Dynamic Parameter Analysis of Spindle Bearing Using 3-Dimension Quasi-Dynamic Model

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1. Introduction

Development of a spindle bearing system can significantly improve the efficiency and accuracy of machine tools. The quasi-dynamic models of the spindle bearing are not only better capable of analyzing bearing dynamic parameters than quasi-static models and could also need less numerical calculation time than the dynamic model. Generally, the steady working duration of ball bearings would last 90 percent of the whole service life under normal condition. Therefore, establishment of perfect bearing quasi-dynamic model plays massive significant role on the analysis of the bearing operational performance [1–3], such as cage slipping, cage impacts, roller skidding, roller-flange contact, and roller skewing.

Many contributions for spindle bearing system were conducted based on the bearing quasi-dynamic modeling, such as Harris [4] and Yuan [5], who established the equation groups about 6 degrees of rolling elements, 3 degrees of cages, and 5 degrees of rings by dropping the hypothesis of the ring control theory and considering the oil film drags of the contacting ellipse between rolling elements and raceways with oil film lubrication. Wang et al. [6] analyzed the boundary conditions of aeroengine bearing by the quasi-dynamic method and then discussed the relationship between status parameters associated with Failure modes (such as lubricating oil film thickness, etc.) and working parameters (such as rotation speed and external load). Ren et al. [7] established a new quasi-dynamic model of the counter-rotating cylindrical roller bearing based on elastohydrodynamic lubrication theory, hydrodynamic lubrication theory, and the quasi-dynamic analysis method to dynamic behaviors of counter-rotating cylindrical roller bearing. However, those researches have not considered the complete degree of freedom of the rolling bearing components which is very significant to determine their dynamic characteristics. Therefore, it is badly necessary to establish the bearing quasi-dynamic model for the analysis of the component complete degree of freedom.

In order to reflect the real operation of high speed angular contact ball bearings, a quasi-dynamic calculation model for the high speed angular contact ball bearing was built by embedded Broyden-Fletcher-GoldFarb-Shanno mutative
scale optimization in quasi-dynamic calculation program [8]. For the spindle bearing with a certain contact angle, gyroscopic moment will inevitably induct large sliding between the balls and raceways and increase the bearing friction moment, which are the main reasons for the bearing heat generation and wear [9–11]. Therefore, accurately analyzing the spin angular velocity between ball and raceway is benefit for the correct calculation of bearing power dissipation, assessment of the rate of heat generation, and appropriate decision of lubrication flow rate, etc. Meanwhile, the bearing cage is a critical component which is often the most Failure part for high speed bearings, typically. With the increase of bearing rotation speed, the analytical precision of the contact force between the cage and the balls or the guide rings is dramatically important for the force characteristics of bearing research [12–14].

To analyze the operating characteristics of rolling bearings at steady working stage, the bearing quasi-dynamic model is developed in this paper. In the optimal model, (i) by the analysis of the relative positions of each bearing component, the normal forces between the bearing components are determined and (ii) the tangential forces between the bearing components are obtained according the study of the directions and magnitudes of the relative velocities of the contact points and then (iii) the force mathematical models of balls’ 6-DOF, cage’s 6-DOF, and the driving ring’s 5-DOF. On the base of those models, the gyroscopic moment model is established to analyze the pitching angles and yaw angles, and the cage forces and operating condition are studied by the contacting force models between the cage and the ball and guide ring. The results of the improved quasi-dynamic model are furthermore compared with the results of the typical rolling bearing force calculation example of Gupta [15]. Studies show that the model is able to accurately describe the dynamic variations of the parameter characteristics of spindle bearings.

2. Mechanics Modeling of Spindle Bearing

2.1. Techniques of Mechanics Modeling. Figure 1 shows the interaction model of bearing components. The interaction forces between these parts are mainly the normal contact forces and the tangential forces. First, by means of determining the position vectors of part B and part A in space, judging whether part B in contact with part A and calculating the normal contact force, the normal force of part B relative to part A can be obtained. Second, according to the velocity vectors of part A and part B in space, the velocity direction of part B relative to part A at the contact point can be determined, and on the basis of Coulomb’s law, the tangential force of part B relative to part A is obtained. Finally, the influence of the lubricant on the force will be considered to solve the force of part B relative to part A. Equilibrium equations of part A and part B will be established and the numerical solution can be done by considering these three factors.

2.2. Interaction Model of Each Parts of Bearing

2.2.1. Interaction of Ball and Ring. The vector position relationship of ball and ring in space is shown in Figure 2.

The points of $O_i$ and $O_e$ are the centers of inner and outer raceway groove curvatures, respectively. The position vectors of $r_{boc}$ and $r_{boc_i}$ are from the center of the ball to the center of groove curvature of the outer ring and the inner ring, respectively.

In the vector triangle of $\Delta O_O' O_i$ (seeing Figure 2), the vector of $r_{boc}$ is the position vector from the ball center to the outer raceway groove curvature center and can be expressed by

$$ r_{boc} = r_b - r_{OOC} \tag{1} $$

Likewise, the vector of $r_{boc_i}$ in the vector triangle of $\Delta O' O' O_i$ is the position vector from the ball center to the inner raceway groove curvature center and can be described as

$$ r_{boc_i} = r_b - r_{OOC_i} \tag{2} $$

The elastic approaches of ball and ring raceways are expressed by

$$ \delta_k = r_{boc_k} - \left( f_k - 0.5 \right) d_w \tag{3} $$

where $k = i, e$ denote the inner and outer ring, respectively. When $\delta_k \leq 0$, the ball and raceway have no contact with each other and contact deformation will not occur, and the normal force is zero, whereas when $\delta_k > 0$, the contact deformation occurred, and the normal contact load can be obtained by (4) using Hertz theory of contact stress.

$$ F_{kHH} = K\delta_k \tag{4} $$

where $k$ is the stiffness coefficient of Hertz contact.

As illustrated in Figure 3, the local coordinate of $o_{xy}$ is established to analyze the contact ellipse between ball and inner raceway. Relative linear velocity can be described in the direction of $x$-axis and $y$-axis as (5) and (6), respectively.

$$ V_{xi} = - \left\{ \sqrt{R_i^2 - x_i^2} - \sqrt{R_i^2 - a_i^2} + \sqrt{\left( \frac{d_w}{2} \right)^2 - a_i^2} \right\} \cdot \omega_j \left( \frac{a_b}{\omega_i} \right) \cos \beta \sin \beta' \tag{5} $$

$$ V_{yi} = - \frac{\omega_j}{2} D_{m} - \left\{ \sqrt{R_i^2 - x_i^2} - \sqrt{R_i^2 - a_i^2} \right. $$

$$ + \left. \sqrt{\left( \frac{d_w}{2} \right)^2 - a_i^2} \right\} \times \left( \frac{\omega_b}{\omega_i} \cos \beta \cos \beta' \cos \alpha_i \right. $$

$$ + \left. \frac{\omega_b}{\omega_i} \sin \beta \sin \alpha_i - \cos \alpha_i \right) \omega_j \tag{6} $$

The angle between the direction of relative linear velocity and the direction of semimajor axis of the contact ellipse is

$$ \theta_{br} = \arctan \left( \frac{-V_{xi}}{V_{yi}} \right) \tag{7} $$
According to Coulomb’s law, tangential friction with respect to semimajor axis and semiminor axis of the contact ellipse is stated as
\[
T_{ix} = -T_i \sin \theta_{br} \\
T_{iy} = T_i \cos \theta_{br}
\]  
(8)

The force of contact point between ball and inner raceway is then obtained as
\[
F_i = [T_{ix}, T_{iy}, F_{iH}]
\]  
(9)

As illustrated by Figure 4, the vectors from the ball center to the contact ellipse centers of inner and outer raceways are \( \mathbf{r}_{obi} \) and \( \mathbf{r}_{obc} \), respectively. The friction torque between the ball and the inner raceway is as follows:
\[
M_{bri} = \mathbf{r}_{obi} \times [T_{ix}, T_{iy}, 0]
\]  
(10)

and the friction torque of the inner raceway is
\[
M_{rbi} = (r_{obe} + r_{abi}) \times [T_{ix}, T_{iy}, 0]
\]  
(11)

where \( \mathbf{r}_{abi} \) is the vector from the ball center to inner ring coordinate center. Similarly, the contact force between ball and outer raceway can be given as
\[
F_e = [T_{ex}, T_{ey}, F_{eH}]
\]  
(12)

2.2.2. Interaction of Ball and Cage. As illustrated in Figure 5, the vectors of \( r_{bp1}^p \) and \( r_{bp2}^p \) are the first and the second components of the vectors from the ball center to the cage pocket center in the pocket coordinates \( x^p y^p z^p \), respectively. The contact point between the ball and the pocket wall and the ball center and the pocket center is aligned in line. The minimum distance from pocket wall to the ball surface can be expressed as
\[
\delta_{bp} = \frac{d_p - d_w}{2} - \sqrt{\left( r_{bp1}^p \right)^2 + \left( r_{bp2}^p \right)^2}
\]  
(13)

When \( \delta_{bp} \leq 0 \), the elastohydrodynamic lubrication state will occur between the ball and cage pocket, and the normal force component between the ball and cage in the contact coordinate system can be obtained based on Brewe formula by [16]
\[
F_{bp} = \frac{UL}{q} \sqrt{\frac{128\alpha_c R_z h}{R_z}}
\]  
(14)

where \( L = 0.131 \arctan(\alpha_c/2) + 1.683, \) \( q = (1 + 2/3\alpha_c)^{-1}, \) and \( \alpha_c = R_c/R_x U = \eta u/E^*R_z. \) And the corresponding tangential force component can be given using Coulomb Law as follows:
\[
T_{bc} = \mu_{bp} |F_{bp}|
\]  
(15)
where $R_x$ is the equivalent radius along semimajor axis of the contact area in the contact coordinate, $R_z$ is the equivalent radius along semiminor axis of the contact area in the contact coordinate, $\eta_0$ is the dynamic viscosity of lubricating oil, $u$ is the relative sliding speed of the ball and the pocket wall, $E^*$ is the equivalent elastic modulus of the ball and the cage, and $\mu_{bp}$ of $0.02 \sim 0.08$ is the friction coefficient between the ball and the pocket wall.

When $\delta_{bp} < 0$, contact deformation will occur between the ball and the pocket wall, and the normal contact load can be obtained by using Hertz theory as

$$F_{bp} = K_c (\delta_{bp})^{3/2} \quad (16)$$

The relative line velocity at the contact point of the ball and the pocket wall is

$$v_{bp} = v_{gb} - v_{gp} \quad (17)$$

In contact coordinate $\omega^\theta \mathbf{x}^\theta \mathbf{y}^\theta \mathbf{z}^\theta$, the angle between $v_{bp}$ with $\mathbf{z}^\theta$ axis is

$$\theta_{bp} = \arctan \left( \frac{v_{bp1}}{v_{bp3}} \right) \quad (18)$$
The tangential friction forces along the directions of \( x^g \) and \( z^g \) are expressed as follows, respectively:

\[
T_{bcx} = T_{bc} \sin \theta_{bp} \\
T_{bcz} = T_{bc} \cos \theta_{bp}
\]  
(19)

The force and the moment subjected to the ball by cage pocket can be stated as follows, respectively:

\[
F_{bc} = [T_{bcx}, F_{bp}, T_{bcz}]^T
\]  
(20)

\[
M_{bc} = r_{gb} \times F_{bc}
\]  
(21)

According to Newton’s third law, the force and the moment subjected to the cage by the ball can be obtained as follows, respectively:

\[
F_{cb} = -F_{bc}
\]  
(22)

\[
M_{cb} = (r_{gb} + r_{pc}) \times F_{cb}
\]  
(23)

where \( r_{gb} \) is the position vector from the contact point of ball and pocket wall to the pocket center and \( r_{pc} \) is the position vector from the pocket center to the cage coordinate center.

2.2.3. Interaction of Cage and Guide Ring. Because of the effect of inertial force during steadily working process, the cage and guide ring exist potential collision, which can be assessed by the minimum clearance between the cage and the guide ring. As shown in Figure 6, the minimum clearance in the vector triangle \( \Delta O_1O_cg \) is given as

\[
\delta_{rc} = \pm \left( R_c - \sqrt{(r_{rg2})^2 + (r_{rg3})^2} \right)
\]  
(24)

where the symbols “+” and “-” are for outer and inner guide rings, respectively. \( r_{rg2} \) and \( r_{rg3} \) are the second and third components of \( r_{rg} \), respectively, which is the vector from the guide ring center to contact point of cage and ring in ring coordinate system \( \alpha', \chi', \psi' \).

When \( \delta_{rc} \leq 0 \), hydrodynamic lubrication will occur between the cage and the guide ring, and the interaction condition can be regarded as short sliding bearing. In the contact coordinate system \( \alpha'x'\beta'y'z'g \) (see Figure 5), the interaction forces of each direction are calculated by the following formula [17, 18]:

\[
F_{crt}^g = \frac{2\eta V_{cr}^g R_c^2 B_c}{C_g}
\]  
(25)

where \( V_{cr}^g \) is the relative speed of the cage with respect to the guide ring at the minimum clearance in the direction of \( x^g \) axis, \( B_c \) is the width of the cage, \( u \) is drag speed of lubricating oil, \( u = R_c (\omega_c + \omega_e) \), \( C_g \) is the cage guide clearance, and \( \epsilon \) is relative offset of the cage center:

\[
\epsilon = \frac{e}{C_g}
\]  
(26)

The friction moment of the moving cage caused by distribution pressure of fluid pressure oil film can be expressed as

\[
M_{crx} = \frac{2\eta V_{cr} R_c^2 B_c}{C_g \sqrt{1 - \epsilon^2}}
\]  
(27)

where \( V_{cr} \) is the relative sliding speed of the cage rail with respect to the ring land and can be stated as follows:

\[
V_{cr}^g = R_c (\omega_c - \omega_e)
\]  
(28)

When \( \delta_{rc} > 0 \), contact deformation will occur between the cage and the guide ring, and the normal contact load can be obtained by using Hertz theory as

\[
F_{cr}^g = K_j \delta_{rc}^{3/2}
\]  
(29)

where \( K_j \) is the Hertz contact stiffness coefficient with finite length.

The relative sliding speed of the ring with respect to the cage at contact point is

\[
v_{rc} = v_{rg} - v_{cg}
\]  
(30)

where \( v_{rg} \) is the velocity at the contact point of the ring and \( v_{cg} \) is the velocity at the contact point of the cage. \( v_{rc1} \) and \( v_{rc2} \) are the first and second components of the tangential relative velocity, respectively, and the angle along \( x^g \) axis is

\[
\theta_{cr} = \arctan \left( \frac{v_{rc2}}{v_{rc1}} \right)
\]  
(31)
Applying Coulomb’s law, the tangential force at the contact point is

\[ F_{cr} = \mu_{cr} F_{crz} \]  \hspace{1cm} (32)

where \( \mu_{cr} \) is the friction coefficient between the cage and the guide ring and is set to 0.02 ~ 0.08.

The tangential forces in the direction of \( x^d \) and \( y^d \) axis of the contact coordinate \( d^x, d^y, d^z \) are as follows, respectively:

\[ T_{crx} = T_{cr} \cos \theta_{cr} \]
\[ T_{cry} = T_{cr} \sin \theta_{cr} \]  \hspace{1cm} (33)

According to Newton’s third law, the force and the moment of the cage caused by the guide ring are

\[ F_{cr} = [T_{crx}, T_{cry}, F_{crz}] \]
\[ M_{cr} = r_{gc} \times F_{cr} \]  \hspace{1cm} (34)

where \( r_{gc} \) is the vector distance from the center of cage coordinate system to the contact point between the cage and the ring.

2.2.4. Drag of Ball from Oil-Gas Mixture. The balls should overcome the viscous friction drag caused by the bearing internal lubrication during orbital motion. The viscous friction drag of a ball shown in [19] can be approximately expressed by

\[ F_d = \frac{\rho_{cf} \pi C_D d_w^2}{32 g} (D_m \omega_m)^{1.95} \]  \hspace{1cm} (35)

where \( \rho_{cf} \) is the lubricant density in the bearing free space divided by the volume of the free space and the drag coefficients \( C_D \) can be determined from [20].

2.3. The Establishment of Equilibrium Equations. The equilibrium equations of the ball are established in the azimuth coordinate system of the ball, in which there are six unknown parameters that need to be calculated, and they are \( x_b, r_b, \omega_m, \omega_{bx}, \omega_{by}, \omega_{bz} \).

The inertial forces on a ball are

\[ F_x = 0 \]
\[ F_y = m_b \omega_x = m_b \omega_m r_b \]  \hspace{1cm} (36)
\[ F_z = m_b \omega_z = m_b \omega_m r_b \]

and the inertial moments on a ball are

\[ M_x = -I_b \omega_{bx} \]
\[ M_y = -I_b (\omega_{by} + \omega_m \omega_{bz}) \]  \hspace{1cm} (37)
\[ M_z = -I_b (\omega_{bz} - \omega_m \omega_{by}) \]

where \( I_b \) is the inertia moment and can be calculated by \( I_b = m_b d_w^2 / 10 \).

The equilibrium equations of ball’s force and moment are

\[ F_{br1} + F_{ca1} = 0 \]  \hspace{1cm} (38)
\[ F_{br2} + F_{ca2} + F_d + F_y = 0 \]  \hspace{1cm} (39)
\[ F_{br3} + F_{ca3} + F_z = 0 \]  \hspace{1cm} (40)
\[ M_{br1} + M_{ca1} + M_x = 0 \]  \hspace{1cm} (41)
\[ M_{br2} + M_{ca2} + M_y = 0 \]  \hspace{1cm} (42)
\[ M_{br3} + M_{ca3} + M_z = 0 \]  \hspace{1cm} (43)

The force equilibrium equations of the cage are established in the inertial coordinate system of the bearing, and the moment equilibrium equation are established in the fixed coordinate system of the cage. There are the six parameters need to be determined as \( x_c, y_c, z_c, \omega_c, \theta_{cx}, \text{and} \ \theta_{cz} \).

The force and moment equilibrium equations of the cage are

\[ F_{cr1} + F_{cb1} = 0 \]  \hspace{1cm} (44)
\[ F_{cr2} + F_{cb2} = 0 \]  \hspace{1cm} (45)
\[ F_{cr3} + F_{cb3} = 0 \]  \hspace{1cm} (46)
\[ M_{cr1} + M_{cb1} = 0 \]  \hspace{1cm} (47)
\[ M_{cr2} + M_{cb2} = 0 \]  \hspace{1cm} (48)
\[ M_{cr3} + M_{cb3} = 0 \]  \hspace{1cm} (49)

The force equilibrium equations of the guide ring are established in the inertial coordinate system of the bearing, and the moment equilibrium equations are established in the ring fixed coordinate system. Similarly, there are also six parameters need to be solved \( x_r, y_r, z_r, \theta_{rx}, \theta_{ry}, \text{and} \ \theta_{rj} \). The force and moment equilibrium equations of the guide ring are

\[ F_x = \sum_{j=1}^{5} (F_{r_{rj1}} + F_{n_{b1j}}) = 0 \]
\[ F_y = \sum_{j=1}^{5} (F_{r_{rj2}} + F_{n_{b1j}}) = 0 \]
\[ F_z = \sum_{j=1}^{5} (F_{r_{rj3}} + F_{n_{b1j}}) = 0 \]  \hspace{1cm} (50)
\[ M_y = \sum_{j=1}^{5} (M_{r_{rj2}} + M_{n_{b2j}}) = 0 \]
\[ M_z = \sum_{j=1}^{5} (M_{r_{rj3}} + M_{n_{b3j}}) = 0 \]

2.4. Numerical Solution. In this paper, six equations of the ball and cage and five equations of the guide ring were solved numerically by Matlab tools. The magnitude order of the
displacement and angular velocity is significantly different; therefore, it is easy to get a nonconvergent solution by combining them to solve. In order to solve this problem, the method from first partial to whole is adopted in this paper. The displacement variables are determined first, and then the angular velocity variables are calculated. Thereafter, the initial value, the displacement, and angular velocity variables will be solved together. The solution process is shown in Figure 7.

3. Validation

In order to verify the accuracy of the updated mathematical model of the angular contact ball bearing, the results of a typical example is illustrated to compare the results with ADORE calculation. The values of bearing parameters are listed in Table 1.

| Table 1: Geometry parameters of ball bearing. |
|-----------------|---------|
| Bearing parameters | value |
| Pitch diameter (mm) | 31 |
| Ball diameter (mm) | 8 |
| The number of balls | 6 |
| Initial contact angle (°) | 24 |
| Inner rolling groove curvature | 0.56 |
| Outer rolling groove curvature | 0.52 |
| Bearing bore diameter (mm) | 20 |
| Bearing outside diameter (mm) | 42 |
| Cage outside diameter (mm) | 35 |
| Cage core diameter (mm) | 30 |
| Cage width (mm) | 14 |
| Elasticity modulus (MPa) | 2e5 |

Bearings working condition is set as $F_a$ of 2000 N, $F_r$ of 400 N, $n_i$ of 120000 r/min with the fixed outer ring. Comparing the results of this paper’s program with Gupta’s ADORE dynamics program at 12.87 ms, the results are compared in Table 2.

As listed in Table 2, the results of this paper and Gupta’s show that the maximum error is about 10%. Such phenomenon is due to the fact that (i) the interference fit generated by bearing installed on the spindle was not considered, which would lead the initial contact angle to decrease, (ii) Gupta’s ADORE dynamics program considered the acceleration influence, but the proposed mechanical model of ball bearing in this paper did not consider such factor.

4. Dynamic Analysis of Spindle Bearing

4.1. Influence of Working Condition Parameters. The changes of $F_a$, $F_r$, and $n_i$ have significant influence on the dynamic characteristics of bearing. Based on the variation of these three working conditions (Table 3), the variation rule of bearing dynamic parameters is analyzed.

4.1.1. Influence of Working Conditions on the Spin-To-Roll Ratio. As an important parameter of the angular contact bearing at high speed, spin-to-roll ratio represents the state of the ball rolling in the raceway. Friction and heating are produced during the spinning of the raceway relative to the ball, and the rotation sliding is more serious with large spin-to-roll ratio, which will lead to the severe heat and wear. Therefore, the spin-to-roll ratio should be reduced as much as possible during operation. The change rule of spin-to-roll ratio is analyzed under three working conditions as tabulated in Table 3.

Figures 8 and 9 show that the variations of the spin-to-roll ratio of ball-inner and ball-outer raceways and increase the spindle bearing axial preload, which will result in spin-to-roll ratio lower, (iii) the ball spin-to-roll ratio is different at any ball azimuth location with the influence of $F_r$, (iv) there exists a certain spin-to-roll ratio between the ball and the outer raceway, which indicates that the high speed spindle bearing of oil lubrication does not conform to the hypothesis of outer raceway control, and (v) when $F_a=1000$ N and $F_a=1500$ N, the ball at the location angle of 120° has the maximum spin-to-roll ratio; with the increase of the load, the spin-to-roll ratio of the ball in this position gradually becomes the minimum. The reason for this phenomenon is that when bearing axial load is relatively small, there is axial clearance between ball and outer ring, which cannot make the ball press outer ring tightly that results in high spin angular velocity.

Figures 10 and 11 show the variation curves of the spin-to-roll ratio of ball-inner and ball-outer raceways with respect to the radial load. Results show that, with constant $F_a$ and $n_i$, the spin-to-roll ratio of the ball-inner raceway decreases in the loading area and increases in the nonloading area, because the radial load can decrease the radial clearance in the loading area and then the ball and the raceway are tightly pressed, which resulted in the spin-to-roll ratio reducing. On the contrary, the spin-to-roll ratio will increase in the nonloading area. It also can be found that the radial load has less effect on the spin-to-roll ratio of the ball-outer raceway than of the ball-inner raceway.

As shown in Figures 12 and 13, under the constant loads $F_a$ and $F_r$, the spin-to-roll ratio of the ball-inner raceway increases obviously with $n_i$ increase; comparatively, the spin-to-roll ratio of the ball-outer raceway increases gradually. Such phenomena are due to the fact that the centrifugal force increased with $n_i$ increasing, which leads to the decrease of the contact load of the ball-inner raceway and the increase of the sliding of the ball-inner raceway and the spin-to-roll ratio obviously. The increase of the centrifugal force can also increase the contact load of the ball and the raceway; restrain the sliding between the ball and the raceway, and slow down the spin-to-roll ratio increase. The ball at location angle of 120° has the maximum spin-to-roll ratio with the inner ring and the minimum spin-to-roll ratio with the outer ring; the
Bearing working condition and geometric mechanism parameter

Static method

Quasi-Static method

Solving displacement equation (44)(45)(46)(48)(49) of cage

Solving displacement equation (38)(40) of ball

Combing and solving displacement equation (38)(39)(44)(45)(46)(48)(49)(50)

Convergent (Y or No)

Solving velocity equation (39)(41)(42)(43) of ball

Solving velocity equation (47) of cage

Combing and solving velocity equation (39)(41)(42)(43)(47)

Convergent (Y or No)

Combing and solving equation (38)-(50)

Convergent (Y or No)

Y

Y

End

Figure 7: Numerical calculation flow chart.

Table 2: Comparison of the computation results at 12.87ms.

<table>
<thead>
<tr>
<th>Calculating parameter</th>
<th>The result of Gupta</th>
<th>The result of this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact angle of ball and inner ring (°)</td>
<td>28.29</td>
<td>33.20</td>
</tr>
<tr>
<td>Contact angle of ball and outer ring (°)</td>
<td>13.87</td>
<td>14.14</td>
</tr>
<tr>
<td>Maximum contact stress of ball and inner ring (Gpa)</td>
<td>2.799</td>
<td>2.883</td>
</tr>
<tr>
<td>Maximum contact stress of ball and outer ring (Gpa)</td>
<td>2.253</td>
<td>2.308</td>
</tr>
<tr>
<td>Semi-major axis of the contact area of ball and inner ring (mm)</td>
<td>0.7971</td>
<td>0.8018</td>
</tr>
<tr>
<td>Semi-major axis of the contact area of ball and outer ring (mm)</td>
<td>1.478</td>
<td>1.512</td>
</tr>
<tr>
<td>Spin-to-roll ratio of ball and inner ring</td>
<td>0.3482</td>
<td>0.4072</td>
</tr>
<tr>
<td>Revolution velocity of ball (r/min)</td>
<td>46660</td>
<td>48961</td>
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<tr>
<td>Rotation velocity of ball (r/min)</td>
<td>228100</td>
<td>240172</td>
</tr>
<tr>
<td>Revolution velocity of cage (r/min)</td>
<td>47660</td>
<td>47950</td>
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</table>

Table 3: Three different conditions.

<table>
<thead>
<tr>
<th>operating condition</th>
<th>constant operating condition</th>
<th>constant operating condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_a=1000:500:3000$ N</td>
<td>$F_r=400N$</td>
<td>$n_i=120000$ r/min</td>
</tr>
<tr>
<td>$F_a=1000:100:1500$ N</td>
<td>$F_r=1000N$</td>
<td>$n_i=100000$ r/min</td>
</tr>
<tr>
<td>$n_i=100000:10000:140000$ r/min</td>
<td>$F_a=2500N$</td>
<td>$F_r=400N$</td>
</tr>
</tbody>
</table>
reason for this phenomenon is that the ball at location angle of 120° is affected by the maximum centrifugal force.

4.1.2. Influence of Working Conditions on $\omega_{by}$. Figure 14 shows the $\omega_{by}$ curves in the direction of $y$-axis in the azimuth coordinate system with the change of $F_a$. Under the condition load $F_r$ and the constant rotational speed $ni$, $\omega_{by}$ gradually increases with $F_a$ increasing. When $F_a=3000$ N, $\omega_{by}$ is basically stable at any ball azimuth location. Such phenomena suggest that $F_a$ can reduce the axial clearance and decrease the influence of $F_r$ on the ball, and the load condition of the ball tends to be steady.

The changing curves of $\omega_{by}$ along $y$-axis in the azimuth coordinate system with the change of $F_r$ are shown in Figure 15. It can be illustrated that when $F_a$ and $ni$ are constant, $\omega_{by}$ gradually decreases with the increase of $F_r$, which is larger in the loading area. Such phenomena are due to the fact that $F_r$ can reduce the radial clearance and inhibits the gyroscopic motion between the ball and the raceway and typically lead to $\omega_{by}$ decreasing significantly in loading area.
As shown in the Figure 16 of $\omega_{by}$ changing curves with the change of $ni$, under the constant load $Fa$ and $Fr$, the value of $\omega_{by}$ gradually increases with $ni$ increasing, and the $\omega_{by}$ of the ball varies obviously with different ball azimuth location. $\omega_{by}$ increases slowly with the increase of $ni$ in the loading area, whereas the increase is obvious in nonloading areas. Such phenomena are due to the fact that the radial load holds the ball and the raceway tight in the loading area and inhibits the changes of the ball in the space position, which causes $\omega_{by}$ increasing slowly. When $ni$ is greater than 110000 r/min, at location angle of 120°, the $\omega_{by}$ value of the ball reaches the minimum. The reason for this phenomenon is that the ball at this position bear the maximum load because of radial load $Fr$, and the increased centrifugal force compacts the ball and the raceway, resulting in the minimum $\omega_{by}$ here.

4.2. Influence of Working Conditions on Cage

4.2.1. Cage Forces. There exits three types of interaction conditions between the ball and the pocket of cage, such as (i) the pocket colliding the balls, (ii) the balls colliding the pocket, and (iii) no contact between the ball and the pocket. In order to study the relative motions between the ball and the
In the loading area, the negative value represents the pocket colliding the ball and conversely the positive is the ball colliding the pocket. (i) It can be illustrated that the normal load between the ball and the pocket of cage is very small during the steady-state working period; (ii) the ball and the pocket collide with each other. It will last until the dynamic equilibrium state is achieved during the bearing steady working state. (iii) Figure 19 shows that as $ni$ increases, the normal load between the cage and the ball gradually decreases in the loading area. The reason for this phenomenon is that the centrifugal force increases with $ni$ increasing in the loading area; it leads to the fact that the ball and the raceway are compressed; and the result is that the ball rotation angular velocity decreases, making the contact force between the ball and the cage decrease. In no-load areas, the opposite is true.

4.2.2. Cage Rotation Speed. In this paper, the relationship between the cage rotational angular speed and the ball
revolution angular speed is studied from the three working conditions listed in Table 2.

\( \omega_c \) and \( \omega_m \) curves of with different \( F_a \), \( F_r \), and \( n_I \) are shown in Figures 20–22, which suggest that \( \omega_c \) and \( \omega_m \) decrease with the increase of \( F_a \), and increase with the increase of \( n_I \), while the change of \( F_r \) has little effect on \( \omega_c \) and \( \omega_m \). The value of \( \omega_c \) lies in the middle position of the values of \( \omega_m \). Therefore, it is reasonable to take the average speed value of the balls as the cage rotational angular speed.

4.3. Influence of Working Conditions on Gyroscopic Moment.

The gyroscopic moment is a dynamic inertial force produced during the process of bearing operation, which is harmful for the bearing operation condition. Especially, under the condition of high speed and light load, it causes the sliding, which will lead to serious friction, makes the bearing internal fever, heats up, and accelerates the bearing degeneration.

Figures 23 and 24 show the curves of \( M_y \) and \( M_z \) with the change of \( F_a \). Under the constant values of \( F_r \) and \( n_I \), (i) \( M_y \) and \( M_z \) increase obviously with the increase of \( F_a \), (ii) when \( F_a = 3000 \) N, \( M_z \) tends to be stable at each azimuth.
location. Such phenomena are due to the fact that $F_a$ reduces the axial clearance and the impact of $F_r$ on balls and results in the same load condition for balls.

Figures 25 and 26 show the curves of $M_y$ and $M_z$ with the change of $F_r$. Under the constant values of $F_a$ and $n$, $M_y$ and $M_z$ gradually decrease with the increase of $F_r$. It is because that the increase of $F_r$ will decrease the radial clearance of the bearing; therefore, the rolling body and the ring are pressed tightly, and the gyroscopic moment is suppressed and reduced.

Figures 27 and 28 show the curves of $M_y$ and $M_z$ with the change of $n$ in the direction of $y$ and $z$ of the azimuth coordinate system of the ball. It is illustrated that $M_y$ and $M_z$ increase significantly with the increase of $n$ under the constant load of $F_a$ and $F_r$.

5. Conclusions

(1) The change of $F_a$, $F_r$, and $n$ can affect the spin-to-roll ratio of ball-inner raceway and ball-outer raceway. In different working conditions, the spin-to-roll ratio of the ball-inner raceway is greater than that of the ball-outer raceway. In high speed, the outer raceway control hypothesis is only a special case. In general,
there is a spin angular velocity between the ball and the outer raceway.

(2) The increase of $F_a$ and $ni$ of spindle bearing can cause $\omega_{by}$ increase; however, the increase of $F_r$ will cause $\omega_{by}$ decrease. A specific $F_a$ can reduce the axial clearance of the bearing, which will make the azimuth of the ball tend to be stable in the space.

(3) The interaction between the ball and the pocket of cage is that not only does the ball collide with the pocket but also the pocket collides with the ball, which can achieve the dynamic equilibrium state of the bearing steady-state work.

(4) It is reasonable to take the average speed value of the balls as the cage rotational angular speed.

(5) Gyroscope moment increases significantly with the increase of $F_a$ and $ni$. The increase of $F_r$ can suppress the gyroscopic movement.

Nomenclature

$O_{XYZ}$: Inertial coordinate system
$O_{xyz}$: Coordinate system
$a$: Contact angle ($^\circ$)
$\alpha$: Semimajor axis of the contact area (mm)
$b$: Semiminor axis of the contact area (mm)
$\beta$: Spiral angle ($^\circ$)
$\beta'$: Yaw angle ($^\circ$)
$D_m$: Pitch diameter (mm)
$F$: Hertz contact force (N)
$F_a$: Thrust load (N)
$F_r$: Radial load (N)
$F_{bc}$: Force act on cage by ball (N)
$f$: Dimensionless curvature radius coefficient
$G$: Dimensionless material parameters

$h$: Center oil film thickness (mm)
$h_0$: Central oil film thickness (mm)
$I_b$: The inertia moment of ball
$I$: Inertia moment
$K$: Hertz contact stiffness coefficients
$m$: Quality (kg)
$M_z$: The gyroscopic moment along $z$-axis in the azimuth coordinate system of ball (N.mm)
$M_y$: The gyroscopic moment along $y$-axis in the azimuth coordinate system of ball (N.mm)
$ni$: The inner ring rotational speed (r/min)
r: Displacement vector
$R$: Radius (mm)
$\rho$: Density (g/cm$^3$)
$\rho_{oa}$: Density of the mixture gas (g/cm$^3$)
$T$: Tangential force (N)
$\omega$: Angular speed (rad/s)
$\omega_c$: The cage rotational angular velocity (rad/s)
$\omega_m$: The ball revolution angular velocity (rad/s)
$\omega_{by}$: The ball rotation angular velocity along $y$-axis in the azimuth coordinate system of ball (rad/s)
$V_r$: Relative sliding velocity (m/s)
$\delta$: Elastic approach (mm)
$\theta$: Relative angular displacement ($^\circ$)
$\mu$: Friction coefficient.

Superscript

$a$: The azimuth coordinates of the ball
$r$: The fixed body coordinate system of inner ring
$i$: Inertial coordinate
$c$: The fixed body coordinate system of the cage
$p$: The pocket coordinate
$g$: Contact coordinate.

Subscript

$b$: Ball
$i$: Inner ring
$c$: Outer ring
$e$: Cage
$p$: Cage pocket
$g$: Contact point
$j$: The $j$th ball
$x, y, z$: The coordinate components.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors confirm that article content has no conflicts of interest.
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