Coordination of Time-Varying Price Supply Chain with Inequity-Averse Retailers

Shuanjun Song and Sheng Hu

School of Mechanical & Electrical Engineering, Xi’an Polytechnic University, Xi’an 710048, China

Correspondence should be addressed to Shuanjun Song; songshuanjun@126.com

Received 23 January 2019; Revised 24 May 2019; Accepted 29 May 2019; Published 17 June 2019

1. Introduction

With the product update cycle shortening and the timeliness of the product becoming increasingly prominent, the competition of supply chain enterprises has changed from cost, quality, and price competition to time competition mode [1]. The response time to market demand has become the key factor for supply chain enterprises to win competitive advantages [2]. In the actual supply chain operation, the market demand, cost, and price of products will change with time, so we must consider the influence of time factors on the decision-making and performance of all members in the supply chain [3–9]. Therefore, supply chain coordination considering price time is more in line with the actual supply chain operation. On the other hand, in the supply chain operation, supply chain members make decisions not only to maximize their own interests, but also to pay attention to the profitability of other stakeholders. Fehr and Schmi propose that inequity aversion can be used to characterize this decision-making behavior [10]. Inequity aversion means rejecting fairness results. When your own benefits are less than the benefits of others, there are fairness and inequity aversion that will result in utility losses. When your own benefits are greater than the benefits of others, there is a favorable inequity aversion, and there will also be utility losses. It can be seen that the supply chain coordination considers the time-varying characteristics of the price and the inequity aversion characteristics of the decision-making members, which helps the actual supply chain operation to achieve an ideal coordination state.

Many scholars have studied the time-varying supply chain. Shen studied the two-level supply chain decision-making model in which market demand fluctuates with time, and analyzed the impact of information sharing on such supply chain decision-making [3]. Bai considered the economic batch model of perishable products with time-varying production costs [5]. References [6–9] studied the supply chain decision-making problem in which price is sensitive to demand and response time.

Supply chain coordination is the ultimate goal of supply chain management. Contract coordination is an effective way and many scholars have studied the issue of contract coordination [11–21]. Wholesale price contract is the simplest form. Spengler believes that “double marginalization” leads to the failure of wholesale price contract to coordinate supply chain [16]. Revenue-sharing contract is considered to coordinate supply chain when there is quantitative competition among retailers [17]. The buy-back contract coordinates the
supply chain by sharing the retailer's unsalable risk through the manufacturer's buy-back strategy [18]. Based on the time competition model, the literature [19] considers the revenue-sharing model with the influence of order time and analyzes the influence of time factor on supply chain decision-making. On the basis of buy-back contract, Su designed a time-based joint contract of price discount sharing and unsalable subsidy to coordinate the time-varying price supply chain with uncertain response time [9]. It can be seen that the above researchers all assume that the decision-maker is self-interested; that is, the decision-maker only makes decisions from the perspective of maximizing his own benefits without considering the benefits of other members.

Guth [22] pointed out the defect of this self-interest hypothesis through the ultimate game model. He believed that, in reality, decision-makers will pay attention to the profit situation of other stakeholders when making decisions, and there are fairness concerns. Since then, many scholars have carried out this research [23–32]. Through empirical analysis, Yang [27] further proves the impact of inequity aversion on the performance of contracts. On this basis, Bi [28] discussed the impact of inequity aversion on wholesale price contract coordination supply chain when market demand is random. Ding [29] takes fairness concerns into account in closed-loop supply chain decision-making and studies the pricing decision-making of closed-loop supply chain in which manufacturers' equity aversion is concerned. However, none of the above studies have considered the time-varying characteristics in the supply chain.

Based on the above analysis, different from the research in [9, 23, 28], this paper comprehensively considers the inequity-averse behavior characteristics of decision-makers and the time-varying characteristics of supply chain. Factors such as market demand, response time, time-varying price, and inequity aversion are introduced into the supply chain buy-back contract to establish a supply chain coordination decision model. Then the impact of inequity aversion and response time for buy-back contracts is analyzed and the conditions for such supply chain coordination are given. Finally, the proposed method is verified through a numerical example.

2. Problem and Symbol Descriptions

This paper considers a two-stage time-varying supply chain consisting of a single manufacturer and a retailer. It produces and sells a short-life-cycle product (such as mobile phones, personal computers, and digital cameras). The market demand of the product is uncertain, the market price falls over time, and the retailer has the characteristics of inequity aversion. It pays more attention to its own and manufacturers' expected profits. The reference value of inequity aversion is the manufacturer's expected profits. The manufacturer organizes production according to the retailer's order. The response time of the manufacturer to the retailer's actual order has a certain randomness. The price of the product in the terminal market decreases with the delay of the order response time. The manufacturer decides the wholesale price of the product and the relevant contract parameters according to the contract relationship established with the retailer. Retailers determine their optimal order quantity according to market demand forecast and their inequity aversion. Assuming that the information is complete, manufacturers and retailers are willing to share private information such as product demand, price, and market demand. In order to establish a decision-making model for such supply chain, the relevant variables are defined as Table 1.

In Table 1, the distribution function $F(x)$ of market demand $x$ is assumed to be continuous, differentiable, monotonically increasing, $F(x) > 0$, $f(x) > 0$; $p$ decreases exponentially with $t$, assuming that the relationship is $p(t) = p_0 e^{-\lambda t}$ (the same expression is used in [9]), $p_0 e^{-\lambda t} > c_r$, $g < p_0 e^{-\lambda t}$. In the following text, the subscript $j$ denotes the centralized system, $f$ denotes the decentralized system, $h$ denotes the cooperative system, and the superscript * denotes the optimal value.

3. Optimal Decision Model of Centralized System

The decision-making goal of the ideal centralized system is to maximize the overall profit of the supply chain. In the two-stage time-varying supply chain system assumed above, when
the retailer’s order quantity is \( q \) and the manufacturer’s actual response time is \( t \), the expected market price of a unit product can be described as follows.

\[
P = E(p) = \int_0^t p_0 e^{-\lambda t} u(t) \, dt	ag{1}
\]

Similarly, the expected profit of supply chain can be described as follows.

\[
\Pi_j = (P - c_j - c_i) q_j - (P - g) \int_0^{q_j} (q_j - x) f(x) \, dx
\]

(2)

Literature [9] has proved that under stochastic response time, the overall optimal expected profit of time-varying price supply chain is a concave function of order quantity, which maximizes the overall profit of the supply chain. At this time, the optimal order quantity can be described as follows.

\[
q_j^* = F^{-1} \left( \frac{P - c_j - c_i}{P - g} \right)
\]

(3)

4. Optimal Decision Model for Decentralized Systems with Inequity-Averse Retailers

In a decentralized decision-making system without contract, when the manufacturer gives a basic wholesale price and the response time is random, the manufacturer’s expected wholesale price can be described as follows.

\[
W = E(w) = \int_0^t w u(t) \, dt
\]

(4)

When the retailer orders are \( q_j \), the expected profits of the manufacturer can be described as follows.

\[
\Pi_s = (W - c_j) q_j
\]

(5)

The expected profits of retailer can be described as follows.

\[
\Pi_r = (P - W - c_j) q_j - (P - g) \int_0^{q_j} (q_j - x) f(x) \, dx
\]

(6)

Under the decentralized decision-making mode, retailers are concerned not only about their own profits, but also about manufacturers’ profits because of their inequity-averse characteristics. Retailers will maximize their own utility according to market demand distribution, manufacturers’ wholesale prices, manufacturers’ response time distribution, and other factors. According to Fehr and Schmidt’s inequity-averse model [18], its utility function can be described as follows.

\[
U_r = \begin{cases} 
(1 + \alpha) \Pi_r - \alpha \Pi_s, & \Pi_r \geq \Pi_s \\
(1 - \beta) \Pi_r + \beta \Pi_s, & \Pi_r < \Pi_s 
\end{cases}
\]

(7)

Here, \( \alpha \geq 0 \), the degree of disadvantageous inequity aversion of retailers increases with the increase of \( \alpha \); \( 0 \leq \beta < 1 \), and when \( \beta \to 1 \), the retailers have extremely advantageous inequity aversion; when \( \alpha = 0 \), \( \beta = 0 \), there is no inequity-averse retailers.

4.1. Decision Model of Retailers with Disadvantageous Inequity Aversion. According to (7), when the retailer’s expected profit is less than the manufacturer’s expected profit, the retailer has disadvantageous inequity-averse behavior, and the retailer’s utility can be described as

\[
U_{r1} = (1 + \alpha) \Pi_r - \alpha \Pi_s
\]

\[
= (1 + \alpha) (P - W - c_j) q_j - (1 + \alpha) (P - g) \int_0^{q_j} (q_j - x) f(x) \, dx
\]

\[
- \alpha (W - c_j) q_j
\]

(8)

so we can get the following.

\[
\frac{\partial U_{r1}}{\partial q_j} = (1 + \alpha) (P - W - c_j) - \alpha (W - c_j)
\]

(9)

\[
\frac{\partial^2 U_{r1}}{\partial q_j^2} = -(1 + \alpha) (P - g) f(q_j) < 0
\]

(10)

From (10), it can be seen that the optimal order quantity of a retailer is a concave function of its utility, and there exists an optimal order quantity to maximize its utility. The optimal order quantity can be described as follows.

\[
q_j^* = F^{-1} \left[ \frac{P - W - c_j}{P - g} - \frac{\alpha (W - c_j)}{(1 + \alpha) (P - g)} \right]
\]

(11)

Proposition 1. In a time-varying price supply chain with stochastic response time, under the decentralized decision-making mode without coordination contract, the optimal order quantity of disadvantageous inequity-averse retailers is less than the optimal order quantity of the whole supply chain, the supply chain cannot be coordinated, and the optimal order quantity of retailers decreases with the increase of the aversion coefficient.

Proof. In general, \( \Pi_s > 0 \); since \( \Pi_s = (W - c_j) q_j \), we can get \( W > c_j \). Also \( P - g > 0 \), \( \alpha > 0 \); we can get \( \alpha (W - c_j)/(1 + \alpha) (P - g) > 0 \). Therefore, \( (P - W - c_j)/(P - g) - \alpha (W - c_j)/(1 + \alpha) (P - g) < (P - W - c_j)/(P - g) - \alpha (W - c_j)/(P - g) \). Comparing formula (3) and (11), since \( f(x) \) is monotonically increasing, \( q_j^* < q_j^* \); that is, the optimal order quantity of the retailer in the decentralized system does not reach the optimal order quantity of the ideal centralized system, so the supply chain cannot be coordinated.

Also \( \partial q_j^* /\partial \alpha = - (W - c_j)/(1 + \alpha)^2 (P - g) f \left[ (P - W - c_j)/(P - g) - \alpha (W - c_j)/(1 + \alpha) (P - g) \right] \), \( W > c_j, P - g > 0 \), \( f(x) > 0 \), so \( \partial q_j^* /\partial \alpha < 0 \), \( q_j^* \) decreases with the increase of \( \alpha \). That is, the optimal order quantity of retailers decreases with the increase of the aversion coefficient.
From the comparative analysis of the above models, we can see that because there is no contract in the supply chain, the retailer bears all the market risks (product unsalable risk caused by uncertain market demand and price falling risk caused by uncertain delivery date), and the retailer has inequity-averse behavior, so the order quantity of the retailer is smaller than that of the optimal supply chain.

4.2. Decision Model of Retailers with Advantageous Inequity Aversion

According to (7), when the retailer’s expected profit is higher than the manufacturer’s expected profit, the retailer has advantageous inequity aversion behavior, and the profit is higher than the manufacturer’s expected profit, the supply chain can be coordinated perfectly; when the retailer’s expected profit is smaller than that of the optimal supply chain, the supply chain cannot be coordinated.

From the above analysis, it can be seen that the inequity-averse retailer’s profit may be lower in a time-varying supply chain system because of full risks (unmarketable risk and price reduction risk) borne by the retailer. Retailer’s inequity aversion aggravates double marginalization of supply chain, which makes the order quantity lower than the optimal order quantity and the supply chain uncoordinated. When the retailer expects his own profit to be more than the manufacturer’s profit, his advantageous inequity-averse behavior increases order quantity. When the aversion reaches a certain value (\(\beta = 1/2\)), the profit of the supply chain reaches the maximum and reaches the coordinated state. However, when the retailer’s aversion is too high (\(\beta > 1/2\)), the order quantity exceeds the optimal value of the centralized system, and the cost of the whole supply chain increases, which leads to the decline of the overall profit of the supply chain and the lack of perfect coordination.

5. Coordination Contract Design of Cooperative System

Since the retailer takes all the risks in the decentralized system, we think that buy-back contract can be used to transfer the risk of the retailer. Under the buy-back contract, the manufacturer sets the wholesale price of the unit product \(w\) and uses the price \(b\) to buy back the unsalable products of the retailer, so as to stimulate the retailer’s ordering behavior. The retailer determines the optimal order quantity according to the distribution of delivery time, wholesale price, buy-back price, market demand forecast, and his own aversion. The decision goal is to maximize the utility of the retailer. Next, let us discuss contract parameters.

When the retailer order quantity is \(q_h\), the expected profit of the manufacturer (\(\Pi^d_h\)) and the retailer (\(\Pi^s_h\)) can be described as follows.

\[
\Pi^d_h = (W - c_j) q_h - (b - g) \int_0^{q_h} (q_h - x) f(x) dx
\]

\[
\Pi^s_h = (P - W - c_j) q_h - (b - g) \int_0^{q_h} (q_h - x) f(x) dx
\]

When the retailer has disadvantageous inequity aversion, the retailer’s utility \(U_{r1}^r\) and optimal order quantity \(q_h^*\) can be described as follows.

\[
q_{h}^{\ast} = F^{-1} \left[ \frac{(1 + \alpha)(P - W - c_j) - \alpha(W - c_j)}{(1 + \alpha)(P - b) - \alpha(b - g)} \right]
\]

Proposition 2. In a time-varying price supply chain with stochastic response time, under the decentralized decision-making mode without coordination contract, the optimal order quantity of advantageous inequity-averse retailers increases with \(\beta\). When 0 \(\leq\) \(\beta\) \(\leq\) 1/2, the optimal quantity is less than the optimal order quantity of the whole supply chain, and the supply chain cannot be coordinated; when \(\beta = 1/2\), the supply chain can be coordinated perfectly; when 1/2 \(\leq\) \(\beta\) \(\leq\) 1, the optimal order quantity is more than the optimal order quantity of the centralized system, and the supply chain cannot be coordinated.
When the retailer has advantageous inequity aversion, the retailer’s utility $U'_{R2}$ and optimal order quantity $q_h^*$ can be described as follows.

$$U'_{R2} = (1 - \beta) \Pi'_{R} + \beta \Pi'_{I}$$

$$= \left[ (1 - \beta)(P - W - c_2) + \beta(W - c_2) \right] q_h$$

$$- \left[ (1 - \beta)(P - b) + \beta(b - g) \right]$$

$$\cdot \int_{0}^{q_h} (q_h - x) f(x) dx$$

$$q_h^* = F^{-1} \left[ \frac{(1 - \beta)(P - W - c_2) + \beta(W - c_2)}{(1 - \beta)(P - b) + \beta(b - g)} \right]$$

Namely, $q_h^* = q_l^*$, $q_h^* = q_l^*$, the optimal contract parameters $W^*$ and $b^*$ can be satisfied as (22).

$$(1 + \alpha) (P - W^* - c_2) - \alpha (W^* - c_2) = P - c_2 - c_2$$

$$(1 + \alpha) (P - b^*) - \alpha (b^* - g) = P - g$$

$$\Pi'_{I} \geq \Pi'_{I}$$

$$(1 - \beta) (P - W^* - c_2) + \beta(W^* - c_2) = P - c_2 - c_2$$

$$(1 - \beta) (P - b^*) + \beta(b^* - g) = P - g$$

$$\Pi'_{I} < \Pi'_{I}$$

$$\Pi'_{I} \geq \Pi'_{I}, \Pi'_{I} \geq \Pi'_{I}$$

Formula (22) is the constraint that is satisfied when the supply chain is coordinated: individual rational constraints and individual compatibility constraints.

### 6. A Numerical Example

Suppose that there is a two-stage supply chain consisting of a manufacturer and a retailer. The products produced by the two-stage supply chain have time-varying price characteristics. The manufacturer is the leader and the retailer has inequity aversion characteristics. $P_0 = 400$ (yuan/pieces), the price is various with time, $p(t) = 400e^{-0.005t}$. The manufacturer’s delivery date (in weeks) follows a uniform distribution in the range $[2, 10]$, $x \sim U [0, 1000]$, $c_2 = 20$ (yuan/pieces), $c_3 = 150$ (yuan/pieces), $g = 50$ (yuan/pieces); suppose that there is no shortage loss. Buy-back contract is used between retailer and manufacturer to coordinate the supply chain.

### 6.1. Expected Profit of Each Member in Different Supply Chain Systems

Firstly, the decision-making results of different supply chain systems are compared, which is shown in Table 2.

<table>
<thead>
<tr>
<th>Supply chain system</th>
<th>$q^*$ (pieces)</th>
<th>$w$ (yuan/pieces)</th>
<th>$b$ (yuan/pieces)</th>
<th>$\Pi_i$ (yuan)</th>
<th>$\Pi_e$ (yuan)</th>
<th>$\Pi$ (yuan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized System</td>
<td>645</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>70391</td>
</tr>
<tr>
<td>Decentralized System $\alpha = 0.5$</td>
<td>251</td>
<td>250</td>
<td>—</td>
<td>19014</td>
<td>25095</td>
<td>44109</td>
</tr>
<tr>
<td>Decentralized System $\beta = 0.2$</td>
<td>512</td>
<td>210</td>
<td>—</td>
<td>36670</td>
<td>30728</td>
<td>67398</td>
</tr>
<tr>
<td>Cooperative System $\alpha = 0.5$</td>
<td>645</td>
<td>250</td>
<td>180</td>
<td>32928</td>
<td>37463</td>
<td>70391</td>
</tr>
<tr>
<td>profit increment</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>13914</td>
<td>12368</td>
<td>26282</td>
</tr>
<tr>
<td>Cooperative System $\beta = 0.2$</td>
<td>645</td>
<td>250</td>
<td>205</td>
<td>38132</td>
<td>32259</td>
<td>70391</td>
</tr>
</tbody>
</table>

**Table 2: Decision-making results of different supply chain systems.**

It can be seen from Table 2 that in the cooperative system under the buy-back contract, the optimal order quantity of the retailer reaches the optimal order quantity of the centralized system, and the overall profit of the supply chain is optimal under the repurchase contract. Under the buy-back contract, as long as the manufacturer sets a reasonable wholesale price and buy-back price, the retailer's order quantity can reach the optimum. In addition, the profits obtained by both sides in the centralized system can satisfy the individual rational constraints and incentive compatibility principles, which shows that the contract can effectively coordinate such supply chains.

### 6.2. The Influence of Retailer’s Disadvantageous Inequity Aversion on Supply Chain

In decentralized system, the retailer decides the optimal order quantity according to the wholesale price given by the manufacturer, market demand, and the degree of its own inequity aversion and takes the maximum self-utility as the decision goal. Suppose the wholesale price given by the manufacturer is $w = 260$. If the retailer does not consider the profit of the manufacturer, the retailer determines the optimal order quantity from its own profit as $q_f^* = 308$. Thus, the manufacturer’s expected profit is 33887 yuan and the retailer's expected profit is 15773 yuan. Correspondingly, when the retailer has disadvantageous inequity aversion behavior, the retailer decides the optimal order quantity $q_f^*$ according to its own utility. Figure 1 shows the curve of retailer's optimal order quantity varying with the degree of aversion in a decentralized system.

As shown in Figure 1, in the decentralized system, the retailer's order quantity is significantly lower than the optimal order quantity of the centralized system due to the disadvantageous inequity aversion. The higher the retailer's aversion to disadvantageous inequity, the lower the retailer’s risk-taking ability, and the smaller the retailer's order quantity, so the supply chain can not be coordinated.

Figure 2 shows the comparison of supply chain profits between cooperative and decentralized systems under buy-back contract. It can be seen that the supply chain profit is optimal through the incentive of buy-back contract, and the supply chain profit does not change with the increase of retailer's aversion coefficient; in decentralized system, the
expected profit of supply chain decreases with the increase of retailer’s aversion coefficient.

Figure 3 shows the effect of $\alpha$ on the profits of retailer and manufacturer in decentralized and cooperative system. From Figure 3, the profit of retailer and manufacturer decreases with the increase of $\alpha$. In the cooperative system, the profit of retailer and manufacturer increases significantly compared with that of decentralized system, which shows that buy-back contract stimulates retailers’ ordering behavior and ultimately benefits all members of the supply chain. In addition, in the cooperative system, when the wholesale price of the manufacturer, the higher the degree of aversion the retailer has, the lower the buy-back price set by the manufacturer (shown in Figure 4). Therefore, the profit of the retailer will decrease and the profit of the manufacturer will increase. In this case, the buy-back contract is beneficial to the manufacturer.

6.3. The Influence of Retailer’s Advantageous Inequity Aversion on Supply Chain. Furthermore, the influence of retailers’ advantageous inequity aversion degree on decision-making parameters and expected profits of supply chain parties is analyzed in different systems.

Figure 5 shows the variation curve of retailer’s order quantity with its advantageous inequity aversion coefficient in a decentralized system. It reveals that the higher the aversion coefficient of advantageous inequity aversion retailers, the more the order quantity of retailers. When the aversion coefficient is 0.5, the optimal order quantity of the retailer is exactly equal to the optimal order quantity of the centralized system, and the overall profit of the supply chain is optimal. Figure 6 shows the variation curve of the overall profit of the supply chain in cooperative and decentralized systems with the aversion coefficient of advantageous inequity-averse retailer. In Figure 6, in the cooperative system, the supply chain profit does not change with the change of the retailer’s aversion coefficient, while in the decentralized system, the supply chain profit first increases and then decreases with the increase of the retailer’s aversion coefficient, reaching the maximum at $\beta = 0.5$.

Figure 7 shows the effect of retailer’s advantageous inequity aversion coefficient on the profits of all parties in the supply chain. It can be seen that when the retailer’s aversion increases in the decentralized system, the retailer’s expected profit decreases and the manufacturer’s expected profit increases. In the cooperative system, the expected profits of retailers and manufacturers do not change with the retailer’s aversion coefficient. However, in order to coordinate the supply chain, the manufacturer should increase wholesale price and reduce buy-back price to ensure its own profit, so
that the supply chain can reach a coordinated state. In this case, the buy-back contract makes the retailer’s profit increase greatly, so it is more beneficial to the retailer.

6.4. The Influence of Response Time on Contract Parameters. For time-varying price supply chain, uncertainty of manufacturer’s response time is an important factor leading to supply chain risk. In the decentralized system, retailer assumes all supply chain risks. Under buy-back contracts, manufacturer and retailer share the risk of price reduction with uncertain order response time. We calculate the value range of buy-back contract parameters when the order response time is evenly distributed among [2, 10], [4, 10], [6, 10], [8, 10]. The results are shown in Table 3.

Table 3 shows the influence of response time on contract parameters. As can be seen from Table 3, the fluctuation of order response time has little effect on the optimal parameters of the buy-back contract. With the increase of the average order response time, the overall profit of the supply chain decreases. Therefore, when the response time is uncertain, the manufacturer should shorten the order response time.

7. Conclusions

In this paper, retailers’ inequity aversion, market demand uncertainty, and price time-varying characteristics are considered comprehensively, and a decision-making model of cooperative system based on buy-back contract is established. The optimal parameters of buy-back contract and the conditions of supply chain coordination are given. The results show that retailers’ disadvantageous inequity aversion aggravates the double marginalization of supply chain, and retailers’ advantageous inequity aversion improves the degree of supply chain coordination; when retailers have disadvantageous inequity aversion, buy-back contracts are beneficial to manufacturers; when retailers have advantageous inequity aversion, buy-back contracts are beneficial to retailers. The uncertainty of manufacturer’s order response time has little effect on the parameters of buy-back contract.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that no conflicts of interest exist.

Acknowledgments

This research was supported by the National Science Foundation of China No. 71271170, Science and Technology Guidance Plan of China Textile Industry Federation No. 2016090, and Doctoral Research Initiation Fund Project of Xi’an Polytechnic University No. BS201834.
References
