricing policies. This may explain why many firms seek to attract consumers using various promotions when first entering a competitive market.

3.2. Effectiveness of Different Pricing Policies. We demonstrate the effectiveness of dynamic pricing (elastic pricing) and fixed-ratio pricing policy. See Figure 3 for illustration. We assume that $\Delta \pi_i(N_{ij}, N_{ij}) = \pi^d_i(N_{ij}, N_{ij}) - \pi^e_i(N_{ij}, N_{ij})$, which denotes the difference in equilibrium profits of firm $i$ between dynamic pricing (elastic pricing) and a fixed-ratio pricing policy, where $i, j = A, B$ and $i \neq j$. In addition, we consider different combinations of $\theta$ and $\beta$ while fixing $N_{ij} = 50$ and $N_{ij} = 100$. When $\beta = 0.4$ and $\theta > 0.5$, the difference in profit of firm $i$ between two pricing policies is less than zero. The result indicates that when the consumers are highly rational and the price discount factor is small (firms mark down sharply), the fixed-ratio pricing policy is a perfect substitute for an elastic pricing policy under the zigzag competition model. Thus, according to Theorems 2 and 6, we can calculate the firms’ expected profits.

Moreover, we evaluate the impact of $N_{ij}$ on the equilibrium profit under different pricing policies to allow for a meaningful and systematic perspective comparison. We consider different combinations of $N_{ij}$ and $N_{ij}$ while fixing $N_{ii} + N_{jj} = 100$. Under this fixed condition, $N_{ij}$ tends to increase the equilibrium profits of firm $i$ under different pricing policies. This may explain why many firms seek to attract consumers using various promotions when first entering a competitive market.

3. Results and Discussion

In this section, we study our model’s results and managerial significances. The experimental design centers around two aspects: (1) the impact of consumer rationality on the equilibrium profits and (2) the effectiveness of different pricing policies. There are three main contributing factors in both models. The first factor is strategic consumer behavior, which is denoted by the parameter $\theta$. The second factor is the initial number of consumers who visit the firms, denoted by the parameter $N_{ij}, N_{ij}$. The third factor is the discount factor, which is denoted by the parameter $\beta$. Based on numerical simulation, this section analyzes the firms’ equilibrium profits under the different pricing policies.

3.1. Impact of Consumer Rationality on the Equilibrium Profits. Let us examine the firms’ equilibrium profit over a range of consumer rationality under different pricing policies, as demonstrated in Figures 2(a) and 2(b). In the figures, consumer rationality, $\theta$, ranges from $\theta = 0.1$ to $\theta = 0.9$. Observe that strategic consumer behavior tends to increase equilibrium profits and then reduce equilibrium profits under different pricing policies. When consumers are highly rational, they are more willing to remain in the market until the second period, which creates competition between the two firms. However, consumers become less rational; thus the firms can obtain higher profits. Therefore, consumers’ rationality does not contribute to the firms’ equilibrium profits.

**Lemma 7.** The firms’ expected profits when both firms choose the dynamic pricing policy are the same as when one firm uses a dynamic pricing policy, while the other firm adopts an elastic pricing policy.

Compared with Theorem 1 and Theorem 5, the price under the dynamic pricing policy is equal to that under the elastic pricing policy. This means the firm that adopts an elastic pricing policy will eventually converge with the firm that uses a dynamic pricing policy under the zigzag competition model. The dynamic pricing policy is a perfect substitute for an elastic pricing policy under zigzag competition. Thus, according to Theorems 2 and 6, we can calculate the firms’ expected profits when both firms choose the dynamic pricing policy as follows:

\[ \pi^f(N_{ij}, N_{ij}) = \pi^d_i(N_{ij}, N_{ij}). \]

![Figure 2: (a) The impact of different parameters on the equilibrium profit under dynamic and elastic pricing. (b) The impact of different parameters on the equilibrium profit under fixed-ratio pricing.](image-url)

**Figure 2:** (a) The impact of different parameters on the equilibrium profit under dynamic and elastic pricing. (b) The impact of different parameters on the equilibrium profit under fixed-ratio pricing.
can effectively attract consumers to purchase units to obtain more profits. This indicates that when more consumers are willing to wait until the second period, firms can obtain more profits by selling more units. Hence, it would be interesting to investigate whether a prisoner’s dilemma situation can be found, where the expected profits of firms under fixed-ratio pricing is higher than the expected profits of firms under dynamic and elastic pricing. Otherwise, a dynamic pricing policy can increase the equilibrium profits by adjusting the price system’s flexibility.

4. Conclusion

In this paper we have presented a two-period model of duopoly competition between two firms that sell an identical product with sufficient inventories, under the assumption that each consumer visits only one of the firms in each period. If consumers’ net utility in the second period is greater than that of consumers in the first period, they are expected to visit the competing firm in the second period. We investigate a firm’s optimal pricing strategy in the presence of strategic consumers under zigzag competition.

Our results indicate that a firm’s equilibrium state of elastic pricing is consistent with that of dynamic pricing. Moreover, our results also lead to several managerial insights. Specifically, under zigzag competition with a finite number of consumers in the market, a firm’s pricing policy should be the same as the single consumer case under different pricing policies. Furthermore, firms’ expected profits increase as the initial number of consumer visits increase, which may explain why many firms seek to attract consumers by various means when entering competitive markets. Competition exerts significant pressure on pricing strategies. Though dynamic pricing is a more complex policy than fixed-ratio pricing, it may lead to decreased equilibrium profits. It is because firms sharply reduce prices and consumers’ rationality is unlimited. Otherwise, dynamic pricing policy is best for maximizing profits.

Our model lays the foundation for analyzing firms’ profit-maximizing pricing strategies, wherein firms may choose different pricing policies and then compete in the market over multiple periods. Clearly, additional variants of the model can be explored, such as allowing for replenishment during the selling seasons, incorporating holding and wholesale costs as well as depreciation of products. Another possible extension is to consider a market that includes myopic consumers and strategic consumers, as different types of consumers can have a significant and varied impact on a firm’s profits. Additionally, future studies may consider two competing firms that engage in marketing efforts in order to raise their initial market share. Finally, one natural extension is to consider heterogeneous products.

Appendix

A. Proof of Theorem 1

The profit of firm $i, j$ can be obtained from equations (1) and (2). This profit of firm $i$ is maximized at $R_{1i}^d = (1 - \theta + R_{2j}^d)/2$; at the same time, this profit of firm $j$ is maximized at $R_{2j}^d = (R_{1i}^d + \theta - \sqrt{(R_{1i}^d)^2 + \theta R_{1i}^d + \theta^2})/3$. We can solve $R_{1i}^d$ and $R_{2j}^d$ jointly, therefore, this profit of firm $i$ is maximized at $R_{1i}^d = (10 - 7\theta - K)/16$, and the maximized profit is given by $n_{1i}^d(1, 0) = (K + 7\theta - 10)^2/256(1 - \theta)$. This profit of firm $j$ is maximized at $R_{2j}^d = 2 + \theta - K)/8$, and the maximized profit is given by $n_{2j}^d(0, 1) = (2 + \theta - K)(6 - 9\theta + K)(7\theta - 2 + K)/1024\theta(1 - \theta)$.

B. Proof of Theorem 2

According to (5)

$$n_{1i}^d(N_{1i}, N_{1j}) = \max_{R_{1i}^d} \sum_{n_{1i}=0}^{N_{1i}} P_{N_{1i}, n_{1i}}^d n_{1i} R_{1i}^d$$

$$+ \max_{R_{2j}^d} \sum_{n_{1i}=0}^{N_{1i}} P_{N_{1i}, n_{1j}}^d R_{2j}^d (N_{1j} - n_{1j}) \left[ 1 - F \left( \frac{R_{2j}^d}{\theta} \right) \right]$$

$$= \max_{R_{1i}^d} \sum_{n_{1i}=0}^{N_{1i}} n_{1i}$$

$$\cdot \frac{N_{1i}!}{n_{1i}!(N_{1i} - n_{1i})!} F \left( \frac{R_{1i}^d - R_{2j}^d}{1 - \theta} \right)^{n_{1i}}$$

$$\cdot \left[ 1 - F \left( \frac{R_{2j}^d}{1 - \theta} \right) \right] + \max_{R_{2j}^d} \sum_{n_{1i}=0}^{N_{1i}} (N_{1j} - n_{1j})$$

$$- F \left( \frac{R_{2j}^d}{\theta} \right) \sum_{n_{1i}=0}^{N_{1i}} (N_{1j} - n_{1j})$$

FIGURE 3: Effectiveness of different pricing policies.
\[
\frac{(K - 2 - \theta)(\beta K + 7\beta - K - 10\beta + \theta + 2)}{64\beta}
\]
therefore, the maximum profit is given by

C. Proof of Theorem 3

The profits of firm \( i,j \) can be obtained from (13) and (14). This profit of firm \( i \) is maximized at \( R_{ij}^f = (1 - \theta + \beta R_{ij}^f)/2 \); the profits of firm \( j \) is maximized at \( R_{ij}^\ell = (R_{ij}^f + \theta - \sqrt{(R_{ij}^f)^2 - \theta R_{ij}^f + \theta^2})/3\beta \). We can solve \( R_{ij}^f = R_{ij}^\ell = (2 + \theta - K)/8\beta \); therefore, the maximized profits are given by \( \pi_i^f(N_{ij}, N_{ij}) = (K - 2 - \theta)(\beta K + 7\beta - K - 10\beta + \theta + 2)/64\beta^2(1 - \theta) \), and the maximized profits are given by \( \pi_j^f(N_{ij}, N_{ij}) = (2 + \theta - K)(K(\beta - 1) - (1 - \beta)(\theta + 2))(7\theta - 2 + K)/512\beta(1 - \theta) \).

D. Proof of Theorem 4

According to (14)

\[
\pi_i^f(N_{ij}, N_{ij}) = \max_{R_{ij}} \left\{ \sum_{n_{ij}=0}^{N_{ij}} P_{N_{ij},n_{ij},n_{ii}} R_{ij} \right\}\\
+ \sum_{n_{ij}=0}^{N_{ij}} P_{N_{ij},n_{ij},n_{ii}} \beta R_{ij} \left[ (N_{ij} - n_{ij}) \left( 1 - F \left( \frac{\beta R_{ij}^f}{\theta} \right) \right) \right]
\]

\[
= \max_{R_{ij}} R_{ij} \sum_{n_{ij}=0}^{N_{ij}} n_{ij}
\]

\[
\frac{N_{ij}!}{n_{ij}!(N_{ij} - n_{ij})!} F \left( \frac{R_{ij}^f - \beta R_{ij}^f}{1 - \theta} \right)^{N_{ij} - n_{ij}}
\]

\[
\left[ 1 - \frac{R_{ij}^f - \beta R_{ij}^f}{1 - \theta} \right]^{n_{ij}} + \max_{R_{ij}} R_{ij} \sum_{n_{ij}=0}^{N_{ij}} (N_{ij} - n_{ij}) \left[ 1 - F \left( \frac{\beta R_{ij}^f}{\theta} \right) \right]
\]

where the above equality follows by the induction step; the last one follows (5) and (6).

We consider the decision of firm \( i \) in detail, and the analysis process will be identical for firm \( j \). Therefore, \( \pi_i^f(N_{ij}, N_{ij}) = N_{ij} \pi_i^f(1,0) + N_{ij} \pi_j^f(0,1) \).

E. Proof of Theorem 5

The profits of firm \( i,j \) can be obtained from equations (16) and (17). This profit of firm \( i \) is maximized at \( R_{ij}^\ell = (1 - \theta + \beta R_{ij}^\ell)/2 \); this profit of firm \( j \) is maximized at \( R_{ij}^\ell = (R_{ij}^f + \theta - \sqrt{(R_{ij}^f)^2 - \theta R_{ij}^f + \theta^2})/3\beta \). We can solve \( R_{ij}^\ell \) and \( R_{ij}^f \) jointly, therefore, this profit of firm \( i \) is maximized at \( R_{ij}^f = (10 - 7\theta - K)/16 \), and the maximized profit is given by \( \pi_i^f(1,0) = (K + 7\theta - 10)^2/256(1 - \theta) \); this profit of firm \( j \) is maximized at \( R_{ij}^\ell = (\theta + 2 - K)/8\beta \), and the maximized profit is given by \( \pi_j^f(0,1) = (2 + \theta - K)(6 - 9\theta + K)(7\theta - 2 + K)/1024\theta(1 - \theta) \).

The profits of firm \( i,j \) can be obtained from equations (18) and (19). This profit of firm \( j \) is maximized at \( R_{ij}^\ell = (1 - \theta + \beta R_{ij}^\ell)/2 \); this profit of firm \( i \) is maximized at \( R_{ij}^f = (R_{ij}^f + \theta - \sqrt{(R_{ij}^f)^2 - \theta R_{ij}^f + \theta^2})/3\beta \). We can solve \( R_{ij}^f \) and \( R_{ij}^\ell \) jointly, therefore, this profit of firm \( i \) is maximized at \( R_{ij}^f = (10 - 7\theta - K)/16 \), and the maximized profit is given by \( \pi_i^f(1,0) = (K + 7\theta - 10)^2/256(1 - \theta) \); this profit of firm \( j \) is maximized at \( R_{ij}^\ell = (2 + \theta - K)/8 \), and the maximized profit is given by \( \pi_j^f(0,1) = (2 + \theta - K)(6 - 9\theta + K)(7\theta - 2 + K)/1024\theta(1 - \theta) \).

F. Proof of Theorem 6

According to (12)

\[
\pi_i^c(N_{ij}, N_{ij}) = \max_{R_{ij}} \left\{ \sum_{n_{ij}=0}^{N_{ij}} P_{N_{ij},n_{ij},n_{ii}} R_{ij} \right\}\\
+ \sum_{n_{ij}=0}^{N_{ij}} P_{N_{ij},n_{ij},n_{ii}} \beta R_{ij} \left[ (N_{ij} - n_{ij}) \left( 1 - F \left( \frac{R_{ij}^c}{\theta} \right) \right) \right]
\]
\[ \frac{N_{ij}}{n_{ij}} \sum_{n_{ij} \neq 0} n_{ij} \left( R_{ij}^q \right)^{N_{ij} - n_{ij}} \\
+ \frac{N_{ij}}{n_{ij}} \left( N_{ij} - n_{ij} \right) \left( 1 - \theta \right)^{N_{ij} - n_{ij}} F \left( \frac{R_{ij}^q - \beta R_{ij}^q}{1 - \theta} \right) \\
+ \frac{N_{ij}}{n_{ij}} \left( N_{ij} - n_{ij} \right) \left( 1 - \theta \right)^{N_{ij} - n_{ij}} F \left( \frac{R_{ij}^q - \beta R_{ij}^q}{1 - \theta} \right) \\
- F \left( \frac{R_{ij}^q}{\theta} \right) \left[ \sum_{n_{ij} \neq 0} n_{ij} \left( N_{ij} - n_{ij} \right) \left( 1 - \theta \right)^{N_{ij} - n_{ij}} F \left( \frac{R_{ij}^q - \beta R_{ij}^q}{1 - \theta} \right) \right] \\
+ \max \left( \frac{R_{ij}^q}{R_{ij}^q} \right) \left[ \sum_{n_{ij} \neq 0} n_{ij} \left( N_{ij} - n_{ij} \right) \left( 1 - \theta \right)^{N_{ij} - n_{ij}} F \left( \frac{R_{ij}^q - \beta R_{ij}^q}{1 - \theta} \right) \right] \\
\times F \left( \frac{R_{ij}^q}{\theta} \right) = \pi_i^v(1, 0) + \max \pi_i^v(0, 1) \\
\] (E1)

The analysis will be identical for firm j. Therefore, \( \pi_j^v(N_{ij}, N_{ij}) = \pi_{ij}^v(1, 0) + \pi_{ij}^v(0, 1) \).

**Data Availability**

The data used to support the findings of this study have not been made available because the date is the simulation data. No other type of data was used to support this study.

**Conflicts of Interest**

The authors declare no conflicts of interest.

**Funding**

This work was supported by the National Natural Science Foundation of China (Grant no. 71402012) and the Natural Science Foundation of Education in Chongqing (KJ130402).

**References**


