Research Article

Locally Most Powerful Test for the Random Coefficient Autoregressive Model

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In this article, we study the problem of testing the constancy of the coefficient in a class of stationary first-order random coefficient autoregressive (RCAR(1)) model. We construct a new test statistic based on the locally most powerful-type (LMP) test. Under the null hypothesis, we derive the limiting distribution of the proposed test statistic. In the simulation, we compare the power between LMP test and empirical likelihood (EL) test and find that the accuracy of using LMP is 6.7%, 28.8%, and 26.1% higher than that of EL test under normal, student's t , and symmetric contamination errors, respectively. A real life data is given to illustrate the practical effectiveness of our test.

1. Introduction

In time series analysis, autoregressive and linear processes are widely used due to their mathematical tractability. In fact, the autoregressive model has two prominently advantages: their estimation procedures are well established and the existence of stationary solutions are easily derived. Therefore, this model has been investigated in the field of signal detection and classification, psychometry, and biomedical engineering. See, for example, Hoque [1], Ogawa et al. [2], Subasi et al. [3], and Maleki et al. [4]. The first-order autoregressive model (AR(1)) is defined as

$$X_t = \alpha X_{t-1} + \varepsilon_t, \quad t \geq 1, \quad (1)$$

where $\{\varepsilon_t\}$ is an independent and identically distributed (i.i.d.) random error sequence with probability density function f_ε , $E(\varepsilon_t) = 0$, $\text{Var}(\varepsilon_t) = \sigma_\varepsilon^2$, and $\{\varepsilon_t\}$ independent of X_0 for all t . However, it has been found that a variety of data sets cannot be modelled precisely by assuming linearity. Financial data, for instance, present heteroscedasticity, and biological data suffer from random perturbations. To address this problem, Conlisk [5] considered a random coefficient autoregressive model defined as follows:

$$X_t = \alpha_t X_{t-1} + \varepsilon_t, \quad t \geq 1, \quad (2)$$

where $\{\alpha_t\}$ is a sequence of i.i.d random variables with $E(\alpha_t) = \alpha$ and $\text{Var}(\alpha_t) = \sigma_\alpha^2$. $\{\varepsilon_t\}$ is an i.i.d. random error sequence with mean zero and variance σ_ε^2 . The random variable X_0 is assumed to be independent of $\{\alpha_t\}$, which is independent of $\{\varepsilon_t\}$. Note that when $\sigma_\alpha^2 = 0$, the RCAR(1) model reduces to the standard AR(1) model.

The RCAR(1) models have been widely applied in mathematical literature due to its strong application value in practice. For instance, Nicholls and Quinn [6–8] derived the least square (LS) estimates of the model parameter and showed that the LS estimates are strongly consistent and obey the central limit theorem. Brandt [9] restated the necessary and sufficient conditions for the existence of a strict stationarity and ergodicity solution in the RCAR(1) model. Wang and Ghosh [10] considered the Bayesian estimation method to estimate RCAR model parameter and explored the frequentist properties of the Bayes estimator. Wang et al. [11] obtained the asymptotic properties of the maximum likelihood estimators of these parameters in RCAR(1) model under the unit root assumption.

More recently, the problem of testing traditional AR(1) model against RCAR(1) model is thus of essential issue in this settings. This detection problem was firstly investigated by Nicholls and Quinn [8], where a Gaussian Lagrange

multiplier test is derived for the problem. However, this likelihood ratio approach has several weaknesses, as the true value of the parameter under the null hypothesis lies on the boundary of the parameter space, the asymptotics will not be easily established. Ramanathan and Rajarshi [12] considered a non-Gaussian test method to handle the change problem for parameters in RCAR(1) case. Their signed rank test, however, require a symmetry assumption on the innovation density, which is highly unnatural in this context. Still for the RCAR(1) model, Lee [13] proposed the cumulative sum test for parameter change in RCAR(1) model. But, it detects a change only in a function of the parameter rather than the parameter themselves. Recently, Moreno and Romo [14] studied robust unit root test with autoregressive errors. The unit root test shows some size distortions and requires many estimates in the construction of test.

To deal with these drawbacks, we construct a locally most powerful-type (LMP) test for testing the constancy of parameters in the RCAR(1) model. The LMP test has been investigated by several authors. Rohatgi et al. [15], Chikkagoudar and Biradar [16], and Manik et al. [17] considered the LMP test for parameter constancy. The test has merit in that its calculation process is less cumbersome and it produces stable sizes especially when parameter nearby the true value. In contrast, the traditional tests mentioned above show some size distortions and require many estimates in the construction of tests, see Ramanathan and Rajarshi [12]. In a general way, our test can conventionally discard correlation effects and enhance the performance of the test. In this paper, we illustrate how our proposed method can be implemented for finite samples under normal, student's t , and symmetric contamination errors (see Huber [18] and Hettmansperger [19]). Through numerical simulation studies, we can see that our test has a stronger power in terms of maintaining the empirical level and power than the empirical likelihood (EL) test suggested by Zhao et al. [20].

The outline of this paper is organized as follows. In Section 2, we introduce our test statistic and derive its limiting distribution under the null hypothesis. Numerical simulations to evaluate the empirical size and power of our test technique are discussed in Section 3. Section 4 presents a real life data example to illustrate the superior of our methods. We provide brief concluding remarks in Conclusions. Appendix provides the proofs of the main results.

2. Methodology and Main Results

In this section, we will construct a test statistic to test whether α_t is a constant. To achieve this task, we set up the null and alternative hypotheses

$$\begin{aligned} H_0: \sigma_\alpha^2 &= 0 \\ \text{vs } H_1: \sigma_\alpha^2 &> 0. \end{aligned} \quad (3)$$

In what follows, we give our main results. Suppose that the time series data $x_1, x_2, \dots, x_n$ are generated from (2). Let $\{\alpha_t\}$ be an i.i.d. sequence of random variables with common probability distribution F_α ; $\{\varepsilon_t\}$ is an i.i.d sequence of random variables with density function f_ε . In this article, we regard X_0

as a given number; alternatively, if X_0 is a random variable, we shall consider only inferences conditionally on X_0 are fixed. Note that X_0 , $\{\alpha_t\}$, and $\{\varepsilon_t\}$ are independent.

Next, we assume the following conditions to establish the asymptotic properties of the test statistic:

$$(C1) \alpha^2 + \sigma_\alpha^2 < 1.$$

$$(C2) \text{ The distribution } F_\alpha \text{ of } \alpha_t \text{ is such that } E|\alpha_t|^3 < \infty.$$

$$(C3) \text{ The third and mixed derivatives of } \log f_\varepsilon \text{ with respect to } \alpha \text{ and } \sigma_\varepsilon^2 \text{ are uniformly bounded on } (\alpha, \sigma_\varepsilon^2).$$

$$(C4) \text{ Differentiation thrice with respect to } (\alpha, \sigma_\varepsilon^2) \text{ of } f_\varepsilon \text{ under the integration is bounded.}$$

It is easy to derive that X_t is a Markov chain on $\{0, 1, 2, \dots\}$ with the following transition probabilities:

$$\begin{aligned} f_{X_t|X_{t-1}}(x_t | x_{t-1}) &= \int f_{X_t|X_{t-1}}(x_t | x_{t-1}, \alpha_t) dF_\alpha \\ &= \int f_\varepsilon(x_t - \alpha_t x_{t-1}) dF_\alpha. \end{aligned} \quad (4)$$

The Markovity follows from (2) and the fact that $\{\alpha_t\}$ is an i.i.d sequence. The conditional density $f_{X_t|X_{t-1}}(x_t | x_{t-1})$ is obtained by noting that conditional on α_t . Conditional on (X_{t-1}, α_t) and $\{\varepsilon_t\}$ is an i.i.d sequence with the density f_ε , hence, the probability density function of X_t is given by $f_\varepsilon(x_t - \alpha_t x_{t-1})$. So, we get the result in the above equation.

Therefore, we can write down the likelihood function L_{H_1} for RCAR(1) model:

$$L_{H_1}(x_1, \dots, x_t) = f_{X_0}(x_0) \prod_{t=1}^n \int f_\varepsilon(x_t - \alpha_t x_{t-1}) dF_\alpha. \quad (5)$$

Furthermore, employing this spirit of LMP approach introduced by Manik et al. [17], we can obtain our test statistic

$$Q_n(\beta) = \sum_{t=1}^n \frac{1}{2} \frac{f_\varepsilon''(x_t - \alpha x_{t-1})}{f_\varepsilon(x_t - \alpha x_{t-1})} = \sum_{t=1}^n R_t(\beta), \quad (6)$$

where

$$R_t(\beta) = \frac{1}{2} \frac{f_\varepsilon''(x_t - \alpha x_{t-1})}{f_\varepsilon(x_t - \alpha x_{t-1})}. \quad (7)$$

And the detailed procedures can be found in Appendix A.

Let $\beta = (\alpha, \sigma_\varepsilon^2)^T$ denote vector of all unknown parameters of the RCAR(1) process. Hereafter, we use the notation $\hat{\beta} = (\hat{\alpha}, \hat{\sigma}_\varepsilon^2)$ to represent the maximum likelihood estimator (MLE) of β . In addition, the actual test statistic can be obtained by replacing β with their MLE $\hat{\beta}$. It can be easily derived that $\{R_t(\beta), \mathcal{F}_t^R\}$ is a zero mean martingale.

Before we state our main results, the following assumptions will be made:

$$(A.1) \sup_t E|R_t(\beta)|^{2+\eta} < \infty, \text{ for some } \eta > 0.$$

Then about the asymptotic distribution of $Q_n(\hat{\beta})$, we have the following theorem.

Theorem 1. *Let $\{x_t\}$ be a sequence of strictly stationary, ergodic, and $\mathcal{F}_t$ -measurable solutions to equation (2) under*

TABLE 1: Empirical sizes of LMP and EL tests at nominal level 0.05 for model N1.

N		(σ_N, α)							
		(0.5, 0.3)	(0.5, 0.5)	(0.5, 0.7)	(0.5, 0.9)	(1, 0.3)	(1, 0.5)	(1, 0.7)	(1, 0.9)
100	MP	0.095	0.105	0.109	0.101	0.116	0.103	0.089	0.098
	EL	0.008	0.004	0.002	0.003	0.003	0.002	0.003	0.002
200	LMP	0.089	0.082	0.085	0.086	0.085	0.082	0.083	0.087
	EL	0.006	0.007	0.004	0.005	0.004	0.004	0.003	0.003
300	LMP	0.074	0.077	0.082	0.069	0.076	0.071	0.076	0.074
	EL	0.006	0.009	0.007	0.006	0.005	0.007	0.008	0.009
400	LMP	0.063	0.070	0.075	0.068	0.067	0.063	0.070	0.072
	EL	0.007	0.009	0.006	0.004	0.007	0.008	0.005	0.004
500	LMP	0.066	0.060	0.077	0.060	0.065	0.062	0.060	0.055
	EL	0.009	0.005	0.006	0.005	0.006	0.009	0.014	0.008
600	LMP	0.062	0.057	0.067	0.053	0.065	0.060	0.059	0.068
	EL	0.009	0.005	0.011	0.007	0.009	0.007	0.008	0.007
700	LMP	0.064	0.059	0.068	0.056	0.057	0.062	0.058	0.063
	EL	0.005	0.010	0.008	0.007	0.007	0.008	0.005	0.003
800	LMP	0.060	0.061	0.060	0.056	0.058	0.055	0.055	0.063
	EL	0.009	0.007	0.007	0.010	0.011	0.012	0.008	0.009
900	LMP	0.055	0.062	0.054	0.048	0.053	0.053	0.062	0.062
	EL	0.015	0.015	0.008	0.010	0.011	0.007	0.010	0.009
1000	LMP	0.063	0.051	0.057	0.052	0.058	0.062	0.057	0.054
	EL	0.019	0.009	0.012	0.009	0.011	0.012	0.007	0.009
5000	LMP	0.051	0.051	0.051	0.053	0.050	0.048	0.049	0.053
	EL	0.020	0.022	0.018	0.020	0.016	0.021	0.024	0.022

conditions of (C1) – (C4) and above (A.1). Then under H_0 , we have asymptotic normality

$$(\sqrt{n})^{-1} Q_n(\hat{\beta}) \xrightarrow{d} N(0, \hat{\omega}^2), \quad (8)$$

here $\hat{\omega}^2 = \sigma^2 - W^T \Sigma^{-1} W$.

(The expressions for σ^2 , and Σ^{-1} are derived in Appendix B).

3. Simulation

In this section, we carry out some simulation studies to compare performances of the locally most powerful-type test and the empirical likelihood test in terms of empirical size and power. The empirical size and power for the two tests in Tables 1–6 are based on 1000 repetitions with the help of R software. Within each study, we set the initial values $X_0 \equiv 1$ and employ the significance level at 0.05. Throughout this simulation, we used notation LMP for locally most powerful-type test by our algorithm, EL for empirical likelihood method.

3.1. Empirical Size. To calculate empirical sizes, we pay our attention to three kinds of model (1) defined as follows:

$$(N1) \alpha_t = \alpha, \varepsilon_t \sim N(0, \sigma_N^2).$$

$$(N2) \alpha_t = \alpha, \varepsilon_t \sim t(m).$$

(N3) $\alpha_t = \alpha, \varepsilon_t \sim F_\varepsilon(x)$. The distribution function of ξ -contamination is

$$F_\varepsilon(x) = \xi \Phi\left(\frac{x}{\sigma_1}\right) + (1 - \xi) \Phi\left(\frac{x}{\sigma_2}\right), \quad (9)$$

where ξ is a fixed constant satisfying $0 < \xi < 1$ and $\Phi(x)$ is the distribution function of standard normal random variable, $\sigma_i > 0, i = 1, 2$.

Let us now describe how the simulated results are obtained. First of all, we use the model N1 to generate data when the $\sigma_N = 0.5, 1$ and $\alpha = 0.3, 0.5, 0.7, 0.9$. Secondly, we generate data from model N2 with the degrees of freedom $m = 8, 10$ and $\alpha = 0.3, 0.5, 0.7, 0.9$. For model N1 and N2, we take the sample size $N = 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000$, and 5000. Thirdly, we simulate samples from model N3 with the $(\xi, \sigma_1, \sigma_2) = (0.8, 1, 3), (\xi, \sigma_1, \sigma_2) = (0.9, 1, 3)$, and $\alpha = 0.3, 0.5, 0.7, 0.9$. For model N3, we set the sample size $N = 100, 300, 500, 1000$ and 5000. The empirical levels of three models are presented in Tables 1–3, respectively. As seen from Tables 1–3, the LMP test give similar results under normal, student's t , and symmetric contamination errors. In addition, the LMP test produces sizes closer to the nominal significance level 0.05 quite satisfactorily, especially for larger sample sizes. However, the EL method has lower levels for each sample size, so one may make wrong decisions in tests based on them. Looking at three models results, we can conclude that our method has a greater effect than the EL test in terms of the empirical level.

TABLE 2: Empirical sizes of LMP and EL tests at nominal level 0.05 for model N2.

N		(m, α)							
		(8, 0.3)	(8, 0.5)	(8, 0.7)	(8, 0.9)	(10, 0.3)	(10, 0.5)	(10, 0.7)	(10, 0.9)
100	MP	0.060	0.065	0.066	0.061	0.065	0.068	0.075	0.067
	EL	0.003	0.003	0.002	0.003	0.004	0.003	0.004	0.001
200	LMP	0.059	0.061	0.065	0.059	0.066	0.063	0.068	0.061
	EL	0.002	0.004	0.002	0.002	0.004	0.004	0.002	0.004
300	LMP	0.058	0.057	0.059	0.055	0.063	0.058	0.063	0.055
	EL	0.005	0.004	0.003	0.009	0.006	0.006	0.004	0.002
400	LMP	0.058	0.055	0.058	0.055	0.062	0.054	0.061	0.058
	EL	0.004	0.003	0.005	0.003	0.004	0.003	0.005	0.006
500	LMP	0.056	0.055	0.056	0.052	0.057	0.059	0.055	0.054
	EL	0.006	0.004	0.005	0.005	0.007	0.005	0.006	0.007
600	LMP	0.060	0.057	0.058	0.060	0.062	0.059	0.056	0.053
	EL	0.003	0.005	0.005	0.005	0.008	0.007	0.009	0.006
700	LMP	0.055	0.056	0.058	0.057	0.060	0.060	0.056	0.058
	EL	0.006	0.005	0.005	0.008	0.006	0.006	0.004	0.008
800	LMP	0.050	0.054	0.054	0.052	0.056	0.054	0.054	0.054
	EL	0.005	0.007	0.008	0.006	0.006	0.008	0.008	0.008
900	LMP	0.051	0.052	0.058	0.056	0.056	0.060	0.049	0.051
	EL	0.005	0.006	0.006	0.009	0.006	0.008	0.010	0.005
1000	LMP	0.052	0.056	0.057	0.055	0.050	0.053	0.050	0.054
	EL	0.007	0.008	0.005	0.006	0.012	0.009	0.009	0.007
5000	LMP	0.052	0.050	0.050	0.050	0.051	0.049	0.054	0.052
	EL	0.015	0.014	0.012	0.016	0.017	0.014	0.012	0.013

TABLE 3: Empirical sizes of LMP and EL tests at nominal level 0.05 for model N3.

N		$(\sigma_1, \sigma_2, \alpha)$								
		(1, 3, 0.1)	(1, 3, 0.2)	(1, 3, 0.3)	(1, 3, 0.4)	(1, 3, 0.5)	(1, 3, 0.6)	(1, 3, 0.7)	(1, 3, 0.8)	(1, 3, 0.9)
$\xi = 0.8$										
100	LMP	0.064	0.068	0.065	0.072	0.053	0.078	0.071	0.053	0.047
	EL	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001
300	LMP	0.045	0.066	0.053	0.038	0.054	0.081	0.051	0.043	0.040
	EL	0.000	0.001	0.000	0.001	0.001	0.002	0.000	0.001	0.000
500	LMP	0.058	0.057	0.057	0.049	0.050	0.060	0.043	0.049	0.044
	EL	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001
1000	LMP	0.050	0.059	0.052	0.051	0.045	0.054	0.036	0.043	0.036
	EL	0.002	0.001	0.001	0.001	0.001	0.001	0.003	0.001	0.001
5000	LMP	0.053	0.050	0.054	0.050	0.048	0.050	0.047	0.050	0.046
	EL	0.003	0.004	0.003	0.007	0.007	0.006	0.008	0.006	0.011
$\xi = 0.9$										
100	LMP	0.078	0.081	0.089	0.069	0.089	0.095	0.083	0.091	0.086
	EL	0.000	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.001
300	LMP	0.080	0.070	0.074	0.070	0.070	0.056	0.063	0.057	0.055
	EL	0.000	0.000	0.001	0.000	0.000	0.001	0.002	0.000	0.001
500	LMP	0.072	0.081	0.068	0.067	0.058	0.059	0.060	0.046	0.053
	EL	0.001	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.004
1000	LMP	0.064	0.061	0.067	0.067	0.055	0.055	0.052	0.051	0.045
	EL	0.000	0.000	0.000	0.001	0.002	0.003	0.003	0.000	0.001
5000	LMP	0.055	0.053	0.050	0.050	0.055	0.046	0.050	0.047	0.050
	EL	0.004	0.002	0.003	0.002	0.002	0.006	0.003	0.003	0.009

TABLE 4: Empirical powers of LMP and EL tests at nominal level 0.05 for model A1.

N		(σ_N, a, b)							
		(0.5, 0.5, 0.5)	(0.5, 0.5, 1)	(0.5, 1, 0.5)	(0.5, 1, 1)	(1, 0.5, 0.5)	(1, 0.5, 1)	(1, 1, 0.5)	(1, 1, 1)
100	LMP	0.165	0.102	0.171	0.110	0.171	0.107	0.174	0.090
	EL	0.055	0.024	0.040	0.023	0.050	0.022	0.041	0.025
200	LMP	0.318	0.143	0.294	0.147	0.320	0.140	0.311	0.160
	EL	0.191	0.070	0.156	0.073	0.202	0.049	0.155	0.067
300	LMP	0.513	0.209	0.494	0.253	0.511	0.175	0.449	0.241
	EL	0.405	0.144	0.332	0.173	0.397	0.124	0.294	0.168
400	LMP	0.674	0.232	0.561	0.330	0.663	0.264	0.590	0.338
	EL	0.578	0.176	0.456	0.272	0.559	0.189	0.450	0.260
500	LMP	0.791	0.377	0.720	0.427	0.770	0.374	0.725	0.438
	EL	0.722	0.298	0.629	0.366	0.705	0.303	0.625	0.374
600	LMP	0.842	0.415	0.792	0.521	0.832	0.424	0.790	0.508
	EL	0.798	0.352	0.717	0.460	0.793	0.365	0.712	0.458
700	LMP	0.912	0.483	0.862	0.598	0.919	0.478	0.855	0.598
	EL	0.882	0.429	0.809	0.541	0.885	0.423	0.794	0.536
800	LMP	0.964	0.560	0.915	0.645	0.947	0.568	0.918	0.643
	EL	0.938	0.512	0.877	0.604	0.928	0.516	0.872	0.599
900	LMP	0.967	0.633	0.936	0.727	0.971	0.623	0.955	0.703
	EL	0.952	0.586	0.910	0.696	0.959	0.586	0.933	0.687
1000	LMP	0.988	0.712	0.963	0.799	0.978	0.702	0.962	0.776
	EL	0.981	0.678	0.942	0.774	0.969	0.676	0.938	0.749
5000	LMP	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	EL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5: Empirical powers of LMP and EL tests at nominal level 0.05 for model A2.

N		(m, a, b)							
		(6, 0.5, 0.5)	(6, 0.5, 1)	(6, 1, 0.5)	(6, 1, 1)	(10, 0.5, 0.5)	(10, 0.5, 1)	(10, 1, 0.5)	(10, 1, 1)
100	LMP	0.289	0.110	0.220	0.149	0.241	0.096	0.183	0.131
	EL	0.045	0.016	0.027	0.019	0.047	0.018	0.027	0.029
200	LMP	0.596	0.252	0.519	0.310	0.525	0.199	0.452	0.272
	EL	0.161	0.052	0.118	0.055	0.186	0.057	0.146	0.063
300	LMP	0.790	0.400	0.722	0.475	0.762	0.351	0.686	0.429
	EL	0.336	0.111	0.276	0.124	0.373	0.124	0.293	0.152
400	LMP	0.899	0.538	0.861	0.644	0.862	0.462	0.815	0.564
	EL	0.486	0.147	0.399	0.196	0.520	0.173	0.440	0.223
500	LMP	0.964	0.624	0.933	0.757	0.957	0.596	0.906	0.699
	EL	0.622	0.218	0.529	0.285	0.695	0.262	0.590	0.345
600	LMP	0.979	0.701	0.951	0.819	0.965	0.657	0.947	0.754
	EL	0.708	0.237	0.606	0.351	0.767	0.301	0.650	0.395
700	LMP	0.995	0.791	0.974	0.862	0.983	0.727	0.978	0.824
	EL	0.814	0.351	0.669	0.404	0.839	0.389	0.773	0.483
800	LMP	0.996	0.837	0.990	0.927	0.994	0.809	0.984	0.882
	EL	0.845	0.404	0.767	0.512	0.895	0.488	0.839	0.585
900	LMP	0.999	0.874	0.995	0.934	0.996	0.831	0.993	0.924
	EL	0.879	0.430	0.801	0.548	0.929	0.514	0.877	0.622
1000	LMP	1.000	0.936	1.000	0.971	1.000	0.889	0.996	0.951
	EL	0.913	0.515	0.887	0.626	0.946	0.601	0.930	0.706
5000	LMP	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	EL	0.992	0.967	0.994	0.985	1.000	1.000	1.000	0.999

TABLE 6: Empirical powers of LMP and EL tests at nominal level 0.05 for model A3 with $(\sigma_1, \sigma_2) = (1, 3)$.

N		(a, b)								
		(0.5, 0.5)	(0.5, 1)	(0.5, 1.5)	(1, 0.5)	(1, 1)	(1, 1.5)	(1.5, 0.5)	(1.5, 1)	(1.5, 1.5)
$\xi = 0.8$										
100	LMP	0.141	0.054	0.034	0.120	0.073	0.038	0.084	0.082	0.046
	EL	0.039	0.009	0.006	0.026	0.012	0.008	0.018	0.012	0.009
300	LMP	0.775	0.388	0.183	0.628	0.493	0.299	0.475	0.448	0.331
	EL	0.325	0.102	0.030	0.231	0.097	0.052	0.143	0.114	0.061
500	LMP	0.959	0.675	0.381	0.880	0.791	0.594	0.754	0.729	0.583
	EL	0.656	0.236	0.070	0.500	0.268	0.143	0.347	0.222	0.131
1000	LMP	1.000	0.967	0.753	0.996	0.992	0.934	0.982	0.982	0.939
	EL	0.945	0.599	0.255	0.891	0.673	0.399	0.720	0.609	0.402
5000	LMP	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	EL	1.000	1.000	0.981	1.000	1.000	0.997	0.999	1.000	0.997
$\xi = 0.9$										
100	LMP	0.118	0.043	0.035	0.071	0.041	0.025	0.049	0.052	0.037
	EL	0.042	0.013	0.015	0.032	0.021	0.014	0.020	0.013	0.011
300	LMP	0.673	0.278	0.101	0.545	0.372	0.190	0.388	0.313	0.220
	EL	0.301	0.077	0.021	0.199	0.101	0.043	0.128	0.093	0.055
500	LMP	0.914	0.558	0.277	0.868	0.681	0.423	0.700	0.622	0.467
	EL	0.571	0.183	0.054	0.486	0.239	0.128	0.353	0.212	0.113
1000	LMP	1.000	0.929	0.627	0.995	0.965	0.849	0.970	0.954	0.865
	EL	0.927	0.503	0.204	0.852	0.637	0.375	0.702	0.548	0.368
5000	LMP	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	EL	1.000	1.000	0.963	1.000	1.000	0.996	1.000	1.000	0.994

3.2. *Empirical Power.* In order to investigate the empirical powers, we consider the alternatives under three versions of model (2):

(A1) $\alpha_t \sim \text{Beta}(a, b)$, $\varepsilon_t \sim N(0, \sigma_N^2)$.

(A2) $\alpha_t \sim \text{Beta}(a, b)$, $\varepsilon_t \sim t(m)$.

(A3) $\alpha_t \sim \text{Beta}(a, b)$, $\varepsilon_t \sim F_\xi(x)$. The distribution function of ξ -contamination is

$$F_\xi(x) = \xi \Phi\left(\frac{x}{\sigma_1}\right) + (1 - \xi) \Phi\left(\frac{x}{\sigma_2}\right), \quad (10)$$

where ξ is a fixed constant satisfying $0 < \xi < 1$ and $\Phi(x)$ is the distribution function of standard normal random variable, $\sigma_i > 0$, $i = 1, 2$.

To calculate the empirical powers of the two tests, in the first, we generate samples from the model A1 in which the parameters $a = 0.5, 1$, $b = 0.5, 1$, and $\sigma_N = 0.5, 1$. In the second setting, we simulate samples from model A2 with the degrees of freedom $m = 6, 10$ the parameters $a = 0.5, 1$ and $b = 0.5, 1$. For model A1 and A2, we take the sample size $N = 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000$, and 5000 . In the last case, we generate data from the model A3 in which the $(\xi, \sigma_1, \sigma_2) = (0.8, 1, 3)$, $(\xi, \sigma_1, \sigma_2) = (0.9, 1, 3)$, and the parameters $a = 0.5, 1, 1.5$, $b = 0.5, 1, 1.5$. For model A3, we employ the sample size $N = 100, 300, 500, 1000$, and 5000 . The empirical power of the above three models are summed up in Tables 4–6. In three cases, the power of two test statistics increase with the sample size, while our test produces relatively better powers

than the EL test. Furthermore, both tests powers are close to 1 at the sample size $N = 5000$. Overall, from these simulations, we conclude that the accuracy of using LMP is 6.7%, 28.8%, 26.1% higher than that of EL test under normal, student's t , and symmetric contamination errors, respectively. As anticipated, our finding shows that the LMP test is a functional tool to detect a parameter change for RCAR(1) model. Therefore, we recommend the LMP for practical use because its computation is not difficult and the overall performance is better than that EL under normal, student's t , and symmetric contamination errors.

4. A Real Life Data Analysis

In this section, we illustrate how the LMP method can be applied to a practical application. This data consist of 78 monthly number of annulus growth rate of the import bill in Australia, starting in December 2010 and ending in May 2017. The data are available online at the CEInet Statistics Database site <http://db.cei.cn/page/Default.aspx>. The mean and variance of the data are found to be 0.2069 and 15.3347. The data are denoted as $y_1, y_2, \dots, y_{78}$. Figure 1 is the sample path plot for the real data $y_t, t = 1, 2, \dots, 78$.

For such a process, the mean function of $\{y_t\}$ is constant and we may assume that the process mean is subtracted out to produce a process $X_t = y_t - E(y_t)$ with zero mean. The plot of the sample path, autocorrelation function (ACF), and the partial autocorrelation function (PACF) for series $\{X_t\}$ are given in Figures 1 and 2, respectively. From Figure 1, we can

FIGURE 1: The left one is the sample path of original series y_t and the right one is the sample plot of centering series X_t for the real data.

FIGURE 2: The sample autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of centering series X_t for the real data.

see X_t may come from a stationary autoregressive time series process. From Figure 2, we conclude that X_t is from first-order autoregressive process. Moreover, we test the normality of the residual data X_t according to the Normal Q-Q plot introduced by Henry C Thode [21]. From Figure 3, we notice that the scatter points on the Normal Q-Q plot are close to the reference line, so the residual data X_t can basically be seen as following the normal distribution. Hence, we derive that the model N1 would be more suitable to model these data.

From the time series plot given in Figure 1, the constancy of the parameter may be suspected, and, therefore, we are interested in testing the hypothesis of constancy of the coefficient parameter. We carry out the test for $H_0 : \sigma_\alpha^2 = 0$ against $H_1 : \sigma_\alpha^2 > 0$. The value of the test statistic in (8) turned out to be 2.1151, with p value 0.0344, which indicates the rejection of the null hypothesis at 5% level of significance. Thus, in this case the RCAR(1) model would be much more appropriate as opposed to the AR(1) model.

5. Conclusions

In this article, we propose a locally most powerful-type test for testing the constancy of the random coefficient parameter in autoregressive model and derive their limiting null distributions under regularity conditions. It is clear from the applications that the coefficient need not remain constant throughout the time. Therefore, it is essential to have such a

FIGURE 3: The normal Q-Q plot of centering series X_t for the real data.

test conducted whenever the random variation is suspected. Through our simulation study and a real-data analysis, we demonstrate that our test succeeds and it performs better than the competitor. Finally, we anticipate that our locally most powerful-type test can be extended to other types of time series model.

Appendix

A. Process of Establish the Test Statistic

In this Appendix A, we present the detailed steps to obtain the test statistics.

Proof. Therefore, we can get the likelihood function

$$\begin{aligned} L_{H_1}(x_1, \dots, x_t) &= f_{X_0}(x_0) \prod_{t=1}^n f_{X_t|X_{t-1}}(x_t | x_{t-1}) \\ &= f_{X_0}(x_0) \prod_{t=1}^n \int f_\varepsilon(x_t - \alpha x_{t-1}) dF_\alpha. \end{aligned} \quad (A.1)$$

Now, the Taylor series expansion of L_{H_1} around α which is the mean of α_t gives that

$$\begin{aligned} L_{H_1}(x_1, \dots, x_t) &= f_{X_0}(x_0) \prod_{t=1}^n \int f_\varepsilon(x_t - \alpha x_{t-1}) \\ &\quad - \alpha_t x_{t-1}) dF_\alpha. \\ &= f_{X_0}(x_0) \prod_{t=1}^n \int \left[f_\varepsilon(x_t - \alpha x_{t-1}) \right. \end{aligned}$$

$$\begin{aligned} &\quad + (\alpha_t - \alpha) f'_\varepsilon(x_t - \alpha x_{t-1}) \\ &\quad + \frac{(\alpha_t - \alpha)^2}{2!} f''_\varepsilon(x_t - \alpha x_{t-1}) \\ &\quad \left. + \frac{(\alpha_t - \alpha)^3}{3!} f'''_\varepsilon(x_t - \alpha x_{t-1}) + \dots \right] dF_\alpha \\ &= f_{X_0}(x_0) \prod_{t=1}^n \left[f_\varepsilon(x_t - \alpha x_{t-1}) + \frac{1}{2} \sigma_\alpha^2 f''_\varepsilon(x_t - \alpha x_{t-1}) \right. \\ &\quad \left. - \alpha x_{t-1}) + f'''_\varepsilon(x_t - \alpha x_{t-1}) \int \frac{(\alpha_t - \alpha)^3}{3!} dF_\alpha \right. \\ &\quad \left. + \dots \right]. \end{aligned} \quad (A.2)$$

Obviously, we can conclude that $P(\alpha_t - \alpha = 0) = 1$ if $\sigma_\alpha^2 = 0$. That is to say, the distribution function of random variable α_t is degenerated. Furthermore, we derive that

$$\int \frac{(\alpha_t - \alpha)^k}{k!} dF_\alpha = 0, \quad k = 1, 2, 3, \dots \quad (A.3)$$

By taking the derivative of $\log L_{H_1}$ with respect to β , at $\sigma_\alpha^2 = 0$, we have the following equation:

$$\begin{aligned} &\left. \frac{\partial \log L_{H_1}}{\partial \sigma_\alpha^2} \right|_{\sigma_\alpha^2=0} \\ &= \left. \frac{\partial \sum_{t=1}^n \log \left[f_\varepsilon(x_t - \alpha x_{t-1}) + (1/2) \sigma_\alpha^2 f''_\varepsilon(x_t - \alpha x_{t-1}) + f'''_\varepsilon(x_t - \alpha x_{t-1}) \int \frac{(\alpha_t - \alpha)^3}{3!} dF_\alpha + \dots \right]}{\partial \sigma_\alpha^2} \right|_{\sigma_\alpha^2=0} \\ &= \left. \frac{\partial \sum_{t=1}^n \log \left[f_\varepsilon(x_t - \alpha x_{t-1}) + (1/2) \sigma_\alpha^2 f''_\varepsilon(x_t - \alpha x_{t-1}) \right]}{\partial \sigma_\alpha^2} \right|_{\sigma_\alpha^2=0} = \frac{1}{2} \sum_{t=1}^n \frac{f''_\varepsilon(x_t - \alpha x_{t-1})}{f_\varepsilon(x_t - \alpha x_{t-1})}. \end{aligned} \quad (A.4)$$

Thus, the LMP test statistic has the form

$$Q_n(\beta) = \sum_{t=1}^n \frac{1}{2} \frac{f''_\varepsilon(x_t - \alpha x_{t-1})}{f_\varepsilon(x_t - \alpha x_{t-1})}. \quad (A.5)$$

This completes the proof. $\square$

B. Proof of Theorem 1

Proof of Theorem 1. In the following, we can write down the log-likelihood function under the null hypothesis:

$$\log L = \sum_{t=1}^n \log f_\varepsilon(x_t - \alpha x_{t-1}). \quad (B.1)$$

According to the maximum likelihood principle, the maximum likelihood estimate $\hat{\beta}$ can be obtained by maximizing

the log-likelihood function $\log L$ with respect to β . Applying the Taylor series expansion to $\partial \log f_\varepsilon / \partial \beta$ gives

$$0 = \frac{\partial \log f_\varepsilon}{\partial \beta} = \frac{\partial \log f_\varepsilon}{\partial \beta} + (\hat{\beta} - \beta) \frac{\partial^2 \log f_\varepsilon(\beta^*)}{\partial \beta \partial \beta'}, \quad (B.2)$$

where β^* is on the line segment between β and $\hat{\beta}$, so $\beta^* \xrightarrow{P} \beta(n \rightarrow \infty)$. Under the null hypothesis, we can write

$$\hat{\beta} - \beta = - \left\{ \sum_{t=1}^n \frac{\partial^2 \log f_\varepsilon}{\partial \beta \partial \beta'} + o_p(n) \right\}^{-1} \sum_{t=1}^n \frac{\partial \log f_\varepsilon}{\partial \beta}. \quad (B.3)$$

Using the similar method as in the above, expanding $(1/\sqrt{n}) \sum_{t=1}^n R_t(\hat{\beta})$ around β gives that

$$\begin{aligned} \frac{1}{\sqrt{n}} \sum_{t=1}^n R_t(\hat{\beta}) &= \frac{1}{\sqrt{n}} \sum_{t=1}^n R_t(\beta) \\ &+ (\hat{\beta} - \beta)^T \frac{1}{\sqrt{n}} \sum_{t=1}^n \frac{\partial R_t(\beta)}{\partial \beta} + o_p(1), \end{aligned} \quad (\text{B.4})$$

with $\partial R_t(\beta)/\partial \beta = (\partial R_t/\partial \alpha, \partial R_t/\partial \sigma^2)^T$. The remainder term is $o_p(1)$ since $\hat{\beta} \xrightarrow{p} \beta$. Under (C3) and (C4), we conclude that $\partial^2 R_t(\beta)/\partial \beta \partial \beta' = O_p(n)$. Therefore,

$$\begin{aligned} &(\hat{\beta} - \beta)^T \left(\frac{1}{\sqrt{n}} \right) \sum_{t=1}^n \frac{\partial R_t(\beta)}{\partial \beta} \\ &= - \left\{ \frac{1}{n} \sum_{t=1}^n \left[\frac{\partial R_t(\beta)}{\partial \beta} \right]^T \right\} \\ &\quad \times \left\{ \frac{1}{n} \sum_{t=1}^n \frac{\partial^2 \log f_\varepsilon}{\partial \beta \partial \beta'} + o_p(1) \right\}^{-1} \\ &\quad \times \frac{1}{\sqrt{n}} \sum_{t=1}^n \frac{\partial \log f_\varepsilon}{\partial \beta}, \end{aligned} \quad (\text{B.5})$$

Now the first term of (B.5) converges to

$$\begin{aligned} \frac{1}{n} \sum_{t=1}^n \frac{\partial R_t(\beta)}{\partial \beta} &= \frac{1}{n} \sum_{t=1}^n \frac{\partial}{\partial \beta} \left\{ \frac{\partial \log f_\varepsilon}{\partial \sigma_\alpha^2} \right\}_{\sigma_\alpha^2=0} \\ &= \left(\frac{1}{n} \sum_{t=1}^n \frac{\partial^2 \log f_\varepsilon}{\partial \alpha \partial \sigma_\alpha^2} \right)_{\sigma_\alpha^2=0} \longrightarrow W \\ &\quad \text{as } n \longrightarrow \infty, \end{aligned} \quad (\text{B.6})$$

with $W = (W_{11}, W_{12})^T$. This is because

$$\begin{aligned} W_{11} &= E \left(\frac{\partial \log f_\varepsilon}{\partial \alpha \partial \sigma_\alpha^2} \right) = E \left(- \frac{\partial \log f_\varepsilon}{\partial \alpha} \frac{\partial \log f_\varepsilon}{\partial \sigma_\alpha^2} \right) \\ &= - \frac{1}{2} E \left[\frac{\partial \log f_\varepsilon}{\partial \alpha} \left\{ \frac{\partial^2 \log f_\varepsilon}{\partial \alpha^2} + \left(\frac{\partial \log f_\varepsilon}{\partial \alpha} \right)^2 \right\} \right], \\ &\quad \text{at } \sigma_\alpha^2 = 0, \\ W_{12} &= E \left(\frac{\partial \log f_\varepsilon}{\partial \sigma_\varepsilon^2 \partial \sigma_\alpha^2} \right) = E \left(- \frac{\partial \log f_\varepsilon}{\partial \sigma_\varepsilon^2} \frac{\partial \log f_\varepsilon}{\partial \sigma_\alpha^2} \right) \\ &= - \frac{1}{2} E \left[\frac{\partial \log f_\varepsilon}{\partial \sigma_\varepsilon^2} \left\{ \frac{\partial^2 \log f_\varepsilon}{\partial \alpha^2} + \left(\frac{\partial \log f_\varepsilon}{\partial \alpha} \right)^2 \right\} \right], \\ &\quad \text{at } \sigma_\alpha^2 = 0. \end{aligned} \quad (\text{B.7})$$

Next, we can obtain the elements of Fisher information matrix

$$\begin{aligned} \Sigma &= \frac{1}{n} \sum_{t=1}^n E \left[\frac{\partial \log f_\varepsilon}{\partial \beta} \left(\frac{\partial \log f_\varepsilon}{\partial \beta} \right)^T \mid \mathcal{F}_{t-1}^\gamma \right] \\ &= \begin{bmatrix} -E \left[\frac{\partial^2 \log f_\varepsilon}{\partial \alpha^2} \right] & -E \left[\frac{\partial^2 \log f_\varepsilon}{\partial \alpha \partial \sigma_\varepsilon^2} \right] \\ -E \left[\frac{\partial^2 \log f_\varepsilon}{\partial \alpha \partial \sigma_\varepsilon^2} \right] & -E \left[\frac{\partial^2 \log f_\varepsilon}{\partial \sigma_\varepsilon^2 \partial \sigma_\varepsilon^2} \right] \end{bmatrix}. \end{aligned} \quad (\text{B.8})$$

Thus (B.5) is asymptotically equivalent to

$$-W \Sigma^{-1} \frac{1}{\sqrt{n}} \sum_{t=1}^n \frac{\partial \log f_\varepsilon}{\partial \beta}. \quad (\text{B.9})$$

We present the following lemma recommended by Billingsley [22] to establish the asymptotic normality of the test statistic.

Lemma B.1. Under assumption of (A.1), we have as $n \rightarrow \infty$,
(i) $\lim_{n \rightarrow \infty} (1/n) \sum_{t=1}^n E(R_t^2(\beta) \mid \mathcal{F}_{t-1}^R) = \sigma^2$, a.s.; (ii) $(1/\sqrt{n}) \sum_{t=1}^n R_t(\beta) \xrightarrow{d} N(0, \sigma^2)$, where $\sigma^2 = (1/4)E\{\partial^2 \log f_\varepsilon / \partial \alpha^2 + (\partial \log f_\varepsilon / \partial \alpha)^2\}$.

Proof. By the ergodic theorem, the first conclusion holds.

Now, let $\mathcal{F}_t = \sigma\{x_0, x_1, \dots, x_t\}$, $t \geq 1$ and $\mathcal{F}_0$ is a sigma field. Note that $Q_n(\beta) = \sum_{t=1}^n R_t(\beta)$. It is easy to see that $[R_n(\beta) \mid \mathcal{F}_{n-1}] = 0$. Then, we have

$$\begin{aligned} E[Q_n(\beta) \mid \mathcal{F}_{n-1}] &= E[Q_{n-1}(\beta) + R_n(\beta) \mid \mathcal{F}_{n-1}] \\ &= Q_{n-1}(\beta), \end{aligned} \quad (\text{B.10})$$

Thus $\{Q_n, \mathcal{F}_n, n \geq 0\}$ is a martingale. We have shown $\sup_t E[R_t(\beta)]^{2+\eta} < \infty$, which means $E[R_t^2(\beta)]$ is uniformly integrable. By Theorem 1.1 of Billingsley [22], we have that as $n \rightarrow \infty$

$$\frac{1}{n} \sum_{t=1}^n R_t^2(\beta) \xrightarrow{a.s.} E[R_n^2(\beta) \mid \mathcal{F}_{n-1}] = \sigma^2. \quad (\text{B.11})$$

Hence, using the following version of Martingale Central Limit Theorem from Hall and Heyde [23], we have $(1/\sqrt{n}) \sum_{t=1}^n R_t(\beta) \xrightarrow{d} N(0, \sigma^2)$. The proof of Lemma B.1 is thus completed. $\square$

Similarly, we can verify that $S_n = \sum_{t=1}^n (\partial \log f_\varepsilon / \partial \beta)$ is a martingale. By ergodic and stationary properties, we obtain that as $n \rightarrow \infty$,

$$\frac{1}{n} \sum_{t=1}^n \frac{\partial^2 \log f_\varepsilon}{\partial \beta \partial \beta^T} \xrightarrow{a.s.} E \left[\frac{\partial \log f_\varepsilon}{\partial \beta} \left(\frac{\partial \log f_\varepsilon}{\partial \beta} \right)^T \right] = \Sigma. \quad (\text{B.12})$$

Therefore, $(1/\sqrt{n})S_n \xrightarrow{d} N(0, \Sigma)$.

In the same way, for any vector $\mathbf{c} = (c_1, c_2, c_3)^T \in R^3 \setminus (0, 0, 0)$ we have

$$\begin{aligned} \frac{1}{\sqrt{n}} \mathbf{c}^T \left(\frac{\sum_{t=1}^n R_t(\boldsymbol{\beta})}{\sum_{t=1}^n \frac{\partial \log f_\varepsilon}{\partial \boldsymbol{\beta}}} \right) &= \frac{1}{\sqrt{n}} \\ &\cdot \sum_{t=1}^n \left(c_1 R_t(\boldsymbol{\beta}) + c_2 \frac{\partial \log f_\varepsilon}{\partial \alpha} + c_3 \frac{\partial \log f_\varepsilon}{\partial \sigma_\varepsilon^2} \right), \\ &\xrightarrow{d} N \left(0, E \left(c_1 R_t(\boldsymbol{\beta}) + c_2 \frac{\partial \log f_\varepsilon}{\partial \alpha} + c_3 \frac{\partial \log f_\varepsilon}{\partial \sigma_\varepsilon^2} \right)^2 \right) \end{aligned} \quad (\text{B.13})$$

By the Cramer-Wold device, we obtain

$$\frac{1}{\sqrt{n}} \left(\frac{\sum_{t=1}^n R_t(\boldsymbol{\beta})}{\sum_{t=1}^n \frac{\partial \log f_\varepsilon}{\partial \boldsymbol{\beta}}} \right) \xrightarrow{d} N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & W^T \\ W & \Sigma \end{pmatrix} \right]. \quad (\text{B.14})$$

where $W = (W_{11}, W_{12})^T$. Then we can make the conclusion that

$$\frac{1}{\sqrt{n}} \sum_{t=1}^n R_t(\hat{\boldsymbol{\beta}}) \xrightarrow{d} N(0, \omega^2), \quad (\text{B.15})$$

where $\omega^2 = \sigma^2 - W^T \Sigma^{-1} W$. The proof of the theorem is completed. $\square$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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